UNIT - I
LINEAR ALGEBRA

1. Define Generalized eigen vector

A Eigen ~~ei~~ vector X_m is called the generalized eigen vector corresponding to the eigen value λ of rank m if

$$(A - \lambda I)^m X_m = 0 \ \& \ (A - \lambda I)^{m-1} X_m \neq 0$$

2. What is meant by least Square Solution?

Least square approximation is the process of finding functional relationship between variables x & $y = f(x)$ by minimizing the residual Σd_i^2, where d_i = observed value of y - functional value of y

3. Define Canonical basis.

Canonical basis of a matrix is the set of linearly Independent eigen vectors.

4. Define Vector Space.

A non-empty set V is said to be a ~~s~~ Vector Space over the field F if

If 1. $(V, +)$ is an abelian group.

2. $\alpha(u+v) = \alpha u + \alpha v$

3. $(\alpha+\beta) u = \alpha u + \beta u$

4. $\alpha(\beta u) = (\alpha\beta) u$

5. $1 \cdot u = u \quad \forall \; u, v \in V \;\&\; \alpha, \beta \in F$.

5. Define norms.

Norm is the length of the vector. It is denoted by $\| \cdot \|$.

Eg: Let $X = (x_1, x_2)$ be a vector.

Then $\| X \| = |x_1| + |x_2|$

6. Define Inner Products.

Inner product is the generalization of dot product.

Inner Product on the Vector Space V

$\langle \; \rangle : V \times V \to R$ s.t

1. $\langle x, x \rangle > 0$ if $x \neq 0$

2. $\langle x+z, y \rangle = \langle x, y \rangle + \langle z, y \rangle$

3. $\langle ax, y \rangle = a \langle x, y \rangle$

4. $\overline{\langle x, y \rangle} = \langle y, x \rangle$.

Why the method of least square is called so?

In this method, the residual Σd_i^2 is minimum. So this method is called least square method.

UNIT V

MULTIVARIATE ANALYSIS

1. Define random vector and random matrix

A random vector is a vector whose elements are random variables.

A random matrix is a matrix whose elements are random variables.

Eg:

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ \vdots & & & & \\ x_{n1} & x_{n2} & x_{n3} & \cdots & x_{nn} \end{bmatrix}$$

is a random matrix where x_{ij} is are random variables

2. Define Mean Vector (or) Expected value of random matrix or population mean vector.

Let $X = [x_1 \ x_2 \ \ldots \ x_p]$ be a $1 \times p$ random matrix.

∴ Then Mean vector of X is

$$E[X] = \begin{bmatrix} E(x_1) \\ E(x_2) \\ \vdots \\ E(x_p) \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix}$$

3. Define multivariate normal density function.

~~Let~~ The p- dimensional normal density for the ~~vector~~ random vector $x' = [x_1\ x_2 \cdots x_p]$ is

$$f(x) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{\frac{-(x-\mu)' \Sigma^{-1} (x-\mu)}{2}}$$

Where $-\infty < x_i < \infty$.

The p-dimensional normal density is denoted by $N_p(\mu, \Sigma)$.

4. State additional properties of the multivariate normal distribution.

1. Linear combinations of the components of X are normally distributed
2. All subsets of the components of X have a normal distribution.
3. Zero covariance implies that the corresponding components are independently distributed.
4. The conditional distributions of the components are normal.

5. Define 1st & 2nd principal components

1st principal component = Linear combination $a_1'x$ that maximizes $Var(a_1'x)$ subject to $a_1'a_1 = 1$

2nd principal component = Linear combination $a_2'x$ that maximizes $Var(a_2'x)$ subject to $a_2'a_2 = 1$ $Cov(a_1'x, a_2'x) = 0$.

6. Compute expected values for discrete random vector $x' = [x_1 . x_2]$ where $E(x_1) = 0.1$ & $E(x_2) = 0.2$.

Expected value $E(x) = \begin{bmatrix} E(x_1) \\ E(x_2) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$

7. Explain covariance matrix & Mean Vector.

Let $x = [x_1\ x_2 \cdots x_p]$ be a $1 \times p$ random matrix. Then Mean Vector

$$E(x) = \begin{bmatrix} E(x_1) \\ E(x_2) \\ \vdots \\ E(x_p) \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix}$$ and covariance

matrix $$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1p} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2p} \\ \vdots & & & & \\ \sigma_{p1} & \sigma_{p2} & \sigma_{p3} & \cdots & \sigma_{pp} \end{bmatrix}$$

Where $\sigma_{ii} = \sum_{x_i} (x_i - \mu_i)^2 \, p_i(x_i)$ &

$$\sigma_{ik} = \sum_{x_i} \sum_{x_k} (x_i - \mu_i)(x_k - \mu_k) \, p_{ik}(x_i, x_k)$$

8. Define Principal component Analysis.

A principal component Analysis is concerned with explaining the variance-covariance structure of a set of variables through a few linear combinations of these variables.

Its general objectives are

1. Data reduction 2. Interpretation.

9. What is the ith principal component of the standardized variables $Z' = [Z_1, Z_2, \ldots Z_p]$ with $\mathrm{cov}(Z) = \rho$ of X.

$$Y_i = e_i' Z = e_i' (V^{1/2})^{-1} (X - \mu)$$

where $i = 1, 2, \ldots p$.

10. What is the proportion of (standardized) population variance due to kth principal component?

proportion of (standardized) population variance due to kth principal component $= \frac{\lambda_k}{p}$

Where p = Total standardized population variance

What is the proportion of total population variance due to k^{th} principal component?

Proportion of total population variance due to k^{th} principal component

$$= \frac{\lambda_k}{\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_p}, \quad k = 1, 2, \ldots p.$$