Task 1:

Note: Ignore the diffraction and diffusion terms in the E differential term. Just calculated the power turn on transient using Explicit Euler's Method.

Chapter 1

Bistability curve

The paraxial equations describing the system dynamics of broad area VCSEL having integrated saturable absorber are:

$$\frac{\partial E}{\partial t} + \frac{1}{T}\frac{\partial E}{\partial z} = [(1-i\alpha)D + (1-i\beta)d - 1]E + i\nabla_{\perp}^{2}E + \underline{d}\frac{\partial^2 E}{\partial z^2} \tag{1.1}$$

$$\frac{\partial D}{\partial t} = -b_1[D(1+|E|^2) - \mu(z)] \tag{1.2}$$

$$\frac{\partial d}{\partial t} == -b_2[d(1+s\,|E|^2) + \mu_s(z)] \tag{1.3}$$

The parameter values used in the simulation including the trivial terms:

The first three parameters electric field, carrier population in the active medium and in the passive medium are initialised in accordance with the bistability curve (see figure 1.1).

In other words, the values 0.05(L) and 0.3 (H) for the electric field correspond to intensity values on the curve at injection level 1.47, similarly values for carrier population in the active medium 1.15(L), 1.43(H) represent the parameter 'D' values for injection current 1.47, similarly values for carrier population in the Saturable Absorber -0.57(L),-0.23(H) represent the parameter 'd' values for injection current 1.47.

The parameter values starting from mirror transmissivity to saturation parameter are taken from the paper [**?**]. The cavity length i.e in the z direction is 1. The chisize is 20 based on diffraction.

1	E	Electric field	0.05 (L), 0.3(H)
2	D	Carrier population in the active medium	1.15(L), 1.43(H)
3	d	Carrier population in the passive medium	-0.57(L),-0.23(H)
4	T	Mirror transmissivity	0.1
5	α	linewidth enhancement factor for active media	2
6	β	linewidth enhancement factor for absorber	0
7	b_1	ratio of photon to carrier liftime for active media	0.05
8	b_2	ratio of photon to carrier lifetime for passive media	0.005
9	$\underline{d}$	field diffusion coefficient	$1.5 * 10^4$
10	s	saturation parameter	10
11	dt	integration time term	10^{-3}
12	Z	cavity length	1
13	X	chip size	20
14	dimx		1 (or)16
15	dimz		16
16	dx		$\frac{chipsize}{dimx-1}$
17	dz		$\frac{cavitylength}{dimz-1}$
18	$\mu(z)$	pump or injection along gain region	1.44 to 1.47
19	$\mu_s(z)$	absorption or injection along SA region	0.5
20	dkx		$\frac{2\pi}{dimx*dx}$
21	dkz		$\frac{2\pi}{dimz*dz}$
22	noise		0.0001

Table 1.1: Parameter Values

Task 2:

Note: Just predicted the behavior of Predator-Prey using Explicit Euler's Method.
The model is Lotka-Volterra Model

CHAPTER 4

Predator-Prey Interaction Models

4.1 The Lotka-Volterra Predator-Prey Interaction Model

One of the most universally recognized models in mathematics is the classic model for the interaction of a single predator species and a single prey specie developed by Alfred Lotka [34] and Vito Volterra [53]. If we let x represent the prey species, and we let y represent the predator species, then the model has the form,

$$\begin{aligned} \dot{x}(t) &= ax - bxy \\ \dot{y}(t) &= cxy - dy, \end{aligned} \tag{4.1}$$

where a, b, c and d are positive constants. We see that this model includes an exponential growth term for prey in the absence of predation, and an exponential decay for predators in the absence of prey. The interaction of the two species is represented by a mass action term, which implicitly assumes that the two species encounter each other at a rate proportional to each population level, and that the effect of predation on each is in turn proportional to the number of encounters.

This system of two ordinary differential equations has two steady state solutions, $(0, 0)$ and $(\frac{d}{c}, \frac{a}{b})$. It is well known that the trivial steady state is a saddle, while the nontrivial steady state is a center, and solutions in the phase plane form an infinite family of periodic orbits (Figure 4.1).

Task 3:

$2\frac{d^2y}{dt^2} = 11e^{-x^3} - 3\frac{dy}{dt} - 5y$

Just calculated the behavior of higher order differential equation using Explicit Euler's Method. The equation is reduced to first order using the transformation $\frac{dy}{dt} = x$.