$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}}_{x} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}}_{b}$$

CX1104: **Linear Algebra for Computing**

Chap. No : **6.1.1**

Lecture : **Orthogonality**

Topic : **Dot Product**

Concept : **Review of Vectors**

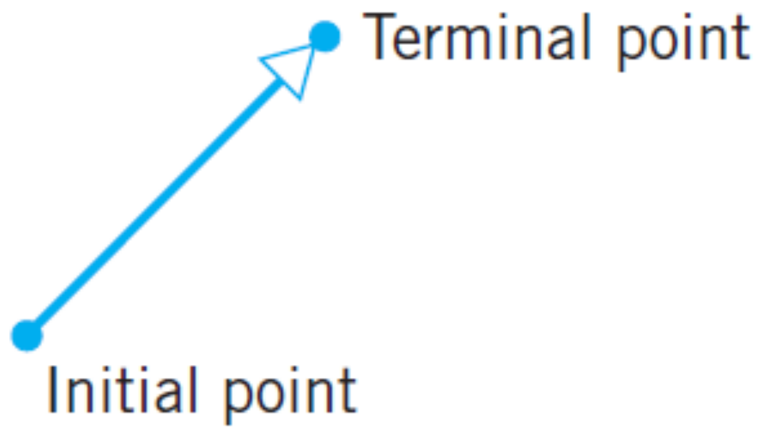
Instructor: **A/P Chng Eng Siong**

TAs: **Zhang Su, Vishal Choudhari**

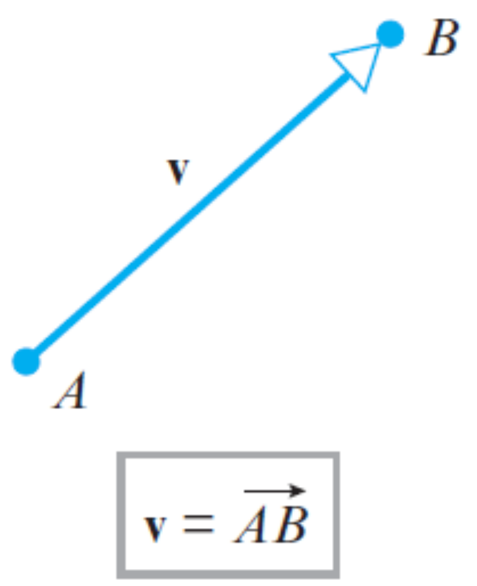
Rev: 25th June 2020

Geometric Vectors

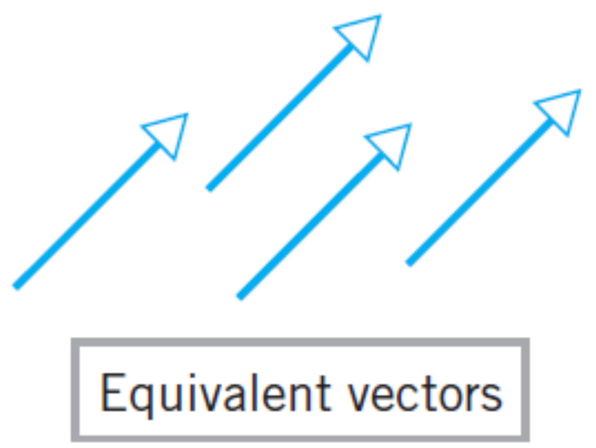
Geometric Vectors



▲ Figure 3.1.1



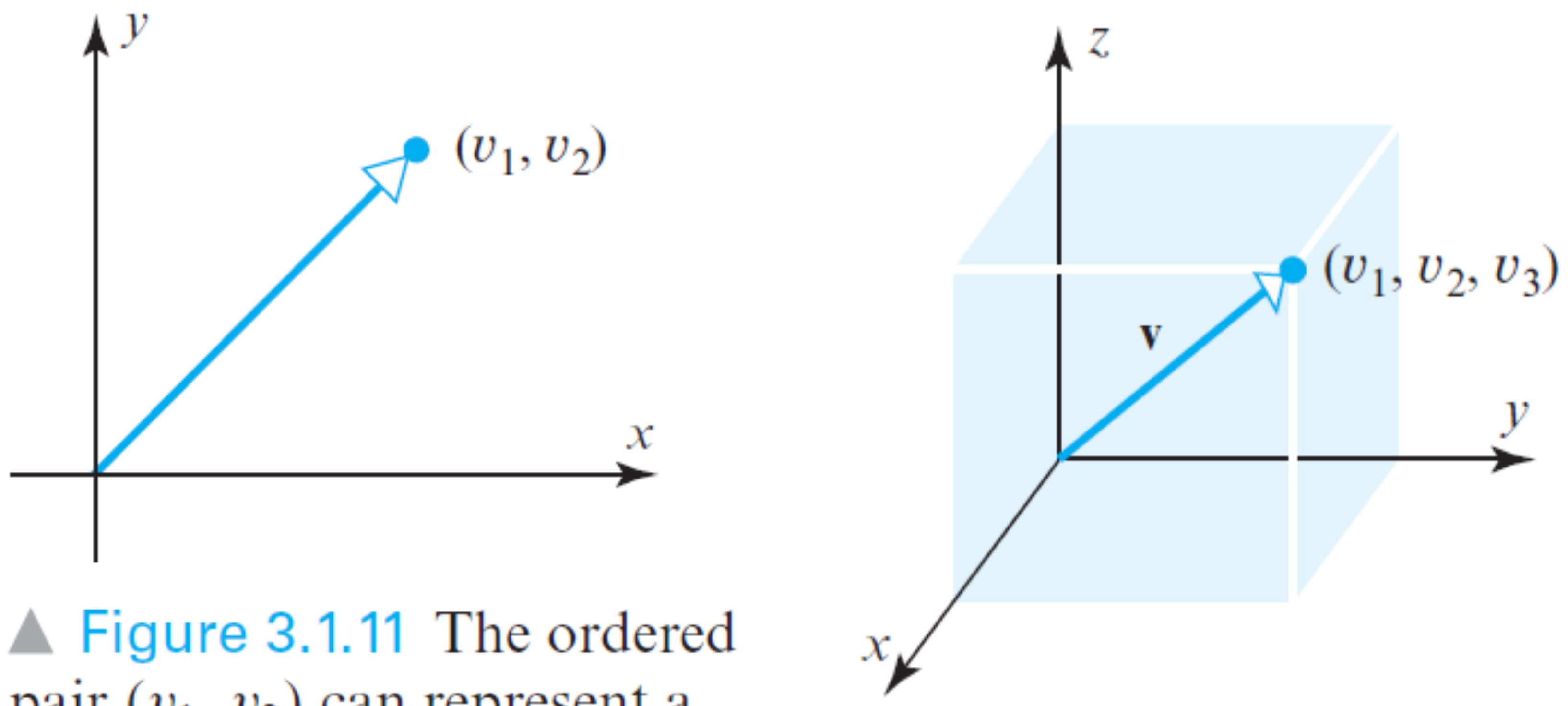
▲ Figure 3.1.2



▲ Figure 3.1.3

Vectors with the same length and direction, such as those in Figure 3.1.3, are said to be ***equivalent***. Since we want a vector to be determined solely by its length and direction, equivalent vectors are regarded as the same vector even though they may be in different positions. Equivalent vectors are also said to be ***equal***, which we indicate by writing

$$\mathbf{v} = \mathbf{w}$$



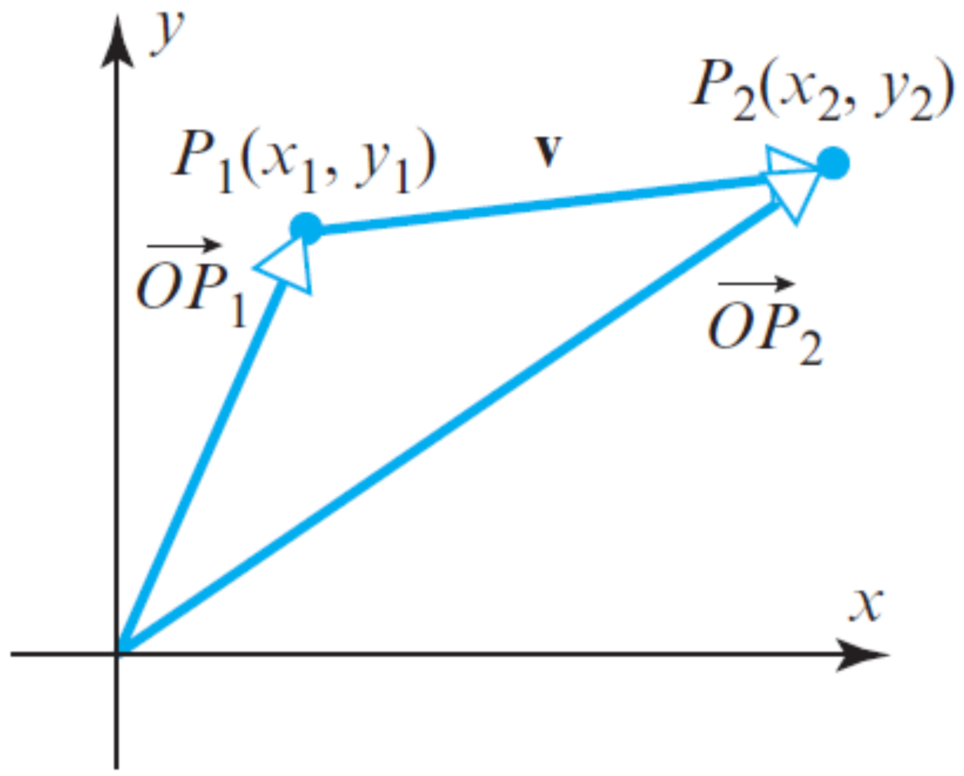
▲ Figure 3.1.11 The ordered pair (v_1, v_2) can represent a point or a vector.

If a vector $\mathbf{v}$ in 2-space or 3-space is positioned with its initial point at the origin of a rectangular coordinate system, then the vector is completely determined by the coordinates of its terminal point (Figure 3.1.11). We call these coordinates the ***components*** of $\mathbf{v}$ relative to the coordinate system. We will write $\mathbf{v} = (v_1, v_2)$ to denote a vector $\mathbf{v}$ in 2-space with components (v_1, v_2), and $\mathbf{v} = (v_1, v_2, v_3)$ to denote a vector $\mathbf{v}$ in 3-space with components (v_1, v_2, v_3).

Remark It may have occurred to you that an ordered pair (v_1, v_2) can represent either a vector with *components* v_1 and v_2 or a point with *coordinates* v_1 and v_2 (and similarly for ordered triples). Both are valid geometric interpretations, so the appropriate choice will depend on the geometric viewpoint that we want to emphasize (Figure 3.1.11).

Geometric Vectors

Vectors Whose Initial Point Is Not at the Origin



$$\boxed{\mathbf{v} = \overrightarrow{P_1P_2} = \overrightarrow{OP_2} - \overrightarrow{OP_1}}$$

▲ Figure 3.1.12

It is sometimes necessary to consider vectors whose initial points are not at the origin. If $\overrightarrow{P_1P_2}$ denotes the vector with initial point $P_1(x_1, y_1)$ and terminal point $P_2(x_2, y_2)$, then the components of this vector are given by the formula

$$\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1) \tag{4}$$

As you might expect, the components of a vector in 3-space that has initial point $P_1(x_1, y_1, z_1)$ and terminal point $P_2(x_2, y_2, z_2)$ are given by

$$\overrightarrow{P_1P_2} = (x_2 - x_1, y_2 - y_1, z_2 - z_1) \tag{5}$$

▶ **EXAMPLE 1 Finding the Components of a Vector**

The components of the vector $\mathbf{v} = \overrightarrow{P_1P_2}$ with initial point $P_1(2, -1, 4)$ and terminal point $P_2(7, 5, -8)$ are

$$\mathbf{v} = (7 - 2, 5 - (-1), (-8) - 4) = (5, 6, -12) \blacktriangleleft$$

DEFINITION 1 If n is a positive integer, then an ***ordered n-tuple*** is a sequence of n real numbers $(v_1, v_2, \ldots, v_n)$. The set of all ordered n-tuples is called ***n-space*** and is denoted by R^n.

Remark You can think of the numbers in an n-tuple $(v_1, v_2, \ldots, v_n)$ as either the coordinates of a *generalized point* or the components of a *generalized vector*, depending on the geometric image you want to bring to mind—the choice makes no difference mathematically, since it is the algebraic properties of n-tuples that are of concern.

Differences between a set and a tuple:

Set: An **unordered** collection of **distinct** objects.
Tuple: An **ordered** collection of objects.

1. A tuple may contain multiple instances of the same element, so tuple $(1, 2, 2, 3) \neq (1, 2, 3)$; but set $\{1, 2, 2, 3\} = \{1, 2, 3\}$.
2. Tuple elements are ordered: tuple $(1, 2, 3) \neq (3, 2, 1)$, but set $\{1, 2, 3\} = \{3, 2, 1\}$.

Ref: https://qr.ae/pNKbD4

Vectors in an Euclidean Space

Euclidean space

A. The basic vector space

We shall denote by $\mathbb{R}$ the field of real numbers. Then we shall use the Cartesian product $\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \ldots \times \mathbb{R}$ of ordered n-tuples of real numbers (n factors). Typical notation for $x \in \mathbb{R}^n$ will be

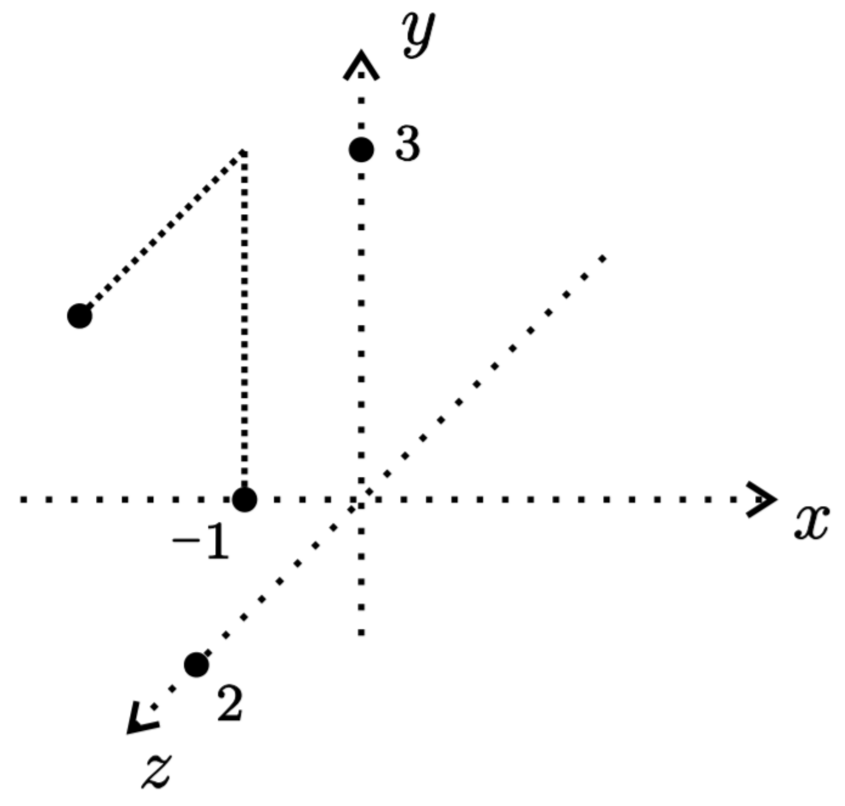
$$x = (x_1, x_2, \ldots, x_n).$$

Here x is called a *point* or a *vector*, and $x_1, x_2, \ldots, x_n$ are called the *coordinates* of x. The natural number n is called the *dimension* of the space. Often when speaking about $\mathbb{R}^n$ and its vectors, real numbers are called *scalars*.

Special notations:

$\mathbb{R}^1$	x
$\mathbb{R}^2$	$x = (x_1, x_2)$ or $p = (x, y)$
$\mathbb{R}^3$	$x = (x_1, x_2, x_3)$ or $p = (x, y, z)$.

We like to draw pictures when $n = 1, 2, 3$; e.g. the point $(-1, 3, 2)$ might be depicted as



Ref: http://www.owlnet.rice.edu/~fjones/chap1.pdf

Different ways to represent a vector in R^n

Up to now we have been writing vectors in R^n using the notation

$$\mathbf{v} = (v_1, v_2, \ldots, v_n) \tag{15}$$

We call this the ***comma-delimited*** form. However, since a vector in R^n is just a list of its n components in a specific order, any notation that displays those components in the correct order is a valid way of representing the vector. For example, the vector in (15) can be written as

$$\mathbf{v} = [v_1 \quad v_2 \quad \cdots \quad v_n] \tag{16}$$

which is called ***row-vector*** form, or as

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \tag{17}$$

which is called ***column-vector*** form.

Ref: Elementary Linear Algebra (11th edition),
Howard Anton, Chris Rorres (pg. 139), Wiley 2019

Vector Spaces

In this course, we deal with R^n, C^n

R^n: Real coordinate space of dimension n
R^n: Also, set of all $n-$ tuples of real numbers

C^n: Complex coordinate space of dimension n
C^n: Also, set of all $n-$ tuples of complex numbers

Examples

Some examples of vector spaces are

$$\mathbb{R}^2 = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| a, b \in \mathbb{R} \right\} \quad \text{or} \quad \{(a,b) \mid a,b \in \mathbb{R}\}$$

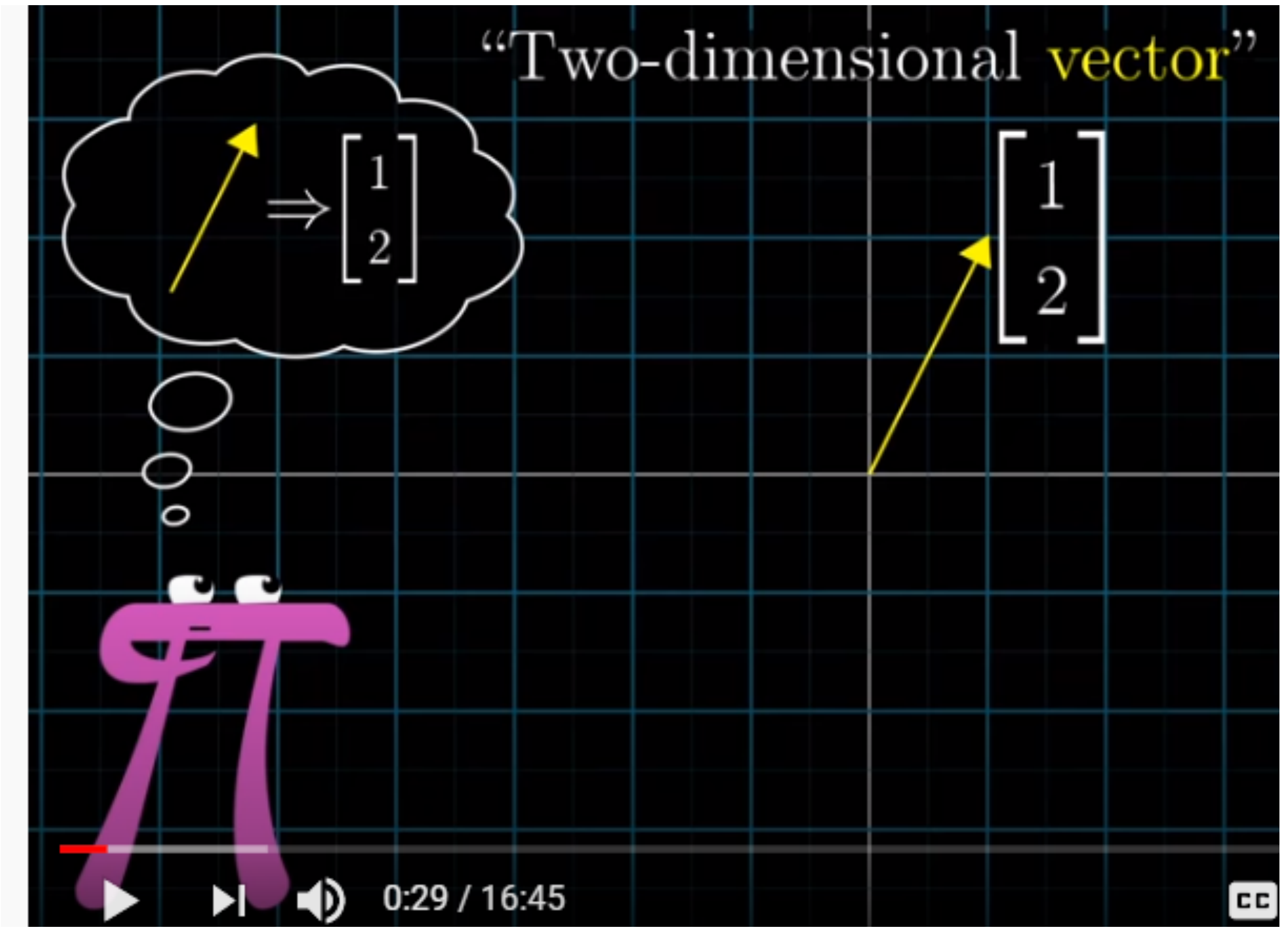
$$\mathbb{R}^3 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \middle| a,b,c \in \mathbb{R} \right\} \quad \text{or} \quad \{(a,b,c) \mid a,b,c \in \mathbb{R}\}$$

$$\mathbb{R}^n = \left\{ \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \middle| x_1,,,,,x_n \in \mathbb{R} \right\} \quad \text{or} \quad \{(x_1,\ldots,x_n) \mid x_1,\ldots,x_n \in \mathbb{R}\}$$

$$\mathbb{C}^n = \left\{ \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \middle| x_1,,,,,x_n \in \mathbb{C} \right\} \quad \text{or} \quad \{(x_1,\ldots,x_n) \mid x_1,\ldots,x_n \in \mathbb{C}\}$$

Ref: Elementary Linear Algebra by James R. Kirkwood, Bessie H. Kirkwood

3Blue1Brown: "What are Vectors?"



Ref: https://www.youtube.com/watch?v=TgKwz5Ikpc8

Ref: https://www.youtube.com/watch?v=ozwodzD5bJM