вKAZAKH-BRITISH TECHNICAL UNIVERSITY

Approved by
Dean of School of Applied Mathematics
__________Assylbek A. Issakhov
On 23.08.2022, protocol № 1

Academic Policy of the Course
Calculus – III, MATH1213
Semester: Fall 2022
2022/2023 Academic Year
3 KZ credits (1/0/2)/5 ECTS

Lecturer: Sinitsa Artem Vitaliyevich, Master of Sciences, Senior Lecturer.

Lecturer/Instructor personal information	**Time and place of classes**		**Contact information**
	Lessons	**Office Hours**	**e-mail**
Senior-Lecturer, Master of Sciences in engineering and technologies of specialty MCM, PhD Candidate	According to the schedule, available on wsp.kbtu.kz	*Will be announced via M-Teams*	a.sinitsa@kbtu.kz

COURSE DURATION:

- 3 credits\5 ECTS, 15 weeks, 45 class hours

COURSE PRE-REQUISITES:

- It is assumed a sufficient knowledge of advanced mathematics in content of the Calculus-I, II courses.

COURSE DESCRIPTION
Course objectives

Calculus or mathematical analysis is the essential part of the mathematical background required of mathematicians, economists, engineers, physicists, and other applied researchers. It is difficult to overestimate importance of the Mathematical Analysis for engineering students. This requirement reflects the importance and wide applications of the subject matter. The course is designed for IT and engineering specialties students as well as applied mathematicians. The primary objective of the designed course is to direct students towards general understanding of basic concepts of advanced mathematical applied tools and familiarize students with varied range of applied techniques to deal with the functions of complex-valued variables.

The objectives of the course are to deepen students' understanding of:

- Mathematical analysis skills - apply appropriate mathematical concepts and operations to interpret data and to solve problems in terms of advance scientific applications.
- Skills to process and evaluate effectively both theoretical and real-life quantitative data, - identify a problem and analyze it in terms of its significant parts and the information needed to solve it.

- Skills to apply the main methods of problem solving to the situations connected with the major.

General Topics

- Advances of algebra and calculus of several variables' functions.
- Approximation and discretization, integration, and differentiation in higher dimensions.
- Geometrical and Physical applications of multidimensional integrals and differentiation operators.
- Fields and Potential functions.
- Parametrization of surfaces.
- Complex variables' functions and their applications towards modeling of physical phenomenon.
- Overview of variational and operational calculus.

Course learning outcomes

The goals of the course are to familiarize students with the important branches of calculus and their applications in computer sciences. During the educational process students should become familiar with and able to apply mathematical analysis tools to solve a variety of applied problems in topics such as: limits of sequences and functions of a single variable, continuity, derivatives, indefinite and definite integrals, functions of several variables, partial derivatives, multiple integrals, line integrals of the first and second kind with its physical applications as well as the vector fields applications. Fields of complex numbers and their applications towards physical phenomenon modeling.

Students successfully completing the course will be able to:

- Enhance their self-study abilities due to combined blended (combined) distance learning format.
- Do the tasks with finding functions with several variables domains, ranges and level curves.
- Implement the tools necessary for calculation of function extreme values by gradient notation and Lagrange multipliers to investigate extreme value of several variable functions.
- Evaluate multiple integrals of several variable functions, its partial derivatives and approximation forms.
- Identify potential function for conservative and gradient fields.
- Parametrize surfaces and calculate its areas.
- Operate with complex-valued variables functions.

Knowledge

After successful completion of the course students gain the knowledge of:

- Tools used to work with functions of several variables like limits, partial and directive derivatives.
- Basic notions of gradient operator.
- Elements of mathematical analysis such as differential and tangent planes, extreme value, and saddle points evaluation.
- Essentials of Lagrange multiplier's optimization methods and multiple integrals.
- Operations over vector fields and their physical applications.
- Physical implementation of iterated integrals.
- Operations over functions with complex variables and their applications.

Literature

- *Main Textbooks*

[1]. Thomas' Calculus. 12th Edition. Addison-Wesley, 2010.
[2]. Vladimir A. Zorich, Mathematical Analysis, Moscow Center for Continuous Mathematical Education (MCCME), 9-th edition, 2019.
[3]. Matthias Beck, Gerald Marchesi, Dennis Pixton, Lucas Sabalka, A First Course in Complex Analysis, Binghamton University (SUNY) and San Francisco State University, Version 1.54, 2018.
[4]. M. Ablowitz and A. Fokas, Complex Variables, Cambridge Univ. Press, Cambridge UK, 2003.

- *Supplementary*

[5] Sinkevich G.I., Functions of the complex variables. Theory and practice: tutorial, Saint Petersburg State University, 2016.
[6] M. A. Lavrent'ev, B. V., Shabat Methods of the theory of functions of a complex variable, 4th ed., Rev. and add. - M .: Science. Ch. ed. physical -mat. lit. 1973 - 749 s
[7]. Dubovin V.T., Theory of functions of the complex variable: theory and practice tutorial: Cazan state University, 2010.
[8]. Grigorii M. Fichtenholz, Course of integral and differential calculus, part 2, Phys. Mat. Lit., 2003, ISBN 5-9221-0157-9.
[9]. Guoning Wu, Integrals Depending on a Parameter, China University of Petroleum-Beijing, 2017.9.

COURSE CALENDAR

Week #	General Information				SIS and assessments
	Lessons' content	**Lecture classes**	**Seminar classes**	**Topics and materials to study**	
1	Line Integrals and Vector Fields. Line integral of multivariable function over parametrically given curve, - evaluation methods. Additivity principle. Calculation of mass, first moments about the coordinate planes, centroids, and moments of inertia about axes and other lines. First and second kinds of integrals. Line integrals in the plane. Line integral of vector fields: Computation of work, circulation, and flux. Work done by a force over a curve in space. Flow integrals and circulation for velocity fields. Flux across a simple\smooth closed plane curve. Revising vector fields and forms in $\mathbb{R}^3$, and some differential operators: $grad, rot, div$ and ∇ (Hamiltonian), ∇^2 (Laplacian).	2	1	**Line Integrals and Vector Fields.** Compulsory Reading: Lecture notes; [1] Ch.16.1, 16.2. Additional Reading: [2] pp. 213 – 218, 240 – 243.	**SIS 1:** [1] Ch.16.1, 16.2 – even numbers.
2	Conservative fields and general integral formulas. Path independence. Potential function. Fundamental theorem for line integrals in conservative fields, connection with gradient fields. Loop property. Component test for conservative fields and exactness of differential form. Green's theorem, - tangential and normal forms.	2	1	**Conservative fields and general integral formulas.** Compulsory Reading: Lecture notes; [1] Ch.16.3, 16.4. Additional Reading: [2] pp. 223 – 227, 269 – 276.	**SIS 2:** [1] Ch.16.3, 16.4 – even numbers.

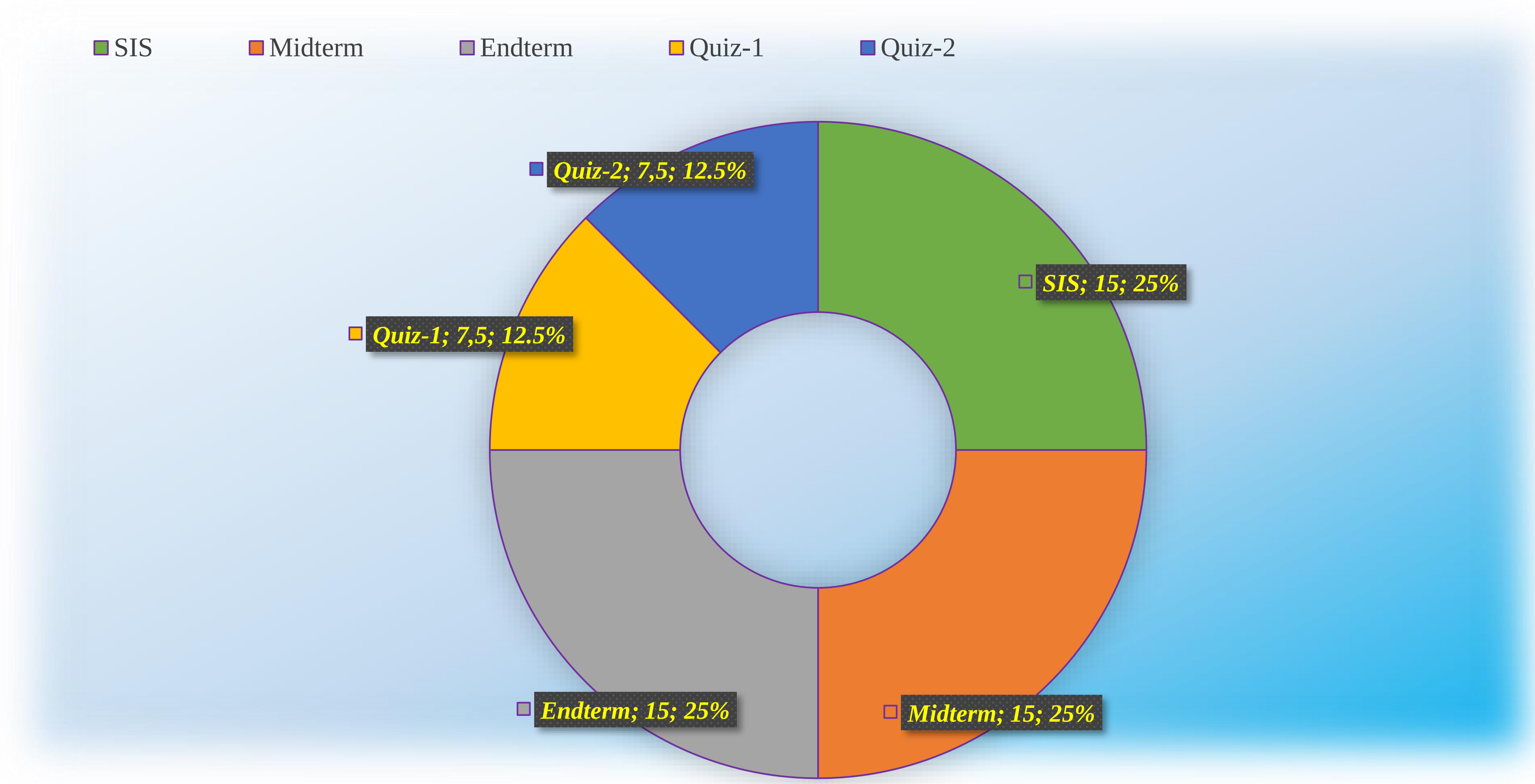
3	Surface Integrals. Parametrization of surfaces. Smooth surface area. Definition of line integral from forms of oriented surface. Surface area differential for a parametrized and implicitly given surfaces. Evaluation of a surface integral for a scalar function and vector fields. Computation of moments and mass of a thin shell.	2	1	**Surface Integrals.** Compulsory Reading: Lecture notes; [1] Ch.16.5, 16.6. Additional Reading: [2] pp.199 - 205.	**SIS 3:** [1] Ch.16.5, 16.6 – even numbers.
4	Divergence theorem. The curl vector field. Gauss's – Ostrogradsky's theorem for conservative fields and various surfaces. Divergence in three dimensions and the curl: example with continuity equation of hydrodynamics. A unifying fundamental theorem of vector integral calculus	2	1	**Divergence theorem.** Compulsory Reading: Lecture notes; [1] Ch.16.7, 16.8. Additional Reading: [2] pp.227 – 232.	**SIS 4:** [1] Ch.16.7, 16.8 – even numbers. ***Quiz-1.***
5	Proper integrals depending on a parameter: definition and continuity, its differentiation and integration. Improper integrals depending on a parameter: Uniform convergence with respect to parameter, Cauchy criterion and sufficient condition. Stirling's formula for approximation.	2	1	**Integrals depending on parameter.** Compulsory Reading: [8] pp.1 – 7. Additional Reading: [9] Ch.14, § 1 – 3 **.**	**SIS 5:** Distrib.
6	The Eulerian integrals. The Beta-function: domain of definition, symmetry, the reduction formula, other forms of representation of the beta function. The Gamma-function: domain of definition, smoothness and the formula for the derivatives, the reduction formula, the Euler-Gauss formula, the component formula. Connection between the Beta and Gamma function.	2	1	**The Eulerian Integrals.** Compulsory Reading: [8] pp.: 9 – 12. Additional Reading: [6] Ch. 7, § 1-7; [9] Ch.14, § 4, 5.	**SIS 6:** Distrib.
7	Operations over complex numbers. Properties of arithmetical operations. Geometrical representations over complex numbers: cartesian and polar complex planes, complex graphs. Notations of modulus and argument operators of complex numbers. Root extraction. Equality, ordering conjugate, reciprocal, exponential function, and complex algorithm. Elementary Topology of the Plane.	2	1	**The algebra and calculus of complex numbers.** Lecture notes; [3] Ch. 1, p.1-18; [7] Ch.1, p.3-7.	**SIS 7:** [7] pp.: 10 – 11, even numbers. **Midterm assessment.**

8	Elementary functions of complex variables. Examples of functions: Mobius transformations, infinity and the cross ratio, stereographic projection, exponential and trigonometric functions of complex variables, logarithms, and complex exponentials. Fractals and their applications.	2	1	**Complex variable functions.** Lecture notes; [3] Ch. 3, p.34-48; [4] Ch.1.2.1, 1.2.2.	**SIS 8:** [7] pp.: 16 – 17, even numbers. Recommended problems: [4] pp.: 18 – 20 (Ex-s: 1 – 12)
9	Limits and continuity, differentiability and holomorphicity. The Cauchy-Riemann equations, analytic functions. Physical applications of complex differentiation.	2	1	**Differentiation of complex variable functions.** Reading: Lecture notes; [3] Ch.2, pp.:19-30; [4] Ch.1.3, pp.:20-32; Ch.2.1.1.	**SIS 9:** [7] p.: 25 (Ex.-s.: 98-101); Recommended problems: [4] pp.: 29 – 31 (Ex-s: 1 – 13)
10	Integration of complex variable function: definition and basic properties. Antiderivatives. Reduction to line integrals, indefinite integrals in complex plane. Cauchy's integral theorem. Indefinite integral in complex plane.	2	1	**Integration of complex variable functions.** Reading: Lecture notes; [3] Ch.4; [4] Ch. 2.4 – 2.6.	**SIS 10:** [7] pp.: 38 – 39, even numbers; p.: 38, Ex.-s.: 102 – 127, even numbers; p. 43, ex.-s.: 128 – 137.
11	Consequences of Cauchy's integral theorem: higher order derivatives of complex variables' functions and Caushy's inequality. Liouville, Morera, and Maximum-Modulus Theorems	2	1	**Consequences of Cauchy's integral theorem.** Reading: Lecture notes; [3] Ch. 5; [4] 2.6.1, 2.6.2.	**SIS 11:** [3] Ch.5, even numbers; [7] p. 48, ex.-s.: 138 – 147. Recommended problems: [4] Ch. 2.6 **Quiz – 2.**

12	Definitions and Basic Properties of Complex Sequences, Series. Power series and holomorphic functions: Taylor and Laurent series. Properties of uniformly convergent functional series. Classifications of zeroes and the identity principle.	2	1	**Representing analytic functions as series.** Reading: Lecture notes; [3] Ch.8; [4] Ch. 3.1 – 3.3. Recommended reading: [4] Ch. 3.4, 3.6, 3.7, 3.8.	**SIS 12:** [3] Ch.8, even numbers; [7] p.53, ex.-s.: 148 – 157. p.56, ex.-s.: 170 – 186; p. 66 – 67, ex.-s.: 200 – 225, even numbers.
13	Classifications of singularities. Cauchy’s residue theorem. Argument principle and Rouche’s theorem. Principal value integrals and integrals with branch points. Fourier and Laplace transforms applications to differential equations.	2	1	**Residue calculus and application of contour integration.** Reading: Lecture notes; [3] Ch.9; [4] Ch. 4. Recommended reading: [6] Ch.6, § 1, 2.	**SIS 13:** [3] Ch. 9, even numbers; [7] pp.:73-74, ex.-s.:226-243, even numbers; p. 80, ex.-s.:244-249; p.83, ex.-s.:273-291, even numbers.
14	Conformal mappings and applications: critical points and inverse mappings, the Schwarz-Christoffel transformation, bilinear transformation. Physical applications: ideal fluid flow, heat flow, electrostatics, the force due to fluid pressure	2	1	**Applications of complex function theory.** Reading: Lecture notes; [4] Ch. 5.1-5.4, 5.6-5.7.	**SIS 14:** [4] Ch. 5.1-5.4, 5.6-5.7. **Endterm Assessment.**
15	Basic variational principles. Variation of the Robin functions under deformation of a domain or a portion of its boundary. Dilation inequalities and comparison of distortions. Behavior of level curves. Physical applications of conformal mappings and harmonic functions: variation of lifting force, waves in heavy liquids, stall in fluid dynamics, groundwater flow.	2	1	**Variational principles of conformal mappings.** Reading: Lecture notes; [6] Ch. 4, § 1 – 3.	**SIS 15:** Distrib.

COURSE ASSESSMENT PARAMETERS

Grading criteria distribution (Final Exam is not included here):



Tentative timetable of exams and tasks:

№	Assessment Criteria	Week №															Total
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	-
1	Seminar's Activity[1]	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	15[1]
2	Midterm Test							15									15
3	Endterm Test														15		15
4	SIS	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	15
5	Quizzes				7.5							7.5					15
6	Special task								5								5
7	Final exam-ination																40
8	Total																100 {15}[2]

[1]*Bonus points – *up to 1 point, depending on the difficulty of solved problem (max possible score: 15 points).*

[2]*Special task – Individual work of student, distributed as a bonus task, to compensate lost points on first attestation.*

[3]*Assume 11,2 points as bonus points (additional)*

Lectures are conducted in the form of explaining the theory given in the course that is why students supplied with handouts uploaded into the intranet. Activity and attendance on lessons is mandatory. Mandatory requirement is preparation for each lesson. Lectures are conducted through online platform.

Grading policy:

Intermediate attestations (on 8th and 15th week) join topics of all lectures, laboratories, homework, quiz and materials for reading discussed to the time of attestation. Maximum number of points within attendance, activity, homework, quiz, and laboratories for each attestation is 30 points.

Final exam joins and generalizes all course materials, is conducted in the complex form with questions and problems. Final exam duration is usually 120 min. Maximum number of points is 40. At the end of the semester, students receive overall total grade (summarized index of students' work during semester) according to conventional KBTU grade scale.

Online examination:

All types of assessments (examinations, quiz, midterm and end of term exams) are carried out using the *Proctor Edu* proctoring system.

ACADEMIC POLICY

Students are required:

- to be respectful to the teacher and other students;
- to switch off mobile phones during classes;
- DO NOT cheat. Plagiarized papers shall be graded with zero points!
- to come to classes prepared and actively participate in classroom work; to meet the deadlines;
- to enter the room before the teacher starts the lesson;
- to attend all classes. No make-up tests or quiz are allowed unless there is a valid reason for missing it;
- to follow KBTU academic policy regarding **W, AW, I, F** grades.
- When students have a score of 29 or less for attestation 1 added to attestation 2, then their grade is F.
- When students have a score of 19 or less (less than 50%) for their final exam, then their grade is F.
- When students do not come for their final exam, then their grade is F.

Students are encouraged to

- consult the teacher on any issues related to the course.
- make up within a week's time for the works undone for a valid reason without any grade deductions.

Considered in meeting of School of Applied Mathematics, minutes №_____ «__» ___________ 2022 year.

Senior-Lecturer, Artem V. Sinitsa _________