Department of Chemical and Process Engineering
Thermodynamics

Tutorial Sheet: Ideal Gas Processes

1. Two moles of an ideal gas undergo a slow isothermal compression at 350 K in a frictionless piston/cylinder arrangement, in which the volume is reduced to one third of the initial volume.

(a) Calculate the total work of compression.

(b) Calculate the heat transferred during the compression.

2. For an adiabatic process involving an ideal gas, the following relationship relating pressure and volume is obeyed:

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

(a) Show that by using the ideal gas equation of state, the relationship can also be written as:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \qquad \frac{T_2}{T_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1-\gamma}{\gamma}}$$

(b) Consider an adiabatic process involving a diatomic gas such as nitrogen, for which we can take γ as constant and equal to 1.4. The value of c_V for nitrogen is 5/2R. Our closed system consists of 1 mole of gas located within the classic piston/cylinder arrangement, and the initial state of the system is 300 K (T_1) and 1 bar (P_1). Calculate the final temperature (T_2) and pressure (P_2) when the compression is carried out to *half* the original volume.

(c) By starting with the definition for work (W), show that for an adiabatic process:

$$W = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} \rightarrow W = n\, c_V\, (T_2 - T_1)$$

and use this to calculate the work term for this compression.

(d) Consider now an adiabatic expansion where the volume is doubled. Calculate the final temperature and suggest how this finding forms the basis of an important domestic and industrial application.

CP203 -Thermodynamics Tutorial Solutions Week 4
Group 17

1.
a)

$$W = -\int_1^2 PdV$$
$$W = -nRT \ln \frac{V_2}{V_1}$$
$$W = -2 \cdot 8.3145 \cdot 350 \left(\ln \frac{1}{3} \right)$$
$$\boxed{W = 6394.1\ J}$$

b)

The heat transferred during the compression is $\boxed{-6394.1\ J}$

2.
a)

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$
$$\frac{\cancel{nR}T_1}{V_1} \cdot V_1^\gamma = \frac{\cancel{nR}T_2}{V_2} \cdot V_2^\gamma$$
$$T_1 \cdot V_1^{\gamma-1} = T_2 \cdot V_2^{\gamma-1}$$
$$\boxed{\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1}}$$

$$P_1^{\frac{1}{\gamma}} V_1 = P_2^{\frac{1}{\gamma}} V_2$$
$$P_1^{\frac{1}{\gamma}} \frac{\cancel{nR}T_1}{P_1} = P_2^{\frac{1}{\gamma}} \frac{\cancel{nR}T_2}{P_2}$$
$$P_1^{\frac{1}{\gamma}-1} T_1 = P_2^{\frac{1}{\gamma}-1} T_2$$
$$\boxed{\frac{T_1}{T_2} = \left(\frac{P_2}{P_1} \right)^{\frac{1-\gamma}{\gamma}}}$$

Note that these are equivalent to the forms given in the question, the only difference is that the fractions have been flipped.

b)

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{\gamma-1}$$
$$T_2 = 300 \div \left(\frac{1}{2} \right)^{1.4-1}$$
$$\boxed{T_2 = 396\ K}$$
$$\left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{1-\gamma}} = \frac{P_2}{P_1}$$
$$\left(\frac{300}{396} \right)^{\frac{1.4}{1-1.4}} = \frac{P_2}{1}$$
$$\boxed{P_2 = 2.64\ Bar}$$

c)

$$W = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$$

$$W = \frac{n\cancel{R}(T_2 - T_1)}{\cancel{1} + \frac{\cancel{R}}{C_v} \cancel{-1}}$$

$$W = nC_v(T_2 - T_1)$$

d)

$$T_2 = 300 \div (2)^{1.4-1}$$

$$\boxed{T_2 = 227\ K}$$

This shows that by undergoing a volume expansion we see cooling as a result. This is a useful thing to understand as it is a key element of engineering. This is how fridges work, the gas undergoes a volume expansion which cools it down and allows us to refrigerate things.