LABORATORY MANUAL

CX1104: Linear Algebra for Computing

SESSION 2020/2021
SEMESTER 1
COMPUTER ENGINEERING COURSE

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY

LAB – Chapter 7

LINEAR ALGEBRA FOR COMPUTING USING PYTHON

1. OBJECTIVE

The objective of this laboratory is to:

a) Solve least-squares problems using pseudo inverse and normal equations, verify orthogonality of residual error, optimality of least squares solution, and evaluate rank of matrix.
b) Find unknown matrix values given a complex algebraic relationship.

2. Expectations of students for the Laboratory sessions.

The following rules and regulations apply for all the laboratories pertaining to CX1104.

You are expected to have completed all the exercises of the laboratory before attending the laboratory. The purpose of the laboratory is to conduct a laboratory quiz.

If you have questions on how to develop the code, check with the TA (Teaching Assistant) before the laboratory class. If you have not complete the laboratory exercises before coming to lab, you are unlikely to finish the quiz questions.

3. Before you come to the Laboratory Session:

1. Ensure that your python environment and jupyter notebook are working fine.
2. Have pandas and dill already installed in your python environment. You can do this by running "pip install dill pandas". Dill is an extension library of pickle which helps us serialize our variables. Pandas is a popular tool for loading files. We will require these libraries to facilitate the lab. Ensure that your pandas version is >=1.1.1 and dill version >= 0.3.2
3. Familiarize yourself with python basics, especially dictionaries, and functions.

During the Laboratory Quiz:

1. At the beginning of the laboratory lesson, you will be provided with **a jupyter notebook** and a **.db file**. Place the .db file in the same directory as the jupyer notebook.
2. It is an open book laboratory quiz, i.e., you can use your notes, and the internet to help answer the questions. You are however **not allowed** to receive help from any other person. The questions will be related to the given tasks of the lab as well as the lectures you have attended.
3. Once you complete the lab quiz, you are required to **submit both the dill file and the notebook**. The code for submission will be provided.
4. If you are absent for the laboratory quiz with valid medical reason, the quiz mark for the absent quiz will be the average of your other quiz marks. Without a valid reason, you will receive 0 marks. Please submit valid MC and exemptions to be away by emailing them to the TA for the laboratory and cc the laboratory technician in charge.

4. Tasks

Q4A) You are give a matrix A (m by n) and a vector b (n by 1). Find x such that Ax = b, or if it does not exist, find the least squares solution. Answer the following questions:

a) Solve for x using pseudo inverse, save x as x1a vector.

b) Solve for x using normal equation, save x as x1b vector.

c) Are the solutions of pseudo inverse and normal equation same? Why?

d) Find $\hat{b}$ (hat_b) using pseudo inverse vs normal_equation solution:

save as hat_b1a = A*x1a (pseudo inverse), hat_b1b = A*x1b (normal equation)
Comment if their results are the same?

e) What is the norm of error to approximate b using the two approaches? Is x1a and x1b different? Why? Which solution is better if different?

f) Is b in the column space of A? Use at least 2 different ways to show this?

g) Is the error vector (produced by normal eq) orthogonal to column space of A?

How can you verify this using your code?

h) Can we use the normal equation to solve for x in this example? Justify your answer? When can we not use the normal equation to solve Least Squares?

i) Construct Q with orthogonal basis spanning the column space of A
using QR (reduced form) decomposition.

i.a) how can you show that columns of Q is in the column space of A?

i.b) solve Qx = b, what is the error of approximating b? is it the same as Question (e)? Why?

j) Evaluate N point (e.g, 10) around the least squares solution (normal equation) when it exist

to show that it gives the least error squares in approximating b. Specifically, new_x = x + delta_x. Additionally make delta_x has norm(1) generated by numpy.random.rand

Q4B) Basic Matrix Operation

Given

$$(AX - A)^{-1} = X^{-1}BC^{-1}$$

a. Manually solve for X. Clearly write the derivation and assumption.
b. Calculate X by first randomly initiating three square matrices A, B, C in R^{3x3} (using **numpy.random.rand**) , and then substituting into the equation you obtained to verify your working is correct.

5. References

[1] https://www.anaconda.com/download/

[2] "A Crash Course in Python for Scientist" http://nbviewer.jupyter.org/gist/rpmuller/5920182

[3] “Scientific Computing with Python”,
http://nbviewer.jupyter.org/url/atwallab.cshl.edu/teaching/QBbootcamp3.ipynb

[4] “I used Matlab. Now I use Python”, Steve Tjoa, Sep 2010. https://stevetjoa.com/305/

[5] http://phillipmfeldman.org/Python/Advantages_of_Python_Over_Matlab.html

[6] “Matlab vs Python: Top reasons to choose Matlab”
https://www.mathworks.com/products/matlab/matlab-vs-python.html

[7] http://www.pyzo.org/python_vs_matlab.html