LC Tank with Resistance

Vinod Kumar Metla

Roll No. : 130010048

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Abstract

The problem is to simulate the behaviour of a LC tank with a resistance applied across a voltage in series with given initial conditions of the charge in the capacitor and current in the inductor.

- Public git repo with open source code is available on https://github.com/vinodm24/sdes_project_1
- Ipython 2.3.0 is used to run the IPython notebook. So the version 2.3.0 or higher is preferrable to run the notebook.
- numpy version 1.8.2 is used. So numpy version 1.8.2 or higher is prefered to run the code.
- matplotlib version 1.4.2 is used. So matplotlib version 1.4.2 or higher is preffered to run the code.

Governing Equation for the problem

The governing equation for this electrical problem is

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{V}{C} \tag{1}$$

where Q is the charge in the capacitor varying with time, R is the resistance in the circuit, L is inductance of the inductor,C is the capacitance of the capacitor and V is the voltage of the source voltage under the initial conditions of $Q(0^+) = Q_0$ the initial charge in the capacitor and $\frac{dQ}{dt}(0^+) = i_0$, where initial inductor current is i_0.

Solving this analytically for two cases of $\Delta = 0$ and $\Delta \neq 0$ where $\Delta = \frac{R^2}{L^2} - \frac{4}{LC}$

$\Delta = 0$:

The solution when $\Delta = 0$ under the given initial conditions is:

$$Q(t) = CV + e^{\frac{-2Lt}{R}}(C_1 t + C_2) \tag{2}$$

where $C_1 = i_0 + \frac{R}{2L}(Q_0 - CV), C_2 = (Q_0 - CV)$

$\Delta \neq 0$:

The solution when $\Delta \neq 0$ under the given initial conditions is:

$$Q(t) = CV + A_1 e^{s_1 t} + A_2 e^{s_2 t} \tag{3}$$

where $s_1 = \frac{-R}{2L} + \frac{\sqrt{\Delta}}{2}$, $s_2 = \frac{-R}{2L} - \frac{\sqrt{\Delta}}{2}$
$A_1 = \frac{i_0-(Q_0-CV)s_2}{s_1-s_2}$, $A_2 = \frac{i_0-(Q_0-CV)s_1}{s_2-s_1}$

The results were plotted showing the voltage across all the elements of the circuit varying with time for three cases of damping.

Under Damped case

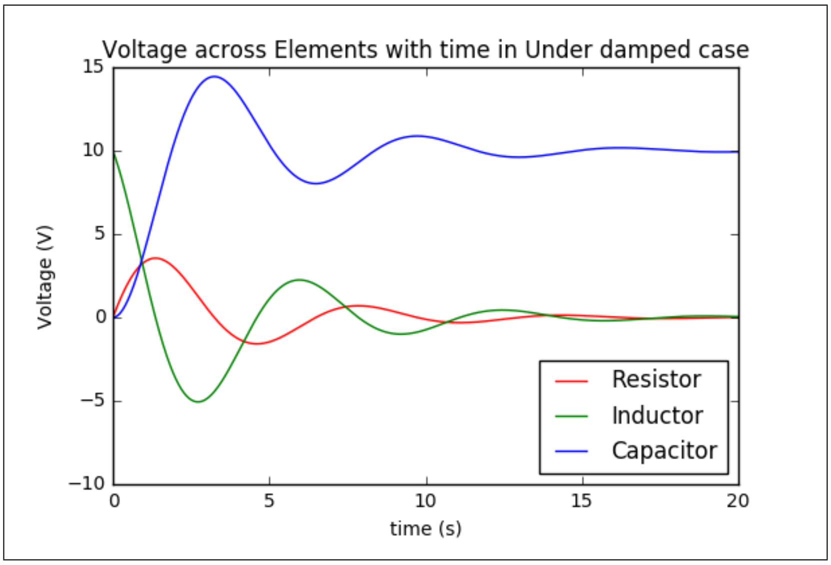

Figure 1: *Voltage vs time* [1].

Critically Damped case

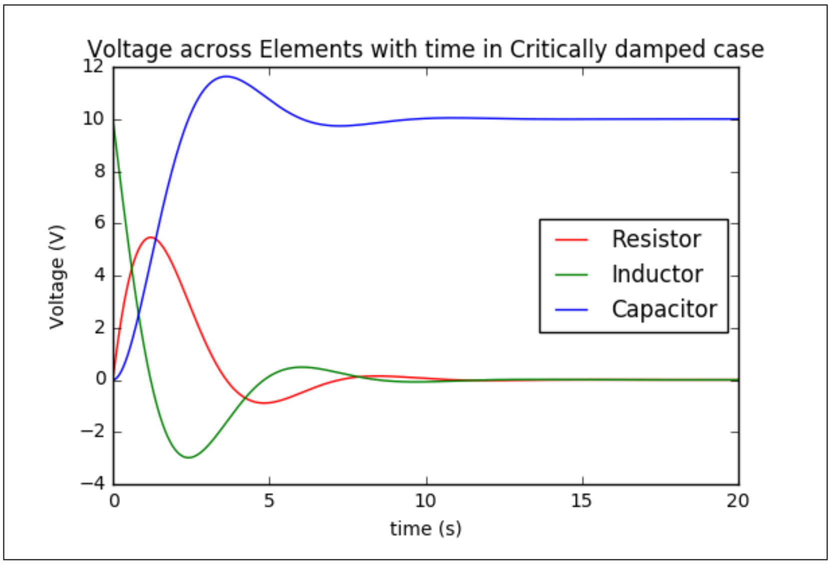

Figure 2: *Voltage vs time* [2].

Over Damped case

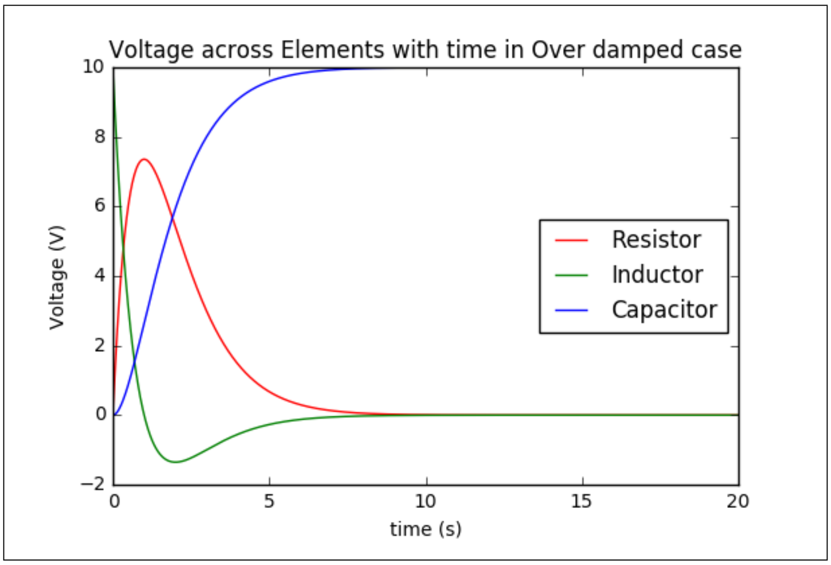

Figure 3: *Voltage vs time* [3].