DEPARTMENT OF CHEMICAL & PROCESS ENGINEERING

Degree of BEng/MEng in Chemical Engineering

CP218 Applied Mechanics

Date: Tuesday, 19/04/2022 **Time: 9:30 am**
Duration: 02:40 (hrs:mins)

Answer ALL Questions

PLEASE PAY CAREFUL ATTENTION TO INSTRUCTIONS ON MYPLACE FOR UPLOAD OF YOUR ANSWERS

Plagiarism statement: By submitting answers to this paper I declare that these answers are entirely my own work and have not been shared, in part or in whole or in any draft form, with any other student, or disseminated in any other way. I understand that infringing this statement would represent a serious academic offence subject to disciplinary action according to the University Regulations and Procedures regarding Plagiarism, with significant consequences for degree progression and final degree outcome.

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Answer ALL Questions

Q1. 50 marks

As shown in Fig. 1 below, a horizontal, spring-supported bar is placed between two vertical walls. The bar is firmly attached to the left vertical wall, and in contact with an ideal, linear spring on its right side. The spring is firmly attached to the right vertical wall. The length of the load-free bar is L. The bar is loaded by a constant distributed body force $b(x) = b_o$. The units of $b(x)$ are force per unit length. Assume that the bar is materially homogeneous, i.e. of constant Young's modulus E, prismatic , i.e. of constant area A, and described by a linear elastic constitutive law. The mass density of the bar is ρ. The elastic spring-constant is k. You can assume that when $b_o = 0$, the spring is in equilibrium.

(a) Based on the loads shown in Fig. 1, state whether the spring will be under compression or extension. What is the magnitude and direction of the force that the spring exerts on the bar?

[4 marks]

(b) For the case of continuous loads, write the one-dimensional form of the equation of momentum conservation in the bar, which is valid for all types of elastic material.

[2 marks]

(c) Derive a second order ordinary differential equation for the displacement field $u(x)$ of a *linearly elastic* bar.

[3 marks]

(d) Integrate the derived differential equation, and write the general solution.

[4 marks]

(e) State the boundary condition on the bar's left and hence determine the first integration constant.

[4 mark]

(f) State the boundary condition on the bar's right and hence determine the second integration constant.

[14 mark]

(g) Consider the case where the bar is placed vertically between two horizontal walls, and *with the spring on top*. All other problem specifications remain the same. For this case, if the gravitational acceleration is g, derive an expression for the bar displacement field $u(x)$.

[3 marks]

(h) For the vertical bar case, assuming that the area $A = 2b_o/(\rho g)$, determine whether the force exerted by the spring on the bar is extensional or compressive.

[2 marks]

(i) For the vertical bar case, assuming that the area $A = b_o/(\rho g)$, determine the axial-deflection of the bar.

[2 marks]

(j) For the vertical bar case, assume $A = b_o/(2\rho g)$. Perform a cut anywhere within the bar, and carry out a free body diagram analysis to derive an equation for the internal force at the cut.

[8 marks]

(k) Employing the equation for the internal force which you derived in the previous question, specify the reaction force at the bottom wall and prove that it is extensional.

[4 marks]

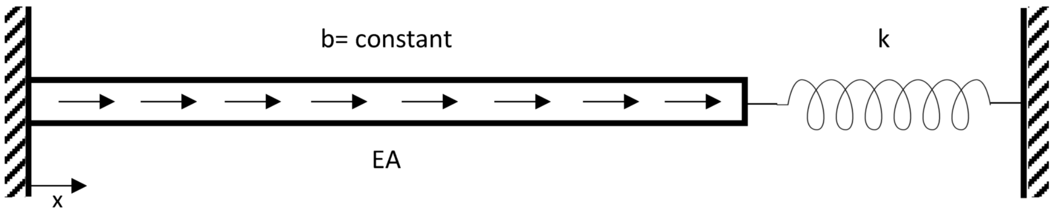

Figure 1: Spring-supported bar between two vertical walls

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Q2. 50 marks

The sprinkler turbine shown in Fig. 2 below has two arms of length $R = 18$ cm, and a total flow rate $Q = 14\ \mathrm{m^3\ h^{-1}}$ of water of mass density $\rho = 998\ \mathrm{kg\ m^{-3}}$. The arms are tube-like with wall thickness δ and inner diameter $D = 0.8$ cm, which is the nozzle exit diameter of each of the two water jets. The flow enters at the centre point along a direction normal to the plane of the figure, and splits evenly into two streams of flow rate $Q/2$. The flow velocity of each of the two water jets *relative to the sprinkler* is V. Friction can be neglected. The two arms rotate with angular velocity ω and do work on a shaft. The fluid pressure immediately after the flow splits and water starts flowing within each arm is equal to $p_i = 2p_a$, where p_a is the atmospheric pressure. The arms are made of elastic material with corresponding *yield point* stress equal to σ_Y.

(a) Write a general equation for the power P delivered by a turbomachine in steady state operation.

[2 marks]

(b) Simplify the general equation for P by inserting the corresponding sprinkler turbine quantities. Justify your choices.

[8 marks]

(c) Derive the relation between angular velocity ω and relative velocity V that maximizes power P.

[6 marks]

(d) Apply control volume analysis to specify the relative velocity V.

[4 marks]

(e) Find the angular velocity ω (in *r/min* units), that corresponds to maximum power P.

[2 marks]

(f) Specify the maximum power P in *W* units.

[2 marks]

(g) Apply control volume analysis to find a *formula (no numerical values or manipulations required)* for the force that loads the elastic material in each arm. To keep formulae short, you can use symbol A for the flow area $\pi D^2/4$ within the arms.

[16 marks]

(h) Find a *formula (no numerical values or manipulations required)* for the inequality that the arm wall-thickness δ needs to satisfy so that the loading of the material in the arms is kept below the yield point.

[10 marks]

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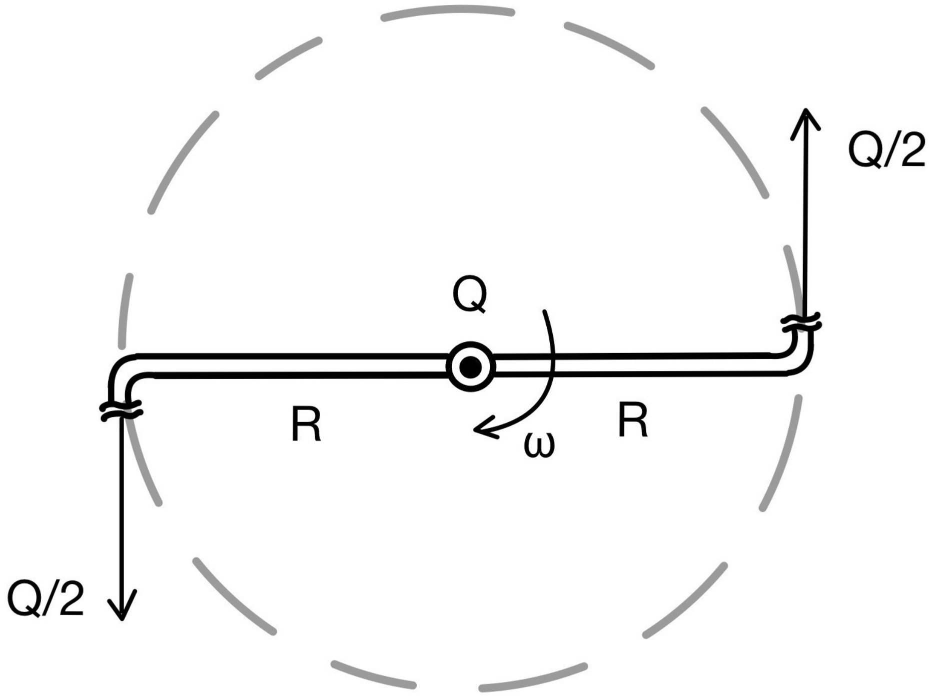

Figure 2: Sprinkler turbine with two arms of length R and flow rate Q

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