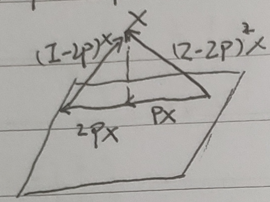
No.
Date.

Lecture 6

6.1 Proof. P is an orthogonal projector, $\Rightarrow P=P^*,\ P^2=P \quad \Rightarrow P=P^*=P^2=PP^*$

$(I-2P)(I-2P)^* = I-2P-2P^*+4PP^* = I$



$(I-2P)^2$ 是恒等变换

6.2 $F=\begin{pmatrix} & & & 1 \\ & & 1 & \\ & ⋰ & & \\ 1 & & & \end{pmatrix}$ $\quad E=\frac{I+F}{2}. \quad F^2=I \quad F=F^T$

$\Rightarrow E^2=E,\ E=E^T$. orthogonal projector.

6.3 A^*A 奇异 $\Leftrightarrow$ 有非零特征向量对应 $\lambda=0$

$\Rightarrow A^*Ax=0 \quad x\neq 0$

$\Rightarrow x^*A^*Ax=0$

$\Rightarrow \|Ax\|^2=0$

$\Rightarrow Ax=0$

$\Rightarrow$ A非满秩

从而A满秩 $\Rightarrow A^*A$ 非奇异

若 A非满秩. $\because Ax=0.\ x\neq 0$

$\Rightarrow A^*Ax=0 \quad x\neq 0$

$\Rightarrow A^*A$ 奇异

从而 A^*A 非奇异 $\Rightarrow$ A满秩

6.4 (a) $P=A(A^TA)^{-1}A^T$

$=\begin{pmatrix}1&0\\0&1\\1&0\end{pmatrix}\begin{pmatrix}2&0\\0&1\end{pmatrix}^{-1}\begin{pmatrix}1&0&1\\0&1&0\end{pmatrix}$

$=\begin{pmatrix}\frac{1}{2}&0&\frac{1}{2}\\0&1&0\\\frac{1}{2}&0&\frac{1}{2}\end{pmatrix}$

$Px=\begin{pmatrix}2\\2\\2\end{pmatrix}$

(b) $P = A(A^TA)^{-1}A^T$

$$= \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

$$= \frac{1}{6} \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix}$$

$$Px = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$$

6.5 $\|Px\| = \|P^2x\| \le \|P\| \cdot \|Px\| \Rightarrow \|P\| \ge 1$

P orthogonal Projector

$\Rightarrow P = P^*$

$P = U\Sigma V^*$. $P^2 = PP^* = U\Sigma^2 V^* = P$

$\Rightarrow \Sigma^2 = \Sigma \Rightarrow \Sigma_{ii} = 1/0$.

P nonzero $\Rightarrow \Sigma_{11} = 1$.

$\Rightarrow \|P\|_2 = 1$

若 P isn't an orthogonal Projector.

$Range(P) \not\perp Range(I-P)$

$\exists a \neq 0$, $a \in Range(I-P)^\perp$ 且 $a \notin Range(P)$

从而 $Pa \neq a$ $a \perp Range(I-P)$

$\|Pa\|_2 = \|a + (P-I)a\|_2 > \|a\|_2$

从而 $\|P\|_2 > 1$. 矛盾.

No.

Date.

7.1 (a) $A=\begin{pmatrix}\frac{\sqrt2}{2} & 0\\ 0 & 1\\ \frac{\sqrt2}{2} & 0\end{pmatrix}\begin{pmatrix}\sqrt2 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}\frac{\sqrt2}{2} & 0 & 0\\ 0 & 1 & 0\\ \frac{\sqrt2}{2} & 0 & 1\end{pmatrix}\begin{pmatrix}\sqrt2 & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}$

(b) $B=\begin{pmatrix}1 & 2\\ 0 & 1\\ 1 & 0\end{pmatrix}=\begin{pmatrix}\frac{\sqrt2}{2} & 0\\ 0 & 1\\ \frac{\sqrt2}{2} & 0\end{pmatrix}\begin{pmatrix}\sqrt2 & 2\sqrt2\\ 0 & 1\end{pmatrix}=\begin{pmatrix}\frac{\sqrt2}{2} & 0 & 0\\ 0 & 1 & 0\\ \frac{\sqrt2}{2} & 0 & 1\end{pmatrix}\begin{pmatrix}\sqrt2 & 2\sqrt2\\ 0 & 1\\ 0 & 0\end{pmatrix}$ $\times$ $\begin{pmatrix}\frac{\sqrt2}{2} & \frac{\sqrt3}{3} & -\frac{\sqrt6}{6}\\ 0 & \frac{\sqrt3}{3} & \frac{\sqrt6}{3}\\ \frac{\sqrt2}{2} & -\frac{\sqrt3}{3} & \frac{\sqrt6}{6}\end{pmatrix}\begin{pmatrix}\sqrt2 & \sqrt2\\ 0 & \sqrt3\\ 0 & 0\end{pmatrix}$

7.2 $\cancel{Q}_{ij}$ $R_{ij}=0$. 当 $i\,j$ 一奇一偶.

7.3 若 A 非满秩 $|\det A|=0$. 自然成立

设 A 满秩. $A=QR$.

$\det A = \det Q\cdot \det R = \det Q \cdot \prod_j r_{jj}$.

$|\det A| = |\det Q|\cdot \prod_j |r_{jj}| = \prod_j |r_{jj}| \le \prod_{j=1}^{m} \|a_j\|_2$.

上述由 $r_{jj} = a_j - \sum_{i=1}^{j-1} (q_i^* a_j) q_i$ 得.

7.4 $P^{(1)} = Q^{(1)}R^{(1)}$ $Q^{(1)} = (Q_1^{(1)}, Q_2^{(1)}, Q_3^{(1)})$

$\langle Q_1^{(1)}, Q_2^{(1)} \rangle = \langle x^{(1)}, y^{(1)} \rangle$, $Q_3^{(1)}$ 是 $x^{(1)}\times y^{(1)}$ 的 α 倍. (外积)

同理 $Q_3^{(2)}$ 是 $x^{(2)}\times y^{(2)}$ 的 α 倍.

$P=(Q_3^{(1)}\ Q_3^{(2)})$

$P=QR$.

Q_3 是 $Q_3^{(1)}\times Q_3^{(2)}$ 的 α 倍, 是 $P^{(1)}\cap P^{(2)}$ 的直线的方向向量. 即一个差.

7.5 (a) A rank n $\Leftrightarrow$ $q_j \notin \langle q_1, \dots, q_{j-1}\rangle$, $\forall j$ $\Leftrightarrow$ $r_{jj}\neq 0$

(b) 至多 k.

$(q_1\ q_2\ q_3)\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix} = (q_1, 0, q_2)$ $\operatorname{rank} A = 2 > k=1$

8.1 $3mn + \frac{(4m-1)}{2}(n-1)n \approx 3mn + 2mn^2$

8.3 $R_1=\begin{pmatrix}\frac{1}{r_{11}} & & & \\ & 1 & & \\ & & 1 \ddots & \\ & & & 1\end{pmatrix}\begin{pmatrix}1 & -r_{12} & -r_{13} & \cdots\\ & 1 & & \\ & & 1 \ddots & \\ & & & 1\end{pmatrix}$

line 5 line 8

Date.

10.1 (a) $F = I - \frac{2vv^*}{v^*v}$

$Fv = -v$. 特征值为 -1 重数为 1

$\forall y \in v^\perp$, 即 $v^* y = 0$

有 $Fy = y - \frac{2vv^*y}{v^*v} = y$. 特征值为 1. 重数为 $m-1$

这由反射的性质也可知

(b) $\det F = \prod \lambda_i = -1$

(c) $F = X\,\mathrm{diag}(\lambda)\,X^*$ 特征分解

$F = X\,\mathrm{sign}(\lambda)\,\mathrm{diag}(|\lambda|)\,X^*$. SVD.

由SVD性质知，奇异值为 1.

10.4 (a) F 反射（翻折）

J 顺时针旋转

(b)