# Assignment 4: Word Embeddings

Welcome to the fourth (and last) programming assignment of Course 2!

In this assignment, you will practice how to compute word embeddings and use them for sentiment analysis.

- To implement sentiment analysis, you can go beyond counting the number of positive words and negative words.
- You can find a way to represent each word numerically, by a vector.
- The vector could then represent syntactic (i.e. parts of speech) and semantic (i.e. meaning) structures.

In this assignment, you will explore a classic way of generating word embeddings or representations.

• You will implement a famous model called the continuous bag of words (CBOW) model.

By completing this assignment you will:

- Train word vectors from scratch.
- Learn how to create batches of data.
- Understand how backpropagation works.
- Plot and visualize your learned word vectors.

Knowing how to train these models will give you a better understanding of word vectors, which are building blocks to many applications in natural language processing.

# Important Note on Submission to the AutoGrader

Before submitting your assignment to the AutoGrader, please make sure you are not doing the following:

- 1. You have not added any *extra* print statement(s) in the assignment.
- 2. You have not added any extra code cell(s) in the assignment.
- 3. You have not changed any of the function parameters.
- 4. You are not using any global variables inside your graded exercises. Unless specifically instructed to do so, please refrain from it and use the local variables instead.
- 5. You are not changing the assignment code where it is not required, like creating *extra* variables.

If you do any of the following, you will get something like, Grader Error: Grader feedback not found (or similarly unexpected) error upon submitting your assignment. Before asking for help/debugging the errors in your assignment, check for these first. If this is the case, and you don't remember the changes you have made, you can get a fresh copy of the assignment by following these instructions.

### Table of Contents

- 1 The Continuous Bag of Words Model
- 2 Training the Model
  - 2.1 Initializing the Model
    - Exercise 1 initialize\_model (UNQ\_C1)
  - 2.2 Softmax
    - Exercise 2 softmax (UNQ C2)
  - 2.3 Forward Propagation
    - Exercise 3 forward\_prop (UNQ\_C3)
  - 2.4 Cost Function
  - 2.5 Training the Model Backpropagation
    - Exercise 4 back\_prop (UNQ\_C4)
  - 2.6 Gradient Descent
    - Exercise 5 gradient\_descent (UNQ\_C5)
- 3 Visualizing the Word Vectors

# 1 - The Continuous Bag of Words Model

Let's take a look at the following sentence:

### 'I am happy because I am learning'.

- In continuous bag of words (CBOW) modeling, we try to predict the center word given a few context words (the words around the center word).
- For example, if you were to choose a context half-size of say C=2, then you would try to predict the word **happy** given the context that includes 2 words before and 2 words after the center word:
  - C words before: [I, am]
  - C words after: [because, I]
- In other words:

$$context = [I, am, because, I]$$
  
 $target = happy$ 

The structure of your model will look like this:

Where  $\dot{x}$  is the average of all the one hot vectors of the context words.

Once you have encoded all the context words, you can use  $\acute{x}$  as the input to your model.

The architecture you will be implementing is as follows:

$$\begin{align} h \&= W_1 X + b_1 \tan{1} \ a \&= ReLU(h) \tan{2} \ z \&= W_2 \ a + b_2 \tan{3} \ hat y \&= softmax(z) \tan{4} \ end{align}$$

```
# Import Python libraries and helper functions (in utils2)
import nltk
from nltk.tokenize import word tokenize
import numpy as np
from collections import Counter
from utils2 import sigmoid, get batches, compute pca, get dict
import w4 unittest
nltk.download('punkt')
[nltk data] Downloading package punkt to /home/jovyan/nltk data...
[nltk data] Unzipping tokenizers/punkt.zip.
True
# Download sentence tokenizer
nltk.data.path.append('.')
# Load, tokenize and process the data
import re
                                                                     #
Load the Regex-modul
with open('./data/shakespeare.txt') as f:
    data = f.read()
                                                                     #
Read in the data
data = re.sub(r'[,!?;-]', '.',data)
                                                                     #
Punktuations are replaced by .
data = nltk.word tokenize(data)
Tokenize string to words
data = [ ch.lower() for ch in data if ch.isalpha() or ch == '.']
Lower case and drop non-alphabetical tokens
print("Number of tokens:", len(data),'\n', data[:15])
print data sample
Number of tokens: 60996
['o', 'for', 'a', 'muse', 'of', 'fire', '.', 'that', 'would',
'ascend', 'the', 'brightest', 'heaven', 'of', 'invention']
# Compute the frequency distribution of the words in the dataset
(vocabularv)
fdist = nltk.FreqDist(word for word in data)
print("Size of vocabulary: ",len(fdist) )
print("Most frequent tokens: ",fdist.most_common(20) ) # print the 20
most frequent words and their freq.
Size of vocabulary: 5778
Most frequent tokens: [('.', 9630), ('the', 1521), ('and', 1394),
('i', 1257), ('to', 1159), ('of', 1093), ('my', 857), ('that', 781),
('in', 770), ('a', 752), ('you', 748), ('is', 630), ('not', 559),
('for', 467), ('it', 460), ('with', 441), ('his', 434), ('but', 417),
('me', 417), ('your', 397)]
```

Mapping words to indices and indices to words

We provide a helper function to create a dictionary that maps words to indices and indices to words.

```
# get_dict creates two dictionaries, converting words to indices and
viceversa.
word2Ind, Ind2word = get_dict(data)
V = len(word2Ind)
print("Size of vocabulary: ", V)
Size of vocabulary: 5778
# example of word to index mapping
print("Index of the word 'king': ",word2Ind['king'])
print("Word which has index 2743: ",Ind2word[2743])
Index of the word 'king': 2745
Word which has index 2743: kindness
```

# 2 - Training the Model

## 2.1 - Initializing the Model

You will now initialize two matrices and two vectors.

- The first matrix  $(W_1)$  is of dimension  $N \times V$ , where V is the number of words in your vocabulary and N is the dimension of your word vector.
- The second matrix  $(W_2)$  is of dimension  $V \times N$ .
- Vector  $b_1$  has dimensions  $N \times 1$
- Vector b<sub>2</sub> has dimensions V × 1.
- $b_1$  and  $b_2$  are the bias vectors of the linear layers from matrices  $W_1$  and  $W_2$ .

The overall structure of the model will look as in Figure 1, but at this stage we are just initializing the parameters.

### Exercise 1 - initialize\_model

Please use numpy.random.rand to generate matrices that are initialized with random values from a uniform distribution, ranging between 0 and 1.

**Note:** In the next cell you will encounter a random seed. Please **DO NOT** modify this seed so your solution can be tested correctly.

```
# UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ C1 GRADED FUNCTION: initialize model
def initialize model(N,V, random seed=1):
    Inputs:
        N:
            dimension of hidden vector
            dimension of vocabulary
        random seed: random seed for consistent results in the unit
tests
     Outputs:
        W1, W2, b1, b2: initialized weights and biases
    ### START CODE HERE (Replace instances of 'None' with your code)
###
    np.random.seed(random seed)
    # W1 has shape (N,V)
    W1 = np.random.rand(N, V)
    # W2 has shape (V,N)
    W2 = np.random.rand(V, N)
    \# b1 has shape (N,1)
    b1 = np.random.rand(N, 1)
    \# b2 has shape (V,1)
    b2 = np.random.rand(V, 1)
    ### END CODE HERE ###
    return W1, W2, b1, b2
# Test your function example.
tmp N = 4
tmp V = 10
tmp W1, tmp W2, tmp b1, tmp b2 = initialize model(tmp N,tmp V)
assert tmp W1.shape == ((tmp N,tmp V))
assert tmp W2.shape == ((tmp V,tmp N))
print(f"tmp_W1.shape: {tmp_W1.shape}")
print(f"tmp W2.shape: {tmp W2.shape}")
print(f"tmp_b1.shape: {tmp_b1.shape}")
print(f"tmp b2.shape: {tmp b2.shape}")
tmp W1.shape: (4, 10)
tmp W2.shape: (10, 4)
tmp b1.shape: (4, 1)
tmp b2.shape: (10, 1)
```

#### **Expected Output**

```
tmp_W1.shape: (4, 10)
tmp_W2.shape: (10, 4)
tmp_b1.shape: (4, 1)
tmp_b2.shape: (10, 1)

# Test your function
w4_unittest.test_initialize_model(initialize_model)

All tests passed
```

### 2.2 - Softmax

Before we can start training the model, we need to implement the softmax function as defined in equation 5:

$$\operatorname{softmax}(z_i) = \frac{e^{z_i}}{\sum_{i=0}^{V-1} e^{z_i}}$$

- Array indexing in code starts at 0.
- *V* is the number of words in the vocabulary (which is also the number of rows of *z*).
- *i* goes from 0 to |V| 1.

### Exercise 2 - softmax

**Instructions**: Implement the softmax function below.

- Assume that the input z to softmax is a 2D array
- Each training example is represented by a vector of shape (V, 1) in this 2D array.
- There may be more than one column, in the 2D array, because you can put in a batch of examples to increase efficiency. Let's call the batch size lowercase m, so the z array has shape (V, m)
- When taking the sum from  $i=1 \cdots V-1$ , take the sum for each column (each example) separately.

#### Please use

- numpy.exp
- numpy.sum (set the axis so that you take the sum of each column in z)

```
# UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ_C2 GRADED FUNCTION: softmax
def softmax(z):
```

```
I \cap I \cap I
    Inputs:
        z: output scores from the hidden layer
    Outputs:
    yhat: prediction (estimate of y)
    ### START CODE HERE (Replace instances of 'None' with your own
code) ###
    # Calculate yhat (softmax)
    yhat = np.exp(z) / np.sum(np.exp(z), axis=0)
    ### END CODE HERE ###
    return yhat
# Test the function
tmp = np.array([[1,2,3],
                [1,1,1]
tmp sm = softmax(tmp)
display(tmp sm)
array([[0.5
                   , 0.73105858, 0.88079708],
       [0.5
                   , 0.26894142, 0.11920292]])
```

### **Expected Ouput**

### 2.3 - Forward Propagation

### Exercise 3 - forward\_prop

Implement the forward propagation z according to equations (1) to (3).

 $\label{light} $$ \left( 1 \ X + b_1 \right) \ \ \\ = \ \ \left( 1 \ X + b_1 \right) \ \ \\ = \ \ \left( 1 \ X + b_2 \right) \ \$ 

For that, you will use as activation the Rectified Linear Unit (ReLU) given by:

$$f(h)=max(0,h)$$

```
# UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ C3 GRADED FUNCTION: forward prop
def forward prop(x, W1, W2, b1, b2):
    Inputs:
        x: average one hot vector for the context
        W1, W2, b1, b2: matrices and biases to be learned
     Outputs:
       z: output score vector
    ### START CODE HERE (Replace instances of 'None' with your own
code) ###
   # Calculate h
    h = np.dot(W1, x) + b1
    # Apply the relu on h,
    # store the relu in h
   h = np.maximum(0, h)
    # Calculate z
    z = np.dot(W2, h) + b2
    ### END CODE HERE ###
    return z, h
# Test the function
# Create some inputs
tmp N = 2
tmp V = 3
tmp x = np.array([[0,1,0]]).T
#print(tmp x)
tmp W1, tmp W2, tmp b1, tmp b2 = initialize model(N=tmp N,V=tmp V,
random seed=1)
print(f"x has shape {tmp_x.shape}")
print(f"N is {tmp N} and vocabulary size V is {tmp V}")
# call function
tmp_z, tmp_h = forward_prop(tmp_x, tmp_W1, tmp_W2, tmp_b1, tmp_b2)
print("call forward prop")
print()
# Look at output
print(f"z has shape {tmp z.shape}")
print("z has values:")
print(tmp z)
print()
```

```
print(f"h has shape {tmp_h.shape}")
print("h has values:")
print(tmp_h)

x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

z has shape (3, 1)
z has values:
[[0.55379268]
   [1.58960774]
   [1.50722933]]

h has shape (2, 1)
h has values:
[[0.92477674]
   [1.02487333]]
```

### **Expected output**

```
x has shape (3, 1)
N is 2 and vocabulary size V is 3
call forward_prop

z has shape (3, 1)
z has values:
[[0.55379268]
  [1.58960774]
  [1.50722933]]
h has shape (2, 1)
h has values:
[[0.92477674]
  [1.02487333]]
# Test your function
w4_unittest.test_forward_prop(forward_prop)
All tests passed
```

### 2.4 - Cost Function

• We have implemented the *cross-entropy* cost function for you.

```
# compute_cost: cross-entropy cost function
def compute_cost(y, yhat, batch_size):
    # cost function
```

```
logprobs = np.multiply(np.log(yhat),y)
    cost = - 1/batch size * np.sum(logprobs)
    cost = np.squeeze(cost)
    return cost
# Test the function
tmp C = 2
tmp N = 50
tmp batch size = 4
tmp word2Ind, tmp Ind2word = get dict(data)
tmp V = len(word2Ind)
tmp x, tmp y = \frac{next}{get} batches(data, tmp word2Ind, tmp V, tmp C,
tmp batch size))
print(f"tmp x.shape {tmp x.shape}")
print(f"tmp y.shape {tmp y.shape}")
tmp W1, tmp W2, tmp b1, tmp b2 = initialize model(tmp N, tmp V)
print(f"tmp W1.shape {tmp W1.shape}")
print(f"tmp W2.shape {tmp W2.shape}")
print(f"tmp_b1.shape {tmp_b1.shape}")
print(f"tmp b2.shape {tmp b2.shape}")
tmp z, tmp h = forward prop(tmp x, tmp W1, tmp W2, tmp b1, tmp b2)
print(f"tmp z.shape: {tmp z.shape}")
print(f"tmp h.shape: {tmp h.shape}")
tmp yhat = softmax(tmp z)
print(f"tmp yhat.shape: {tmp yhat.shape}")
tmp cost = compute cost(tmp y, tmp yhat, tmp batch size)
print("call compute cost")
print(f"tmp cost {tmp cost:.4f}")
tmp x.shape (5778, 4)
tmp y.shape (5778, 4)
tmp W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp b1.shape (50, 1)
tmp b2.shape (5778, 1)
tmp z.shape: (5778, 4)
tmp h.shape: (50, 4)
tmp yhat.shape: (5778, 4)
call compute cost
tmp cost 10.5788
```

#### **Expected output**

```
tmp_x.shape (5778, 4)
tmp_y.shape (5778, 4)
tmp_W1.shape (50, 5778)
tmp_W2.shape (5778, 50)
tmp_b1.shape (50, 1)
tmp_b2.shape (5778, 1)
tmp_z.shape: (5778, 4)
tmp_h.shape: (50, 4)
tmp_yhat.shape: (5778, 4)
call compute_cost
tmp_cost 10.5788
```

### 2.5 - Training the Model - Backpropagation

### Exercise 4 - back\_prop

Now that you have understood how the CBOW model works, you will train it. You created a function for the forward propagation. Now you will implement a function that computes the gradients to backpropagate the errors.

**Note**: z1 is calculated as  $w1 \cdot x + b1$  in this function. In practice, you would save it already when making forward propagation and just re-use here, but for simplicity, it is calculated again in back prop.

As reference, below are the equations of backpropagation as taught in the lecture:

```
### START CODE HERE (Replace instances of 'None' with your code)
###
    # Compute l1 as W2^T (Yhat - Y)
    l1 = np.dot(W2.T, yhat - y)
    # if z1 < 0, then l1 = 0
    # otherwise 11 = 11
    # (this is already implemented for you)
    l1[z1 < 0] = 0 # use "l1" to compute gradients below
    # compute the gradient for W1
    grad_W1 = \frac{1}{batch_size} * (np.dot(l1, x.T))
    # Compute gradient of W2
    grad_W2 = 1/batch_size * np.dot(yhat - y, h.T)
    # compute gradient for b1
    grad b1 = 1/batch size * (np.sum(l1, axis=1, keepdims=True))
    # compute gradient for b2
    grad b2 = 1/batch size * np.sum((yhat - y), axis=1, keepdims=True)
    ### END CODE HERE ####
    return grad_W1, grad_W2, grad_b1, grad_b2
# Test the function
tmp C = 2
tmp N = 50
tmp batch size = 4
tmp word2Ind, tmp Ind2word = get dict(data)
tmp V = len(word2Ind)
# get a batch of data
tmp x, tmp y = \frac{next}{get}  batches(data, tmp word2Ind, tmp V, tmp C,
tmp_batch_size))
print("get a batch of data")
print(f"tmp_x.shape {tmp_x.shape}")
print(f"tmp_y.shape {tmp_y.shape}")
print()
print("Initialize weights and biases")
tmp W1, tmp W2, tmp b1, tmp b2 = initialize model(tmp N,tmp V)
print(f"tmp W1.shape {tmp W1.shape}")
print(f"tmp W2.shape {tmp W2.shape}")
```

```
print(f"tmp b1.shape {tmp b1.shape}")
print(f"tmp b2.shape {tmp b2.shape}")
print()
print("Forwad prop to get z and h")
tmp z, tmp h = forward prop(tmp x, tmp W1, tmp W2, tmp b1, tmp b2)
print(f"tmp_z.shape: {tmp_z.shape}")
print(f"tmp h.shape: {tmp h.shape}")
print()
print("Get yhat by calling softmax")
tmp yhat = softmax(tmp z)
print(f"tmp_yhat.shape: {tmp_yhat.shape}")
tmp m = (2*tmp C)
tmp grad W1, tmp grad W2, tmp grad b1, tmp grad b2 = back prop(tmp x,
tmp yhat, tmp y, tmp h, tmp W1, tmp W2, tmp b1, tmp b2,
tmp batch size)
print()
print("call back prop")
print(f"tmp grad W1.shape {tmp grad W1.shape}")
print(f"tmp grad W2.shape {tmp grad W2.shape}")
print(f"tmp grad bl.shape {tmp grad bl.shape}")
print(f"tmp grad b2.shape {tmp grad b2.shape}")
get a batch of data
tmp x.shape (5778, 4)
tmp y.shape (5778, 4)
Initialize weights and biases
tmp W1.shape (50, 5778)
tmp W2.shape (5778, 50)
tmp b1.shape (50, 1)
tmp b2.shape (5778, 1)
Forwad prop to get z and h
tmp z.shape: (5778, 4)
tmp h.shape: (50, 4)
Get yhat by calling softmax
tmp yhat.shape: (5778, 4)
call back prop
tmp grad W1.shape (50, 5778)
tmp grad W2.shape (5778, 50)
tmp grad bl.shape (50, 1)
tmp grad b2.shape (5778, 1)
```

#### **Expected output**

```
get a batch of data
tmp x.shape (5778, 4)
tmp y.shape (5778, 4)
Initialize weights and biases
tmp W1.shape (50, 5778)
tmp W2.shape (5778, 50)
tmp bl.shape (50, 1)
tmp_b2.shape (5778, 1)
Forwad prop to get z and h
tmp z.shape: (5778, 4)
tmp h.shape: (50, 4)
Get yhat by calling softmax
tmp yhat.shape: (5778, 4)
call back prop
tmp grad W1.shape (50, 5778)
tmp grad W2.shape (5778, 50)
tmp grad b1.shape (50, 1)
tmp grad b2.shape (5778, 1)
# Test your function
w4 unittest.test back prop(back prop)
All tests passed
```

### 2.6 - Gradient Descent

### Exercise 5 - gradient\_descent

Now that you have implemented a function to compute the gradients, you will implement batch gradient descent over your training set.

**Hint:** For that, you will use initialize\_model and the back\_prop functions which you just created (and the compute\_cost function). You can also use the provided get\_batches helper function:

```
for x, y in get_batches(data, word2Ind, V, C, batch_size):
...
```

Also: print the cost after each batch is processed (use batch size = 128)

```
# UNIT TEST COMMENT: Candidate for Table Driven Tests
# UNQ C5 GRADED FUNCTION: gradient descent
def gradient_descent(data, word2Ind, N, V, num_iters, alpha=0.03,
                     random seed=282,
initialize model=initialize model,
                     get batches=get batches,
forward prop=forward prop,
                     softmax=softmax, compute cost=compute cost,
                     back prop=back prop):
    This is the gradient descent function
     Inputs:
        data:
                  text
        word2Ind: words to Indices
                  dimension of hidden vector
        V:
                  dimension of vocabulary
        num iters: number of iterations
        random seed: random seed to initialize the model's matrices
and vectors
        initialize model: your implementation of the function to
initialize the model
        get batches: function to get the data in batches
        forward prop: your implementation of the function to perform
forward propagation
        softmax: your implementation of the softmax function
        compute cost: cost function (Cross entropy)
        back prop: your implementation of the function to perform
backward propagation
     Outputs:
        W1, W2, b1, b2: updated matrices and biases after num iters
iterations
    1.1.1
    W1, W2, b1, b2 = initialize model(N,V, random seed=random seed)
\#W1=(N,V) and W2=(V,N)
    batch size = 128
    batch size = 512
    iters = 0
    C = 2
    for x, y in get_batches(data, word2Ind, V, C, batch size):
        ### START CODE HERE (Replace instances of 'None' with your own
code) ###
        # get z and h
        z, h = forward prop(x, W1, W2, b1, b2)
       # get yhat
```

```
yhat = softmax(z)
        # get cost
        cost = compute cost(y, yhat, batch size)
        if ((iters+1) % 10 == 0):
            print(f"iters: {iters + 1} cost: {cost:.6f}")
        # get gradients
        grad W1, grad W2, grad b1, grad b2 = back prop(x, yhat, y, h,
W1, W2, b1, b2, batch size)
        # update weights and biases
        W1 -= alpha * grad W1
        W2 -= alpha * grad W2
        b1 -= alpha * grad_b1
        b2 -= alpha * grad b2
        ### END CODE HERE ###
        iters +=1
        if iters == num iters:
            break
        if iters % 100 == 0:
            alpha *= 0.66
    return W1, W2, b1, b2
# test your function
C = 2
N = 50
word2Ind, Ind2word = get dict(data)
V = len(word2Ind)
num iters = 150
print("Call gradient descent")
W1, W2, b1, b2 = gradient descent(data, word2Ind, N, V, num iters)
Call gradient descent
iters: 10 cost: 9.686791
iters: 20 cost: 10.297529
iters: 30 cost: 10.051127
iters: 40 cost: 9.685962
iters: 50 cost: 9.369307
iters: 60 cost: 9.400293
iters: 70 cost: 9.060542
iters: 80 cost: 9.054266
iters: 90 cost: 8.765818
iters: 100 cost: 8.516531
iters: 110 cost: 8.708745
iters: 120 cost: 8.660616
```

```
iters: 130 cost: 8.544338
iters: 140 cost: 8.454268
iters: 150 cost: 8.475693
```

#### **Expected Output**

```
iters: 10 cost: 9.686791
iters: 20 cost: 10.297529
iters: 30 cost: 10.051127
iters: 40 cost: 9.685962
iters: 50 cost: 9.369307
iters: 60 cost: 9.400293
iters: 70 cost: 9.060542
iters: 80 cost: 9.054266
iters: 90 cost: 8.765818
iters: 100 cost: 8.765818
iters: 110 cost: 8.708745
iters: 120 cost: 8.660616
iters: 130 cost: 8.544338
iters: 140 cost: 8.454268
iters: 150 cost: 8.475693
```

Your numbers may differ a bit depending on which version of Python you're using.

# 3 - Visualizing the Word Vectors

In this part you will visualize the word vectors trained using the function you just coded above.

```
pyplot.annotate(word, xy=(result[i, 0], result[i, 1]))
pyplot.show()
```

You can see that man and king are next to each other. However, we have to be careful with the interpretation of this projected word vectors, since the PCA depends on the projection -- as shown in the following illustration.

```
result= compute_pca(X, 4)
pyplot.scatter(result[:, 3], result[:, 1])
for i, word in enumerate(words):
    pyplot.annotate(word, xy=(result[i, 3], result[i, 1]))
pyplot.show()
```

