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Question Paper Code : 12194

M.E./M.Tech. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

Computer Science and Engineering

MA 4151 — APPLIED PROBABILITY AND STATISTICS FOR COMPUTER SCIENCE ENGINEERS

(Common to M.E. Computer Science and Engineering (With Specialization in Artificial Intelligence and Machine Learning/M.E. Computer Science and Engineering (With Specialization in Networks)/M.E. E-Learning Technologies/ M.E. Multimedia Technology

(Regulations 2021)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Let V be the vector space of functions from R into R. Then show that the functions $f(t)=\sin t$, $g(t)=e^t$ and $h(t)=t^2$ are linearly independent.

2. Find $\cos\theta$, where θ is the angle between

$$A=\begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{pmatrix} \text{ and } B=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}, \text{ where } \langle A, B\rangle = tr\left(B^T A\right).$$

$0.65 = A + B -$

3. Two events A and B are such that $P(A\cap B)=0.15$, $P(A\cup B)=0.65$, and $P(A|B)=0.5$. Find $P(B|A)$. $P(A\cap B)$

$= A + 0.$

$= 0.A +$

$A = ..$

$\frac{P(A\cap B)}{P(B)} = 0.5$

$\frac{0.15}{0.5} = P(B) \Rightarrow 0.3$

4. Let X be a random variable with the probability density function $f(x)$ and mean $E[X]$. Let a and b be constants. Then, if Y is the random variable defined by $Y=(aX+b)$, find the expected value of Y.

5. X and Y are two continuous random variables whose joint probability density function is given by $f_{XY}(x, y) = \begin{cases} e^{-(x+y)} & 0 \le x < \infty;\ 0 \le y < \infty \\ 0 & \text{otherwise} \end{cases}$

Find the marginal density function of X.

6. The joint probability density function (PDF) of the random variables X and Y is defined as follows:

$$f_{XY}(x, y) = \begin{cases} Ke^{-5y} & 0 \le x < 0.2;\ y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of K.

7. Define type-I and type-II errors in hypothesis testing.

8. What do you mean by simple and composite hypothesis?

9. Define a covariance matrix.

10. What are the advantages of multivariate analysis?

PART B — (5 × 13 = 65 marks)

11. (a) (i) Consider the vector space P(t) with inner product $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$. By applying the Gram-Schmidt algorithm to the set $\{1, t, t^2\}$, derive the orthogonal set $\{f_0, f_1, f_2\}$. (7)

(ii) Let $A = \begin{pmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, then find the Jordan canonical form of A. (6)

Or

(b) (i) Let $T: C^3 \to C^3$ be defined by $T = L_A$, where

$$A = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{pmatrix}$$

Find a basis for each eigenspace and each generalized eigenspace of T. (7)

(ii) Let T: $P_2(C) \to P_2(C)$ be defined by $T(f) = -f - f'$. Find a basis for each eigenspace and generalized eigen space of T. (6)

12. (a) (i) There are three identical urns containing white and black balls. The first urn contains 2 white and 3 black balls, the second urn contain 3 white and 5 black balls, and the third urn contains 5 white and 2 black balls. An urn is chosen at random, and a ball is drawn from it. If the ball drawn is white, what is the probability that the second urn is chosen? (7)

(ii) If X is a standard normal variate, then prove that $Y=\frac{1}{2}X^2$ is a Gama distribution with parameter 1/2. (6)

Or

(b) (i) Let X be uniformly distributed over the interval $(-2, 2)$. Find the mean, variance and moment generating function of X. (7)

(ii) Three marksmen can hit a target with probabilities $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ respectively. They shoot simultaneously, and two hits are registered. Find the probability that atleast one of the three marksmen hits the target. (6)

13. (a) The joint probability density function of two random variable X and Y is $K(1-x-y)$ inside the triangle formed by the axes and the line $x+y=1$ and zero elsewhere. Find the value of K and $P\left(X<\frac{1}{2}, Y>\frac{1}{4}\right)$. Also find all the marginal and conditional distributions and determine whether the random variables are independent or not. (13)

Or

(b) The joint probability density function of two random variables X, Y is given by $f(x, y)=\begin{cases}\frac{6-x-y}{8} & 0<x<2,\ 2<y<4\\ 0 & \text{elsewhere}\end{cases}$

Find the coefficient of correlation between X and Y. (13)

14. (a) (i) A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance. (6)

(ii) The specimen of copper wires drawn form a large lot have the following breaking strength (in kg. weight) :

578, 572, 570, 568, 572, 578, 570, 572, 596, 544.

Test (using Student's t-statistic) whether the mean breaking strength of the lot may be taken to be 578 kg. weight (Test at 5 per cent level of significance). (7)

Or

(b) A sample of 400 cases are used to determine whether there is a relationship between an employee's performance and the company's training program. The results are shown below

		Performance in training program		
		Below Average	Average	Above Average
Success in Job	Poor	23	69	29
	Good	28	79	60
	Very good	9	49	63

Use 0.01 level of significance to test whether the performance in training program and success in the job are independent.

15. (a) Suppose p = 2 and n = 1, and consider the random vector $X'=[X_1, \ X_2]$. Let the discrete random variables X_1, X_2 have the following probability functions, respectively

x_1	–1	0	1
$p_1(x_1)$	.3	.3	.4

x_2	–0	1
$p_2(x_2)$	.8	.2

The joint probability function $p_{12}(x_1, x_2)$ is represented by the entries in the following table.

x_1 \ x_2	0	1	$p_1(x_1)$
–1	.24	.06	0.3
0	.16	.14	0.3
1	.40	.00	0.4
$p_2(x_2)$	.8	.2	1

Find the covariance and correlation matrix for the two random variables X_1, X_2. (13)

Or

(b) (i) Suppose the random variables $X_1\,X_2$ and X_3 have the covariance matrix $\Sigma=\begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

Calculate the population principal components of the random variables X_1, X_2 and X_3. (7)

(ii) Show that the principal components obtained from covariance matrix, Σ and correlation matrix, ρ are different, when

$$\Sigma=\begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix} \text{ and } \rho=\begin{pmatrix} 1 & .4 \\ .4 & 1 \end{pmatrix}.$$ (6)

PART C — ($1 \times 15 = 15$ marks)

16. (a) (i) Let T be a linear operator on a finite-dimensional vector space V. Then prove that, T is diagonalizable if and only if the minimal polynomial for T is of the form $p(t)=(t-\lambda_1)(t-\lambda_2)\dots(t-\lambda_k)$, where $\lambda_1, \lambda_2, \dots \lambda_k$ are distinct scalars. (7)

(ii) Suppose U and W are finite-dimensional subspaces of a vector space V. Then prove that, $U+W$ has finite dimension and

$$\dim(U+W)=\dim U+\dim W-\dim(U\cap W).$$ (8)

Or

(b) The following data gives the daily number of power failures in a city:

No. of Power Failures :	0	1	2	3	4	5	6	7	8	9
No. of days :	9	43	64	62	42	36	22	14	6	2

Test at 5% level of significance if the data can be fitted by a Poisson distribution.