Bi-Section laser having saturable absorber threshold calculations

$$\frac{\partial F}{\partial t} = \left[(1 - i\alpha) D + (1 - i\beta) d - 1\right] F \tag{0.1}$$

In the above equation [0.1], the diffraction is neglected.

$$\frac{\partial D}{\partial t} = -b_1 \left[D \left(1 + |F|^2\right) - \mu\right] \tag{0.2}$$

$$\frac{\partial d}{\partial t} = -b_2 \left[d \left(1 + s\,|F|^2\right) + \gamma\right] \tag{0.3}$$

Using the complex notation of electric field

$$F(t) = A(t)\, exp\left\{i\phi(t)\right\} \tag{0.4}$$

$$\Rightarrow \frac{\partial F}{\partial t} = \frac{\partial A}{\partial t} exp\left\{i\phi(t)\right\} + iA(t)\, exp\left\{i\phi(t)\right\} \frac{\partial \phi}{\partial t} \tag{0.5}$$

Equating the electric field equations [0.1] and [refe5]we get:

$\left[(1 - i\alpha) D + (1 - i\beta) d - 1\right] A(t)\, exp\left\{i\phi(t)\right\} = \frac{\partial A}{\partial t} exp\left\{i\phi(t)\right\} + iA(t)\, exp\left\{i\phi(t)\right\} \frac{\partial \phi}{\partial t}$

$\Rightarrow$

$\left[(1 - i\alpha) D + (1 - i\beta) d - 1\right] A(t) = \frac{\partial A}{\partial t} + iA(t) \frac{\partial \phi}{\partial t}$

Equating the similar terms in the above equations we get:

$$\frac{\partial A}{\partial t} = \left[D + d - 1\right] A(t) \tag{0.6}$$

$$\frac{\partial \phi}{\partial t} = -D\alpha - d\beta \tag{0.7}$$

The steady state solutions from Equations 0.6, 0.7, 0.2 and 0.3 are simply by doing $\frac{\partial A}{\partial t}$ =0, $\frac{\partial \phi}{\partial t}$ =0, $\frac{\partial D}{\partial t}$ =0 and $\frac{\partial d}{\partial t}$ =0:

From equation [0.6]

$$D = 1 - d \tag{0.8}$$

Similarly from equation [0.7]

$$D = -\frac{d\beta}{\alpha} \tag{0.9}$$

Similarly from equation [0.2]

$$\mu = D\left(1 + |A|^2\right) \tag{0.10}$$

Similarly from equation [0.3]

$$\gamma = -d\left(1 + s\,|A|^2\right) \tag{0.11}$$

Solving equation [0.10] and [0.11] and by simply avoiding to pump the passive (saturable absorber) material, we get:

$$\mu = \frac{D}{s}\left[-\frac{\eta_2}{d} - 1 + s\right] \tag{0.12}$$

and the $s = \frac{a_2 b_1}{a_1 b_2}$ value

Therefore we can use two values different values of D obtained from equations [0.8] and [0.9]

Substituting equation [0.8] in equation [0.12] we get:

$$\mu = \frac{(1-d)(a_1 b_2)}{(a_2 b_1)}\left[-\frac{\eta_2}{d} - 1 + \frac{a_2 b_1}{a_1 b_2}\right] \tag{0.13}$$

Similarly substituting equation [0.8] in equation [0.12] we get:

$$\mu = -\frac{(d\beta)(a_1 b_2)}{\alpha(a_2 b_1)}\left[-\frac{\eta_2}{d} - 1 + \frac{a_2 b_1}{a_1 b_2}\right] \tag{0.14}$$

Therefore equations [0.13] and [0.14] represents the threshold current for the bi-section laser having saturable absorber.