Kazakh-British Technical University
School Of Applied Mathematics

«APPROVED BY»
Acting Dean of School of Applied Mathematics
Kenzhebayev T.S.
#5 «10» *January* 2025

SYLLABUS

Discipline: **Algebra**
Number of credits: **3 credits (2/0/1)**
Terms: **Spring 2025**
Instructor: Iskakov Alibek Masgutovich.

Personal Information about the Instructor	**Time and place of classes**		**Contact information**
	Lessons	**Office Hours**	**e-mail**
MSc, Lecturer	According to the schedule, wsp.kbtu.kz	*TBA*, *o.* #313	a.iskakov@kbtu.kz

COURSE DURATION: 3 credits, 15 weeks, 45 class hours

COURSE PRE-REQUISITES: Linear Algebra and Analytic Geometry, Linear Algebra, Discrete Mathematics and Mathematical Logic.

COURSE DESCRIPTION
The goals of the course are to familiarize students with the important branches of Abstract Algebra. This class will provide an introduction to abstract algebraic structures, including groups, rings, and fields. We will see many examples coming from number theory, linear algebra, geometry, combinatorics, and computer science.

Course objectives
This course is designed to meet the following program objectives:

a) To use mathematically correct language and notation for Abstract Algebra.
b) To become computationally proficient in involving procedures in working with different algebraic structures.
c) To understand the axiomatic structure of a modern mathematical subject and learn to construct simple proofs.
d) To solve problems that apply notions of Abstract Algebra to Chemistry, Economics, and Engineering.

The topics that will enable this course to meet its objectives are:

1. Definition of a group and its properties;
2. Notions of the subgroup, order of a group;
3. Cyclic groups, permutation groups, crystallographic groups;
4. The abstract notions of Rings and Fields;
5. Finite fields and Ring Ideals;
6. Applications of Finite Fields to cryptography;
7. basic proof and disproof techniques, including mathematical induction, verifying that axioms are satisfied, standard "uniqueness" proofs, proof by contradiction, and disproof by counterexample.

Course outcomes

Upon successful completion of this course, students will:

1. Construct and understand properties of groups.
2. Proficiently move between abstract groups and concrete examples.
3. Understand mathematical structures deeply.
4. Verify proofs and understand their value.
5. Write proofs clearly and with proficiency.
6. Present and defend mathematical arguments.
7. Think abstractly and appreciate the need for abstraction.
8. Communicate mathematics with greater clarity and more confidence.

Knowledge: During the study of this course, students must obtain knowledge about how to explain with examples the basic terminology of various algebraic structures such as groups, rings, and finite fields as well as their properties. Many of the topics we will learn in this abstract algebra course can be applied within other areas of mathematics, including algebraic topology (my field of expertise,) algebraic geometry, algebraic number theory, and others. The notions of abstract algebra can also be applied in many other scientific realms, including physics, chemistry, coding theory, quantum mechanics, neuroscience, anthropology, and many others. While we won't be exploring these applications much, if at all, in this course, it is important to realize that just because something is studied from an abstract, theoretical point of view, that doesn't mean that it is not useful. There are wide ranging applications throughout all of science and engineering and mathematics.

Skills: As a result of studying this course, students must be able to

- Assess properties implied by the definitions of a group and rings.
- Use various canonical types of groups (including cyclic groups and groups of permutations) and canonical types of rings (including polynomial rings and modular rings).
- Analyze and demonstrate examples of subgroups, normal subgroups and quotient groups.
- Analyze and demonstrate examples of ideals and quotient rings.
- Use the concepts of isomorphism and homomorphism for groups and rings.
- Produce rigorous proofs of propositions arising in the context of abstract algebra.

REFERENCES

Main:

1. William J. Gilbert, W. Keith Nicholson, Modern Algebra with Applications, John Wiley & Sons, 2004, ISBN 0471469890, 9780471469896, 352 pages.
2. Joseph A. Gallian, Contemporaty Abstract Algebra, Brooks/Cole Cengage Learning, [Boston, MA], 2013, ISBN-13: 978-1-133-59970-8, ISBN-10: 1-133-59970-2, 664 pages.
3. Fraleigh, J.B., A first course in Abstract Algebra, Pearson Education Limited, 2014, ISBN-10: 1-292-02496-8, ISBN-13: 978-1-292-02496-7, 461 pages.

Supplementary

4. E. B. Vinberg, A course in Algebra, American Mathematical Society, 2003, 511 pages
5. V. A. Romankov, Introduction to cryptography, Forum, Moscow, 2012 [In Russian], 243 pages

COURSE CALENDAR

Week №	General Information				SIS and assessments
	Lessons' content	**Lecture classes**	**Seminar classes**	**Topics and materials to study**	
1	Algebraic structures, Algebraic operations, Groups and Symmetries, Subgroups	2	1	Lecture notes and [1, Chapter 3]: 3.1, 3.2, 3.3, 3.4	SIS 1
2	Permutation Groups, Cyclic Groups and Dihedral Groups, Morphisms, Even and Odd Permutations, Cayley's Representation Theorem	2	1	Lecture notes and [1, Chapter 3]: 3.5, 3.6, 3.7, 3.8	SIS 2
3	Quotient Groups. Equivalence Relations, Cosets and Lagrange's Theorem, Normal Subgroups and Quotient Groups, Morphism Theorem, Direct Products, Groups of Low Order, Action of a Group on a Set	2	1	Lecture notes and [1, Chapter 4]: 4.2, 4.3, 4.4, 4.5	SIS 3
4	Symmetry Groups in Three Dimensions, Translations and the Euclidean Group, Matrix Groups, Finite Groups in Two Dimensions, Proper Rotations of Regular Solids, Finite Rotation Groups in Three Dimensions, Crystallographic Groups, Isomorphism theorems	2	1	Lecture notes and [1, Chapter 5]: 5.1, 5.2, 5.3, 5.6	**Quiz 1**
5	p-subgroups, Sylow theorems	2	1	Lecture notes and [3, Chapter 7]	SIS 4
6	Rings, Subrings: Definitions, examples, Ring Thorems, Morphisms of Rings	2	1	Lecture notes and [1, Chapter 8]: 8.1, 8.2, 8.3	SIS 5
7	Zero divisors, Units, Integral Domains	2	1	Lecture notes and [2, Chapter 3] 12, 13	**Midterm assessment**

8	Ring Ideals, Factor Rings, Examples: Euclidean Rings, Prime and Maximal Ideals	2	1	Lecture notes and [2, Chapter 3] 14, 15	SIS 6
9	Polynomial Rings, Factorization of Polynomials, Irreducibility tests	2	1	Lecture notes and [2, Chapter 3] 16, 17	SIS 7
10	Fields: Defintions and Examples, Ordered Fields, The Field of Quotients of an Integral Domain	2	1	Lecture notes and [3, Chapter 4] 18, 19	SIS 8
11	Finite Fields, Polynomials over Finite Fields, Euclidean division Algorithm for polynomials	2	1	Lecture notes and [3, Chapter 4] 20, 22	**Quiz 2**
12	Construction of a Field with p^m elements, Multiplicative groups of Finite Fields	2	1	Lecture notes and [5] Lecture 8	SIS 9
13	Introduction to Extension Fields, Algebraic and Transcendental Elements	2	1	Lecture notes and [1, Chapter 11] 11.1, 11.2	SIS 10
14	Insolvability of quintic polynomial equations in radicals	2	1	Lecture notes and [5] Lectures 9, 10	SIS 11
15	Applications of Multiplicative groups of Finite Fields to cryptography	2	1	Lecture notes and [3, Chapter 10]	**Endterm assessment**

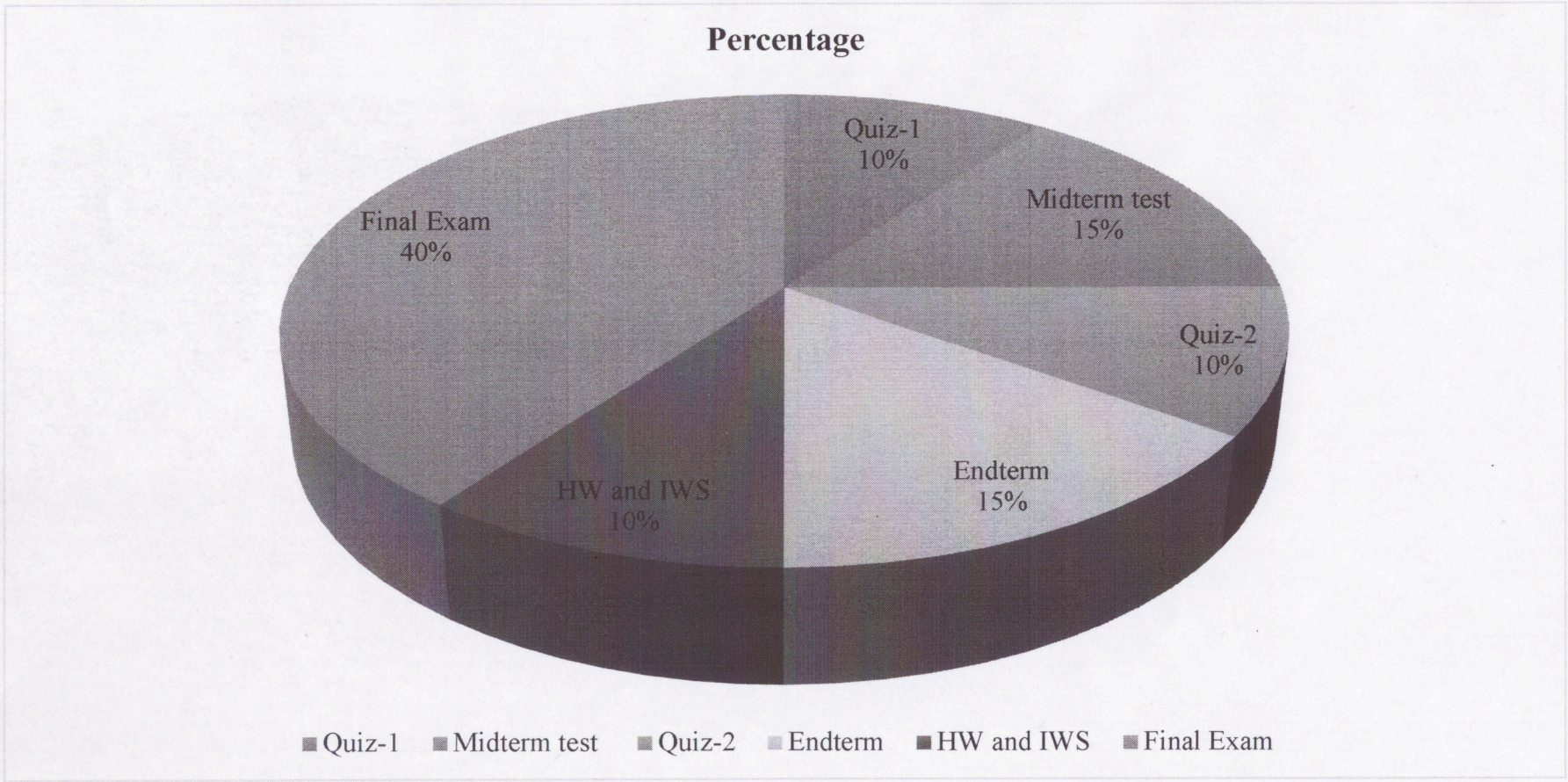
COURSE ASSESSMENT PARAMETERS

The tentative timetable of exams and tasks:

№	Assessment Criteria	Week №															Total
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	-
1	Activity on practice classes	DW[1]	0.5	0.5	Q-1	0.5	0.5	MT	0.5	0.5	0.5	Q-2	0.5	0.5	ET	0.5	**5**[2]
2	Homeworks and IWS[2]	DW[1]	1	1	*	1	1	*	1	1	1	*	1	1	*	1	10
3	Midterm Test	*	*	*	*	*	*	15	*	*	*	*	*	*	*	*	15
4	Endterm Test	*	*	*	*	*	*	*	*	*	*	*	*	*	15	*	15
5	Quizzes	*	*	*	10	*	*	*	*	*	*	10	*	*	*	*	20
6	Final examination	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	40
5	Total	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	100(**5**)[4]

[1]DW – Drop Week
[2]Seminar activity bonus points
[3]IWS – Individual Work of Student
[4]Total bonus points

Grading criteria:



Lectures are conducted in the form of explaining the theory given in the course, that is why students are supplied with handouts uploaded onto the intranet. Activity and attendance in lessons is mandatory. Mandatory requirement is preparation for each lesson.

Grading policy:

Intermediate attestations (on 7th and 14th week) join topics of all lectures, laboratories, homework, quiz and materials for reading discussed to the time of attestation. The maximum number of points within attendance, activity, homework, quiz and laboratories for each attestation is 30 points. Every intermediate attestation and quiz are held during practice hours in the university.

Final exam joins and generalizes all course materials, is conducted in the complex form with questions and problems. Final exam duration is 120 min. Maximum number of points is 40. At the end of the semester, you receive an overall total grade (summarized index of your work during semester) according to conventional KBTU grade scale.

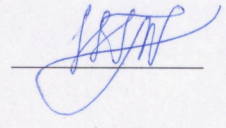
ACADEMIC POLICY

Students are required:

- to be respectful to the teacher and other students;
- to switch off mobile phones during classes;
- DO NOT cheat. Plagiarized papers shall be graded with zero points!
- to come to classes prepared and actively participate in classroom work; to meet the deadlines;
- to enter the room before the teacher starts the lesson;
- to attend all classes. No make-up tests or quiz are allowed unless there is a valid reason for missing it;
- to follow KBTU academic policy regarding **W, AW, I, F** grades.
- When students are absent for 30% of the lessons or more, then their grade is F.
- When students have a score of 29 or less for attestation 1 added to attestation 2, then their grade is F.
- When students have a score of 19 or less (less than 50%) for their final exam, then their grade is F.
- When students do not come for their final exam, then their grade is F.

Students are encouraged to

- consult the teacher on any issues related to the course;
- make up within a week's time for the works undone for a valid reason without any grade deductions;

Lecturer of School of Applied Mathematics  *Iskakov A.M.*