Week 9 Classwork/Homework Assignment-Answers

John Vinson

10/17/2017

For the assignment, you only need to turn in a completed pdf. You may need to attach a hand written document showing your work for these problems. Make sure that the compiled documents will display all the required code to get your results.

The assignment is due Tuesday October 17 at 3:30 PM.

1. Suppose that when a sense organ receives a stimulus at time t, the total number of action potentials is $P(t)$. If the rate at which action potentials are produced is $t^3 + 4t^2 + 6$, and if there are 0 potentials when t=0, find the formula for $P(t)$.

$$\frac{dP}{dt} = t^3 + 4t^2 + 6 \tag{1}$$

We want to find P(t).

$$\begin{aligned}
\frac{dP}{dt} &= t^3 + 4t^2 + 6 && (2)\\
dP &= t^3 + 4t^2 + 6dt && (3)\\
\int dP &= \int t^3 + 4t^2 + 6dt && (4)\\
P &= \frac{1}{4}t^4 + \frac{4}{3}t^3 + 6t + C && (5)
\end{aligned}$$

To find C we need to solve the initial value problem. We know that $P(0) = 0$.

$$\begin{aligned}
P(t) &= \frac{1}{4}t^4 + \frac{4}{3}t^3 + 6t + C && (6)\\
P(0) &= \frac{1}{4}0^4 + \frac{4}{3}0^3 + 6 * 0 + C && (7)\\
0 &= C && (8)\\
& && (9)
\end{aligned}$$

So the solution is

$$P(t) = \frac{1}{4}t^4 + \frac{4}{3}t^3 + 6t + C \tag{10}$$

2. A factory is dumping pollutants into a river at a rate given by $\frac{dx}{dt} = \frac{t^{\frac{3}{4}}}{600}$, where t is time in weeks since the dumping began and x is the number of tons of pollutants.

- Find the equation for total tons of pollutants dumped.

- How many tons were dumped during the first year?

3. The rate of growth of the world population can be modeled by

$$\frac{dn}{dt} = N_0(1+r)^t ln(1+r) \tag{11}$$

with $r < 1$ and t is the time and years from the present and N_0 and r are constants. What function describes world population if the present population is N_0. (Hint: We can show that $\int (a^u lna)u^{'} dx = a^u + C$.)

4. Suppose that the growth of a certain species of bacteria is described by

$$\frac{dy}{dt} = ky \tag{12}$$

where y is the number of individuals and t is the number of hours.

- If initially there are 10,000 organisms and the number triples after 2 hours, how long will it be before there is 100 times the original population?
- If the doubling rate depends on temperature, find how long it takes for the number of bacteria to reach 50 times the original number at each given temperature in:
  - at 90 $°F$, the number doubles after 30 minutes.
  - at 40 $°F$, the number doubles after 3 hours.

5. If x and y are measurements of certain parts of an organism, then the rate of change of y with respect to x is proportional to the ratio of y to x. We can describe this relationship as

$$\frac{dy}{dx} = k\frac{y}{x} \tag{13}$$

which is referred to as an allometric law of growth. Solve this differential equation.