VEENZ Vocational Engineering Education | SOUTHERN INSTITUTE OF TECHNOLOGY | Te Pūkenga

NEW ZEALAND DIPLOMA IN ENGINEERING

(Core Paper)

DE4102: Engineering Mathematics 1

2023 S1 – Test 2 : Calculus, Trigonometry and Complex Numbers

Time Allowed:	**One and a Half (1.5) hours plus 10 minutes reading time**
Total Marks:	**40**

QUESTION AND ANSWER BOOK

Please Note:

1. This examination will be held in accordance with institutional examination procedures and in conjunction with the rules and regulations imposed by Vocational Engineering Education New Zealand.
2. Any candidate who aids, attempts to aid, obtains aid or attempts to obtain aid from another candidate will be disqualified and further dealt with under the institutional disciplinary procedures for cheating. This could result in the offender's expulsion from the institution.
3. Candidates are permitted to use approved scientific calculators. This explicitly excludes lap-top computers, cell phones, i-phones, i-pads or any other device capable of connecting by wireless to the Internet or other electronic media, or source of potential aid. Violations of this rule will also be dealt with as acts of cheating.
4. This is a CLOSED BOOK examination. You may not be in possession of books, notes, memoranda or reference materials other than those supplied in the examination.
5. You may make use of the information in the Formula Sheet attached to this paper.

Instructions:

1. Enter your Student ID number in the space provided below.
2. All answers must be written in this book, in the spaces available, or on extra sheets supplied on request.
3. Write your student ID and the question number on each extra page, and enclose it in the book adjacent to the relevant question.
4. CROSS OUT any work that you do not wish to have marked.
5. Show ALL working in calculations to obtain full marks. Where answers require rounding, use 3 significant figures, unless otherwise specified. Rounding may be specified correct to so many significant figures (sfs) or decimal places (dps).
6. This examination counts towards 15% of your final mark for the course.
7. Answer ALL questions

For Official Use Only:

Question	5	6	Total
Maximum Mark	20	20	40
Student Mark			

Student ID: **Model Answers**

SECTION B: CALCULUS TECHNIQUES [20 marks]

Question Five

Given $f(x) = x + x^{-1}$,

5(a) i. Determine $f'(x)$. (1 mark)

$1 - x^{-2}$ ✓

Part-marks: ½ per term.

ii. Give the coordinates of the point on the curve of $f(x)$, where $x = -2$. (2 marks)

Substitute $x = -2$ in $y = f(x) = x + x^{-1}$✓$= -2 - \frac{1}{2} = -2\frac{1}{2}$. Hence (-2, -2$\frac{1}{2}$)✓ falls on f.

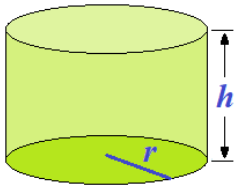
Part-marks: 1 for substituting into the original function. Accept decimals. ECF applies.

iii. Find the slope of the tangent to the curve of $f(x)$ at the point calculated in ii. (2 marks)

Slope = $f'(-2) = 1 - x^{-2}$✓$= 1 - \frac{1}{4} = \frac{3}{4}$✓

Part-marks: 1 for substituting into the derivative. Accept decimals. ECF applies.

5(b) A cylindrical storm water storage tank has a volume of 30 m^3. It is closed both top and bottom, and has radius r and height h, as shown to the right.



i. Show that: $h = \frac{30}{\pi r^2}$ (1 mark)

$30 = \pi r^2 h \Rightarrow \boldsymbol{h = \frac{30}{\pi r^2}}$✓

ii. If the tank is constructed of negligibly thick plastic material, show that the area A of the material is given by:
$A = 60r^{-1} + 2\pi r^2$. (2 marks)

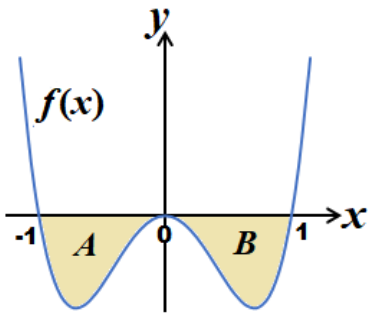
$A = 2\pi rh + 2\pi r^2 = 2\pi r\frac{30}{\pi r^2} + 2\pi r^2$✓$= \frac{60}{r} + 2\pi r^2 = \boldsymbol{60r^{-1} + 2\pi r^2}$ ✓ ½ per term.

i. Correct to the nearest mm, determine the value of r that minimises the area of plastic material, which goes into the construction of the tank. (3 marks)

$A' = -60r^{-2} + 4\pi r$✓$= 0 \Rightarrow 4\pi r = \frac{60}{r^2}$✓$\Rightarrow r^3 = \frac{60}{4\pi} \Rightarrow \boldsymbol{r = 1.684\ m}$; ✓

Part-marks: as shown. Accept 1 684 mm.

5(c) **The figure to the right shows a part of the curve of the function $f(x) = 4x^4 - 4x^2$. Determine:**



i. $\int_{-1}^{1} f(x)\,dx$, **correct to 3 dps;** **(2 marks)**

ii. **The shaded area A, to the same level of accuracy;** **(2 marks)**

iii. **The total shaded area $A + B$.** **(1 mark)**

i. $\int_{-1}^{1}(4x^4 - 4x^2)\,dx = \frac{4}{5}x^5 - \frac{4}{3}x^3 \,|_{-1}^{1}$ ✓$= \frac{4}{5} - \frac{4}{3} - \left(-\frac{4}{5} + \frac{4}{3}\right) = -1.067$ unit^2✓

Lose ½ for incorrect sign. Lose another ½ for incorrect accuracy.

ii. $A = \left|\int_{-1}^{0}\left(4x^4 - 4x^2\right)dx\right| = \left|\frac{4}{5}x^5 - \frac{4}{3}x^3 \,|_{-1}^{0}\right|$ ✓$= 0.533$ unit^2 . ✓

Accept a value with reason; —e.g. half by symmetry. Lose ½ for giving a negative area.

iii. 1.067 unit^2 ✓

Part-marks: Lose ½ for giving a negative area. ECF applies.

5(d) **If the velocity (v) in m/s, of a test trolly starting from rest is given by: $v = 3t^2 + 2$, after t seconds,**

i. **Find a formula for the trolley's acceleration after t seconds.** **(1 mark)**

ii. **Determine a formula for the trolley's displacement after t seconds, if the initial displacement is taken as 0.** **(1 mark)**

iii. **Find the displacement and acceleration of the trolley after the first 3 s of motion.** **(2 marks)**

i. $a = 6t$ ✓

Part-marks: ½ for derivative expression, if correctly shown.

ii. $s = t^3 + 2t$ ✓

Part-marks: ½ for integral expression, if correctly shown.

iv. $a = 6t = 18$ m/s^2 ✓; $s = 3^3 + 6 = 33$ m ✓

Lose half a mark per missing/incorrect unit. ECF applies from above formulae.

SECTION C: TRIGONOMETRY AND COMPLEX NUMBERS [20 marks]

Question Six

6(a) **The graph shown to the right has an equation of the form:**

$$y = a\sin(bx + c).$$

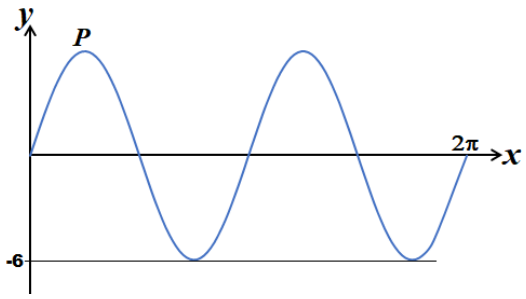
Give:

i. the amplitude; and (1 mark)

ii. the period. (1 mark)

iii. Write down an equation for the graph free of unknown constants. (2 marks)

iv. Give the coordinates of the turning point, P. (2 mark)



i. 6 ✓

Part-marks: ½ off for -ve sign.

ii. π ✓

No part-marks.

iii. $y = 6\sin(2x)$✓✓

Part-marks: 1 for each of the 6 and the $2x$.

iv. $\left(\frac{\pi}{4}, 6\right)$ ✓✓ (✓per coordinate. Also accept x-coordinate of 0.785.)

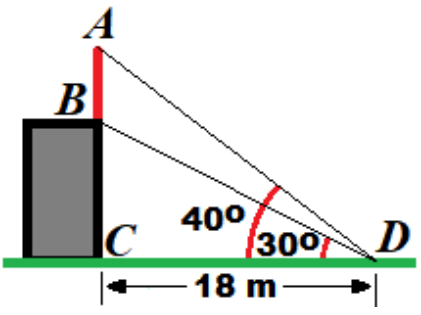
ECF applies.

6(b) **The sketch (right) shows a vertical mast AB mounted on top of a building, immediately above the wall BC. From a point D, horizontally 18 m from the base of the building, the angles of elevation of the bottom and top of the mast are 30° and 40° respectively.**

i. Correct to 1 dp, find the combined height of the mast and the building, AC. (2 marks)

ii. To the same level of accuracy, find the height of the building BC. (1 mark)

iii. Determine the length of the mast AB. (1 mark)



i. $\frac{AC}{CD} = \tan 40°$✓$\Rightarrow AC = 18\tan 40° = \mathbf{15.1\ m}$ ✓

Part-marks: as shown. Lose ½ for incorrect rounding and another ½ for no/incorrect units.

ii. $\frac{BC}{CD} = \tan 40° \Rightarrow BC = 18\tan 30° = \mathbf{10.4\ m}$ ✓

Part-marks: as above.

iii. $AB = 15.1 - 10.4 = \mathbf{4.7\ m}$✓

6(c) Solve for x: $2\cos(2x - 30°) = -1$, given $0 \leq x \leq 180°$ (4 marks)

$2x - 30° = 120° + 360°n \; or \; 240° + 360°n$ ✓ ½ for each option

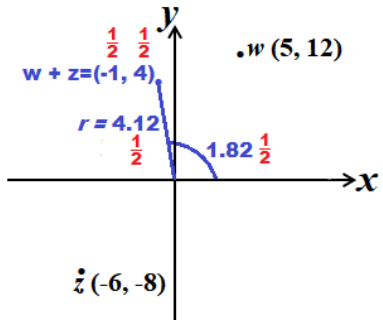
$2x = 150° + 360°n \; or \; 270° + 360°n$

$x = 75° + 180°n \; or \; 135° + 180°n$ ✓ ½ for each option

When $n = 0$, $x = 75° \; or \; 135°$ ✓½ for each; only allowable answers within restrictions ✓

Part marks as shown. ECF applies.

6(d) Given the following Argand diagram:



i. Write down the two complex numbers shown, in re-im form. (1 mark)

$w = 5 + 12j$ ½ $z = -6 - 8j$ ½

ii. Determine $w + z$ in re-im form. (1 marks)

$(5 + 12j) + (-6 - 8j) = -1 + 4j$ ✓ ½ for each term

iii. Give $w + z$ in polar form, expressing the argument in radians. (2 marks)

Argument is in 2nd quad. $\boldsymbol{\theta = \pi - \tan^{-1}\frac{4}{1}}$ ½= $\mathbf{1.82}$ ½ . ECF applies.

$r = \sqrt{1^2 + 4^2} = 4.12$ ½; hence $\boldsymbol{wz = 4.12\angle 1.82}$ ½ . ECF applies.

i. Insert $w + z$ on the Argand diagram above, showing its real and imaginary parts, as well as its modulus and its argument in radians. (2 marks)

See blue inserts in Argan diagram above.