unit 4.

1. Theory
2. Theory

3. $H_0: \mu = 40$ $\quad \bar{x} = 44.9$ $\quad s = 8.9$ $\quad n = 15$

$H_1: \mu > 40$ $\quad \mu = 40$ $\quad df = n - 1 = 15 - 1 = 14$

test statistic $= t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}} = \dfrac{44.9 - 40}{(8.9/\sqrt{15})}$

$= \dfrac{4.9}{2.2980}$

$= 2.1322$

$t_{\alpha, n-1} = t_{0.05, 14} = 1.761$

$2.132 > 1.761$ Reject Null Hypothesis.

$\bar{x} = 4.949$

OCTOBER						2001
M	1	8	15	22	29	
T	2	9	16	23	30	
W	3	10	17	24	31	
T	4	11	18	25		
F	5	12	19	26		
S	6	13	20	27		
S	7	14	21	28		

DECEMBER						2001
M	31	3	10	17	24	
T		4	11	18	25	
W		5	12	19	26	
T		6	13	20	27	
F		7	14	21	28	
S	1	8	15	22	29	
S	2	9	16	23	30	

4. $\bar{x} = 25.51$; $s = 2.1933$; $n = 20$; $df = n-1 = 19$

$\alpha = 0.05$; It is a two tailed test.

$\alpha/2 = \frac{0.05}{2} = 0.025$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{25.51 - 25}{\frac{2.1933}{\sqrt{20}}} = \frac{0.51}{0.4904} = 1.039$

$t_{\alpha/2, n-1}$; $t_{0.025, 19} = \pm 2.093$

The observed t-value is 1.039. The null hypothesis is not rejected.

Null Hypothesis is rejected if the observed t-value is less than -2.093 or greater than $+2.093$ (two tail)

5. $\bar{x} = 16.45$; $s = 3.59$; $n = 20$; $df = 19$

$\alpha = 0.05$; It is two tailed test.

$\alpha/2 = \frac{0.05}{2} = 0.025$

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{16.45 - 16}{3.59/\sqrt{20}} = \frac{0.45}{0.8027} = 0.5606$

$t_{\alpha/2, n-1}$; $t_{0.025, 19} = \pm 2.093$

The observed t-value is 0.5606. The null hypothesis is rejected.

6. $H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$

It is 2 tail because $\alpha/2 = 0.05/2 = 0.025$.

$df : n_1 + n_2 - 2$

$= 15 + 12 - 2$

$= 25$

$t_{0.025, 25} = \pm 2.060$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(47.73 - 56.5) - (0-0)}{\sqrt{\frac{19.495 \times 14 + 18.273 \times 11}{25}} \times \sqrt{\frac{1}{15} + \frac{1}{12}}}$$

$$t = \frac{-8.77 - 0}{\sqrt{\frac{272.93 + 201.003}{25}} \times \sqrt{\frac{4+5}{60}}}$$

$$t = \frac{-8.77}{\sqrt{\frac{473.933}{25}} \times \sqrt{\frac{9}{60}}} = \frac{-8.77}{4.3540 \times 0.3872}$$

$$= \frac{-8.77}{1.6858}$$

$$= -5.20$$

The Null hypothesis is rejected.

7. $H_0 : \mu_1 - \mu_2 = 0$ $\quad \alpha = 0.01$

$H_a : \mu_1 - \mu_2 < 0$ $\quad df = n_1 + n_2 - 2 = 8 + 11 - 2 = 17$

For one tail test, $\alpha = 0.01$, Critical $t_{0.01, 19} = -2.567$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$t = \frac{(24.56 - 26.42) - 0}{\sqrt{\frac{12.4 \times 7 + 15.8 \times 10}{8+11-2}} \sqrt{\frac{1}{8} + \frac{1}{11}}}$$

$$= \frac{-1.86}{\sqrt{\frac{2496.4}{17}} \sqrt{0.2159}}$$

$$= \frac{-1.86}{12.1180 \times 0.4646} = -1.05$$

Since $t = -1.05 > t_{0.19, 17} = -2.567$, the decision is fail to reject null hypothesis.

8. $\bar{x}_{diff}$: sample mean diff $= -0.95$

S : sample S.D diff $= 1.317$

n : sample size $= 20$; df $= n-1 = 19$

H_0 : $D = 0$ OR $(\mu_1 - \mu_2) = 0$

H_1 : $D > 0$ OR $(\mu_1 - \mu_2) > 0$

$$t = \frac{d'}{sd/\sqrt{n}} = \frac{-0.95}{(1.317)/\sqrt{20}} = -3.226$$

$t_{0.05,19} = 1.729$

$-3.226 < 1.729$

Accept Null hypothesis.

9. H_0 : $D = 0$; $\alpha = 0.1$; Two tail ; $\alpha/2 = 0.1/2 = 0.05$

H_a : $D \neq 0$; $t_{\alpha/2, n-1} = t_{0.05,8} = \pm 3.355$

$$t = \frac{\bar{d} - D}{sd/\sqrt{n}}$$

df $= n-1 = 8$; $n = 9$

Company	Year 1 P/E	Year 2 P/E	d	d-d'	$(d-d')^2$
1	8.9	12.7	-3.8	1.233	1.520
2	38.1	45.4	-7.3	-2.267	5.139
3	43	10	33	38.033	1446.509
4	34	27.2	6.8	11.833	140.020
5	34.5	22.8	11.7	16.733	279.993
6	15.2	24.1	-8.9	-3.867	14.954
7	20.3	32.3	-12	-6.967	48.539
8	19.9	40.1	-20.2	-15.167	230.038
9	61.9	106.5	-44.6	-39.567	1565.547
			-45.3		3732.26

mean of Sample diff $d' = -5.033$

variance $= 466.5325$

S.D $= 21.599$

$$t = \frac{d' - D}{sd/\sqrt{n}} = \frac{-5.033 - 0}{21.599/\sqrt{9}} = -0.70$$

10. $H_0: D = 0$

$H_a: D > 0$

Pair	Sample1	Sample2	d
1	38	22	16
2	27	28	-1
3	30	21	9
4	41	38	3
5	36	38	-2

$n = 9$; $\bar{d} = \frac{\Sigma d}{n} = \frac{25}{5} = 5$; $sd = 6.45$; $\alpha = 0.1$;

$df = 4$.

critical $t_{0.01,4} = 3.747$

$$t = \frac{\bar{d} - D}{s/\sqrt{n}} = \frac{5 - 0}{6.45/2.7360} = \frac{5}{2.8846} = 1.7333$$

∴ Reject Null Hypothesis.

11. $H_0: D = 0$

$H_a: D \neq 0$

Before	After	d
107	102	5
99	98	1
110	100	10
113	108	5
96	89	7

$n = 5$; $\bar{d} = \frac{\Sigma d}{n} = \frac{28}{5} = 5.6$

$sd = 3.4789$ $\alpha = 0.05$

$df = 4$.

critical $t_{0.025,4} = \pm 2.776$

$$t = \frac{\bar{d} - D}{sd/\sqrt{n}} = \frac{5.6 - 0}{3.4789/2.2360} = \frac{5.6}{1.5558} = 3.5994$$

Since observed $t = 3.5994 > t_{0.025,12} = 2.776$

the decision is to Reject the Null Hypothesis.

12.

f_o	f_e	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
21	16.56	19.7136	1.1904
109	97.29	137.1241	1.4094
62	70.38	70.2244	0.9977
15	22.77	60.3724	2.6514
			6.2489 = 6.25

H_0: The observed distribution is the same as the expected distribution.

H_1: The observed distribution is not the same as the expected distribution.

$\alpha = 0.05$; $n = 4$; $df = 3$.

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$= 6.25$$

$$\chi^2_{0.05,3} = 7.8147$$

Not Reject Null Hypothesis.

13.

f_o	f_e	$\frac{(f_o - f_e)^2}{f_e}$
53	68	3.309
37	42	0.595
32	33	0.030
28	22	1.636
18	10	6.400
15	8	6.125

H_0: The observed distribution is the same as the expected distribution.

H_1: The observed distribution is not the same as the expected distribution.

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$= 18.095$$

$$\chi^2_{0.05,5} = 11.07$$

Reject the Null Hypothesis.

14.

f_o	f_e	$(f_o-f_e)^2/f_e$
19	18	0.056
17	18	0.056
14	18	0.889
18	18	0.000
19	18	0.056
21	18	0.500
18	18	0.000
18	18	0.000
		1.557

$\bar{x} = \frac{\sum f_o}{k} = \frac{144}{8} = 18$

H_0: The observed frequencies are uniformly distributed.

H_1: The observed frequencies are not uniformly distributed.

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

$$= 1.557$$

$\chi^2_{0.17,7} = 18.4753$

Since $\chi^2 = 1.557 < \chi^2_{0.17,7} = 18.4756$, the decision is to fail to reject the null hypothesis.

15.

Month	f_o	f_e	$\frac{(f_o-f_e)^2}{f_e}$
Jan	1610	1537.25	3.4428
Feb	1585	"	1.4832
March	1649	"	8.1236
Apr	1590	"	1.8100
may	1540	"	0.0049
June	1397	"	12.7974
July	1410	"	10.5334
Aug	1350	"	22.8086
Sept	1495	"	1.1612
Oct	1564	"	0.4656
Nov	1602	"	2.7273
Dec	1655	"	9.0193
			74.3771

H_0: The monthly figures for milk sales are uniformly distributed.

H_1: The monthly figures for milk sales are not uniformly distributed.

The critical value $\chi^2_{0.01,11} = 24.7250$

To find expected frequency.

If the frequencies are uniformly distributed the same no. of gallons of milk is expected to be sold each month = $\frac{18,447}{12} = 1537.25$ gallons

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = 74.37$$

Reject Null Hypothesis

17.

1	2	3
29	32	25
27	33	24
30	31	24
27	34	25
28	30	26

* Critical f value

Observed 6.93

T_j : $T_1 = 141$ $T_2 = 160$ $T_3 = 124$, $T = 425$

n_j : $n_1 = 5$ $n_2 = 5$ $n_3 = 5$ $N = 15$

$\bar{x}_j$: $\bar{x}_1 = 28.2$ $\bar{x}_2 = 32.0$ $\bar{x}_3 = 24.8$ $\bar{x}_{..} = 28.33$

$$SSC = \sum_{j=1}^{k} n_j (\bar{x}_j - \bar{x})^2$$

$$= 5(28.2 - 28.33)^2 + 5(32.0 - 28.33)^2 + (5(24.8 - 28.33)^2$$

$$= 129.73$$

$$SSE = \sum_{i=1}^{n} \sum_{j=1}^{c} (x_{ij} - \bar{x}_j)^2$$

$$= 19.60$$

$$SST = SSC + SSE = 149.33$$

$$SST = \sum_{i=1}^{i} \sum_{j=1}^{c} (x_{ij} - \bar{x})^2$$

$$= 149.33$$

$df_c = 3 - 1 = 2$ $df_e = 15 - 3 = 12$ $df_t = 15 - 1 = 14$

	SS	df	MS	F
Between	129.73	2	64.87	39.80
Error	19.60	12	1.63	
Total	149.33	14		

Reject Null Hypothesis

18.

1	2	3	4
6.33	6.26	6.44	6.29
6.26	6.36	6.38	6.23
6.31	6.23	6.58	6.19
6.29	6.27	6.54	6.21
6.40	6.19	6.56	
	6.50	6.34	
	6.19	6.58	
	6.22		

$T_j : T_1 = 31.59 \quad T_2 = 50.22 \quad T_3 = 45.42 \quad T_4 = 24.92$

$n_j : n_1 = 5 \quad n_2 = 8 \quad n_3 = 7 \quad n_4 = 4$

$\bar{x}_j : \bar{x}_1 = 6.318 \quad \bar{x}_2 = 6.2775 \quad \bar{x}_3 = 6.488 \quad \bar{x}_4 = 6.230$

$T = 152.15$

$N = 24$

$\bar{x} = 6.3395$

$SSC = 0.23658 \quad df_C = 3 ; \; df_E = 20 ; \; df_T = 23$

$SSE = 0.15492 \quad MSC = 0.078860$

$SST = 0.39150 \quad MSE = 0.007746$

$F = 10.18$

	df	SS	MS	F
Between	3	0.2365	0.0788	10.18
Error	20	0.1549	0.0077	
Total	23	0.3915		

19. $H_0 : p = 0.17 \quad n = 550$

$H_1 : p > 0.17 \quad \hat{p} = 115/550 = 0.2090 \; ; p = 0.17$

The test is one tail

$\alpha = 0.05 \; ; \; Z_{0.05} = 1.645.$

The sample result must yield a observed test value greather than 1.645 to reject null hypothesis.

$$z = \frac{\hat{p} - p}{\sqrt{p \cdot q / n}} = \frac{0.209 - 0.17}{\sqrt{\frac{0.17 \times 0.83}{550}}} = \frac{0.039}{0.016} = 2.437$$

Because $z = 2.437$ is beyond $Z_{0.05} = 1.645$ we Reject the Null Hypothesis.

$$z_{0.05} = \frac{\hat{p}_e - p}{\sqrt{\frac{p.q}{n}}} \quad 1.645 = \frac{\hat{p}_e - 0.17}{0.016}$$

$$1.645 \times 0.016 = \hat{p}_e - 0.17$$

$$0.02632 + 0.17 = \hat{p}_e$$

$$\hat{p}_e = 0.19632$$

20. $H_0 : p = 0.08$

$H_a : p \neq 0.08$

$z_{0.05} = \pm 1.645 \quad \hat{p} = 33/200 = 0.165$

Observed z value

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p.q}{n}}} = \frac{0.165 - 0.080}{\sqrt{\frac{(0.08)(0.92)}{200}}} = \frac{0.085}{0.019} = 64.43$$

Reject Null Hypothesis

$$z_{\alpha/2} = \frac{\hat{p}_e - p}{\sqrt{\frac{p.q}{n}}} = \pm 1.645 = \frac{\hat{p}_e - 0.08}{\sqrt{\frac{0.08 \times 0.92}{200}}}$$

$$\hat{p}_e = 0.08 \pm 1.645 \sqrt{\frac{0.08 \times 0.92}{200}} = 0.08 \pm 0.32$$

$$= 0.048 \text{ and } 0.112$$

23. } Notes.
24. }

25. } Theory
26. }

27.

		$x - \bar{x}$	$y - \bar{y}$	
X	Y	x^2	y^2	xy
11	50	9	64	24
13	50	1	64	8
14	55	0	9	0
16	60	4	4	4
16	65	4	49	14
15	65	1	49	7

15	65	1	49	7
14	60	0	4	0
13	60	1	4	-2
13	50	1	64	8
140	580	22	360	70

$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = \frac{70}{\sqrt{22 \times 360}}$

$\bar{x} = \Sigma x / n = 14$

$\bar{y} = \Sigma y / n = 58$

$\frac{70}{\sqrt{7920}} = 0.7865$

There exist a higher degree of correlation.

28.

x	y	x^2	y^2	xy
7.43	221	55.2049	48841	1642.03
7.48	222	55.9504	49284	1660.56
8.00	226	64	51076	1808
7.75	225	60.0625	50625	1743.75
7.60	224	57.76	50176	1702.4
38.26	1118	292.9778	250002	8556.74

Karl Pearson's product moment of coff $= r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$

$= \frac{5 \times 8556.74 - 38.26 \times 1118}{\sqrt{5 \times 292.9778 - (38.26)^2 \times 5 \times 250002 - (1118)^2}}$

$= \frac{42783.7 - 42774.68}{\sqrt{1464.889 - 1463.8276 \times 1250010 - 1249924}}$

$= \frac{9.02}{\sqrt{1.0614 \times 85}}$

$= \frac{9.02}{9.4983}$

$= 0.9496$

∴ The Distribution is Positively Skewed.

29.

x	y	xy	x^2	y^2
7	8	56	49	64
4	5	20	16	25
6	4	24	36	16
9	8	72	81	64
3	3	9	9	9
8	6	48	64	36
37	34	229	255	214

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \times \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{6 \times 229 - 37 \times 34}{\sqrt{[6 \times 255 - (37)^2][6 \times 214 - (34)^2]}} = \frac{1374 - 1258}{\sqrt{161 \times 128}}$$

$$= \frac{116}{143.5548}$$

$$= 0.8080$$

Positive correlation of higher degree.

30.

x	y	xy	x^2	y^2
4	18	72	16	324
6	12	72	36	144
7	13	91	49	169
11	8	88	121	64
14	7	98	196	49
17	7	119	289	49
21	4	84	441	16
80	69	624	1148	815

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{7 \times 624 - 80 \times 69}{\sqrt{(8036 - 6400)(5705 - 4761)}} = \frac{4368 - 5520}{\sqrt{1636 \times 944}}$$

$$= \frac{-1152}{1242.7324} = -0.9269$$

Negative correlation.

31.

x	y	xy	x^2	y^2
158	349	55142	24964	121801
296	510	150960	87616	260100
87	301	26187	7569	90601
110	322	35420	12100	103684
436	550	239800	190096	302500
1087	2032	507509	322345	878686

$$r_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$$

$$= \frac{2537545 - 2208784}{\sqrt{(1611725 - 1181569)(4393430 - 4129024)}}$$

$$= \frac{328761}{\sqrt{430156 \times 264406}} = \frac{328761}{337247.4274}$$

$$= 0,9748$$

Positive correlation.

Question Bank.

Date / /

Unit 4 32.

x	y	x^2	y^2	xy
10	15	100	225	150
12	18	144	324	216
16	23	256	529	368
11	14	121	196	154
15	20	225	400	300
14	17	196	289	238
20	25	400	625	500
22	28	484	784	616
120	160	1926	3372	2542

Regression equation of x on y:

$\bar{x} = \frac{\Sigma x}{n} = 15$ $\quad \bar{y} = \frac{\Sigma y}{n} = 20$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \frac{20336 - 19,200}{26976 - 25600}$$

$$= \frac{1136}{1372} = 0.8279$$

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$x - 15 = 0.8279(y - 20)$$

$$x = 0.8279y - 1.558$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{20336 - 19,200}{15408 - 14400}$$

$$= \frac{1136}{1008} = 1.1269$$

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$y - 20 = 1.1269(x - 15)$$

$$y = 1.1269x + 3.0965$$

33.

x	y	xy	x^2	y^2
2	18	36	4	324
4	12	48	16	144
5	10	50	25	100
6	8	48	36	64
8	7	56	64	49
11	5	55	121	25
36	60	293	266	706

$$\bar{x} = \frac{\Sigma x}{n} = 6 \qquad \bar{y} = \frac{\Sigma y}{n} = 10$$

Regression Equation y on x

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{1758 - 2160}{1596 - 1296}$$

$$= \frac{-402}{300} = -1.34$$

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$y - 10 = -1.34(x - 6)$$

$$y = -1.34x + 18.04$$

34.

x	y	x^2	y^2	xy
3	3	9	9	9
6	2	36	4	12
5	3	25	9	15
4	5	16	25	20
4	3	16	9	12
6	6	36	36	36
7	6	49	36	42
5	4	25	16	20
40	32	212	144	166

$\bar{x} = 5$

$\bar{y} = 4$

Date / /20

Regression equation x on y

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \frac{1328 - 1280}{1152 - 1024}$$

$$= \frac{48}{128} = 0.375$$

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 5 = 0.375(y - 4)$$

$$x = 0.375y + 3.5$$

When $y = 9$,

$$x = 0.375(9) + 3.5$$

$$x = 6.875$$