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11.1 Proof.

$A=\hat{Q}\hat{R}=\begin{pmatrix}\hat{Q}_1\\ \hat{Q}_2\end{pmatrix}\hat{R}$ $\hat{Q}_1\in C^{n\times n}$, $\hat{Q}_2\in C^{m\times n}$ $\hat{R}\in C^{n\times n}$

$A_1=\hat{Q}_1\hat{R}$ $A_2=\hat{Q}_2\hat{R}$

$A^+=(A^*A)^{-1}A^*=(\hat{R}^*\hat{Q}^*\hat{Q}\hat{R})^{-1}\hat{R}^*\hat{Q}^*=\hat{R}^{-1}\hat{Q}^*=$ 多余 $A_1^{-1}\hat{Q}_1\hat{Q}^*$.

$\|A^+\|_2\le\|A_1^{-1}\|_2\cdot\|\hat{Q}_1\hat{Q}^*\|_2$

只需证明 $\|\hat{Q}_1\hat{Q}^*\|_2\le 1$

将 $\hat{Q}$ 补成 $(\hat{Q},H)$ 是酉矩阵. $H\in C^{m\times(m-n)}$

$H=\begin{pmatrix}H_1\\ H_2\end{pmatrix}$ $H_1\in C^{n\times(m-n)}$ $H_2\in C^{(m-n)\times(m-n)}$

$\|\hat{Q}_1\hat{Q}^*b\|_2\le\left\|\begin{pmatrix}\hat{Q}_1 & H_1\\ \hat{Q}_2 & H_2\end{pmatrix}\begin{pmatrix}\hat{Q}^*b\\ 0\end{pmatrix}\right\|_2=\left\|\begin{pmatrix}\hat{Q}^*b\\ 0\end{pmatrix}\right\|_2\le\left\|\begin{pmatrix}\hat{Q}^*b\\ H^*b\end{pmatrix}\right\|_2=\|(\hat{Q}\ H)^*b\|_2=\|b\|_2$

$\Rightarrow\|\hat{Q}_1\hat{Q}^*\|_2\le 1$

$\Rightarrow\|A^+\|_2\le\|A_1^{-1}\|_2$

11.2 (a) $F=a_1\sin x+a_2e^x+a_3T(x)-\frac{1}{x}\perp$ Range$(\sin x, e^x, T(x))$

即 $\int_1^2 F\sin x=0$ $\int_1^2 Fe^x=0$ $\int_1^2 FT(x)=0$

即 $\int_1^2[a_1\sin x+a_2e^x+a_3T(x)]\sin x\,dx=\int_1^2$ [illegible] $\frac{\sin x}{x})\,dx$

$\int_1^2(a_1\sin x+a_2e^x+a_3T(x))e^x\,dx=\int_1^2\frac{e^x}{x}dx$

$\int_1^2(a_1\sin x+a_2e^x+a_3T(x))T(x)\,dx=\int_1^2\frac{T(x)}{x}dx.$

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12.1 令 $A=U\Sigma V^*$，$U,\Sigma,V^*\in C^{202\times 202}$

则 $\|A\|_2=\Sigma_{11}=100$，$\|A\|_F=\sqrt{\sum_i \Sigma_{ii}^2}=101$

$K(A)=\frac{\Sigma_{11}}{\Sigma_{202}}\geq\frac{100}{\frac{1}{201}}=20100$.

当 $\Sigma_{22}=\cdots=\Sigma_{202}=\frac{1}{201}$ 时 成立.