$$
\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}_{m\times n}}_{A}
\underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}}_{x}
=
\underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m\times 1}}_{b}
$$

CX1104: **Linear Algebra for Computing**

Chap. No : **7.2.1**

Lecture : **Least Squares**

Topic : **Reviewing Basic Matrix Algebra**

Concept : **Binary Matrix Operations**

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Binary Matrix Operations

Consider two matrices:

$$A, B \in R^{M \times N}$$

The matrices have:

- M rows
- N columns

Product with a scalar [edit]

If $\mathbf{A}$ is a matrix and c a scalar, then the matrices $c\mathbf{A}$ and $\mathbf{A}c$ are obtained by left or right multiplying all entries of $\mathbf{A}$ by c. If the scalars have the commutative property, then $c\mathbf{A} = \mathbf{A}c$.

If the product $\mathbf{AB}$ is defined (that is the number of columns of $\mathbf{A}$ equals the number of rows of $\mathbf{B}$, then

$$c(\mathbf{AB}) = (c\mathbf{A})\mathbf{B} \text{ and } (\mathbf{AB})c = \mathbf{A}(\mathbf{B}c).$$

If the scalars have the commutative property, then all four matrices are equal. More generally, all four are equal if c belongs to the center of a ring containing the entries of the matrices, because in this case $c\mathbf{X} = \mathbf{X}c$ for all matrices $\mathbf{X}$.

Ref: https://see.stanford.edu/materials/lsoeldsee263/Additional1-notes-matrix-primer.pdf

What are some of the rules of binary matrix operations?

Commutative law of addition

If $[A]$ and $[B]$ are $m \times n$ matrices, then

$$[A]+[B]=[B]+[A]$$

Associative law of addition

If [A], [B] and [C] are all $m \times n$ matrices, then

$$[A]+([B]+[C])=([A]+[B])+[C]$$

Associative law of multiplication

If $[A]$, $[B]$ and $[C]$ are $m \times n, n \times p$ and $p \times r$ size matrices, respectively, then

$$[A]([B][C])=([A][B])[C]$$

and the resulting matrix size on both sides of the equation is $m \times r$.

Distributive law

If $[A]$ and $[B]$ are $m \times n$ size matrices, and $[C]$ and $[D]$ are $n \times p$ size matrices

$$[A]([C]+[D])=[A][C]+[A][D]$$
$$([A]+[B])[C]=[A][C]+[B][C]$$

and the resulting matrix size on both sides of the equation is $m \times p$.

In general, $AB \neq BA$ even when the sizes allows for the operation.