VEENZ Vocational Engineering Education — SOUTHERN INSTITUTE OF TECHNOLOGY — Te Pūkenga

NEW ZEALAND DIPLOMA IN ENGINEERING

(Core Paper)

DE4102: Engineering Mathematics 1

November 2022: Final Examination

Time Allowed:	Three (3) hours plus 10 minutes reading time
Total Marks:	100

QUESTION AND ANSWER BOOK

MARKING INSTRUCTIONS

- Alternative methods resulting in correct answers attract full marks.
- Each ✓ on the marking schedule indicates one mark.
- ½ -marks may be used to acknowledge partial working.
- Marks are allocated for the correct use of an appropriate technique rather than for the exact replication of solutions as given in this marking schedule.
- In general, the policy of Error Carried Forward (ECF), applies. If a candidate makes correct use of an incorrect prior solution, full follow-on marks are to be awarded.

Students have been instructed to answer ALL questions in the ENTIRE PAPER.

Maximum mark allocation is as follows:

Section A:	Basic Numeracy and Algebra Answer ALL Questions	60 marks
Section B:	Calculus Answer the entire section	20 marks
Section C:	Trigonometry and Complex Numbers Answer the entire section	20 marks

For Official Use Only:

	SECTION A				SECTION B	SECTION C	
Question	1	2	3	4	5	6	Total
Maximum Mark	15	15	15	15	20	20	100
Student Mark							

Student ID: **Marking Schedule**

SECTION A: BASIC NUMERACY AND ALGEBRA 60 marks

Question One [15 marks]

1(a) The charge on 1 electron is $-1.602176634 \times 10^{-19}$ C (coulomb).

i. A metal sphere holds a net charge of -1 C. In scientific notation and correct to 3sfs, determine how many electrons contribute to this charge.

(2 marks)

$$n = \frac{-1\text{ C}}{-1.602176634\times10^{-19}\text{ C}} = 6.24 ✓ \times 10^{18} ✓$$

Part-marks: Lose ½ if not correct to 3sfs. Lose ½ if quantity is given as negative.

ii. An oil film is 0.000 000 000 231 m thick. Express this quantity in engineering notation.

(2 marks)

$\mathbf{0.231} ✓ \times \mathbf{10^{-9}} ✓$ (Also accept 231×10^{-12})

Part-marks: Lose ½ if correct but not in engineering notation.

1(b) 1 mile = 1,760 yards, and 1 yard = 0.9144 metre. How long would a 20 mile tramping trail be in kilometres? Give the answer correct to 1dp. (2 marks)

$1760 \times 0.9144 / 1000 \times 20 ✓ = \mathbf{32.2\ km} ✓$

Part-marks: ½ off if rounding is wrong. Lose ½ if units are missing/wrong.

1(c) **An engineering contractor predicts that a customer project will cost them $450,000,000.**

i. What did it charge the customer to make a projected profit of 23%? (1 mark)

ii. The project is carried out, and the following costs were recorded:

Material costs	**$123,000,000**
Labour	**$232,000,000**
Administration costs	**$ 23,000,000**

If the customer paid the charge calculated in i, what profit or loss did this project make? (2 marks)

iii. What percentage (to the nearest 1%) profit or loss did it make compared with the actual costs incurred? (2 marks)

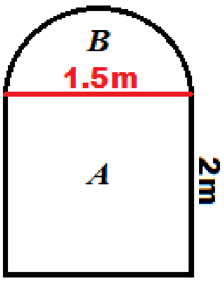
i. $1.23 \times \$450\text{ m} = \mathbf{\$553.5\ million}$ ✓

ii. Sum of costs = **378 million** ✓ $\mathbf{\$553.5\ m - 378\ m = \$175.5\ million}$ ✓

iii. % profit = $\frac{175.5}{378}$ ✓ $\times\ 100\% = \mathbf{46}\ \%$ ✓

Part-marks: As shown. ECF applies throughout.

1(d) **The diagram to the right shows a hall window.**
Shape A is a rectangle.
Shape B is a semicircle.



i. Correct to 2dps, find the area of A. (1 mark)

ii. To the same degree of accuracy, determine the area of B. (2 marks)

iii. Find the area of the window. (1 mark)

i. $A = lb = 2 \times 1.5 = \mathbf{3.00\ m^2}$ ✓

Part-marks: ½ off for no units or wrong nr decimals or both.

ii. $A = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 0.75^2$ ✓ $= \mathbf{0.88\ m^2}$ ✓

Part-marks: ½ off for no units. ECF applies.

iii. $\mathbf{3.88\ m^2}$ ✓

Part-marks: ½ off for no units. ECF applies.

Question Two **[15 marks]**

2(a) If $2a - 3 = 7a - 38$, find a. **(1 mark)**

$2a - 3 = 7a - 38 \Rightarrow 35 = 5a$ ½ $\Rightarrow a = \mathbf{7}$ ½

Part-marks as shown.

2(b) Solve for x: $x^2 = x$ **(2 marks)**

$x^2 = x \Rightarrow x(x-1) = 0$ ✓ $\Rightarrow x = \mathbf{0}$ ½ or $\mathbf{1}$ ½

Part-marks as shown.

2(c) Solve the following simultaneous equations for p and q:

$$2p - 3q = p + q + 11$$
$$3p + 2q = p + 2$$

(4 marks)

$2p - 3q = p + q + 11 \Rightarrow p - 4q = 11$ [1] ✓

$3p + 2q = p + 2 \Rightarrow 2p + 2q = 2$ [2] ✓

Eliminate either p or q, for example, [1] +2[2]: $5p = 15 \Rightarrow \boldsymbol{p} = \mathbf{3}$ ✓

Substituting back into say, [1]: $3 - 4q = 11 \Rightarrow -4q = 8 \Rightarrow \boldsymbol{q} = \mathbf{-2}$ ✓

Other correct approaches accepted. Part-marks as shown.

2(d) Correct to 2dps, determine x if $x^2 - x = 1$. **(3 marks)**

$x^2 - x = 1 \Rightarrow x^2 - x - 1 = 0 \Rightarrow \Delta = 5 \Rightarrow x = \frac{1 \pm \sqrt{5}}{2}$ ✓ $\Rightarrow \boldsymbol{x = 1.62}$ ✓ $\boldsymbol{or - 0.62}$ ✓

✓ for identifying $a = 1, b = -1, c = -1$, and ✓ each for the roots, as shown.

2(e) Solve for x and y:

$\boldsymbol{x^2 + y^2 = 20}$ [1]

$\boldsymbol{y = 2x}$ [2] **(5 marks)**

Subst [2] into [1]: $x^2 + (2x)^2 = 20 \Rightarrow 5x^2 = 20$ ✓ $\Rightarrow \boldsymbol{x = \pm 2}$ ✓✓(one for each root)

Substituting $\Rightarrow \boldsymbol{x = -2}$ into [2]: $\boldsymbol{y = -4}$ ✓

Substituting $\Rightarrow \boldsymbol{x = +2}$ into [2]: $\boldsymbol{y = +4}$. ✓

Question Three **[15 marks]**

3(a) Fully factorise: $a^2r - r$. **(2 marks)**

$r(a^2 - 1)$ ✓ $= r(a+1)(a-1)$ ✓

Part marks: as shown.

3(b) Simplify expressing the answers free of negative exponents: $\frac{p}{q} \div \frac{p^2}{q^3} \times \frac{3q^2}{4qp^2}$. **(2 marks)**

$\frac{p}{q} \div \frac{p^2}{q^3} \times \frac{3q^2}{4qp^2} = \frac{p}{q} \times \frac{q^3}{p^2} \times \frac{3q^2}{4qp^2}$ ✓ $= \frac{1}{1} \times \frac{q^3}{p} \times \frac{3}{4p^2} = \frac{3q^3}{4p^3}$ ✓

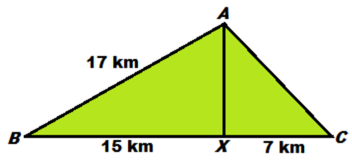
Part marks: as shown.

3(c) Simplify $\left(\frac{2x^3y^5}{54x^9y^2}\right)^{\frac{2}{3}}$, without fractional and negative exponents, and without brackets. **(3 marks)**

$\left(\frac{2x^3y^5}{54x^9y^2}\right)^{\frac{2}{3}} = \left(\frac{y^3}{27x^6}\right)^{\frac{2}{3}}$ ✓ $= \left(\frac{y}{3x^2}\right)^2$ ✓ $= \frac{y^2}{9x^4}$ ✓

Part marks: as shown. If correct, lose 1 if fractional exponents/brackets still present.

3(d) In the diagram to the right, ABC represents a piece of land set aside for development.
$AB = 17$ km, $BX = 15$ km and $XC = 7$ km.
AX is perpendicular to BC.



i. Find AX. **(2 marks)**

ii. Determine the area of the land. **(2 marks)**

i. $AX^2 = BA^2 - BX^2 = 17^2 - 15^2 = 64$✓. Hence $\boldsymbol{AX = 8}$ **km**✓

ii. $A = \frac{1}{2} BC \times AX = 11 \times 8$✓$=$ **88 km^2** ✓

Part marks: Lose ½ for no units or incorrect units. Otherwise as shown.

Other correct approaches, such as determining the area of each triangle, to be accepted.

3(e) Given the formula: $F = G\frac{m_1 m_2}{r^2}$

i. Change the subject of the formula to G. **(1 mark)**

ii. Determine G in scientific notation to 3sfs, given $m_1 = 1$, $m_2 = 1$, $r = 0.05$ and $F = 2.6696 \times 10^{-8}$ **(3 marks)**

i. $\boldsymbol{G = \frac{Fr^2}{m_1 m_2}}$✓ ½ each for numerator and denominator.

ii. $G = \frac{2.6696\times10^{-8}\times0.05^2}{1\times1}$✓$= \mathbf{6.67}$✓$\times \mathbf{10^{-11}}$✓

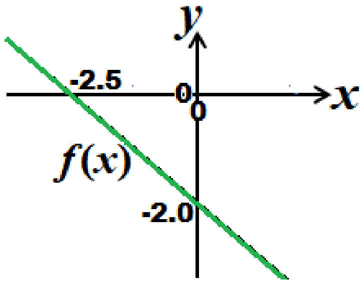
Part marks: as shown. ECF for the formula applies. Lose a mark for wrong answer format.

Question Four **[15 marks]**

4(a) The diagram to the right shows the graph of the function, $f(x)$, which has intercepts as shown.

i. Find the slope of this function correct to 3sfs. (2 marks)

ii. Write down an equation for this function. (2 marks)



i. Slope $= -\frac{2}{2.5} = -0.800$✓✓ (one mark for the correct sign)

Part marks: ½ off for incorrect rounding.

ii. $y = -0.800x - 2$✓✓

4(b) Solve for x: $\ln(x+1) + \ln(x-1) = 0$. Check the validity of your answer(s).

(5 marks)

$\ln(x+1) + \ln(x-1) = 0 \Rightarrow \ln(x+1)(x-1)$✓$= \ln 1$✓

$\Rightarrow x^2 - 1 = 1$ ✓ $x^2 = 2 = 0 \Rightarrow x = \pm\sqrt{2}$.✓ Discard $-\sqrt{2}$ **or** accept $\sqrt{2}$ only.✓

Part marks: ½ off if not in surd form.

4(c) The charge on a capacitor in coulombs (C), falls according to the formula:

$$Q = Q_0 e^{kt},$$

where Q_0 is the initial charge and Q is the residual charge after t seconds. The capacitor is initially charged to 0.25 C.

i. Write down the value of Q_0. (1 mark)

ii. After three seconds, the residual charge is found to be 0.18 C. Correct to 6 decimal places, find the value of the constant, k. (3 marks)

iii. Determine the charge which remains on the capacitor after one minute. Express the answer in engineering notation correct to 3sfs. (2 marks)

i. $\mathrm{Q}_0 = 0.25$ C✓

ii. $0.18 = 0.25e^{3k}$✓$\Rightarrow k = \frac{1}{3}\ln\frac{0.18}{0.25}$✓$= -0.109501$✓

Part marks: ½ off if dps are wrong.

iii. $Q = 0.25e^{-0.109501\times 60}$✓$= 0.350 \times 10^{-3}$ C✓ (accept 350×10^{-6} C)

Part marks: ½ off if sfs are wrong. ½ off for wrong/omitted units. ECF applies.

SECTION B: CALCULUS TECHNIQUES [20 marks]

Question Five

Given $f(x) = x + x^{-1}$,

5(a) i. Determine $f'(x)$. **(1 mark)**

$1 - x^{-2}$ ✓

Part-marks: ½ per term.

ii. Give the coordinates of the point on the curve of $f(x)$, where $x = -2$. **(2 marks)**

Substitute $x = -2$ in $y = f(x) = x + x^{-1}$✓$= -2 - \frac{1}{2} = -2\frac{1}{2}$ · Hence (-2, -2$\frac{1}{2}$)✓ falls on f.

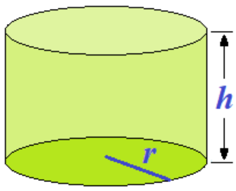
Part-marks: 1 for substituting into the original function. Accept decimals. ECF applies.

iii. Find the slope of the tangent to the curve of $f(x)$ at the point calculated in ii. **(2 marks)**

Slope = $f'(-2) = 1 - x^{-2}$✓$= 1 - \frac{1}{4} = \frac{3}{4}$✓

Part-marks: 1 for substituting into the derivative. Accept decimals. ECF applies.

5(b) A cylindrical storm water storage tank has a volume of 30 m^3. It is closed both top and bottom, and has radius r and height h, as shown to the right.



i. Show that: $h = \frac{30}{\pi r^2}$ **(1 mark)**

$30 = \pi r^2 h \Rightarrow \boldsymbol{h} = \frac{30}{\pi r^2}$✓

ii. If the tank is constructed of negligibly thick plastic material, show that the area A of the material is given by:

$A = 60r^{-1} + 2\pi r^2$. **(2 marks)**

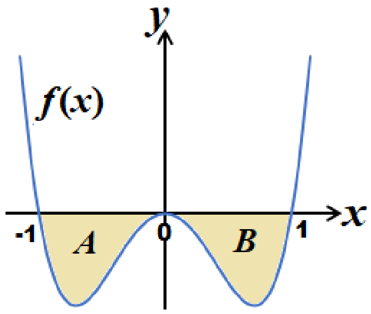
$A = 2\pi rh + 2\pi r^2 = 2\pi r\frac{30}{\pi r^2} + 2\pi r^2$✓$= \frac{60}{r} + 2\pi r^2 = \boldsymbol{60r^{-1} + 2\pi r^2}$ ✓ ½ per term.

iii. Correct to the nearest mm, determine the value of r that minimises the area of plastic material, which goes into the construction of the tank. **(3 marks)**

$A' = -60r^{-2} + 4\pi r$✓$= 0 \Rightarrow 4\pi r = \frac{60}{r^2}$✓ $\Rightarrow r^3 = \frac{60}{4\pi} \Rightarrow \boldsymbol{r = 1.684}$ **m**; ✓

Part-marks: as shown. Accept 1 684 mm.

5(c) The figure to the right shows a part of the curve of the function $f(x) = 4x^4 - 4x^2$. Determine:



i. $\int_{-1}^{1} f(x)\,dx$, correct to 3 dps; **(2 marks)**

ii. The shaded area A, to the same level of accuracy; **(2 marks)**

iii. The total shaded area A + B. **(1 mark)**

i. $\int_{-1}^{1}(4x^4 - 4x^2)\,dx = \frac{4}{5}x^5 - \frac{4}{3}x^3 \Big|_{-1}^{1}$ ✓ $= \frac{4}{5} - \frac{4}{3} - \left(-\frac{4}{5} + \frac{4}{3}\right) = -1.067\ \text{unit}^2$ ✓

Lose ½ for incorrect sign. Lose another ½ for incorrect accuracy.

ii. $A = \left|\int_{-1}^{0}\left(4x^4 - 4x^2\right)dx\right| = \left|\frac{4}{5}x^5 - \frac{4}{3}x^3 \Big|_{-1}^{0}\right|$ ✓ $= 0.533\ \text{unit}^2$. ✓

Accept a value with reason; $-$e.g. half by symmetry. Lose ½ for giving a negative area.

iii. $1.067\ \text{unit}^2$ ✓

Part-marks: Lose ½ for giving a negative area. ECF applies.

5(d) If the velocity (v) in m/s, of a test trolly starting from rest is given by:

$$v = 3t^2 + 2, \text{ after } t \text{ seconds,}$$

i. Find a formula for the trolley's acceleration after t seconds. **(1 mark)**

ii. Determine a formula for the trolley's displacement after t seconds, if the initial displacement is taken as 0. **(1 mark)**

iii. Find the displacement and acceleration of the trolley after the first 3 s of motion. **(2 marks)**

i. $a = 6t$ ✓

Part-marks: ½ for derivative expression, if correctly shown.

ii. $s = t^3 + 2t$ ✓

Part-marks: ½ for integral expression, if correctly shown.

iv. $a = 6t = 18\ \text{m/s}^2$ ✓; $s = 3^3 + 6 = 33\ \text{m}$ ✓

Lose half a mark per missing/incorrect unit. ECF applies from above formulae.

SECTION C: TRIGONOMETRY AND COMPLEX NUMBERS [20 marks]

Question Six

6(a) The graph shown to the right has an equation of the form:

$$y = a\sin(bx + c).$$

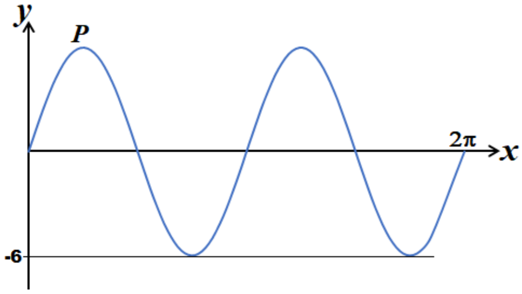
Give:

i. the amplitude; and (1 mark)

ii. the period. (1 mark)

iii. Write down an equation for the graph free of unknown constants. (2 marks)

iv. Give the coordinates of the turning point, P. (2 mark)



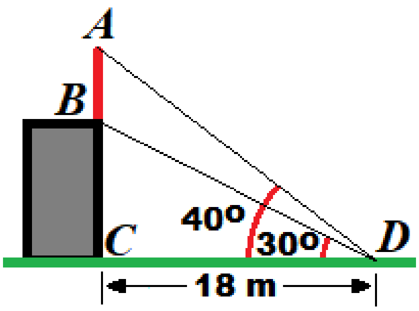
i. 6 ✓
Part-marks: ½ off for -ve sign.
ii. π ✓
No part-marks.
iii. $y = 6\sin(2x)$✓✓
Part-marks: 1 for each of the 6 and the $2x$.
iv. $\left(\frac{\pi}{4}, 6\right)$ ✓✓ (✓per coordinate. Also accept x-coordinate of 0.785.)
ECF applies.

6(b) The sketch (right) shows a vertical mast AB mounted on top of a building, immediately above the wall BC. From a point D, horizontally 18 m from the base of the building, the angles of elevation of the bottom and top of the mast are 30° and 40° respectively.

i. Correct to 1 dp, find the combined height of the mast and the building, AC. (2 marks)

ii. To the same level of accuracy, find the height of the building BC. (1 mark)

iii. Determine the length of the mast AB. (1 mark)

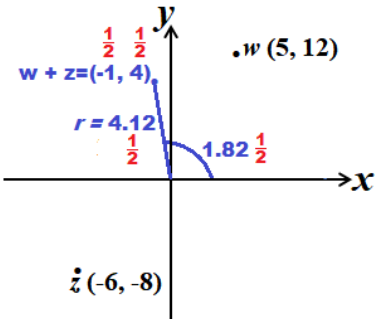


i. $\frac{AC}{CD} = \tan 40°$✓$\Rightarrow AC = 18\tan 40° = \mathbf{15.1\ m}$ ✓
Part-marks: as shown. Lose ½ for incorrect rounding and another ½ for no/incorrect units.
ii. $\frac{BC}{CD} = \tan 40° \Rightarrow BC = 18\tan 30° = \mathbf{10.4\ m}$ ✓
Part-marks: as above.
iii. $AB = 15.1 - 10.4 = \mathbf{4.7\ m}$✓

6(c) **Solve for x: $2\cos(2x - 30°) = -1$, given $0 \le x \le 180°$** **(4 marks)**

$2x - 30° = 120° + 360°n \text{ or } 240° + 360°n$ ✓ ½ for each option
$2x = 150° + 360°n \text{ or } 270° + 360°n$
$x = 75° + 180°n \text{ or } 135° + 180°n$ ✓ ½ for each option
When $n = 0$, $x = 75° \text{ or } 135°$✓½ for each; only allowable answers within restrictions ✓
Part marks as shown. ECF applies.

6(d) **Given the following Argand diagram:**



i. **Write down the two complex numbers shown, in re-im form.** **(1 mark)**

$w = 5 + 12j$ ½ $z = -6 - 8j$ ½

ii. **Determine $w + z$ in re-im form.** **(1 marks)**

$(5 + 12j) + (-6 - 8j) = -1 + 4j$ ✓ ½ for each term

iii. **Give $w + z$ in polar form, expressing the argument in radians.** **(2 marks)**

Argument is in 2nd quad. $\boldsymbol{\theta = \pi - \tan^{-1}\frac{4}{1}}$ ½= $\mathbf{1.82}$ ½ . ECF applies.
$r = \sqrt{1^2 + 4^2} = 4.12$ ½; hence $\boldsymbol{wz = 4.12\angle 1.82}$ ½ . ECF applies.

iv. **Insert $w + z$ on the Argand diagram above, showing its real and imaginary parts, as well as its modulus and its argument in radians.** **(2 marks)**

See blue inserts in Argan diagram above.