LAB 1

Exercise 1

Question 1

In [1]:
```
import numpy as np
from scipy import linalg
import math
import random
import matplotlib
import matplotlib.pyplot as plt
%matplotlib inline
from mpl_toolkits import mplot3d
from itertools import combinations
```

In [2]:
```
# Password
x = np.array([1,0,1,1,1])

# Challenge Vectors
c1, c2 = np.array([0,1,0,1,1]), np.array([1,1,1,1,0])

# Correct Response Function
def challenge_response(x, c):
    response = np.dot(x, c) % 2
    return response
b_1 = challenge_response(x, c1)
b_2 = challenge_response(x, c2)

print(f"β1 = {b_1}")
print(f"β2 = {b_2}")
```

```
β1 = 0
β2 = 1
```

Question 2

Since Eve knows β1, β2, c1 and c2, when she received a challenge vector of (c1 + c2), she can deduce that the response to the challenge is simply (β1 + β2) = 0 + 1 = 1.

In [3]:
```
def add_vectors(v1, v2):
    return [(bit1+bit2)%2 for bit1, bit2 in zip(v1,v2)]
eve_response = challenge_response(x, add_vectors(c1, c2))
print(f"Eve's response = {eve_response}")
```

```
Eve's response = 1
```

Question 3

In [4]:
```
def str_to_vector(s):
    vector = []
    for i in s:
        vector.append(int(i))
    return vector

# Known challenges

Ch = {}
Ch['110011'] = 0
Ch['101010'] = 0
Ch['111011'] = 1
Ch['001100'] = 1


# New challenges
c_a, c_b = str_to_vector('011001'), str_to_vector('110111')
def derive_response(Ch, c):
    for key1 in Ch:
        for key2 in Ch:
            v1 = str_to_vector(key1)
            v2 = str_to_vector(key2)
            if add_vectors(v1, v2) == c:
                return (Ch[key1]+Ch[key2])%2
print(f"Response to c_a = {derive_response(Ch, c_a)}")
print(f"Response to c_b = {derive_response(Ch, c_b)}")
```

```
Response to c_a = 0
Response to c_b = 0
```

Question 4

Cx = b

In [5]:
```
Ch['011011'] = 0
Ch['110100'] = 1
matrix_C, vector_b = [], []

for key in Ch:
    matrix_C.append(str_to_vector(key))
    vector_b.append(Ch[key])

# np.vstack or not, the outcome is still the same
# inputs must be a square matrix
x = [int(y%2) for y in linalg.solve(np.vstack(matrix_C), np.vstack(vector_b)).
flatten()]
print(f"Password x = {x}")
```

```
Password x = [1, 0, 1, 0, 0, 1]
```

Optional Question 4a

Gaussian Elimination

Incomplete, ignore.

In [6]:
```
'''def gauss_elim(augmented):
    #check row by row
    #forward operations, anything below pivot = 0
    pivot = 0
    for row in range(len(augmented)):
        for column in range(len(augmented[0])):
            if pivot == 0 and row == column:
                #this is a pivot position
                pivot = 1
            elif pivot == 1 and row <= column:
                continue
            else:
                augmented[row][column]
    #backward operations, pivot = 1, anything above pivot = 0'''
```

Out[6]:
```
'def gauss_elim(augmented):\n    #check row by row\n    #forward operations,
anything below pivot = 0\n    pivot = 0\n    for row in range(len(augmente
d)):\n        for column in range(len(augmented[0])):\n            if pivot =
= 0 and row == column:\n                #this is a pivot position\n
pivot = 1\n            elif pivot == 1 and row <= column:\n                co
ntinue\n            else:\n                augmented[row][column]\n    #backw
ard operations, pivot = 1, anything above pivot = 0'
```

Exercise 2

Question 5

From the data, matrix A is the column x, vector b is the column y.

Since matrix A is a square matrix, it is row equivalent to I. Therefore we can deduce that the matrix A is invertible.

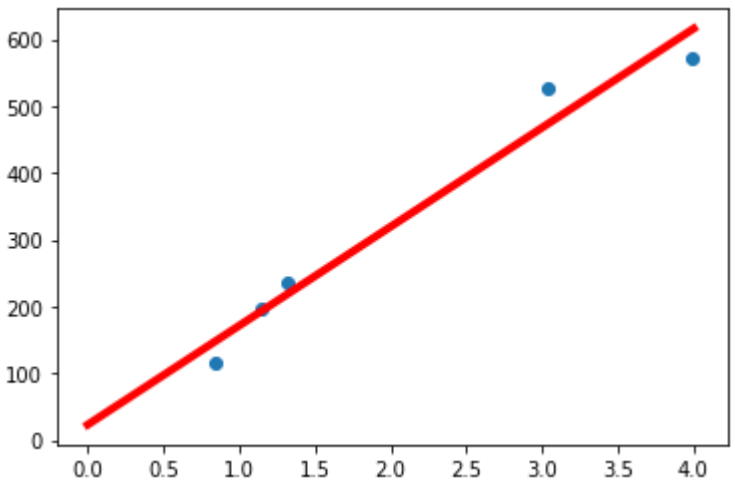
In [7]:
```
house = {}
house['1'] = [0.846, 115.00]
house['2'] = [1.324, 234.50]
house['3'] = [1.150, 198.00]
house['4'] = [3.037, 528.00]
house['5'] = [3.984, 572.50]

# Find summation of x_i, y_i, x_i**2, x_i*y_i
# Solve for m & c

x_i, y_i = [], []
for n in range(1,6):
    x_i.append(house[str(n)][0])
    y_i.append(house[str(n)][1])
x_i, y_i = np.array(x_i), np.array(y_i)

x_1 = x_i.sum()
y_1 = y_i.sum()
x_2 = (x_i@x_i).sum()
x_y = (x_i@y_i).sum()
n = len(x_i)

matrix_A = [[x_2, x_1],
            [x_1, n]]
vector_b2 = [x_y,
             y_1]
# Solve for x = [m, c] with x = (A**-1)b
q = linalg.inv(matrix_A)@vector_b2
m, c = q[0], q[1]
xs = np.linspace(0,4,5)
ys = c + m*xs
plt.plot(xs, ys, 'r', linewidth=4)
plt.scatter(x_i, y_i)
plt.show()
```



Question 6

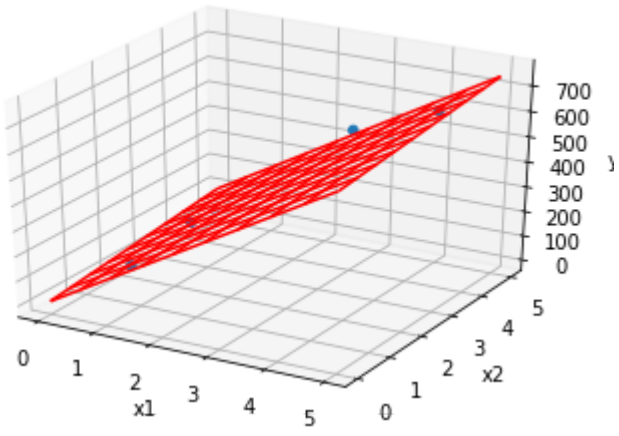
In [8]:
```
# number of bedrooms vector
x_j = np.array(str_to_vector("12345"))

matrix_X = []
for row in range(len(x_i)):
    matrix_X.append([])
    matrix_X[row].append(1)
    matrix_X[row].append(x_i[row])
    matrix_X[row].append(x_j[row])
matrix_X = np.vstack(matrix_X)
vector_a = linalg.solve((matrix_X.T)@(matrix_X), (matrix_X.T)@(y_i))
print(f"vector a = {vector_a}")
```

```
vector a = [  9.97566234 130.67172705  16.45635726]
```

In [9]:
```
fig = plt.figure()
ax = plt.axes(projection='3d')
ax.set_xlabel('x1')
ax.set_ylabel('x2')
ax.set_zlabel('y')

# Create a 3D array
# meshgrid produces all combinations of given x and y
# x goes from 0 to 5, with 10 steps
xR = np.linspace(0, 5, 10)
# y goes from 0 to 5, with 10 steps
yR = np.linspace(0, 5, 10)
# combine all x with all y
xx, yy = np.meshgrid(xR, yR)
zz = vector_a[0] + vector_a[1]*xx + vector_a[2]*yy
ax.scatter3D(x_i, x_j, y_i)
ax.plot_wireframe(xx, yy, zz, color = 'red')
plt.show()
```



Exercise 3

Question 7

In [10]:
```
a_0 = np.array(str_to_vector('110101'))
b_0 = np.array(str_to_vector('110011'))

def rand_key(p):
    key = []
    for i in range(p):
        key.append(random.randint(0, 1))
    return(key)

def random_vector(s, t):
    while True:
        u = rand_key(6)
        if a_0@u == s and b_0@u == t:
            return u
            break

for s in range(2):
    for t in range(2):
        vector_u = random_vector(s,t)
        print(f"vector u = {vector_u}, for s = {s} and t = {t}")
```

```
vector u = [0, 0, 0, 0, 0, 0], for s = 0 and t = 0
vector u = [0, 0, 1, 0, 1, 0], for s = 0 and t = 1
vector u = [0, 0, 0, 1, 0, 0], for s = 1 and t = 0
vector u = [1, 0, 0, 0, 0, 0], for s = 1 and t = 1
```

In [11]:
```
# Function to check if the generated vectors fulfil
# the independency requirements

# input two lists a & b, each contains 4 vectors
# Return "True" if any 3 pairs of vectors from
# (a1,b1),(a2,b2),(a3,b3),(a4,b4) are linearly independent
def check_dependency(a,b):
    for v1 in range(1,3): #1st vector from 1 to 2
        for v2 in range(v1+1,4):
            for v3 in range(v2+1,5):
                squareMatrix = np.vstack((a[v1], b[v1], a[v2], b[v2],a[v3], b[
v3]))
                determinant = np.linalg.det(squareMatrix)
                if determinant == 0: #if determinant is 0, the vectors are not
linearly dependent
                    return False

    # check if a0,b0 and any two random selected pairs of vectors are linearly
independent
    for v1 in range(1,4): #1st vector from 1 to 3
        for v2 in range(v1+1,5):
            squareMatrix = np.vstack((a[0], b[0], a[v1], b[v1], a[v2], b[v2]))
            determinant = np.linalg.det(squareMatrix)
            if determinant == 0: #if determinant is 0, the vectors are not lin
early dependent
                return False
    return True

# Generate 4 a,b pairs using rand_key
# Any 3 pairs must pass check_dependency
def generate_ab():
    while True:
        alist = [a_0]
        blist = [b_0]
        # Creates a temporary list of vectors a and b
        for i in range(4):
            alist.append(rand_key(6))
            blist.append(rand_key(6))
        if check_dependency(alist, blist) == False:
            continue
        else:
            a, b = alist[1:], blist[1:]
            return[np.vstack(a), np.vstack(b)]
            break
ab = generate_ab()
a, b = ab[0], ab[1]
print(a)
print(b)
```

```
[[1 0 0 1 1 1]
 [1 1 0 0 1 0]
 [1 0 0 0 1 1]
 [1 0 1 0 0 1]]
[[0 1 1 1 0 1]
 [1 1 1 1 0 0]
 [1 0 1 0 0 0]
 [1 0 1 1 1 1]]
```

In [12]:
```
# Function to converting String to binary array
def str_to_bits(s):
    res = ''.join(format(ord(i), 'b') for i in s)
    bitsArray = []
    for i in res:
        bitsArray.append(int(i))
    return bitsArray
password_raw = str_to_bits('Potter')
# 2xn matrix password
password = np.vstack(np.array_split(password_raw, len(password_raw)/2))
u_i = []
for st in password:
    u_i.append(random_vector(st[0], st[1]))


#secret vector u
print(np.vstack(u_i))

# Function to converting binary array to string
def bits_to_str(b):
    NumOfChar = int(len(b)/7)
    string = ''
    for i in range(NumOfChar):
        # 7 digits represents 1 char
        bitsChar = ''.join(str(j) for j in b[7*i:7*i+7])
        # convert binary to decimal
        decimalChar = int(bitsChar,2)
        #convert decimal to string
        string = string + chr(decimalChar)
    return string
s_i = np.dot(u_i, a_0)
t_i = np.dot(u_i, b_0)
p = []
for i in range(len(s_i)):
    p.append(s_i[i])
    p.append(t_i[i])

# Just to verify u_i, s_i and t_i are correct, we check to see
# if we get back the original string
print(bits_to_str(p))
```

```
[[0 0 0 1 0 0]
 [0 0 0 1 0 0]
 [0 0 0 0 0 0]
 [0 0 1 0 1 0]
 [0 0 1 1 0 0]
 [0 1 0 0 0 0]
 [1 0 0 0 0 0]
 [0 0 0 0 0 1]
 [0 0 1 1 0 0]
 [0 0 0 1 0 0]
 [0 0 1 0 1 0]
 [0 0 1 0 0 1]
 [0 0 0 0 1 0]
 [0 0 1 0 0 0]
 [0 0 1 1 1 0]
 [0 0 0 0 0 0]
 [0 0 1 1 0 0]
 [1 0 1 0 0 0]
 [0 1 0 0 0 0]
 [0 0 0 0 0 0]
 [0 0 1 1 0 0]]
Potter
```

In [13]:

```
# β1 = a1·u
# γ1 = b1·u

# Function takes in a_i, b_i and  β and γ
def secret_bit(a, b, u):
    beta = [i%2 for i in np.dot(a, u)]
    gamma = [j%2 for j in np.dot(b, u)]
    return [beta, gamma]

ab = generate_ab()
a, b = ab[0], ab[1]
beta_gamma = secret_bit(a, b, vector_u)
beta_i, gamma_i = beta_gamma[0], beta_gamma[1]
print(f"β_i = {beta_i}")
print(f"γ_i = {gamma_i}")
```

```
β_i = [1, 1, 0, 0]
γ_i = [1, 0, 0, 0]
```

In [14]:
```
# Function to generate all the 4 combinations
def comb(a, b, beta, gamma):
    # Generates a list of combinations
    arr = [0, 1, 2, 3]
    # x = [(0, 1, 2), (0, 1, 3), (0, 2, 3), (1, 2, 3)]
    x = list(combinations(arr, 3))
    combination = []
    for i in range(4):
        n = x[i]
        combi = []
        secret =  []
        for j in range(3):
            combi.append(a[n[j]])
            combi.append(b[n[j]])
            secret.append(beta[n[j]])
            secret.append(gamma[n[j]])
        combination.append([combi, secret])
    return combination

c = comb(a, b, beta_i, gamma_i)
u_j = []
for j in range(4):
    u_j.append([int(n) for n in linalg.solve(c[j][0], c[j][1])])
```

In [15]:
```
s_j = np.dot(u_j, a_0)
t_j = np.dot(u_j, b_0)
p_raw = []
for i in range(len(s_i)):
    p_raw.append(s_i[i])
    p_raw.append(t_i[i])

# Return check if the resultant bits in p_raw equates to the string
print(f"raw password bits = {p_raw}")
print()
print(bits_to_str(p_raw))
```

```
raw password bits = [1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 0, 1,
0, 0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0]

Potter
```