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DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EEEN60321/40321 POWER SYSTEM OPERATION AND ECONOMICS

Economic Dispatch, Optimal Power Flow and Security Constrained OPF Laboratory Report

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1 Introduction

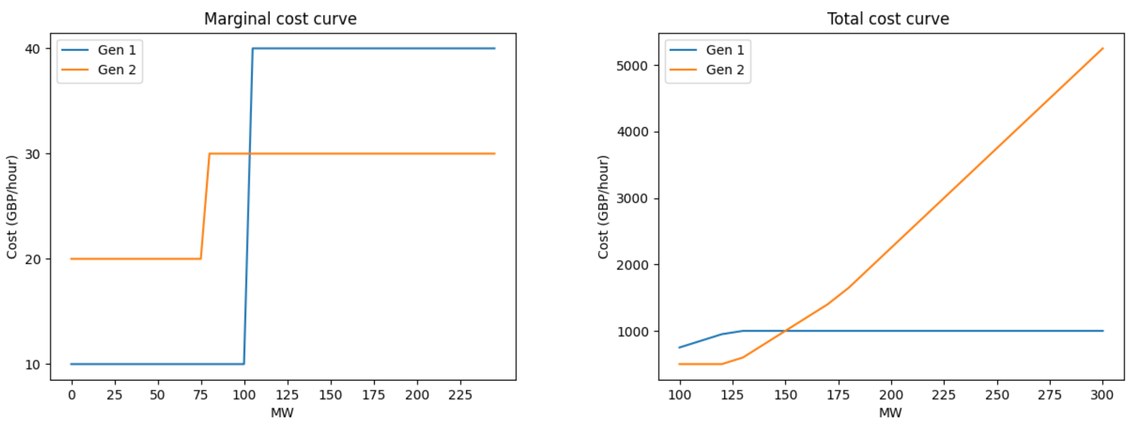
In Electrical Power System, energy as a commodity is define as “MWh at a given time and a given location, with a given level of supply security”. (Levi, 2020). Power System Operational planning includes energy trade economics and safety parameters. Selection of generators and level of security is carried out using three major techniques called Power Flow(PF), Economic Dispatch(ED) and Optimal Power Flow(OPF). This report contains usage and explanation of these techniques, its outcomes used for decision making, and the key differences between them.

2 Economic Dispatch

The cost of generating a unit of power at different circumstances has to be minimized to maximize the profit. When multiple generators of varying specifications are part of the system, the choice of generators to provide the ever changing consumer load is decided by economic dispatch. Ideally, the cheapest generator should provide for all the loads in the system, but practically, there are numerous constraints that does not allow this, and hence increases the cost. For e.g. the upper limit of generation capacity. The purpose of economic dispatch is to minimize costs or maximise the profit. ED does not concern with transmission line network capabilities. This section looks at economic dispatch for a two generator system.

2.1 Marginal Costs and Total Costs

The marginal costs of a generator represents the cost of next unit of energy to be produced when demanded by load. Marginal Costs could be represented by a linear function or a piece-wise linear function. In the case below, marginal cost of two generators are represented as piece-function. Generator 1 is cheaper until 100MW load. The total cost plateaus at £1000 for Generator 1 and ED restricts its generation in normal generation condition. The total costs for Generator 2 is steadily increasing as it is providing the increase in load.



2.2 Normal Conditions

For the two generator system, the load is increased from 100MW to 300MW in steps of 10. The generator output and total costs at each step are recorded in the table below. Under normal conditions, as seen from the marginal cost curve, Generator 1 is cheap until 100MW load, after that point, Generator 2 takes over the supply of load. Inequality constraints with respect to the lower limit of Generators are considered at all steps.

2.3 Reduced Generation Capacity

For the two generator system, the load is increased from 100MW to 300MW in steps of 10. The generator output and total costs at each step are recorded in the table below. Under reduced generation conditions, Generator 2 has a upper limit of 150MW. Once it hits the limit, Generator 2 is asked to supply the remaining load even though it is pricier. Inequality constraints with respect to the lower limit of Generators are considered at all steps.

2.4 Simulation Results

Load (MW)	Gen 1 (MW)	Gen 2 (MW)	G1 (£/hr)	G2 (£/hr)
100	75	25	750	500
110	85	25	850	500
120	95	25	950	500
130	100	30	1000	600
140	100	40	1000	800
150	100	50	1000	1000
160	100	60	1000	1200
170	100	70	1000	1400
180	100	80	1000	1650
190	100	90	1000	1950
200	100	100	1000	2250
210	100	110	1000	2550
220	100	120	1000	2850
230	100	130	1000	3150
240	100	140	1000	3450
250	100	150	1000	3750
260	100	160	1000	4050
270	100	170	1000	4350
280	100	180	1000	4650
290	100	190	1000	4950
300	100	200	1000	5250

ED Normal conditions

Gen 1 (MW)	**Gen 2** (MW)	**G1** (£/hr)	**G2** (£/hr)
75	25	750	500
85	25	850	500
95	25	950	500
100	30	1000	600
100	40	1000	800
100	50	1000	1000
100	60	1000	1200
100	70	1000	1400
100	80	1000	1650
100	90	1000	1950
100	100	1000	2250
100	110	1000	2550
100	120	1000	2850
100	130	1000	3150
100	140	1000	3450
100	150	1000	3750
110	150	1400	3750
120	150	1800	3750
130	150	2200	3750
140	150	2600	3750
150	150	3000	3750

ED Reduced Generation

2.5 Analysis

Given the cost function and constraints for generators:

$$C1 = 430 - 12.5P_1 + 0.15P_1^2 \Rightarrow 50.00 \leq P_1 \leq 250.00$$
$$C2 = 150 + 12.0P_2 + 0.08P_2^2 \Rightarrow 25.00 \leq P_2 \leq 200.00$$

2.5.1 Marginal Costs

The marginal costs of the quadratic function is calculated by the first derivative of the cost function as shown in the figure. Constant Marginal costs or Piece-wise linear marginal costs does not affect the ED solution at the every increment, It changes at thresholds or limits. Meanwhile quadratic cost curves generate a ramp curve for marginal costs. In the example shown below, the curves overlap at certain point. From that point the generators swap roles from an ED perspective. A single MW increment after the overlapping point(175MW) on G1 axis would require change in generator outputs at both sides. Part of the reason this involves both generators is because of the lower limits or minimum required generation by the unit.

ED_Analysis_Marginal_Costs.png ED_Analysis_Costs.png

2.5.2 Graphical Representation

The equality and inequality constraints can be graphically represented by the following diagram. The solution curve for both scenarios of load is also represented.

ED_Graphical_Solution.png

Figure 1: ED Graphical Solution

2.5.3 Lagrangian Equations

Economic Dispatch is carried out to determine the optimal generator output with respect to the objective function. Transmission line network and its output is neglected as it does not take part in ED. Hence, to supply load L, by Generator 1 @ Bus1 generating P_1 power and Generator 2 @ Bus2 generating P_2 power, the cost functions are given by:

$$C1 = 430 - 12.5P_1 + 0.15P_1^2$$
$$C2 = 150 + 12P_2 + 0.08P_2^2$$

Total Cost Objective Function(C = C1 + C2):

$$C = 580 - 12.5P_1 + 0.15P_1^2 + 12P_2 + 0.08P_2^2$$

Subject to:

$$L - P_1 - P_2 = 0$$
$$50 - P_1 \leq 0$$
$$P_1 - 250 \leq 0$$
$$25 - P_2 \leq 0$$
$$P_2 - 200 \leq 0$$

2.5.4 Scenario 1 : Load = 200.00MW

The Lagrangian with binding constraints is given by:

$$l = (580 - 12.5P_1 + 0.15P_1^2 + 12P_2 + 0.08P_2^2) + \lambda(200 - P_1 - P_2) + \mu(25 - P_2)$$

$$\frac{\partial l}{\partial P_1} = -12.5 + 0.3P_1 - \lambda = 0$$
$$\frac{\partial l}{\partial P_2} = 12 + 0.16P_2 - \lambda - \mu = 0$$
$$\frac{\partial l}{\partial \lambda} = 200 - P_1 - P_2 = 0$$
$$\frac{\partial l}{\partial \mu} = 25 - P_2 \leq 0$$

1. Complimentary Slackess - μ > 0; g_j(P) = 0

   Applying this condition in the above equation:

   $$P1 = 175; P_2 = 25; = 40; \mu = -24$$

   This fails to satisfy the Complimentary Slackess condition because μ < 0
2. Complimentary Slackess - μ = 0; g_j(P) < 0

   Applying this in the above equations:

   $$P1 = 122.83; P_2 = 77.174; = 24.35; \mu = 0$$

   This satisfies the condition (25 - P_2) $\leq$ 0
3. Solution

P_1 (MW)	P_2 (MW)	λ {£/ MWh}	Nodal prices(all buses) {£/ MWh}
123	77	24.35	24.35

2.5.5 Scenario 2 : Load = 400.00MW

Here, it is assumed that inequality constraints are non-binding and Economic Dispatch is solved with no inequality constraints.The Lagrangian is given by:

$$l = (580 - 12.5P_1 + 0.15P_1^2 + 12P_2 + 0.08P_2^2) + \lambda(400 - P_1 - P_2)$$

$$\frac{\partial l}{\partial P_1} = -12.5 + 0.3P_1 - \lambda = 0$$
$$\frac{\partial l}{\partial P_2} = 12 + 0.16P_2 - \lambda = 0$$
$$\frac{\partial l}{\partial \lambda} = 400 - P_1 - P_2 = 0$$

Solving the above equations for P1 and P2 gives:

$$P1 = 192.39MW; P_2 = 207.39; = 45.217;$$

This fails to satisfy the inequality constraint for Generator 2 with and upper limit of 200MW. To bring *the solution under constraints, increase lambda or reduce the generation of Generator 2 under the 200MW limit.*

The modified Lagrangian with P_2 = 199MW is :

$$l = (580 - 12.5P_1 + 0.15P_1^2 + 2388 + 3168.1) + \lambda(400 - P_1 - 199)$$
$$= 6136.08 - 12.5P_1 + 0.15P_1^2$$

$$\frac{\partial l}{\partial P_1} = -12.5 + 0.3P_1 - \lambda = 0$$

Solving this in the above equations:

$$P1 = 201; P_2 = 199; \lambda = 47.8; \mu = 0$$

P_1 (MW)	P_2 (MW)	λ {£/ MWh}	Nodal Prices(all buses) £/MWh
201	199	47.8	47.8

2.6 Conclusion

For Economic Dispatch the equality constraint is generation-load balance, and the inequality constraints are generator minimum and maximum limits. Minimum limits of all generators are considered and the load is then served with the cheapest generator until it hits its maximum limit. The marginal cost of system and nodal prices are identified by the Lagrangian coefficient.

3 Optimal Power Flow

Power Flow or Load Flow studies gives a snapshot of the network. It is conducted to find out the line flows in the system, to verify if any lines are getting overloaded. One generator in the system is assumed to have infinite power which provides for the next load increment. Power flow never concerns about money. Economic dispatch finds maximum profit by minimising cost and does not concern with transmission lines. Optimal Power Flow is marriage between Power Flow and Economic Dispatch. It finds the optimum solution with generation, price and availability.

3.1 Marginal costs

The marginal costs of the generators are given by:

OPF_Analysis_Marginal_Costs.png

Figure 2: OPF Marginal Costs of generator

3.2 Power flow, ED and OPF simulation

By disabling the transmission constraints, Power Flow Analysis and OPF is performed on the system. OPF is run again by enabling transmission constraints. The generator and line outputs are recorded at steps by increasing the load from 200MW to 300MW in steps of 10MW. The results are tabulated below:

Load	Gen 1	Gen 2	L12	L13	L23
200	150	50	33	117	83
210	160	50	37	123	87
220	170	50	40	130	90
230	180	50	43	137	93
240	190	50	47	143	97
250	200	50	50	150	100
260	210	50	53	157	103
270	220	50	57	163	107
280	230	50	60	170	110
290	240	50	63	177	113
300	250	50	67	183	117

Table 1: Power flow without thermal limits(Units in MW)

Load	Gen 1	Gen 2	L12	L13	L23
200	50	150	33	83	117
210	50	160	37	87	123
220	50	170	40	90	130
230	50	180	43	93	137
240	50	190	47	97	143
250	50	200	50	100	150
260	50	210	53	103	157
270	50	220	57	107	163
280	50	230	60	110	170
290	50	240	63	113	177
300	50	250	67	117	183

Table 2: OPF without thermal limits(Units in MW)

Load	Gen 1	Gen 2	L12	L13	L23
200	50	150	33	83	117
210	50	160	37	87	123
220	50	170	40	90	130
230	50	180	43	93	137
240	50	190	47	97	143
250	50	200	50	100	**150**
260	70	190	40	110	150
270	90	180	30	120	150
280	110	170	20	130	150
290	130	160	10	140	150
300	150	150	0	150	150

Table 3: OPF with thermal limits(Units in MW)

3.3 Observations from simulation results

Following are the observations from the simulation results table Table 1, Table 2, Table 3 :

1. In Power Flow Analysis, slack bus generator, provides all the extra load in the system. In reality, this load is shared by all generating units in the system. As the load increases, the slack bus output increases linearly. The transmission line flows are not a concern for Power flow analysis. The primary objective is to find the state of power flow in the system as a snapshot

2. In Economic Dispatch, the objective is to minimise costs. ED overlooks network constraints to an assumption that all generators and load is connected to a single bus. The cheapest generator cuts the slack in ED. It provides the load until its limit is reached. PF and ED would look like a mirror image with load less than the generation limits

3. OPF considers all constraints including generator and network limits. The solution of OPF is often costlier than ED and never the other way around. From Table 3 , even though Gen 2 is proffered by ED, OPF brings in Gen 1 due to the thermal limit of Line 2-3

3.4 Analysis

$$Gen1costs(£/h) = 100.00 + 40.00 * P1 \Rightarrow 50.00 <= P1 <= 200.00$$
$$Gen2costs(£/h) = 1000.00 + 30.00 * P2 \Rightarrow 50.00 <= P2 <= 300.00$$

subject to:

$$Line_1 - 2 \leq 140.00MW$$
$$Line_1 - 3 \leq 150.00MW$$
$$Line_2 - 3 \leq 160.00MW$$

3.4.1 Scenario 1 - Load = 200.00MW

The Equality Constraint ==> 200 - P1 - P2 = 0

Considering the impedance of all three lines are same, by using Superposition theorem in the linear system, power flow in the lines can be calculated using:

$$P_{12} = \frac{P1}{3} - \frac{P2}{3}$$
$$P_{23} = \frac{P1}{3} + \frac{2 * P2}{3}$$
$$P_{13} = \frac{2 * P1}{3} + \frac{P2}{3}$$

The Marginal Cost of Generator 2 is less than that of Generator 1. Generator 1 output is set to minimum, and Generator 2 is allowed to generated the remaining load:

P1 = 50 MW and P2 = 150MW

Power flow in lines are calculated from the equations above:

$$P12 = 33MW(reverse - flow)$$
$$P23 = 116.67MW$$
$$P13 = 83.33MW$$

All the lines are within its capacity and the generator limit constraints are satisfied.

G1(MW)	G2(MW)	Cost of Security (£/hr) OPF_{price} - ED_{price}
50	150	0(6500 -6500)

3.4.2 Scenario 2 - Load = 275.00MW

1. Graphical solution

OPF_Analysis_Solution.png

Figure 3: OPF Graphical Solution

The red and orange dashed lines are not binding. The green dashed line representing L23 is the only binding transmission line constraint, which should be taken into consideration in selecting the generation. The intersection point with the solution(blue solid line) corresponds to G2=206MW and G1=69MW on the x and y axis respectively. The same has been verified by simulations

2. Lagrangian method - marginal costs of the transmission constraint The Lagrangian is with binding constraints is given by:

$$l = (1100 + 40P_1 + 30P_2) + \lambda(275 - P_1 - P_2) + \mu(0.333P_1 + 0.667P_2 - 160)$$

$$\frac{\partial l}{\partial P_1} = 40 - \lambda + 0.333\mu = 0$$
$$\frac{\partial l}{\partial P_2} = 30 - \lambda + 0.667\mu = 0$$
$$\frac{\partial l}{\partial \lambda} = 275 - P_1 - P_2 = 0$$
$$\frac{\partial l}{\partial \mu} = 0.333P_1 + 0.667P_2 - 160 \leq 0$$

   (a) Complimentary Slackess - μ = 0; g_j(P) < 0 Applying the generator values from graphical solution, P_1 = 69MW and P2=206MW,

$$0.333 * 69 + 0.667 * 206 - 160 = 0.3172$$

   This does not satisfy the condition. *Adjusting the values to P1=70 and P2=205:*

$$0.333 * 70 + 0.667 * 205 - 160 = -0.016$$

   satisfies the condition and is the proposed solution

G1(MW)	G2(MW)	Cost of Security (£/hr) OPF_{price} - ED_{price}
70	205	200(8950 -8750)

   Marginal Costs at Bus 1, Bus 2 and Bus 3 is 40£/hr, 30£/hr, 50£/hr respectively. Bus 3 has a congestion price at 10£/hr.

   Max load that can be served is 310MW at 10700£/hr

4 Contingency Analysis

4.1 OPF Solution

Only Transmission Line contingencies are considered: Following are the contingency cases and the line flows(same impedance lines) with respect to Generator output(P1 and P2) are calculated as:

The cost has remained same but the maximum load that can be served changes with respect to each contingency.

	Line 1-2 Open	Line 1-3 open	Line 2-3 open
P12 (max 140MW)	0	P1	-P2
P23 (max 160MW)	P2	P1 + P2	0
P13 (max 150MW)	P1	0	P1 + P2

Table 4: Line flows from Superposition method

	Line 1-2 Open	Line 1-3 open	Line 2-3 open
P12 (max 140MW)	0	50	-75 (75 in opp. direction))
P23 (max 160MW)	75	125	0
P13 (max 150MW)	50	0	125
ED cost(£/hr)	4250	4250	4250
OPF cost(£/hr)	4250	4250	4250

Table 5: N-1 case for Load = 125MW; P1 = 50MW; P2 = 75MW

5 Seven bus example

5.1 Thermal Constraints

Following table shows the results of 7-bus system with transmission line limits disabled and enabled in PowerWorld simulation:

Network Limits	Hourly Cost (£/hr)	Top Area (£/hr)	Left Area (£/hr)	Right Area (£/hr)	G1 MW	G2 MW	G4 MW	G6 MW	G7 MW
Disabled	16416	12591	3325	501	220	290	127	200	200
Enabled	16666	9493	4738	2435	100	150	200	232	200

Observations:

1. The lines L1-2, L1-3 and L2-5 are overloaded in the system when constraints were disabled
2. L2-5 was at 150% of its capacity.
3. After enabling the limits, all lines were within limits
4. The hourly cost increased by £200
5. The zonal price at Right Area went up four times after enabling the limits whereas Top area had an advantage with £3000 reduction. Left had moderate increase in price

5.2 Contingency Analysis

Following are the observations after contingency analysis:

1. At normal run(with all lines functioning), the system seems to be stable within limits
2. The overall cost is 15612 £/hr

	Line 1-2 Open	Line 1-3 open	Line 2-3 open
P12 (max 140MW)	0	50	-100(100 in opp. direction))
P23 (max 160MW)	100	150	0
P13 (max 150MW)	50	0	150
ED cost(£/hr)	5000	5000	5000
OPF cost(£/hr)	5000	5000	5000
Maximum serviceable load(MW)	310	160	150

Table 6: N-1 case for Load = 150MW; P1 = 50MW; P2 = 100MW

3. Two contingencies, Line 1-2 and Line 1-3 makes the system vulnerable; as identified by contingency analysis
4. Disabling either of these lines will cause the other to run at nearly full load(120MW), as verified by Power Flow solution
5. There is a marginal rise in cost at 15629 and 15632 respectively.

5.3 Security Constrained OPF

By inserting contingencies and running SCOPF has identified two violations at L1-3 and L1-2, both exceeding the limits of 120MW. With normal OPF and SCOPF run on the seven bus system, following are the observations:

1. The marginal costs remained same in both cases at all nodes
2. The control Generators changed at buses. In normal OPF, Generator 1 was controlling the changes at multiple nodes, with SCOPF, the load response control was done by Generator 4, which was not present in normal OPF
3. The control was also distributed among Gen 5 and Gen 7 at their respective nodes in both cases
4. By running the power flow in the system after inserting contingencies separately at L1-2 and L1-3, it was noticed that the **Security of the system improved** because when L1-2 was open, line L1-3 was not overloaded in SCOPF

6 Conclusion

As mentioned in the introduction, energy as a commodity is define as “MWh at a given time and a given location, with a given level of supply security”. (Levi, 2020). From the lessons learned through the course and laboratory, Economic dispatch identifies the MWh at a given time based on the load curve and the marginal costs of generators involved. Optimal power flow identifies energy at a given location based on transmission constraints and nodal prices. The security of the system is confirmed by Contingency analysis and SCOPF.

7 References

[1] Victor Levi, EEEN60321 Concepts of Operation & Economics, 2020, The University of Manchester.