CS 221 HW 8 - Logic

Name: Vinod Kumar Senthil Kumar
Sunet ID: vinodkum
Contributors: Joe Fan

4 a) $KB = \{(A \vee B) \rightarrow C,\ A\}$

$$
\begin{aligned}
(A \vee B) \rightarrow C &= \neg(A \vee B) \vee C \\
&= (\neg A \wedge \neg B) \vee C \\
&= (\neg A \vee C) \wedge (\neg B \vee C) \\
&= (A \rightarrow C) \wedge (B \rightarrow C)
\end{aligned}
$$

As proved by the derivation above, we can introduce a new generalized rule as below:

$$
\begin{aligned}
&(A_1 \vee A_2 \vee \dots \vee A_n) \rightarrow B \\
= \quad &(A_1 \rightarrow B) \wedge (A_2 \rightarrow B) \wedge \dots \wedge (A_n \rightarrow B)
\end{aligned}
$$

Using the new rule above and modus ponens, we can derive C from KB as follows.

$$(A \vee B) \rightarrow C = (A \rightarrow C) \wedge (B \rightarrow C)$$

The above result can be split and put back into KB.

$KB = \{A \rightarrow C,\ B \rightarrow C,\ A\}$

Using modus ponens we get,

$$\frac{A \rightarrow C,\ A}{C}$$

~~Thus~~ The KB now is $\{A \rightarrow C, B \rightarrow C, A, C\}$

Thus we derived C.

4b) $KB = \{A \vee B,\ B \rightarrow C,\ (A \vee C) \rightarrow D\}$

$= \{A \vee B,\ \neg B \vee C,\ \neg(A \vee C) \vee D\}$

$= \{A \vee B,\ \neg B \vee C,\ (\neg A \wedge \neg C) \vee D\}$

$= \{A \vee B,\ \neg B \vee C,\ (\neg A \vee D) \wedge (\neg C \vee D)\}$

Resolving A and $\neg A$,

$= \{B,\ \neg B \vee C,\ D \wedge (\neg C \vee D)\}$

Resolving B and $\neg B$,

$= \{C,\ D \wedge (\neg C \vee D)\}$

Resolving C and $\neg C$,

$= \{D \wedge D\}$

$= \{D\}$

Thus we derived D from KB ~~using~~ on converting to CNF and using resolution rule.

5b) Let us consider the finite model below for the original 6 constraints:

$\{1, 2\}$

1's successor is 2
2's successor is 1
$2 > 1 > 2$
All constraints regarding odd | even are satisfied.

Thus the finite model above is consistent for the original set of 6 constraints.

Now if we add the 7^{th} constraint "A number is not larger than itself", the finite model ~~is~~ above is not consistent as the transitive property on $2 > 1 > 2$ would derive $2 > 2$ violating the new 7^{th} constraint.

Generalizing from the finite model above, any finite model satisfying the original 6 constraints can be reduced to a standoff b/w two numbers using the transitive property as "$x > y > x$".

Now, enforcing the new 7^{th} constraint on the reduced form "$x > y > x$" would violate the consistency.

However, ~~to~~ the 7 constraints can be applied successfully on an infinite model as it ~~cannot~~ may not be reduced to a form "$x > y > x$".