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Question Paper Code : 90214

M.C.A. DEGREE EXAMINATIONS, APRIL/MAY 2022.

First Semester

MA 4151 — APPLIED PROBABILITY AND STATISTICS FOR COMPUTER SCIENCE ENGINEERS

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Statistical Table may be permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Verify whether the set of vectors {(0, 1), (2, 0)} is ortho normal or not.

2. What is the special feature of pseudo inverse?

3. Test whether $f(x) = \begin{cases} |x|, & -1 \le x \le 1 \\ 0, & otherwise \end{cases}$ can be the probability density function of a continuous random variable.

4. If X is a Poisson random variable with parameter $\lambda > 0$, then prove that $E(X^2)=\lambda E(X+1)$.

5. The joint pdf of a two-dimensional random variable (X,Y) is given by $f(x,y) = \begin{cases} ke^{-(x+y)}; 0 \le x \le y, 0 \le y \le \infty \\ 0 \quad ; otherwise \end{cases}$ Find the value of 'k'.

6. If the covariance between X and Y is 36 and the standard deviation of X and Y are 16 & 9 respectively, find the coefficient of correlation.

7. What do you mean by degrees of freedom in testing of hypothesis?

8. Distinguish between parameter and statistic.

9. What do you mean by random vectors and random matrix?

10. Write down the principal components from standardized variables.

PART B — ($5 \times 13 = 65$ marks)

11. (a) Find a QR factorization of $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Or

(b) Find the least squares solution of the linear system $Ax = b$ given by $x_1 - x_2 = 4; 3x_1 + 2x_2 = 1; -2x_1 + 4x_2 = 3$

12. (a) (i) For a certain binary, communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (1) a '1' is received and (2) a '1' was transmitted given that a '1' was received. (7)

(ii) Find the moment generating function of Geometric distribution. (6)

Or

(b) (i) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events: (1) exactly two messages arrive within one hour (2) no message arrives within one hour (3) at least three messages arrive within one hour. (7)

(ii) If X = N(3,9), which means that X is Normal random variable with mean 3 and variance 9, find the probability that X lies between 2 and 5. (6)

13. (a) Find the bivariate probability distribution of (X,Y) given below, find $P(X \le 1), P(Y \le 3), P(X \le 1, Y \le 3), P(X \le 1 / Y \le 3)$ *and* $P(X + Y \le 4)$.

Y / X	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

Or

(b) Find the coefficient of correlation between X and Y, using following data:

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

14. (a) (i) The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient? (7)

(ii) The SD of a random sample of 1000 is found to be 2.6 and the SD of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from populations with the same SD. (6)

Or

(b) (i) The following data give the number of air-craft accidents that occurred during the various days of a week:

Day	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week. (Use Chi- square test) (7)

(ii) A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance? (6)

15. (a) For X distributed as $N_3(\mu, \Sigma)$, find the distribution of

$$\begin{bmatrix} x_1 - x_2 \\ x_2 - x_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = AX$$

Or

(b) Let the random variables X_1, X_2 and X_3 have the covariance matrix $\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Find all the Principal components.

PART C — (1 × 15 = 15 marks)

16. (a) Let X and Y have the joint probability density function $f(x,y)=\begin{cases} kxye^{-(x^2+y^2)}; x>0, y>0 \\ 0 \quad ; otherwise \end{cases}$ (i) Find the value of k (ii) Find the marginal PDFs of X and Y (iii) Are X and Y independent? Justify.

Or

(b) Is the set of all pairs of real numbers (x,y) with the operations (x, y)+ (x', y') = (x + x' + 1, y + y' + 1) and *k*(x. y) = (*k*x, *k*y) a vector space? Verify all the axioms and list the axioms that fail to hold if it is not a vector space.