model Selection methods

Vinoth Aryan Nagabosshanam March 25, 2017

Model Selection

```
library(ISLR)
## Warning: package 'ISLR' was built under R version 3.3.3
summary(Hitters)
##
        AtBat
                          Hits
                                        HmRun
                                                          Runs
##
           : 16.0
                     Min.
                                    Min.
                                           : 0.00
                                                     Min.
                                                            :
                                    1st Qu.: 4.00
##
    1st Qu.:255.2
                     1st Qu.: 64
                                                     1st Qu.: 30.25
    Median :379.5
                     Median: 96
                                    Median: 8.00
                                                     Median: 48.00
           :380.9
                                           :10.77
                                                            : 50.91
##
    Mean
                     Mean
                            :101
                                    Mean
                                                     Mean
    3rd Qu.:512.0
                     3rd Qu.:137
                                    3rd Qu.:16.00
                                                     3rd Qu.: 69.00
##
    Max.
           :687.0
                     Max.
                            :238
                                    Max.
                                           :40.00
                                                     Max.
                                                            :130.00
##
##
         RBI
                          Walks
                                            Years
                                                              {\tt CAtBat}
           : 0.00
                             : 0.00
                                                                      19.0
##
    Min.
                      Min.
                                        Min.
                                               : 1.000
                                                          Min.
                                                                 :
    1st Qu.: 28.00
##
                      1st Qu.: 22.00
                                        1st Qu.: 4.000
                                                          1st Qu.: 816.8
    Median : 44.00
                      Median: 35.00
                                        Median : 6.000
                                                          Median: 1928.0
          : 48.03
                             : 38.74
                                               : 7.444
                                                                  : 2648.7
##
    Mean
                      Mean
                                        Mean
                                                          Mean
##
    3rd Qu.: 64.75
                      3rd Qu.: 53.00
                                        3rd Qu.:11.000
                                                          3rd Qu.: 3924.2
##
    Max.
           :121.00
                             :105.00
                                        Max.
                                               :24.000
                                                                  :14053.0
                      Max.
                                                          Max.
##
                                                               CRBI
##
        CHits
                          CHmRun
                                            CRuns
##
                4.0
                             : 0.00
                                                                      0.00
    Min.
                      Min.
                                        Min.
                                                    1.0
                                                          Min.
    1st Qu.: 209.0
                      1st Qu.: 14.00
                                        1st Qu.: 100.2
                                                          1st Qu.:
                                                                    88.75
    Median : 508.0
                      Median : 37.50
                                        Median : 247.0
                                                          Median: 220.50
##
    Mean
          : 717.6
                      Mean
                             : 69.49
                                        Mean
                                               : 358.8
                                                          Mean
                                                                  : 330.12
##
    3rd Qu.:1059.2
                      3rd Qu.: 90.00
                                        3rd Qu.: 526.2
                                                          3rd Qu.: 426.25
##
    Max.
           :4256.0
                      Max.
                             :548.00
                                        Max.
                                               :2165.0
                                                          Max.
                                                                  :1659.00
##
##
        CWalks
                                            PutOuts
                                                              Assists
                       League
                               Division
##
    Min.
           :
               0.00
                       A:175
                               E:157
                                         Min.
                                                :
                                                     0.0
                                                           Min.
                                                                  : 0.0
    1st Qu.: 67.25
                       N:147
                               W:165
                                         1st Qu.: 109.2
                                                           1st Qu.: 7.0
    Median: 170.50
##
                                         Median : 212.0
                                                           Median: 39.5
##
    Mean
           : 260.24
                                         Mean
                                                : 288.9
                                                           Mean
                                                                  :106.9
##
    3rd Qu.: 339.25
                                         3rd Qu.: 325.0
                                                           3rd Qu.:166.0
##
    Max.
           :1566.00
                                         Max.
                                                :1378.0
                                                           Max.
                                                                   :492.0
##
##
        Errors
                         Salary
                                       NewLeague
##
    Min.
           : 0.00
                            : 67.5
                                       A:176
##
    1st Qu.: 3.00
                     1st Qu.: 190.0
                                       N:146
##
    Median: 6.00
                     Median: 425.0
          : 8.04
##
    Mean
                     Mean
                            : 535.9
    3rd Qu.:11.00
                     3rd Qu.: 750.0
           :32.00
                            :2460.0
##
    Max.
                     Max.
```

```
## NA's :59
```

affter see the summary its clear that the na values in the salary variable to remove that we using the foolowing code

```
Hitters=na.omit(Hitters)
summary(Hitters)
```

```
AtBat
##
                          Hits
                                          HmRun
                                                             Runs
##
    Min.
            : 19.0
                     Min.
                             : 1.0
                                      Min.
                                              : 0.00
                                                       Min.
                                                               : 0.00
##
    1st Qu.:282.5
                     1st Qu.: 71.5
                                      1st Qu.: 5.00
                                                       1st Qu.: 33.50
##
    Median :413.0
                     Median :103.0
                                      Median: 9.00
                                                       Median : 52.00
##
    Mean
            :403.6
                     Mean
                            :107.8
                                      Mean
                                              :11.62
                                                       Mean
                                                               : 54.75
    3rd Qu.:526.0
                     3rd Qu.:141.5
                                      3rd Qu.:18.00
                                                       3rd Qu.: 73.00
            :687.0
##
    Max.
                     Max.
                             :238.0
                                      Max.
                                              :40.00
                                                       Max.
                                                               :130.00
##
         RBI
                          Walks
                                             Years
                                                               CAtBat
##
    Min.
            : 0.00
                                0.00
                                        Min.
                                                : 1.000
                                                           Min.
                                                                      19.0
    1st Qu.: 30.00
                      1st Qu.: 23.00
                                        1st Qu.: 4.000
                                                           1st Qu.: 842.5
##
##
    Median : 47.00
                      Median : 37.00
                                        Median : 6.000
                                                           Median: 1931.0
                                                : 7.312
##
    Mean
           : 51.49
                      Mean
                              : 41.11
                                        Mean
                                                           Mean
                                                                  : 2657.5
    3rd Qu.: 71.00
                      3rd Qu.: 57.00
                                        3rd Qu.:10.000
                                                           3rd Qu.: 3890.5
           :121.00
                              :105.00
                                                :24.000
##
    Max.
                      Max.
                                        Max.
                                                           Max.
                                                                  :14053.0
##
        CHits
                          CHmRun
                                             CRuns
                                                                CRBI
##
    Min.
                4.0
                      Min.
                              : 0.00
                                        Min.
                                                    2.0
                                                           Min.
                                                                      3.0
    1st Qu.: 212.0
                      1st Qu.: 15.00
                                        1st Qu.: 105.5
                                                           1st Qu.:
                                                                     95.0
##
    Median : 516.0
                      Median : 40.00
                                        Median : 250.0
                                                           Median : 230.0
           : 722.2
                              : 69.24
                                                : 361.2
##
    Mean
                      Mean
                                        Mean
                                                           Mean
                                                                  : 330.4
##
    3rd Qu.:1054.0
                      3rd Qu.: 92.50
                                        3rd Qu.: 497.5
                                                           3rd Qu.: 424.5
##
    Max.
           :4256.0
                      Max.
                              :548.00
                                        Max.
                                                :2165.0
                                                           Max.
                                                                  :1659.0
##
        CWalks
                      League Division
                                            PutOuts
                                                              Assists
##
    Min.
            :
                1.0
                      A:139
                              E:129
                                        Min.
                                                :
                                                    0.0
                                                           Min.
                                                                     0.0
##
    1st Qu.: 71.0
                      N:124
                              W:134
                                        1st Qu.: 113.5
                                                           1st Qu.:
                                                                     8.0
    Median : 174.0
                                        Median : 224.0
                                                           Median: 45.0
                                                : 290.7
##
    Mean
           : 260.3
                                        Mean
                                                           Mean
                                                                  :118.8
##
    3rd Qu.: 328.5
                                        3rd Qu.: 322.5
                                                           3rd Qu.:192.0
##
    Max.
            :1566.0
                                        Max.
                                                :1377.0
                                                           Max.
                                                                  :492.0
##
        Errors
                                        NewLeague
                          Salary
##
           : 0.000
                             : 67.5
    Min.
                      Min.
                                        A:141
    1st Qu.: 3.000
                      1st Qu.: 190.0
                                        N:122
##
   Median : 7.000
                      Median: 425.0
            : 8.593
##
    Mean
                      Mean
                              : 535.9
##
    3rd Qu.:13.000
                      3rd Qu.: 750.0
            :32.000
                              :2460.0
    Max.
                      Max.
```

Best Subset regression

We will now use the package leaps to evaluate all the best-subset models.

```
library(leaps)
## Warning: package 'leaps' was built under R version 3.3.3
regfit.full=regsubsets(Salary~.,data=Hitters)
summary(regfit.full)
```

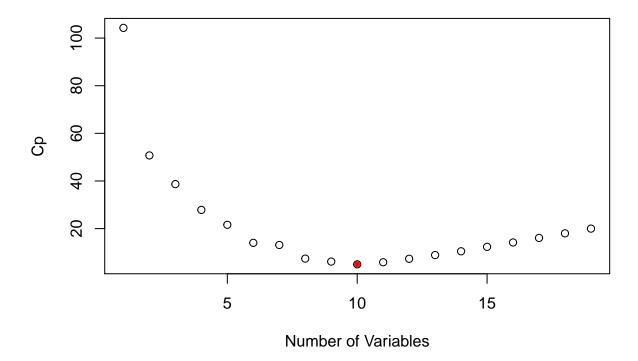
Subset selection object

```
## Call: regsubsets.formula(Salary ~ ., data = Hitters)
## 19 Variables (and intercept)
              Forced in Forced out
##
## AtBat
                   FALSE
                              FALSE
## Hits
                   FALSE
                              FALSE
## HmRun
                   FALSE
                              FALSE
## Runs
                   FALSE
                              FALSE
## RBT
                              FALSE
                   FALSE
## Walks
                   FALSE
                              FALSE
## Years
                  FALSE
                              FALSE
## CAtBat
                   FALSE
                              FALSE
## CHits
                   FALSE
                              FALSE
## CHmRun
                   FALSE
                              FALSE
## CRuns
                   FALSE
                              FALSE
## CRBI
                   FALSE
                              FALSE
## CWalks
                   FALSE
                              FALSE
## LeagueN
                   FALSE
                              FALSE
## DivisionW
                   FALSE
                              FALSE
## PutOuts
                   FALSE
                              FALSE
## Assists
                   FALSE
                              FALSE
## Errors
                   FALSE
                              FALSE
## NewLeagueN
                   FALSE
                              FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##
            AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
     (1)""
## 1
                                    11 11 11 11
## 2
     (1)""
                   "*"
## 3
     (1)
            11 11
                   "*"
                                              11 11
     (1)""
## 4
     (1)
     (1)
            "*"
                   "*"
## 6
                                        "*"
                                              11 11
## 7
      (1)""
                   "*"
                        11 11
                               11 11
                                                     "*"
                                    11 11 11 *11
      (1)"*"
                                                                   "*"
##
            CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
            "*"
                         11 11
                                 11 11
                                            11 11
                                                     11 11
## 1
      (1)
            "*"
                         11 11
                                  11 11
                                            11 11
                                                     11 11
                                                             11 11
                                                                     11 11
      (1)
                                  11 11
            "*"
                         11 11
                                            "*"
## 3
     (1)
                                            "*"
## 4
     (1)
                         11 11
                                  "*"
                         11 11
                                  "*"
                                            11 * 11
## 5
      (1)
                                  "*"
## 6
     (1)"*"
                                            "*"
                                                             11 11
## 7 (1)""
                                  "*"
                                            "*"
                                                     11 11
## 8 (1)""
                  "*"
                                  "*"
                                            "*"
```

It gives by default best-subsets up to size 8; lets increase that to 19, i.e. all the variables

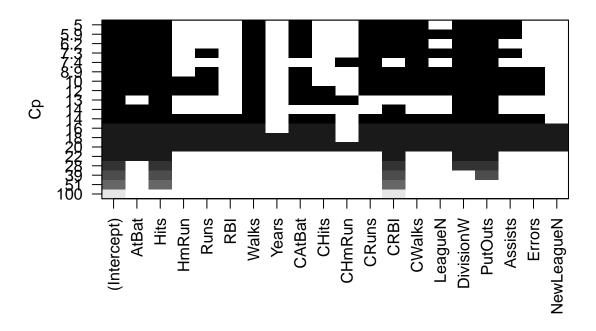
```
regfit.full=regsubsets(Salary~.,data=Hitters, nvmax=19)
reg.summary=summary(regfit.full)
names(reg.summary)

## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"
plot(reg.summary$cp,xlab="Number of Variables",ylab="Cp")
which.min(reg.summary$cp)
```



There is a plot method for the regsubsets object

```
plot(regfit.full,scale="Cp")
```



coef(regfit.full,10)

| ## | (Intercept) | AtBat | Hits | Walks | \mathtt{CAtBat} |
|----|-------------|------------|------------|--------------|-------------------|
| ## | 162.5354420 | -2.1686501 | 6.9180175 | 5.7732246 | -0.1300798 |
| ## | CRuns | CRBI | CWalks | DivisionW | PutOuts |
| ## | 1.4082490 | 0.7743122 | -0.8308264 | -112.3800575 | 0.2973726 |
| ## | Assists | | | | |
| ## | 0.2831680 | | | | |

Forward Stepwise Selection

Here we use the regsubsets function but specify the 'method="forward" option:

```
regfit.fwd=regsubsets(Salary~.,data=Hitters,nvmax=19,method="forward")
summary(regfit.fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
## 19 Variables (and intercept)
##
              Forced in Forced out
## AtBat
                  FALSE
                             FALSE
                             FALSE
## Hits
                  FALSE
## HmRun
                  FALSE
                             FALSE
## Runs
                  FALSE
                             FALSE
## RBI
                  FALSE
                             FALSE
## Walks
                  FALSE
                             FALSE
```

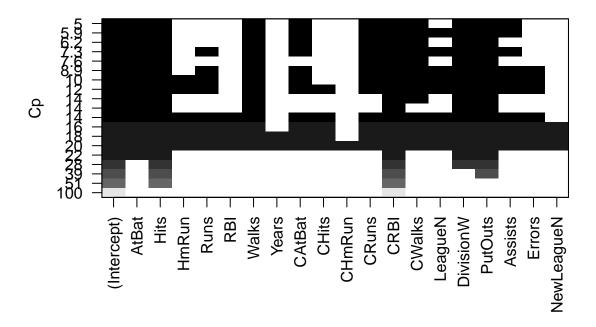
```
## CAtBat
                       FALSE
                                     FALSE
                                     FALSE
## CHits
                       FALSE
## CHmRun
                       FALSE
                                     FALSE
## CRuns
                       FALSE
                                     FALSE
## CRBI
                       FALSE
                                     FALSE
## CWalks
                       FALSE
                                     FALSE
## LeagueN
                                     FALSE
                       FALSE
## DivisionW
                       FALSE
                                     FALSE
## PutOuts
                       FALSE
                                     FALSE
## Assists
                       FALSE
                                     FALSE
## Errors
                       FALSE
                                     FALSE
## NewLeagueN
                       FALSE
                                     FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##
                 AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
## 1
                        11 11
                               11 11
                                      11 11
                                             \Pi=\Pi=\Pi=\Pi
                                                          11 11
                                                                  11 11
                                                                           11 11
                                                                                   11 11
                                                                                            11 11
       (1)
                                       11 11
                                             11 11 11
                                                                                            11 11
                11 11
                        "*"
##
      (1)
                                                  11
                                                                                            11 11
##
   3
       (1)
                 11 11
                        "*"
## 4
       (1)
                                       11 11
                                                                           11
                                                                             11
## 5
       ( 1
            )
                 "*"
                                                                           11 11
## 6
       (1)
                 "*"
                               11 11
                                       11 11
                                                                                            11 11
## 7
       (1)
                 "*"
                        "*"
                               11 11
                                       11 11
                                                                           11 11
                                                                                   11 11
## 8
        (1
            )
                 "*"
                                                                                            "*"
                "*"
                        "*"
                                                                                            "*"
## 9
        (1)
                                                                           11 11
## 10
         (1)
                "*"
                                       11 11
                                                          11 11
                                                                  "*"
                                                                                   11 11
                                                                                            "*"
                                                                           11 11
                                                                                            "*"
##
   11
         ( 1
             )
                "*"
                        "*"
                                                                  "*"
##
   12
         (1
              )
                               11 11
                                       "*"
                                             اايداا اا اا
                                                          .. ..
                                                                  "*"
                                                                           11 11
                                                                                   11 11
                                                                                            "*"
                "*"
                        "*"
                                       "*"
                                                                  "*"
                                                                                            "*"
##
   13
         (1
              )
                                                                           11 11
                                       "*"
                                             11 11
                                                          11 11
                                                                  "*"
                                                                                   11 11
                                                                                            "*"
## 14
         (1
                 "*"
                                       "*"
                                                                           "*"
                                                                                            "*"
                        "*"
                               " * "
                                                                  "*"
## 15
         (
           1
              )
##
   16
         (1
              )
                 "*"
                        "*"
                               "*"
                                       "*"
                                             "*"
                                                          11 11
                                                                  "*"
                                                                           "*"
                                                                                   11 11
                                                                                            "*"
                                       "*"
         (1
                "*"
                        11 🕌 11
                               الياا
                                             11 4 11
                                                                  11 🕌 11
                                                                           11 🕌 11
                                                                                            "*"
##
   17
             )
##
                "*"
                        "*"
                                       "*"
                                             "*" "*"
                                                                                   11 11
                                                                                            "*"
         (1)
   18
                                             "*" "*"
                                                                                            "*"
                        "*"
                               "*"
                                       "*"
                                                          "*"
                                                                           "*"
                                                                                   "*"
                                                                  "*"
##
   19
         (
##
                CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
                                11 11
                                          11 11
                                                        11 11
                                                                            11 11
                                                                                     11 11
## 1
       (1)
                "*"
                       11 11
## 2
       (1)
                 "*"
                                          11 11
                                                        11 11
                                                                                        11
## 3
                                          11 11
                                                        11 * 11
        (1
            )
                                                        "*"
## 4
                 "*"
                                          "*"
       ( 1
            )
                                11 11
                                                        "*"
## 5
       (1
                 "*"
                                          "*"
            )
                 "*"
                                          "*"
                                                        "*"
## 6
       ( 1
            )
##
        (1
                 "*"
                                          "*"
                                                        "*"
                                                                    11
## 8
                 "*"
                                          "*"
                                                        "*"
       ( 1
            )
                                11 11
                                                        "*"
                                                                                        "
## 9
        (1)
                 "*"
                                          "*"
                       "*"
                                          "*"
                                                        "*"
                                                                  "*"
         (1)
                 "*"
## 10
                                                                            11 11
              )
                       "*"
                                "*"
                                          "*"
                                                        "*"
                                                                  "*"
                                                                                     11 11
##
   11
         (
           1
                                          "*"
                                                        "*"
##
   12
         (1
                "*"
                       "*"
                                "*"
                                                                  "*"
              )
                "*"
                                          "*"
                                                        "*"
                                                                  "*"
                                                                                     11 11
##
   13
         (1)
                                          "*"
                                                        "*"
                 "*"
                       11 🕌 11
                                اليواا
                                                                  11 🕌 11
                                                                            11 🕌 11
##
   14
         (
           1
              )
                                "*"
                                          "*"
                                                        "*"
                                                                  "*"
                                                                            "*"
                                                                                     11 11
##
   15
         (1
              )
                                                        "*"
                                                                                     11 11
                       11 * 11
                                11 * 11
                                          11 * 11
                                                                  11 * 11
                                                                            11 * 11
## 16
         (1
              )
                                "*"
                                          "*"
                                                        "*"
                                                                  "*"
                                                                            "*"
                                                                                     "*"
## 17
         (1)
         (1)
                "*"
                       11 * 11
                                11 * 11
                                          11 * 11
                                                        11 * 11
                                                                  11 * 11
                                                                            "*"
                                                                                     "*"
## 18
```

Years

FALSE

FALSE

```
## 19 ( 1 ) "*" "*" "*" "*" "*" "*" "*" "*" plot(regfit.fwd,scale="Cp")
```



Model Selection Using a Validation Set

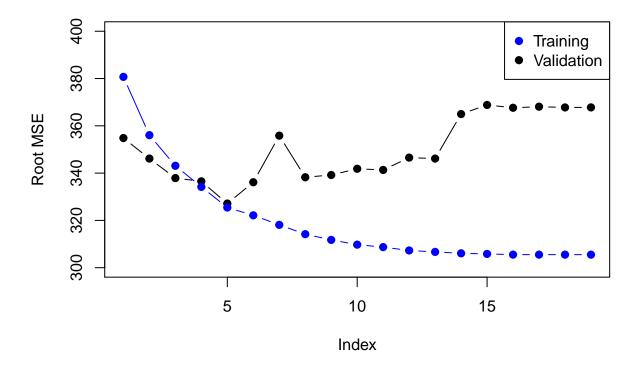
Lets make a training and validation set, so that we can choose a good subset model. We will do it using a slightly different approach from what was done in the the book.

```
dim(Hitters)
## [1] 263
set.seed(1)
train=sample(seq(263),180,replace=FALSE)
train
##
     [1]
          70
              98 150 237
                           53 232 243 170 161
                                                16 259
                                                        45 173
                                                                 97 192 124 178
    [18] 245
              94 190 228
                           52 158
                                   31
                                        64
                                            92
                                                 4
                                                    91 205
                                                             80 113 140 115
##
                       25 163
                               93 184 144 174 122 117
##
    [35] 244 153 181
                                                       251
                                                              6 104 241 149 102
    [52] 183 224 242
                                                    60 211
##
                       15
                           21
                               66 107 136
                                           83 186
                                                             67 130 210
          17 256 207 162 200 239 236 168 249
                                                73 222 177 234 199 203
##
    [69]
                                                                         59 235
                                                        77
##
    [86]
          37 126
                  22 230 226
                               42
                                   11 110 214 132 134
                                                             69 188 100 206
##
   [103]
          44
             159 101
                       34 208
                               75 185 201 261 112
                                                    54
                                                        65
                                                             23
                                                                  2 106 254 257
                  71 166 221 105
   [120] 154 142
                                   63 143
                                            29 240 212 167
                                                           172
                                                                  5
                                                                     84 120 133
                                                             13
          72 191 248 138 182
                               74 179 135
                                            87 196 157 119
                                                                 99 263 125 247
   [137]
  [154]
          50
              55
                  20
                       57
                            8
                               30 194 139 238
                                                46
                                                    78
                                                        88
                                                                  7
                                                                     33 141
```

```
## [171] 180 164 213 36 215 79 225 229 198 76
regfit.fwd=regsubsets(Salary~.,data=Hitters[train,],nvmax=19,method="forward")
```

Now we will make predictions on the observations not used for training. We know there are 19 models, so we set up some vectors to record the errors. We have to do a bit of work here, because there is no predict method for regsubsets.

```
val.errors=rep(NA,19)
x.test=model.matrix(Salary~.,data=Hitters[-train,])# notice the -index!
for(i in 1:19){
    coefi=coef(regfit.fwd,id=i)
        pred=x.test[,names(coefi)]%*%coefi
    val.errors[i]=mean((Hitters$Salary[-train]-pred)^2)
}
plot(sqrt(val.errors),ylab="Root MSE",ylim=c(300,400),pch=19,type="b")
points(sqrt(regfit.fwd$rss[-1]/180),col="blue",pch=19,type="b")
legend("topright",legend=c("Training","Validation"),col=c("blue","black"),pch=19)
```



As we expect, the training error goes down monotonically as the model gets bigger, but not so for the validation error.

This was a little tedious - not having a predict method for regsubsets. So we will write one!

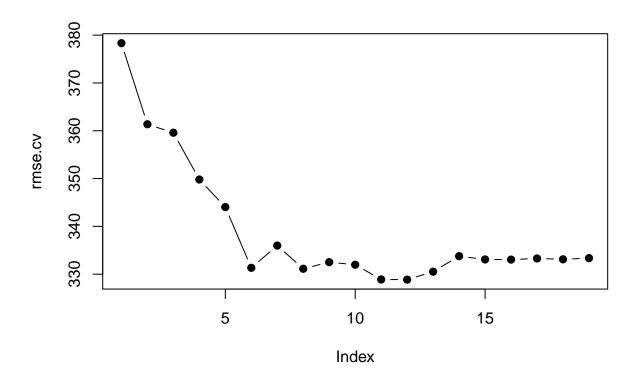
```
predict.regsubsets=function(object,newdata,id,...){
  form=as.formula(object$call[[2]])
  mat=model.matrix(form,newdata)
  coefi=coef(object,id=id)
  mat[,names(coefi)]%*%coefi
```

}

Model Selection by Cross-Validation

We will do 10-fold cross-validation. Its really easy!

```
set.seed(11)
folds=sample(rep(1:10,length=nrow(Hitters)))
folds
##
     [1]
                            3
                               5
                                  5
                                      2
                                        5
                                            2
                                               8
                                                  3
                                                     3
                                                        3
                                                           9
                                                              2
                                                                    8 10
##
    [24]
         5 5 5
                   5 10 10
                            4
                               4
                                  7
                                      6
                                        7
                                            7
                                               7
                                                  3
                                                     4
                                                        8
                                                           3
                                                              6
                                                                8 10
                                                                          3
                                                                             9
##
    [47]
             3
               4
                   9
                      8
                         7 10
                               6 10
                                      3
                                        6
                                            9
                                                  2
                                                     8
                                                        2
                                                           5
                                                              6 10
                                                                    7
   [70]
                   2
                      5
                         8
                                  2
                                                  2
                                                           6
                                                              5
##
             3
                6
                            1
                               1
                                     8
                                        1 10
                                               1
                                                     3
                                                       6
                                                                 8
         1
                            7
                                  7
##
    [93]
         6
             1
                7
                      8
                         3
                               8
                                      1 10
                                            1
                                               6
                                                  2
                                                     9
                                                       10
                                                           1
                                                              7
                                                                 7
                   6
                      6
                         9
                            8 10
                                     7
                                        9
                                            6
                                               7
                                                        2
                                                           2
## [116]
         3
             6 10
                                  6
                                                  1 10
                                                              5
## [139]
         2
             9
                4 10
                      5
                         3
                            7
                               7 10 10
                                        9
                                            3
                                               3
                                                  7
                                                     3
                                                        1
                                                              6
                                                                 6 10
                                                        7
## [162]
                6
                   8 10
                         8
                            5
                               4
                                     6
                                        2
                                            9 10
                                                     7
                                                           6
                                                                 2
         1
             3
                                  5
                                                  3
                                                              6
                                                                    3
                                                                       2
## [185]
         4
             4
                8
                   2
                      3
                         5
                            9
                               9 10
                                     2
                                        1
                                            3
                                               9
                                                  6
                                                     7
                                                        3
                                                           1
                                                              9
                                                                 4 10 10
                                  2
## [208]
          8
             2 5
                   9
                      8 10
                            5
                                     4
                                        1
                                            4
                                               4
                                                  5
                                                        2
                                                          1
                                                              9
                                                                 5
                               8
                                                     5
## [231]
             2
                   9 1
                         7
                            2
                               5 8
                                     1
                                        1
                                            7
                                                        5 10
                                                              5
          3
                1
                                               6
## [254]
             2
               5
                   7 1
                         3
                           1
                               3
                                  1
          1
table(folds)
## folds
## 1 2 3 4 5 6 7 8 9 10
## 27 27 27 26 26 26 26 26 26 26
cv.errors=matrix(NA,10,19)
for(k in 1:10){
  best.fit=regsubsets(Salary~.,data=Hitters[folds!=k,],nvmax=19,method="forward")
  for(i in 1:19){
    pred=predict(best.fit,Hitters[folds==k,],id=i)
    cv.errors[k,i]=mean( (Hitters$Salary[folds==k]-pred)^2)
  }
}
rmse.cv=sqrt(apply(cv.errors,2,mean))
plot(rmse.cv,pch=19,type="b")
```



Ridge Regression and the Lasso

We will use the package glmnet, which does not use the model formula language, so we will set up an x and y.

```
library(glmnet)

## Warning: package 'glmnet' was built under R version 3.3.3

## Loading required package: Matrix

## Warning: package 'Matrix' was built under R version 3.3.3

## Loading required package: foreach

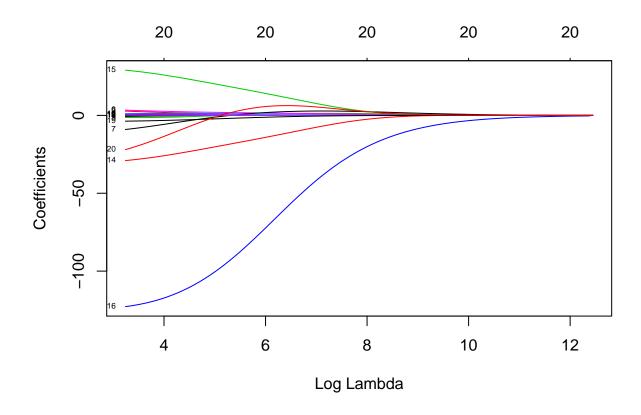
## Warning: package 'foreach' was built under R version 3.3.3

## Loaded glmnet 2.0-16

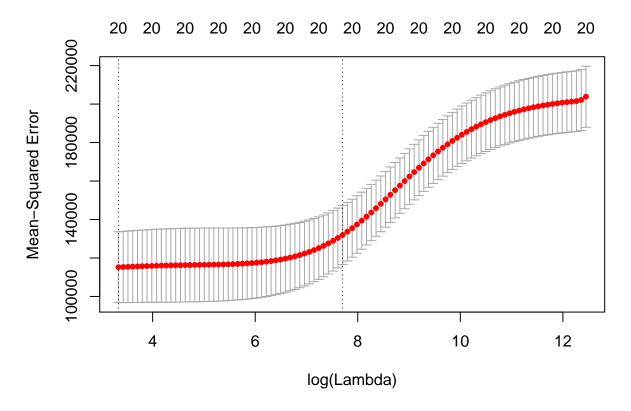
x=model.matrix(Salary~.-1,data=Hitters)
y=Hitters$Salary
```

First we will fit a ridge-regression model. This is achieved by calling glmnet with alpha=0 (see the helpfile). There is also a cv.glmnet function which will do the cross-validation for us.

```
fit.ridge=glmnet(x,y,alpha=0)
plot(fit.ridge,xvar="lambda",label=TRUE)
```

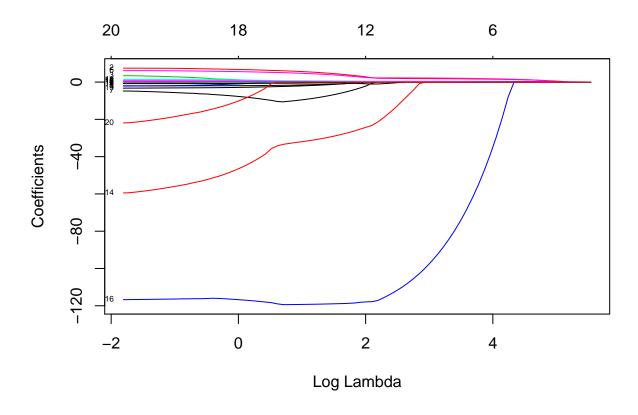


cv.ridge=cv.glmnet(x,y,alpha=0)
plot(cv.ridge)



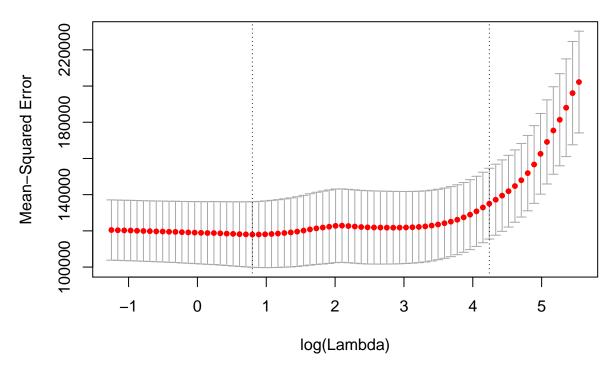
Now we fit a lasso model; for this we use the default alpha=1

```
fit.lasso=glmnet(x,y)
plot(fit.lasso,xvar="lambda",label=TRUE)
```



cv.lasso=cv.glmnet(x,y)
plot(cv.lasso)





coef(cv.lasso)

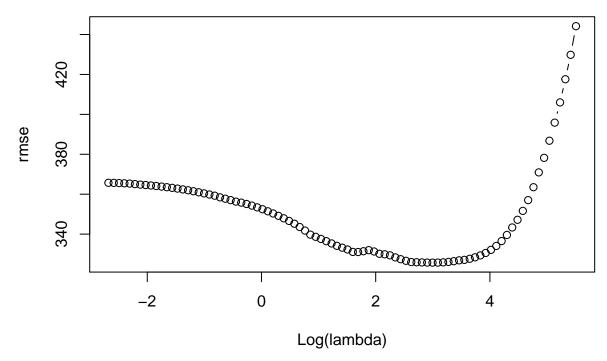
```
## 21 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept) 127.95694771
## AtBat
## Hits
                 1.42342566
## HmRun
## Runs
## RBI
                 1.58214110
## Walks
## Years
## CAtBat
## CHits
## CHmRun
                 0.16027975
## CRuns
## CRBI
                 0.33667715
## CWalks
## LeagueA
## LeagueN
                -8.06171247
## DivisionW
## PutOuts
                 0.08393604
## Assists
## Errors
## NewLeagueN
```

Suppose we want to use our earlier train/validation division to select the lambda for the lasso. This is easy

to do.

```
lasso.tr=glmnet(x[train,],y[train])
lasso.tr
##
## Call: glmnet(x = x[train, ], y = y[train])
##
##
         Df
               %Dev
                       Lambda
    [1,] 0 0.00000 246.40000
##
    [2,] 1 0.05013 224.50000
    [3,] 1 0.09175 204.60000
##
    [4,] 2 0.13840 186.40000
##
   [5,] 2 0.18000 169.80000
   [6,] 3 0.21570 154.80000
##
   [7,] 3 0.24710 141.00000
   [8,] 3 0.27320 128.50000
##
  [9,]
         4 0.30010 117.10000
## [10,]
         4 0.32360 106.70000
## [11,]
         4 0.34310
                    97.19000
## [12,]
         4 0.35920
                     88.56000
## [13,]
         5 0.37360
                     80.69000
## [14,]
         5 0.38900
                     73.52000
## [15,]
         5 0.40190
                     66.99000
## [16,]
         5 0.41260
                     61.04000
## [17,]
         5 0.42140
                     55.62000
## [18,]
         5 0.42880
                     50.67000
## [19,]
         5 0.43490
                     46.17000
## [20,]
         5 0.43990
                     42.07000
## [21,]
         5 0.44410
                     38.33000
## [22,]
         5 0.44760
                     34.93000
## [23,]
         6 0.45140
                     31.83000
## [24,]
         8 0.45480
                     29.00000
## [25,]
         8 0.45770
                     26.42000
## [26,]
         8 0.46010
                     24.07000
## [27,]
         9 0.46220
                     21.94000
## [28,]
         9 0.46380
                     19.99000
## [29,]
         9 0.46520
                     18.21000
## [30,]
         9 0.46630
                     16.59000
## [31,] 9 0.46730
                     15.12000
## [32,] 9 0.46810
                     13.78000
## [33,] 10 0.47110
                     12.55000
## [34,] 10 0.47380
                     11.44000
## [35,] 10 0.47620
                     10.42000
## [36,] 11 0.48050
                      9.49500
## [37,] 10 0.48450
                      8.65200
## [38,] 11 0.48770
                      7.88300
## [39,] 11 0.49360
                      7.18300
## [40,] 12 0.49890
                      6.54500
## [41,] 13 0.50450
                      5.96300
## [42,] 13 0.51010
                      5.43400
## [43,] 14 0.51470
                      4.95100
## [44,] 14 0.51850
                      4.51100
## [45,] 14 0.52170
                      4.11000
## [46,] 15 0.52440
                      3.74500
```

```
## [47,] 15 0.52670
                      3.41200
## [48,] 15 0.52870
                      3.10900
## [49,] 15 0.53030
                      2.83300
## [50,] 15 0.53160
                      2.58100
## [51,] 16 0.53280
                      2.35200
## [52,] 17 0.53420
                      2.14300
## [53,] 18 0.53580
                      1.95300
## [54,] 18 0.53760
                      1.77900
## [55,] 18 0.53890
                      1.62100
## [56,] 18 0.54000
                      1.47700
## [57,] 18 0.54090
                      1.34600
## [58,] 18 0.54160
                      1.22600
## [59,] 17 0.54220
                      1.11700
                      1.01800
## [60,] 17 0.54280
## [61,] 17 0.54320
                      0.92770
## [62,] 17 0.54360
                      0.84530
## [63,] 17 0.54380
                      0.77020
## [64,] 18 0.54410
                      0.70180
## [65,] 19 0.54430
                      0.63940
## [66,] 19 0.54450
                      0.58260
## [67,] 19 0.54470
                      0.53090
## [68,] 19 0.54490
                      0.48370
## [69,] 20 0.54510
                      0.44070
## [70,] 20 0.54520
                      0.40160
## [71,] 20 0.54530
                      0.36590
## [72,] 20 0.54540
                      0.33340
## [73,] 20 0.54550
                      0.30380
## [74,] 20 0.54560
                      0.27680
## [75,] 20 0.54570
                      0.25220
## [76,] 20 0.54570
                      0.22980
## [77,] 20 0.54580
                      0.20940
## [78,] 20 0.54580
                      0.19080
## [79,] 20 0.54590
                      0.17380
## [80,] 20 0.54590
                      0.15840
## [81,] 20 0.54590
                      0.14430
## [82,] 20 0.54590
                      0.13150
## [83,] 20 0.54600
                      0.11980
## [84,] 19 0.54600
                      0.10920
## [85,] 19 0.54600
                      0.09948
## [86,] 19 0.54600
                      0.09064
## [87,] 19 0.54600
                      0.08259
## [88,] 20 0.54600
                      0.07525
## [89,] 20 0.54600
                      0.06856
pred=predict(lasso.tr,x[-train,])
dim(pred)
## [1] 83 89
rmse= sqrt(apply((y[-train]-pred)^2,2,mean))
plot(log(lasso.tr$lambda),rmse,type="b",xlab="Log(lambda)")
```



```
lam.best=lasso.tr$lambda[order(rmse)[1]]
lam.best
## [1] 19.98706
coef(lasso.tr,s=lam.best)
## 21 x 1 sparse Matrix of class "dgCMatrix"
##
## (Intercept) 106.2272607
## AtBat
## Hits
                  0.1591945
## HmRun
## Runs
## RBI
                  1.7340977
## Walks
                  3.4655873
## Years
## CAtBat
## CHits
## CHmRun
## CRuns
                  0.5386938
## CRBI
## CWalks
## LeagueA
                -28.3582245
## LeagueN
                  1.8006762
               -113.8285692
## DivisionW
## PutOuts
                  0.2915437
```

Assists

Errors

NewLeagueN 1.9431535