A Review Survey of Optimisation Models for the Unit Commitment Problem

Vincent Kipkemoi Langat (vincentl@aims.ac.za)
African Institute for Mathematical Sciences (AIMS)

Supervised by: Dr Franklin Djeumou Fomeni
University of Quebec at Montreal,
Co-supervised by: Prof Montaz Ali
University of the Witwatersrand at Johannesburg

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Abstract

In every country in the world, the power industry plays a critical role as electricity is essential not only to our everyday life but also acts as source of power to other sectors that are pillars to the economy. An optimal scheduling of power plants gives a better chance for power companies to save on money which can then be channelled to other key sectors that accelerates the economy. The Unit Commitment (UC) problem is a class of optimisation problem which aims at providing decisions for the short-term scheduling of electrical power plants (e.g. coal, gas, hydro or nuclear) based on generation costs and operational decisions. Power grids have to solve the UC problems every day in order to minimize their operational costs as well as to determine the optimal schedule of generating units needed to satisfy the power demand. The problem amounts to determining an optimal supply portfolio (mix of power plants) based on functions representing economic decisions and energy generation physics.

This problem has received a lot of attention over the past decades from researchers in fields including management science, operations research, industrial and electrical engineering. Various optimisation models for the UC problems have been proposed in the literature. The most interesting models are best formulated as Mixed Integer Linear Programs (MILPs). In this essay, we focused on reviewing and synthesizing some of the mathematical models and formulations for the UC problem. We also present a snapshot of some of the exact and meta-heuristic solution schemes that have been extensively used in solving the UC problems. Eventually, numerical simulations obtained through implementation of MILP is presented. The available Sage package for MILP has made it easier to implement both small scale and large scale UC problems. It has also improved on its adaptability since it is easy to add a new coupling constraint. When a classical example of UC problem is implemented, an optimal solution is obtained within a reasonably small computational time. The results also suggest that when the number of units in the system is fixed, the UC problem is solvable in a polynomial time.

Declaration

I, the undersigned, hereby declare that the work contained in this research project is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

Langat Kipkemoi Vincent, 24 May 2018

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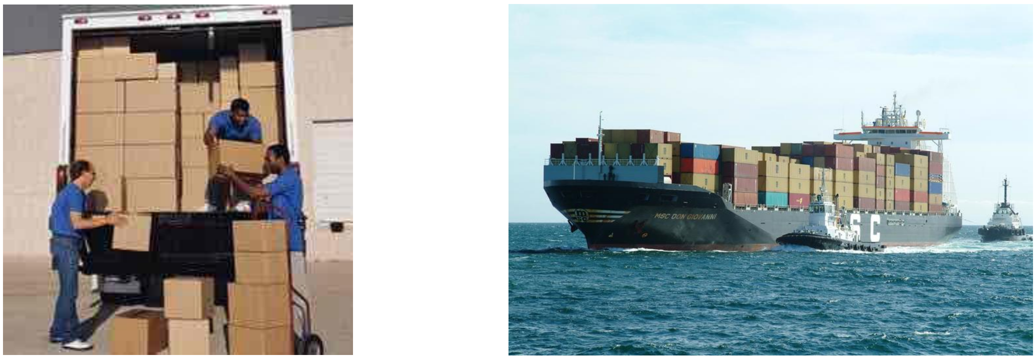
1. Introduction

1.1 Introduction to Optimisation

Optimisation as a branch of operation research has a wide scope of application not only in science but also in engineering and management science. Depending on the performance indicator of the system, optimisation is seen as either maximisation or minimisation of profit, cost, return or even time. It is a crucial tool in analysing the performance of the physical systems and industrial processes. For example, a mining industry may wish to determine the best strategy of extracting the stopes from the ore-body so as to maximize the net present value derived out of it. A power generation company may wish to determine how to schedule their generators and the output of each generator at each time period so as to minimize their total operational costs. Optimisation also has application in shipping industry.



(a) Cargo loading

(b) A ship loaded with containers

Figure 1.1: Cargo loading and loading of ship in shipping industry.

A shipping company may wish to determine the optimal strategy of not only loading their cargo into the containers but also loading the packed containers into the ship so as to maximize the total value of the loaded items. Figure 1.1 illustrates cargo loading and container loading in shipping industry. This therefore suggest that optimisation at large has enormous applications in many industries and diverse sectors of the economy.

An initial stage in optimisation is setting a performance indicator called objective function. The objective function is associated with decision variables. In optimisation, the goal is therefore to determine the values of the decision variables whose overall contribution results in an optimal value of the objective function. It is also worth noting that most of the time the values of these decision variables are restricted by some constraints. Therefore, in formulation of the optimisation problem, we define both the objective function and the constraints. The decision variables may assume continuous values or discrete values giving rise to two categories of optimisation: the continuous optimisation and the discrete optimisation. Discrete optimisation is sometimes referred to as integer programming based on the fact that integer values are discrete values. An example of integer programming problem is the knapsack optimization problem which involves determining the optimal strategy of packing the items into a knapsack such that the weight of the items packed does not exceed the weight the knapsack can carry.

Some class of optimisation problems assume a mixture of both continuous and discrete values. They are termed as mixed integer programming problems. An example is a problem in power generation industry called Unit Commitment (UC) problem. In the UC problem, some variables which denotes the power output of each generator at each time period assumes continuous values while the other variables that denotes on and off status, start-up status and shut-down status of the generators assumes only binary values. Another class of optimisation is the combinatorial optimisation. It is a special type of discrete optimisation which involves an exhaustive search for optimal solution over a finite set of object. In combinatorial optimisation, the whole set of feasible configurations is generated and the configuration that gives the optimal solution is considered.

Optimisation problems may also be classified based on the nature of the objective function and the constraints. If the objective function is a linear function and the constraints are linear equality or linear inequality, then problem is classified as linear programming problem. On the other hand, some problems have a non-linear objective function or non-linear constraints or both. They are termed as non-linear programming problems. Based on the convexity nature of the objective function and the constraints, optimisation problems may be further classified into convex and non-convex optimisation problems. An optimisation problem is classified as convex optimisation if the objective function is concave or convex for maximization and minimization respectively, and the constraints are the convex functions. On the other hand, an optimisation problem is a non-convex if the constraints and/or objective function are non-convex.

Now, after discussing a broader view of optimisation, we will narrow down to the UC optimisation problem in the next section.

1.2 The Unit Commitment Problem

In every country in the world, power industry plays a critical role as electricity is essential not only to our everyday life but also acts as source of power to other sectors that are pillars to economy. An optimal scheduling of power plants therefore gives a better chance for power companies to save on money which can then be channelled to other key sectors that accelerate economy (Roque, 2014) . It also acts as a planning process that ensures that the projected demand is met at lowest cost while ensuring reliability in electricity supply. In the past, most electricity generating companies used to enjoy the monopoly power and controlled wider power market. With tremendous increase in population and more production industries sprouting around the world, the demand for electricity has also increased. This has seen a situation where electricity market has become liberalized and deregulated. The result is that power industries have been compelled to adopt a more robust strategy to generate electricity that will meet the prevailing market demand not only at favourable prices but also ensuring reliability in the supply. Attention has therefore shifted to scheduling of units in the power generation so as to meet the forecasted demand at a minimum cost while satisfying the technical constraints. This is termed as UC problem.

The UC problem is a scheduling but combinatorial optimization problem that involves determining the activation levels and optimal schedule for generating units at each time period (usually 1 hour) over a large time horizon so as to meet the demand at a minimum cost (Delarue and D'haeseleer, 2006). Figure 1.2 shows a power generating unit.

Figure 1.2: A thermal power generating unit

The UC problem has been viewed as a two-stage optimisation problem. The first stage involves determining the optimal scheduling plan for each unit at each time period over the whole time horizon. After the units have been committed for some time, the second stage involves determining the output levels of each unit and optimal way of allocating the generated power to meet the forecasted demand. This stage is termed as economic dispatch optimisation. A challenge to power generation and distribution is the fact that the demand for electricity keeps fluctuating. The demand during the day is generally higher than that of the night. The same case applies to the demand during the weekday as compared to that of the weekend. Due to this stochastic nature of the demand, the power plants are therefore compelled to produce enough power to not only meet the prevailing demand but also meet the spanning reserve requirement. Spanning reserve requirement is introduced to caution the system against failure at an instant where where the demand exceeds the power supply and also helps to mitigate the uncertainty in the system.

In formulation of UC optimisation problem, both the objective function and the constraints are defined by both continuous variables and binary variables (status variables). Due to this fact, the UC problem has been formulated as mixed integer programming that allows incorporation of both continuous and binary variables. The problem is large in dimension and coupled with both demand and technical constraints such as spanning reserve requirement, ramp up and ramp down constraints, minimum and maximum uptime constraints. Essentially, the problem involves minimization of operating costs subject to demand and technical constraints. The standard operating costs includes power generation costs, start up costs and shut down costs. In literature, researchers have found that the power generational cost function is quadratic in nature (Hemmati and Saboori, 2016) . This therefore creates non-linearity in the objective function and hence adding on complication to the solution to the problem.

The fact that the problem is large in dimension has made it to be classified as an NP-hard problem (Bendotti et al., 2017). Due to its size, non-linearity of the objective function, NP-hardness and mixed integer features, the problem is computationally challenging to solve.

1.3 Structure of the Essay

The main objective of this essay is to review optimisation models for UC problem. It is also worth to have a holistic review of UC problem, its relevance in power generation and some of the solution method that have been applied in solving the problem. Therefore Chapter 2 is devoted to literature review and will be solely an overview of power generation, UC problem as a whole and solution schemes that have been extensively used in solving UC problems. In Chapter 3, we will review the mathematical formulation for UC problem as described in (Ostrowski et al., 2012), (Hemmati and Saboori, 2016) and (Delarue and D'haeseleer, 2006). We will also do a synthesis of the three models to check on their similarities and their differences. In Chapter 4, we will start by looking at an overview of solution methods that have been developed to solve UC problem. Then we will present and discuss the numerical results obtained through implementation of MILP using the available Sage package for MILP. Chapter 5 is devoted for conclusion and therefore we will give a brief overview of what we discussed.

2. Literature Review

The UC problem has been in existence since the time when the power producing companies used to enjoy monopolistic power in production and distribution of electricity (Ostrowski et al., 2012). Power industry used to be vertically integrated and all the activities involving generation, transmission and distribution of power were carried out by one company. In the recent past, most of the power industries have been restructured and segmented into generation companies, transmission companies and distribution companies (Roque, 2014). With many countries growing and becoming more productive, the usage of electricity has been rising. This has necessitated a mechanism to develop a more economic, reliable and secure power supply (Roque, 2014). Environmental impacts and the question of sustainability has also been a subject of discussion and most of the power generation industries are turning to a more clean renewable sources of energy such as wind and solar radiation. The drawback with these sources is that they are intermittent in nature and therefore posing a lot of uncertainty in power generation.

Though the use of fuels such as coal in power generation is less costly, their environmental impacts are immense. They contribute to emission of greenhouse gases such as carbon dioxide that have an impact in global warming. Figure 2.1 illustrates gas emission in power industry.

Figure 2.1: Gas emission in power generation.

Environmental policies have been put in place to ensure that companies including power companies emit these environmentally unfriendly gases only within an allowable limit set out by environmental agencies such as United Nation Environmental Programme (UNEP). The policies provides a framework for taxing the companies that fails to adhere to the limit requirement. Taxes on emission of pollutants increases the total operational costs of any power generating company which largely depend on fuel as a source of power. A trade-off between cost of power generation and the level of pollutant emissions has therefore been a subject of discussion. Roque (2014) addresses a bi-objective UC model that is able to capture minimization of cost of running the system and the costs that may arise due to taxation on pollutants. Some policies have also been put in place to create an internal market for pollution allowances. Companies can now buy and sell their pollution allowances among themselves.

With the electricity market becoming liberalized, power generation companies have considered the UC optimisation as a yardstick to reducing their total operational costs and thus positioning themselves well in the market. The UC problem is classified as an NP-hard problem (Bendotti et al., 2017) and this has therefore posed a difficulty in trying to come up with a more adaptive and comprehensive solution scheme that will guarantee optimal solution within a reasonable computational time. Many of the solution methods such as Lagrangian Relaxation (LR) that have been extensively used in the past considered meta-heuristic and do not guarantee optimal solution. Nonetheless, these methods have been used because they can generate a near optimal solution within a reasonable computational time. The dimensionality, non-linearity in the objective function, combinatorial nature and the mixed integer feature has become a big impediment in trying to come up with a more robust mathematical models and solution schemes for the UC problem. Several mathematical models have addressed a corrective measure to remove non-linearity in the objective function. In Hemmati and Saboori (2016) a non-linear power generational cost function is transformed into linear approximation using a sequence of piece-wise linear functions. This transformation has seen a UC problem formulated as MILP problem which can easily be solved by available commercial solvers such as CPLEX. The MILP has been widely adopted is due to the fact that it is appropriate to use in practically large networks, easily adaptable and guarantee convergence of optimal solution within a finite amount of time (Hemmati and Saboori, 2016). Cutting plane and branch and bound algorithms have been widely used in solving the MILP problems (Salam, 2007). Though branch and bound is considered as exact algorithm that guarantees optimal solution, it is only limited to a small scale UC problem (Park et al., 2014).

Other solution methods such as Priority listing, Unit decommitment and dynamic programming have also been applied in solving the UC problem (Delarue and D'haeseleer, 2006). Moreover, many meta-heuristic algorithms such as simulated annealing, genetic algorithms, tabu search and greedy randomized adaptive search procedure have also been used (Salam, 2007). Dynamic programming (DP) and Lagrangian relaxation (LR) have been extensively used due to the fact that they require a reasonable computational time compared to other solution schemes (Salam, 2007). Furthermore dynamic programming is considered exact algorithm and thus guarantees convergence in optimal solution. The only drawback to dynamic programming is that it is iterative in nature and at each iteration, a solution deemed sub-optimal is stored hence taking a lot of memory. Singhal and Sharma (2011) describes three dynamic approaches for solving UC problem: conventional dynamic programming, sequential dynamic programming and truncation dynamic programming. Though the Lagrangian Relaxation method have a relatively small computational time, it is considered to be meta-heuristic and therefore does not guarantee convergence of optimal solution within a finite amount of time (Singhal and Sharma, 2011).

The demand for electricity keeps fluctuating at different hours of the day and at different days of the week. This has necessitated adaptation in power generation to allow storage of electricity when the demand is lower than the supply and the stored electricity discharged in times of excess demand. Several devices capable of storing electricity during charging period has been adopted. Hemmati and Saboori (2016) described a UC model that incorporates Energy Storage System (ESS) in power generation system. ESS not only increases the reliability in power generation but also make the system more economical by reducing the system operational costs through load levelling. Load levelling is a process that involves smoothing the load pattern by decreasing the on-peak and increasing the off-peak (Hemmati and Saboori, 2016). Another form of energy storage referred to as a pumping unit is described in Delarue and D'haeseleer (2006). A pumping unit is capable of storing electricity when the demand is lower than the supply and the stored electricity is used when the demand exceeds the supply. An increase in adoption of renewable sources of energy such as solar radiation and wind power has posed uncertainty in power generation. An energy storage system has therefore been seen as a measure to prevent the power systems against instability caused by intermittent renewable sources. Several models that addresses uncertainty caused by fluctuation of electricity demand and the integration of wind power into the power generation have also been addressed. Park et al. (2014) considers probabilistic and stochastic UC model capable of handling the uncertainty caused by wind power. Wind power as been found to closely follow Weibull distribution and hence adoption of probabilistic UC problem. The paper also considers a stochastic UC model where the autoregressive moving average method (ARMA) is used to simulate a predication of wind power at different times of the day based on a pre-determined standard deviation.

3. Three Mathematical Models for Unit Commitment

The UC problem consists of a set of generating units each with different operational costs as defined by their source of power such as coal, wind, thermal, hydro electric power or even solar radiation. The problem involves making two decisions: (i) which unit to be online or offline at a given time period, and (ii) what is the level of output of all the online units that will ensure that the electricity demand is met at a minimum cost. Usually, a time period is considered to be an interval of 1 hour and if a schedule is to be done for one week for example, then this translates to 168 time periods. The level of output of each unit at a given time is restricted by some technical constraints such as generational limits, spanning reserve, ramp up, ramp down, minimum uptime and minimum downtime constraints (Ostrowski et al., 2012). In this chapter, we will discuss three different formulations for the UC problem.

3.1 Notations

The following notations for variables and parameters will be adopted in the rest of this document.

- N: Total number of units.
- T: Total number of time periods.
- PG_n^C: Power generation cost of unit n per unit of output.
- $PG_{n,t}$: Power generation output of unit n at time period t.
- SU_n^C: Start-up cost of unit n.
- SD_n^C: Shut-down cost of un it n.
- $U_{n,t}$: A binary variable indicating whether unit n is starting up at time period t (1 if it is a start-up and 0 if not).
- $V_{n,t}$: A binary variable that indicate whether unit n is shutting down at time period n or not (1 if it is shutting down and 0 otherwise).
- Dd_t: Demand at time period t
- $W_{n,t}$: A binary variable indicating whether unit n is online or offline at time t (0 if not and 1 if it is online).
- S_t^R: Spanning reserve required at time period t.
- MAX_n^{prod}: Maximum producible power for unit n.
- PG_n^{min}: Minimum power generation limit of unit n when it is online.
- PG_n^{max}: Maximum power generation limit of unit n when it is online.
- R_n^U: Ramp-up rate of unit n.
- R_n^D: Ramp-down rate of unit n.
- SU_n^R: Start-up ramp rates of unit n.

- SD_n^R: Shut-down ramp rates of unit n.
- UT_n^{min}: Minimum uptime of unit n.
- DT_n^{min}: Minimum downtime of unit n.
- F_n: Number of periods during which unit n must remain online after start-up.
- U_n^0: Number of periods unit n has been online before the first period of the time span.
- H_n: Number of time periods unit n should be offline after shut-down.
- S_n^0: The time periods unit n has been offline before the first period of the time span.
- LS_i^C: Load shedding cost of bus i .
- $LS_{i,t}$ Load shedding of bus i at time period t.
- $I_{i,t}^P$: Power injected to the lines connected to bus i, at time period t.
- $L_{i,j,t}^P$: The flow of line j connected to bus i at time period t.
- PG_n^{cap}: Power generation capacity of unit n.
- R_t^{min}: Minimum spanning reserve requirement at time period t.
- OP_n^C: Operating cost of unit n.
- Eff: Efficiency of the pumping unit while pumping down.
- PD_t: Power produced by a pumping unit at time period t.
- PU_t: Power used while pumping water into the storage reservoir at time period t.
- X_t: A binary variable indicating if a pumping unit is pumping down or not (0 if not and 1 if pumping down).
- Y_t: A binary variable indicating if a pumping unit is pumping up water up the reservoir or not (0 is not and 1 if pumping up).
- $Pnlo$: Lower limit around the nominal power level while a pumping unit is pumping.
- $Pnup$: Upper limit around the nominal power level while a pumping unit is pumping.
- PD^{max}: Maximum power level of the turbine when pumping water from the reservoir.
- PC_0: Energy content of the pumping unit at the start of the pumping process.
- PC_t: Total energy content of the pumping unit at the end of period t.
- PC^{max}: Maximum energy that a pumping unit can store.

3.2 The Ostrowski, Anjos and Vanelli Model

In Ostrowski et al. (2012), the UC problem is modelled as a large scale non-convex problem. It is formulated as MILP problem with both continuous and binary variables. The continuous variables are used to describe the power output of each generator at each time period. The binary variables are use to indicate on/off status of generators, start up status and shut-down status. The problem formulation is as defined by the objective function in Subsection 3.2.1 and constraints in Subsection 3.2.2

3.2.1 Objective function.

The objective function is described as minimization of total operational costs. The operational costs consists of power generational costs and start-up costs. The objective function is defined as:

$$Min \sum_{t \in T} \sum_{n \in N} PG_n^C \cdot PG_{n,t} + SU_n^C \cdot U_{n,t}. \tag{3.2.1}$$

3.2.2 Constraints.

The power balance constraint

The sum of the power generated by the online units at each time period must be able to meet the forecasted demand. This is declared by the constraint:

$$\sum_{n \in N} PG_{n,t} \cdot W_{n,t} \geq Dd_t, \quad \forall t \in T. \tag{3.2.2}$$

Spanning reserve requirement

The demand for electricity keep on fluctuating from time to time during the day and during the night. Figure 3.1 shows the nature of fluctuation of power demand at different time of the planning horizon.

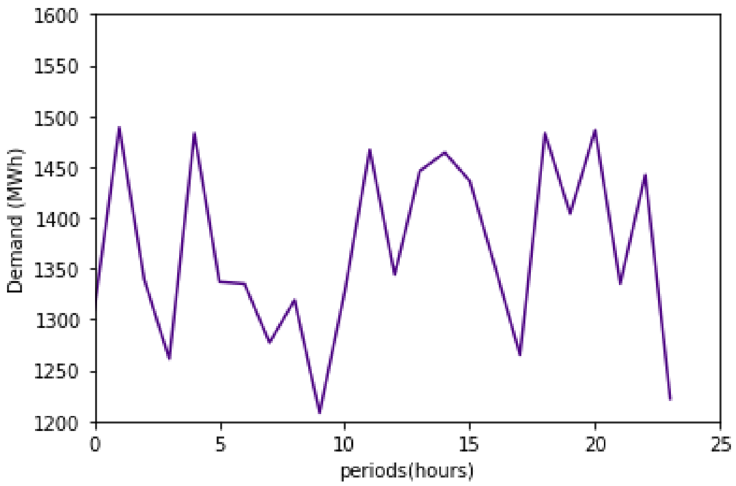


Figure 3.1: A plot to illustrate the stochastic nature of demand for 24hour scheduling period.

When the demand exceeds the cumulative power output, there is high chances that the system might become unstable and fail. The spanning reserve requirement is therefore introduce to caution the system against failure whenever the demand exceeds the supply. With this requirement, the total maximum producible power at each time period should be able to meet both the forecasted demand and the spanning reserve as shown by :

$$\sum_{n \in N} MAX_n^{prod} \cdot W_{n,t} \geq Dd_t + S_t^R, \quad \forall t \in T. \tag{3.2.3}$$

The generation limit constraint

When online, the power output of each unit at each time period is bounded below and above by the minimum and maximum power generation. This is described by:

$$PG_n^{min} \cdot W_{n,t} \leq PG_{n,t} \leq PG_n^{max} \cdot W_{n,t}, \quad \forall n \in N, \quad \forall t \in T. \tag{3.2.4}$$

Ramp-up and ramp-down constraints

The constraints restrict the fluctuation in power output between two consecutive time periods. The variables $PG_{n,t}$ are constraint by both ramp-up and ramp-down rates as well as start-up and shut-down ramp rates. This is described by constraints (3.2.5) and (3.2.6):

$$PG_{n,t} - PG_{n,t-1} \leq R_n^U \cdot W_{n,t-1} + SU_n^R \cdot U_{n,t}, \quad \forall n \in N, \quad \forall t \in T. \tag{3.2.5}$$

If the generator was online in the previous period $(t-1)$, then $W_{n,t-1} = 1$ and thus the increase in power output cannot be larger than the ramp rate. If the generator is started up at time t, then $U_{n,t} = 1$ hence the generator can produce a maximum of SU_n^R at time t.

A similar analogy applies to the ramp-down and shut-down ramp rates constraints.

$$PG_{n,t-1} - PG_{n,t} \leq R_n^D \cdot W_{n,t-1} + SD_n^R \cdot V_{n,t}, \quad \forall n \in N, \quad \forall t \in T. \tag{3.2.6}$$

Minimum uptime and downtime constraints

To avoid a frequent on and off status of each unit at a given time horizon, the minimum downtime and uptime constraints are added to the model. The minimum uptime constraints stipulates that once a given unit is started up, it has to remain in online state for a period not less than the minimum uptime. Equivalently, the minimum downtime constraint asserts that once a given unit shuts down, it must remain in an offline state for a period not less than the minimum downtime. The constraints serves to reduce the total operational costs of the system. Having frequent on and off states of the system means both the start up and shut down costs is high in the system hence increasing the operational cost.

The constraints, (3.2.7) to (3.2.10) below implements the minimum uptime:

$$\sum_{t=1}^{F_n} (1 - W_{n,t}) = 0, \quad \forall n \in N, \tag{3.2.7}$$

$$\sum_{t'=t}^{t+UT_n^{min}-1} W_{n,t'} \geq UT_n^{min} \cdot U_{n,t}, \quad \forall n \in N, \quad \forall t = F_n + 1, ..., T - UT_n^{min} + 1, \tag{3.2.8}$$

$$\sum_{t'=t}^{T} \{W_{n,t'} - U_{n,t}\} \geq 0, \quad \forall n \in N, \quad \forall t = T - UT_n^{min} + 2, ..., T, \tag{3.2.9}$$

$$F_n = \min\{T, (UT_n^{min} - U_n^0) \cdot W_{n,0}\}. \tag{3.2.10}$$

Analogous to the minimum uptime, the constraints (3.2.11) to (3.2.14) implements the minimum downtime:

$$\sum_{t=1}^{H_n} W_{n,t} = 0, \quad \forall n \in N, \tag{3.2.11}$$

$$\sum_{t'=t}^{t+DT_n^{min}-1} (1 - W_{n,t'}) \geq DT_n^{min} \cdot V_{n,t}, \quad \forall n \in N, \quad \forall t = H_n + 1, ..., T - DT_n^{min} + 1, \tag{3.2.12}$$

$$\sum_{t'=t}^{T} \{1 - W_{n,t'} - V_{n,t}\} \geq 0, \quad \forall n \in N, \quad \forall t = T - DT_n^{min} + 2, ..., T, \tag{3.2.13}$$

$$H_n = \min\{T, (DT_n^{min} - S_n^0) \cdot (1 - W_{n,0})\}. \tag{3.2.14}$$

3.3 The Hemmati and Saboori Model

In Hemmati and Saboori (2016), the UC problem is also formulated as MILP with the objective function and the constraints as follows.

3.3.1 Objective function.

The objective function is the sum of the total operational costs. As oppose to Ostrowski et al. (2012), Hemmati and Saboori (2016) defines the total operational costs to comprise of power generation costs, start-up costs, shut-down costs and load shedding costs. A key modification in the objective function is the inclusion of load shedding costs.

3.3.1.1 Load shedding

The uncertainty of the power demand has been seen as a setback to the stability of the power system. When the prevailing power demand exceeds the power generated in a given time period, there is high chances that the power system becomes unstable. This may sometimes result in a trip in the system hence causing power blackout country-wide. A situation like this arising is costly not only to the company but also to the economy. To mitigate the possibility of of this occurring, power generation companies normally agree and notify the huge power consumers to voluntarily reduce their loads so as to bring down the demand.

If after such directive the system is still unstable, the power utility company will have no option but to disconnect part of their customers to a level that the demand balances the power supply. This procedure of disconnecting part of the customers is termed as load shedding. As much as it brings stability to the system, load shedding costs increases the overall operation costs to the company. A strategy should therefore be adopted as to the manner in which the load shedding schedule be done so as to minimize the load shedding costs. A UC problem that incorporates the load shedding costs in the objective function is as shown below:

$$Min \sum_{t\in T} \sum_{n\in N} \sum_{i\in I} PG_n^C \cdot PG_{n,t} + SU_n^C \cdot U_{n,t} + SD_n^C \cdot V_{n,t} + LS_i^C \cdot LS_{i,t}. \tag{3.3.1}$$

The power generation cost of unit n, PG_n^C is described as a quadratic function of $PG_{n,t}$ as represented by equation (3.3.2) below:

$$PG_n^C = \alpha_n \times W_{n,t} + \beta_n \times PG_{n,t} + \gamma_n \times PG_{n,t}^2, \quad \forall n \in N, \quad \forall t \in T. \tag{3.3.2}$$

The parameters α_n, β_n and γ_n are the cost coefficients of the quadratic function. Now the fact that the power generation cost function is quadratic in nature creates non-linearity in the objective function. To guarantee a convergence to an optimal solution, the power generation cost function is approximated by a sequence of piecewise linear functions Hemmati and Saboori (2016).

The start-up status of unit n at time period t is set correctly by the Equation (3.3.3) below. Depending on the values of $W_{n,t}$ and $W_{n,t-1}$, the resulting $U_{n,t}$ may be positive or negative. Because the start-up status assumes two values (0 and 1), only non-negative values of $U_{n,t}$ is considered. This is implemented by (3.3.4):

$$U_{n,t} = W_{n,t} - W_{n,t-1}, \quad \forall n \in N, \quad \forall t \in T, \tag{3.3.3}$$

$$U_{n,t} \geq 0, \quad \forall n \in N, \quad \forall t \in T. \tag{3.3.4}$$

Recall that $U_{n,t}$ indicates whether unit n is a start-up or not at time period t while $W_{n,t}$ is a binary variable representing the on/off status of unit n at time period t. If for example, $W_{n,t} = 1$ and $W_{n,t-1} = 0$, then we have that $U_{n,t} = 1$ and thus implying that unit n is starting-up at time period t. On the other hand if $W_{n,t} = W_{n,t-1} = 0$ or $W_{n,t} = W_{n,t-1} = 1$, then $U_{n,t} = 0$. This implies that unit n is not a start-up at time t.

Like the start-up, the shut-down status of each unit at each time period is set correctly by the equation (3.3.5) below. The shut-down status of each unit at each time period is either 0 or 1. Thus only non-negative values of $V_{n,t}$ is considered as defined by (3.3.6):

$$V_{n,t} = W_{n,t-1} - W_{n,t}, \quad \forall n \in N, \quad \forall t \in T, \tag{3.3.5}$$

$$V_{n,t} \geq 0, \quad \forall n \in N, \quad \forall t \in T. \tag{3.3.6}$$

Whenever $W_{n,t-1} = 1$ and $W_{n,t} = 0$, we have that $V_{n,t} = 1$. This is an indication that unit n is shutting down at time period t. On the other hand, if $W_{n,t-1} = W_{n,t} = 0$ or $W_{n,t-1} = W_{n,t} = 1$, then $V_{n,t} = 0$ which indicate that unit n is not shut down at time period t.

3.3.2 Constraints.

The power balance Constraint

The constraint imposes that the total power generated at each time period plus the load shedding at bus i should be equal to the forecasted demand at time t plus power injected to the lines connected to bus i. This is described by:

$$\sum_{n \in N} \sum_{i \in I} PG_{n,t} \cdot W_{n,t} + LS_{i,t} = Dd_t + I^P_{i,t}, \quad \forall t \in T. \tag{3.3.7}$$

If $W_{n,t} = 1$ at time t, then unit n is online and hence $PG_{n,t} \cdot W_{n,t} > 0$. On the other hand, if we have that $W_{n,t} = 0$, then unit n at time t is offline and hence $PG_{n,t} \cdot W_{n,t} = 0$.

Now power injected into the lines connected to bus i is equal to the sum of power in each flow of the lines connected to the bus. This is described by equation (3.3.8) below:

$$I_{i,t}^{P} = \sum_{j \in J} L_{i,j,t}^{P}, \quad \forall i \in I, \quad \forall t \in T. \tag{3.3.8}$$

The generation limit constraint

The description of this constraint is the same as that Ostrowski et al. (2012) model in Section (3.1). When online, the power output of each unit at each time period is bounded below and above by the minimum and maximum power generation. This is described as:

$$PG_{n}^{min} \cdot W_{n,t} \leq PG_{n,t} \leq PG_{n}^{max} \cdot W_{n,t}, \quad \forall n \in N, \quad \forall t \in T. \tag{3.3.9}$$

The maximum power output of unit n at time period t is non-negative and bounded above by the the capacity of unit n:

$$0 \leq PG_{n}^{max} \leq PG_{n}^{cap} \cdot W_{n,t}, \quad \forall n \in N, \quad \forall t \in T. \tag{3.3.10}$$

Spanning reserve requirement

The total maximum producible power at each time period should be able to meet both the forecasted demand and the spanning reserve as shown by:

$$\sum_{n \in N} MAX_{n}^{prod} \cdot W_{n,t} \geq Dd_{t} + S_{t}^{R}, \quad \forall t \in T. \tag{3.3.11}$$

The total spanning reserve, S_{t}^{R} at time t is found by adding the spanning reserves of all the units online at time t as shown in equation (3.3.12). S_{t}^{R} should be greater or equal to the minimum reserve, R_{t}^{min} at time t as declared in (3.3.13):

$$S_{t}^{R} = \sum_{n \in N} S_{n,t}^{R} \cdot W_{n,t}, \quad \forall t \in T, \tag{3.3.12}$$

$$S_{t}^{R} \geq R_{t}^{min}, \quad \forall t \in T. \tag{3.3.13}$$

Ramp up and ramp down constraints

The constraints impose that the variation in power output between two consecutive time periods should be within a certain lower and upper limit. It allows the fluctuation to be within a certain threshold so as to allow the stability of the system. Now, the the maximum power generation of each unit at any time period is bounded above by ramp-up, generation capacity and start up ramp rates as well as shut-down ramp rates. This is declared by (3.3.14) and (3.3.15) :

$$PG_{n,t}^{max} \leq PG_{n,t-1} + R_{n}^{U} \cdot W_{n,t-1} + SU_{n}^{R} \cdot (W_{n,t} - W_{n,t-1}) + PG_{n}^{cap} \cdot (1 - W_{n,t}), \quad \forall n \in N, \quad \forall t \in T, \tag{3.3.14}$$

$$PG_{n,t}^{max} \leq PG_{n}^{cap} \cdot W_{n,t+1} + SD_{n}^{R} \cdot (W_{n,t} - W_{n,t+1}) \quad \forall n \in N, \quad \forall t = 1, ..., T-1. \tag{3.3.15}$$

Similarly, when there is a decrease in power generation between two consecutive time periods, then the change is is bounded by ramp-down, shut-down ramp rates and and the generation capacity of unit n as described by:

$$PG_{n,t-1} - PG_{n,t} \leq R_n^D \cdot W_n, t + SD_n^R \cdot (W_{n,t-1} - W_{n,t}) + PG_n^{cap} \cdot (1 - W_{n,t-1}) \quad \forall n \in N, \quad \forall t \in T. \tag{3.3.16}$$

Minimum uptime and downtime constraints

The minimum uptime and downtime constraints are formulated the same way as that of Ostrowski et al. (2012) model (refer to Section (3.2.2) for description):

$$\sum_{t=1}^{F_n} [1 - W_{n,t}] = 0, \quad \forall n \in N, \tag{3.3.17}$$

$$\sum_{t'=t}^{t+UT_n^{min}-1} W_{n,t'} \geq UT_n^{min} \cdot U_{n,t}, \quad \forall n \in N, \quad \forall t = F_n + 1, ..., T - UT_n^{min} + 1, \tag{3.3.18}$$

$$\sum_{t'=t}^{T} \{W_{n,t'} - U_{n,t}\} \geq 0, \quad \forall n \in N, \quad \forall t = T - UT_n^{min} + 2, ..., T, \tag{3.3.19}$$

$$\sum_{t=1}^{H_n} W_{n,t} = 0, \quad \forall n \in N, \tag{3.3.20}$$

$$\sum_{t'=t}^{t+DT_n^{min}-1} [1 - W_{n,t'}] \geq DT_n^{min} \cdot V_{n,t}, \quad \forall n \in N, \quad \forall t = H_n + 1, ..., T - DT_n^{min} + 1, \tag{3.3.21}$$

$$\sum_{t'=t}^{T} \{1 - W_{n,t'} - V_{n,t}\} \geq 0, \quad \forall n \in N, \quad \forall t = T - DT_n^{min} + 2, ..., T. \tag{3.3.22}$$

3.4 The Delarue and D'haeseleer Model

This model explores a power generation system that incorporates a pumping unit. A pumping unit is a form of energy storage system capable of storing the electricity when the demand is lower than the supply. The stored electricity is then used to meet the demand in periods when the demand is higher than the total power outputs. The process involves pumping water up and down the the storage reservoir. When the demand is below the supply, electricity is used to pump up water into the reservoir. The process is termed as pump up. On the other hand, at an instant where the the power demand exceeds the power supply, the water is released from the storage reservoir over the turbines hence producing the electricity. The process is referred to as pump down and operates with some degree of efficiency.

3.4.1 Objective function.

The UC problem is formulated as MILP where the objective function is the minimization of total costs. The total costs in this case comprises of power generational costs (fuel costs), start-up costs and operating costs (sometimes referred to as fixed costs). The objective function is given by:

$$Min \sum_{t \in T} \sum_{n \in N} PG_n^C \cdot PG_{n,t} + SU_n^C \cdot U_{n,t} + OP_n^C \cdot W_{n,t}. \tag{3.4.1}$$

3.4.2 constraints.

Equation (3.4.2) describes a relationship between the power output and demand in a system with a pumping unit:

$$\sum_{n \in N} PG_{n,t} \cdot W_{n,t} + Eff \cdot PD_t \cdot X_t = Dd_t + PU_t \cdot Y_t, \quad \forall t \in T. \tag{3.4.2}$$

When pumping water up into the reservoir, the power level has to be around the nominal point. This is defined as:

$$Y_t \cdot Pnlo \leq PU_t \leq Y_t \cdot Pnup, \quad \forall t \in T. \tag{3.4.3}$$

Also while pumping water down from the storage reservoir, the generated power is bounded by the maximum output level of the turbines. This is described by:

$$PD_t \leq PD^{max} \cdot X_t, \quad \forall t \in T. \tag{3.4.4}$$

At the end of each time period, the energy content of the reservoir is described by the following equations:

$$PC_{t=1} = PU_{t=1} - PD_{t=1} + PC_0, \tag{3.4.5}$$

$$PC_t = PU_t - PD + PC_{t-1}, \quad t = 2, ..., T. \tag{3.4.6}$$

Now, the energy content of each pumping unit is constraint by the maximum capacity it can store. This is described by:

$$PC_t \leq PC^{max}, \quad \forall t \in T. \tag{3.4.7}$$

To ensure the feasibility of the solutions, the decision variables X_t and Y_t must be set correctly. It make sense to say that at each time, the pumping unit is either pumping up or pumping down or no pump up and pump down at all.The following inequality is introduce for this:

$$X_t + Y_t \leq 1, \quad \forall t \in T \tag{3.4.8}$$

3.5 Synthesis of the Three Models

The three models discussed are formulated as MILP where the objective function and the associated constraints are defined. The difference arise in what entails the objective function and the constraints. This is due to the fact that different power utility companies adopt different components in their power generation. In the three models, there is a slight variation in the objective function. Ostrowski et al. (2012) considers total operational costs to comprise of power generational costs and start-up costs. In the Hemmati and Saboori (2016) model, the total operation cost also includes shut-down costs and load shedding costs. Delarue and D'haeseleer (2006) model considers the total operational costs to include fixed costs in addition to power generation and start-up cost.

For Ostrowski et al. (2012) and Hemmati and Saboori (2016) the formulation of constraints involved are quite similar and comprise of power balance constraint, spanning reserve constraint, generational limit constraint, ramp up, ramp down, minimum uptime and and minimum downtime. The slight difference is that for Hemmati and Saboori (2016), the power balance constraint is modified to take into account load shedding. The Delarue and D'haeseleer (2006) model incorporates a pumping unit in power generation. This modification therefore give rise to additional constraints associated with the pumping unit. In this case, the power balance constraint is modified to include power generated and power used during pump down and pump up process. Furthermore, technical constraints associated with pumping unit such as required power level during pumping, the output level of turbines and constraints involving the energy content of each unit are included. Therefore the formulation of the Delarue and D'haeseleer (2006) model is quite different as compared to the two other models.

4. Solution Schemes and MILP Numerical Results

In this chapter we start by looking at some of the solutions methods that have been applied in solving UC problem. Next we will narrow down to numerical results and simulations obtained through implementation of MILP using the available Sage package for MILP.

4.1 Overview of Solution Schemes

In the past four decades, various solution algorithms have been devised to solve the UC problem. The algorithms are classified as either exact or meta-heuristic. Exact algorithms are more accurate and guarantees convergence to optimal solution especially when dealing with small scale UC problems. Examples are Dynamic programming (DP) (Singhal and Sharma, 2011) and Branch and Bound (BnB)(Delarue and D'haeseleer, 2006). Though they guarantee convergence to optimal solution, exact algorithms are computationally inefficient for solving a large dimension UC problem. Meta-heuristic algorithms such as Genetic Algorithms (GA) (Salam, 2007), Simulated Annealing (SA) (Salam, 2007), Particle Swarm Optimization (PSO) (Abujarad et al., 2017) and Artificial Neural Network (ANN) (Saravanan et al., 2013) have also been used in solving UC problems. They generate a near optimal solution within a reasonable computational time. The only disadvantage is that they do not guarantee an optimal solution.

With the demand of electricity rising due to increase in population, there is an urgent need for power companies to adopt a more recent and emerging technologies for their power generation. Researchers in this field are working on developing hybrid algorithms capable of merging exactness and heuristics in solving UC problem (Saravanan et al., 2013).

4.1.1 Priority listing. Priority listing is one of the fastest and simplest method used in solving UC problems. The generating units are ranked based on their marginal costs. Units that have low marginal costs are prioritized while ensuring that the total power outputs of the online units satisfy the forecasted demand. The drawback to the method is that it does not take into account some coupling constraints such as minimum uptime and minimum downtime. Thus it does not guarantee an optimal solution (Delarue and D'haeseleer, 2006).

4.1.2 Dynamic Programming. Dynamic programming (DP) is a methodological solution scheme where the UC problem is segmented into sub-problems of lower dimension. An optimal solution for each sub problem is obtained, stored and used whenever the same problem arises. The process of storing and using the same results in future computation is termed as memoization. A global optimum is obtained through merging the solutions of the sub-problems obtained. Dynamic programming is easily adaptable and one can easily add constraints such as power balance constraints (Salam, 2007). It is also classified as an exact algorithm and is computationally efficient only for small scale UC problem. As the dimension of the problem increases, the algorithm becomes computationally inefficient based on the fact that the memoization process takes a lot memory and the running time grows exponentially.

4.1.3 Branch and Bound. Branch and bound is one of the method for solving integer programming and mixed integer programming problems. The method is an implementation of mixed integer linear programming (Delarue and D'haeseleer, 2006). An initial solution is obtained first by solving an associated relaxed linear programming by linear programming approaches such as simplex method. If the solution does not satisfy the integer requirement, an upper or lower bound is created depending on whether the problem is minimization or maximization respectively.The problem is then branched into sub-spaces and the solution in each sub-space is then examined and compared with the upper (or lower) bound. If a solution in the search space is better than the existing bound, this solution becomes the new bound. The process is repeated until a feasible and optimal solution is obtained. Though the method is exact and guarantees optimal solution, it is only efficient when dealing with a smaller dimension UC problem. As the dimension of the problem increases, the method becomes computationally inefficient.

4.1.4 Lagrangian Relaxation. Lagrangian Relaxation (LR) method has been extensively used in solving UC problem because of its capability of handling the non-linearity of objective function and the constraints involved. All the constraints involving inequalities are relaxed and Lagrange multipliers are introduced for each constraint. The Lagrangian function is then formed by adjoining the objective function together with the constraints as modified by relaxing the inequalities and introducing the multipliers (Park et al., 2014). A starting values values of the multipliers is then chosen and the relaxed problem is solved with the current values of multipliers. If the solution does not attain the desired optimal, the multipliers are updated and the process repeats until an optimal solution is achieved. The most difficult part of the process is how to choose the starting values of the multipliers and the best approach on updating the multipliers (Abujarad et al., 2017). LR is suitable for a larger dimension UC problem and can easily be modified to allow addition of more coupling constraints. The downside is that it does not guarantee feasible and optimal solution.

4.1.5 Genetic algorithms. Genetic algorithms (GA) are one of the latest adopted meta-heuristic algorithms for solving UC problem. They are based on the theory of natural selection, crossover and mutation (Saravanan et al., 2013). The solution of the UC problem is determined by both the on-off schedule $W_{n,t}$ of the units at each time period and the power outputs, $PG_{n,t}$ generated by each unit at each time period. Genetic algorithms are the powerful global search algorithms due to the fact that the method explores different region of solution space (Roque, 2014). To improve the solution of genetic algorithms especially around the the local optimal, local search algorithms have been incorporated into GA. Valenzuela and Smith (2002) considers a memetic algorithm which is an hybrid of genetic algorithm and the local search. The new algorithm serves to improve on the local optimum of UC solution. Genetic algorithms are highly suitable for solving non-linear and stochastic UC problems. They are also computationally fast and able to handle the mixed integer feature of the UC problems. Nevertheless, GA are meta-heuristic and thus does not guarantee the optimal solution.

4.2 MILP Numerical Results and Discussions

In the recent past, MILP has been widely adopted in modelling UC problem. MILP is a special case of integer programming where some decision variables are allowed to assume non-integer values (Delarue and D'haeseleer, 2006). The method involves defining an objective function and the associated constraints. The three models described in Chapter 3 of this essay were formulated as MILP where the decision variables that denotes the power output of each unit at each time period are continuous while the decision variables that denotes start-up state, shut-down state and on-off status of the units are binary variables. The solution process for MILP starts by solving the associated relaxed linear program by using linear programming solution schemes such as simplex method. If in the initial solution obtained some values of the decision variables are not integers, then from the initial solution, an integer solution is obtained using integer programming solution methods such as Branch and Bound, Cutting Plane Algorithm and Branch and Cut.

The availability of large scale commercial solvers such as CPLEX has eased the solution of UC problems using MILP (Ostrowski et al., 2012). The method is considered exact and hence guarantees convergence of solution to optimal within a finite amount of time (Hemmati and Saboori, 2016). The method is suitable for both small scale and large scale UC problems. It is easily adaptable and hence one can easily modify by adding an extra constraint. However, one disadvantage of the method is that it is not able to solve problems with non-linear objective function or constraints.

Now, we are going to present and discuss the numerical results obtained through implementation of Ostrowski et al. (2012) model where in addition to power generation cost and start-up cost, a shut-down cost is also put into consideration. The assumption is that both the objective function and constraints are linear. We took advantage of the available Sage package for MILP which provides a framework for easy addition of constraints. In running our simulations, a classical UC problem with the data as presented in Table 4.1 was considered. The data is obtained from Dop and slight modification is applied. The system under consideration uses 10 units using coal, gas and diesel as a source of energy. Based on the data, the objective function is a minimization of total operational costs.

Unit	shutdown cost	initial	max-gen	min-downtime	min-gen	min-uptime	ramp-down	ramp-up	start-cost	gen cost	Spanning res
coal1	3000	400.0	440.0	2.0	50.0	4.0	40.0	75.0	5200.0	43.5	50
coal2	2500	345.0	365.0	2.0	70.0	5.0	50.0	74.0	4700.0	43.0	44
gas1	2670	180.0	220.0	3.0	78.0	4.0	40.0	65.0	1320.0	50.5	36
gas2	3132	60.0	210.0	1.0	52.0	3.0	40.0	56.0	1291.0	59.0	29
gas3	2020	165.0	165.0	2.0	54.2	4.0	30.0	67.0	1280.0	37.1	43
gas4	1940	98.0	158.0	2.0	39.0	3.0	60.0	70.0	1105.0	44.8	35
diesel1	2980	70.0	90.0	2.0	17.4	3.0	34.0	60.0	560.0	40.2	33
diesel2	2544	32.0	87.0	2.0	15.2	3.0	55.0	80.0	554.0	46.5	40
diesel3	3220	0.0	20.0	1.0	4.0	1.0	40.0	40.0	300.0	66.3	43
diesel4	1560	9.0	12.0	1.0	2.4	1.0	22.0	22.0	250.0	57.6	23

Table 4.1: The Unit Commitment data

The data also allow implementation of constraints such as power balance, generation limit, ramp up and ramp down, spanning reserve requirement, minimum uptime and minimum downtime (see formulation and description in Section 3.2). Our simulation starts with a system which is already at a certain configuration. Column 3 of the data in Table 4.1 indicates the initial states of the units, where all units except diesel3 are online and each with a given output as indicated.

The data in Table 4.1 is first represented by a matrix shown by equation (4.2.1). This will allow easy access to each element of a matrix in a row or a column. Four sets of decision variables are then defined, three of which are binary variables that indicate the start-up status, shut-down status and on-off status of each unit at each time period. The other set is a continuous variable that defines the output of each unit at each time period.

$$\begin{pmatrix} 3000.0 & 400.0 & 440.0 & 2.0 & 50.0 & 2.0 & 400.0 & 40.0 & 75.0 & 5200.0 & 43.5 & 50.0 \\ 2500.0 & 345.0 & 365.0 & 2.0 & 70.0 & 3.0 & 350.0 & 50.0 & 74.0 & 4700.0 & 43.0 & 44.0 \\ 2670.0 & 180.0 & 220.0 & 2.0 & 78.0 & 2.0 & 205.0 & 40.0 & 65.0 & 1320.0 & 50.5 & 36.0 \\ 3132.0 & 61.0 & 210.0 & 2.0 & 52.0 & 3.0 & 197.0 & 40.0 & 56.0 & 1291.0 & 59.0 & 29.0 \\ 2020.0 & 165.0 & 165.0 & 2.0 & 54.2 & 2.0 & 155.0 & 30.0 & 67.0 & 1280.0 & 37.1 & 43.0 \\ 1940.0 & 158.0 & 158.0 & 2.0 & 39.0 & 2.0 & 150.0 & 60.0 & 70.0 & 1105.0 & 44.8 & 35.0 \\ 2980.0 & 0.0 & 90.0 & 2.0 & 17.4 & 2.0 & 98.0 & 34.0 & 60.0 & 560.0 & 40.2 & 33.0 \\ 2544.0 & 32.0 & 87.0 & 3.0 & 15.2 & 2.0 & 76.0 & 55.0 & 80.0 & 554.0 & 46.5 & 40.0 \\ 3220.0 & 0.0 & 20.0 & 2.0 & 4.0 & 3.0 & 20.0 & 40.0 & 40.0 & 300.0 & 66.3 & 43.0 \\ 1560.0 & 9.0 & 12.0 & 2.0 & 2.4 & 2.0 & 12.0 & 22.0 & 22.0 & 250.0 & 57.6 & 23.0 \end{pmatrix} \tag{4.2.1}$$

Figure 4.1 presents the output of each unit at each time period and the forecasted demand for the 24 hour schedule. It suggest that the total power output for the online units at each time period was slightly higher than the demand. The difference accounts for the spanning reserve that was impose to the system. The optimal value of the objective function obtained is $\$1,875,030$. The average output of each unit for 24 hour schedule is shown by a bar graph in Figure 4.2.

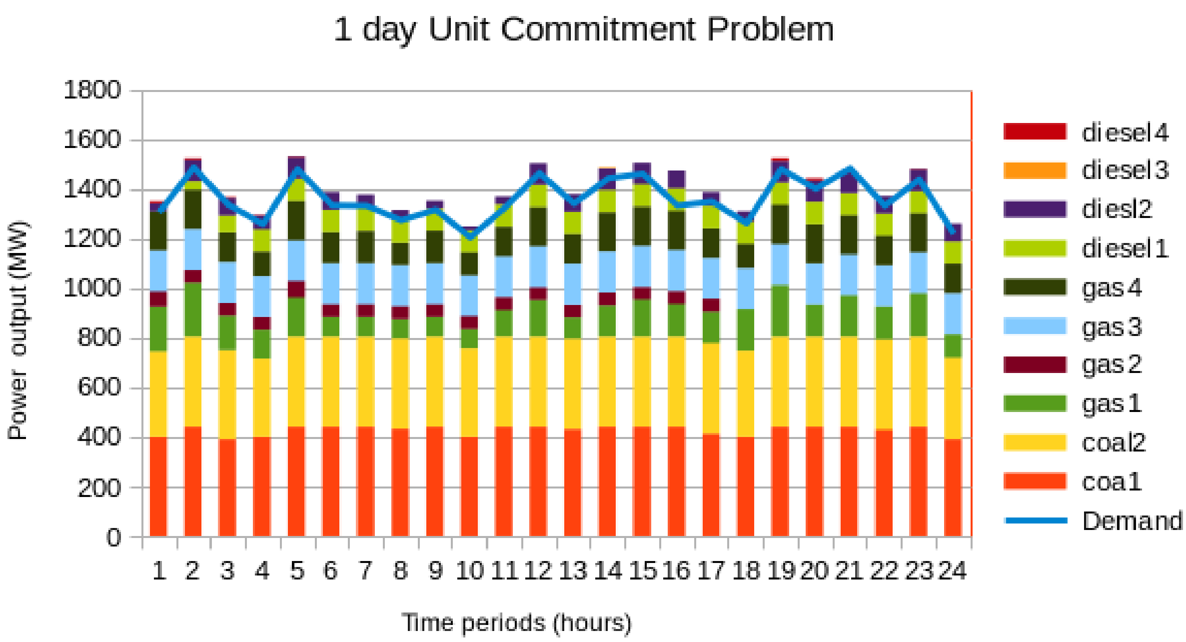


Figure 4.1: Power output of each unit and the forecasted demand at each time period for the 24 hour schedule.

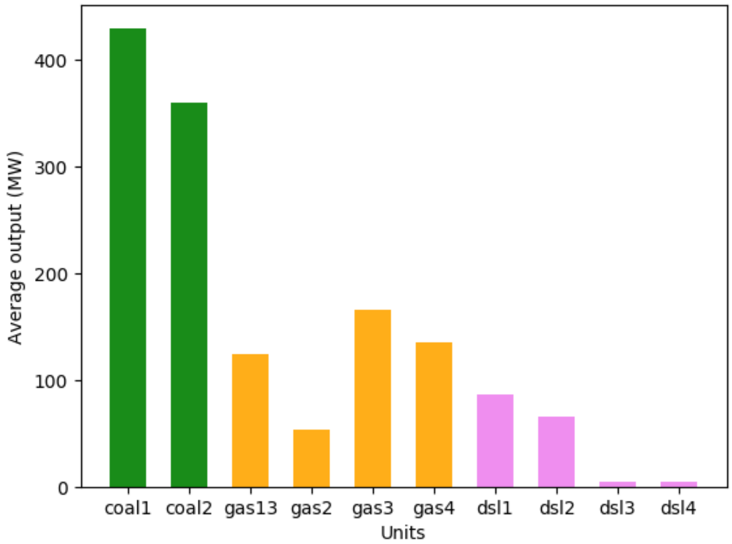


Figure 4.2: A bar graph illustrating an average power output of each unit for a 24 hour scheduling period.

The graph further suggest that for the system under consideration, the power generation is highly dependent on coal as a source power.

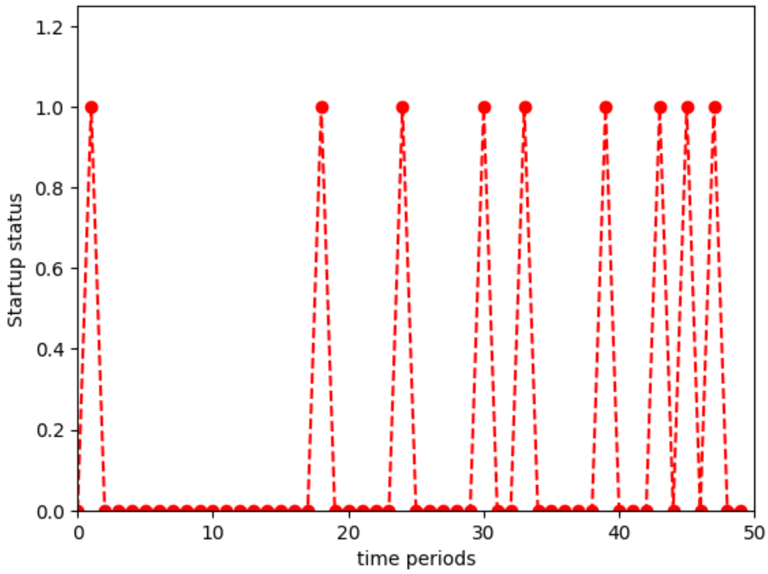


Figure 4.3: A graph illustrating the point of start-up of the units in a 50 hour schedule

Figure 4.3 shows the start-up status of the units for a 50 hour schedule. The dots indicates whether a unit is starting up or not at a given time period. The dots at y-axis equals to 1 indicates the start-up of a unit at a given time while the dots at y-axis equals to 0 indicates that the units under consideration are not starting up at the indicated time. The graph therefore suggest that in a 50 hour schedule, there were 9 start-ups. The number of start-ups per unit is further indicated by Figure 4.4 below where dsl

stands for diesel.

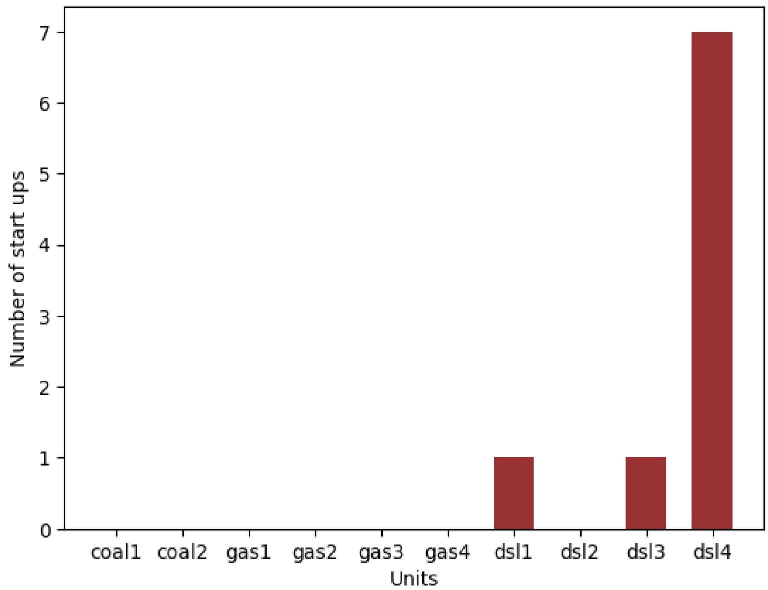


Figure 4.4: A bar graph illustrating the number of start-up per unit in 50 hour schedule.

Similarly, Figure 4.5 illustrates the shut-down status for the units in a 50 hour schedule. The dots at y-axis equal to 1 indicates the shut-down of a given unit at an indicated time while the dots when y-axis is equal to 0 indicates that the units are not shutting down at the designated time. From the graph, it is easy to infer that for a 50 hour schedule, there are 9 shut-downs and the number of shut-downs per unit is indicated by Figure 4.6.

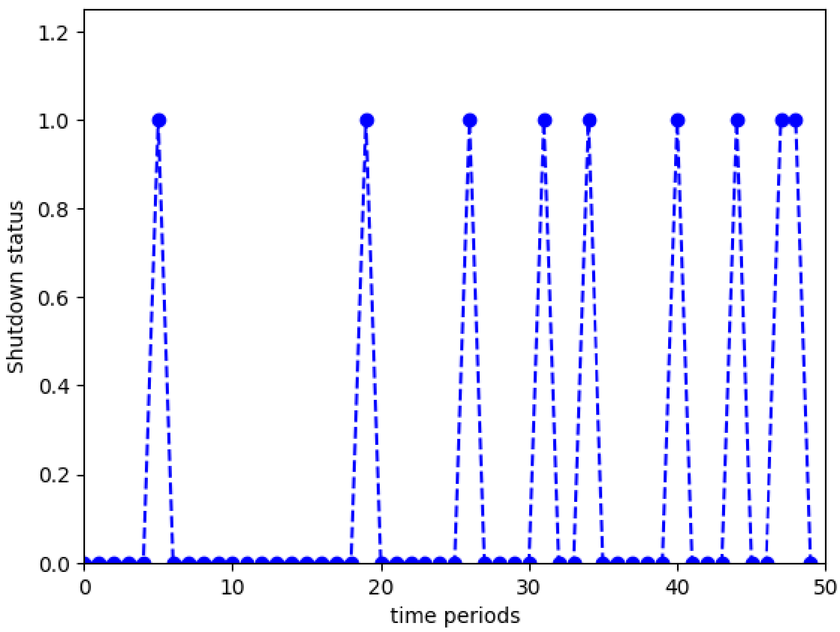


Figure 4.5: A graph illustrating the point of shut-down of the units for a 50 hour schedule

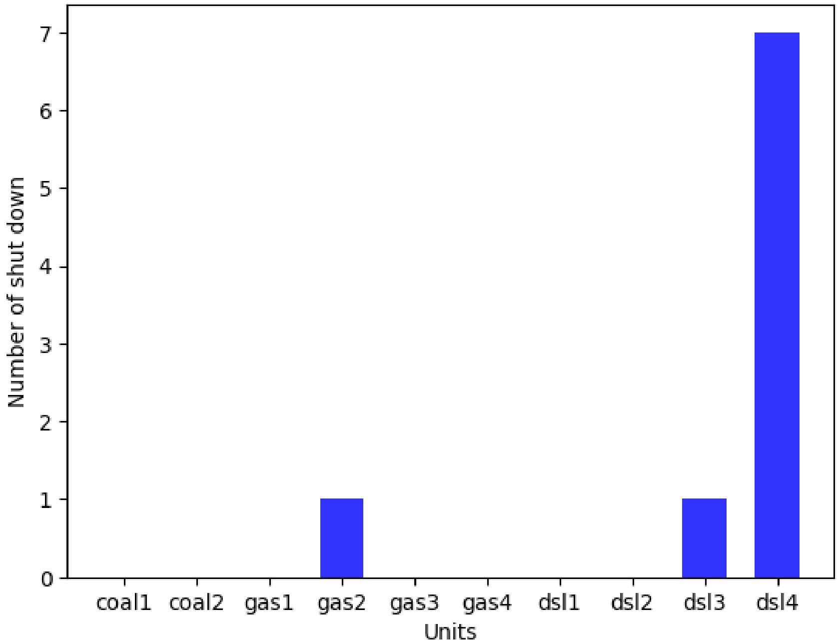


Figure 4.6: A bar graph illustrating the number of shut-downs per unit in 50 hour schedule.

Figure 4.7 shows the total operational costs per unit for a 50 hour schedule. Since the objective function was a minimization, the figure therefore suggest that the operational cost for the units using coal as a source of power is lower as compared as compared to other units. This is further illustrated by the pie chart in Figure 4.8.

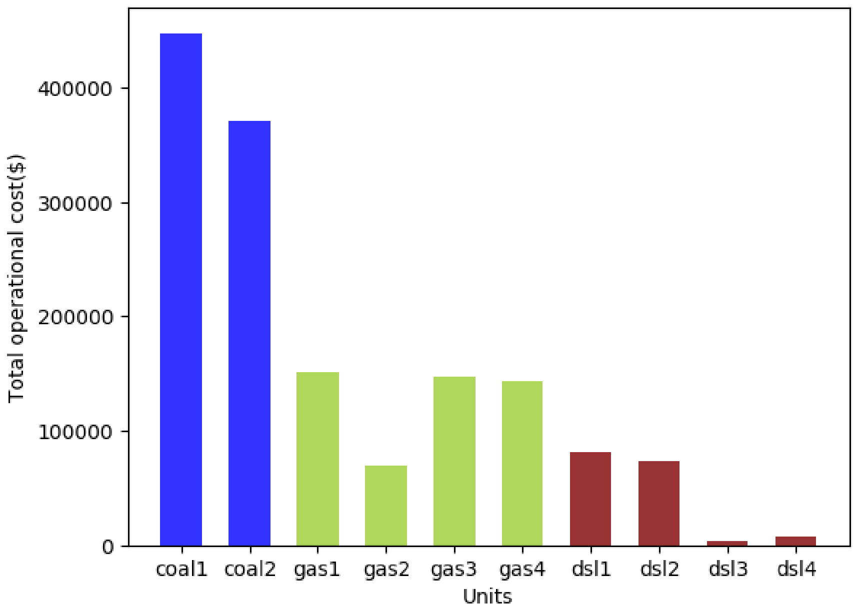


Figure 4.7: A bar graph illustrating the total operational costs of units online in a 50 hour schedule.

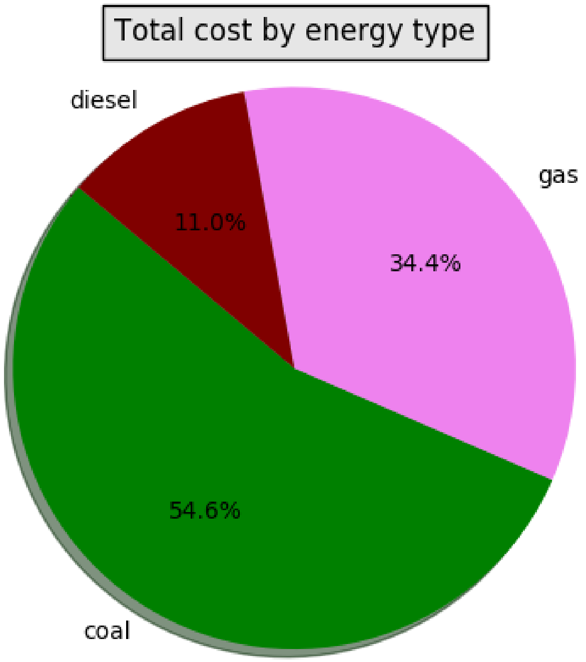


Figure 4.8: A pie chart illustrating the energy type used in power generation and its cost proportion for 50 hour period schedule.

Figure 4.9 indicates the number of units online for 24 hour schedule. It suggest that for the system under consideration, between 7 and 10 units are on average online.

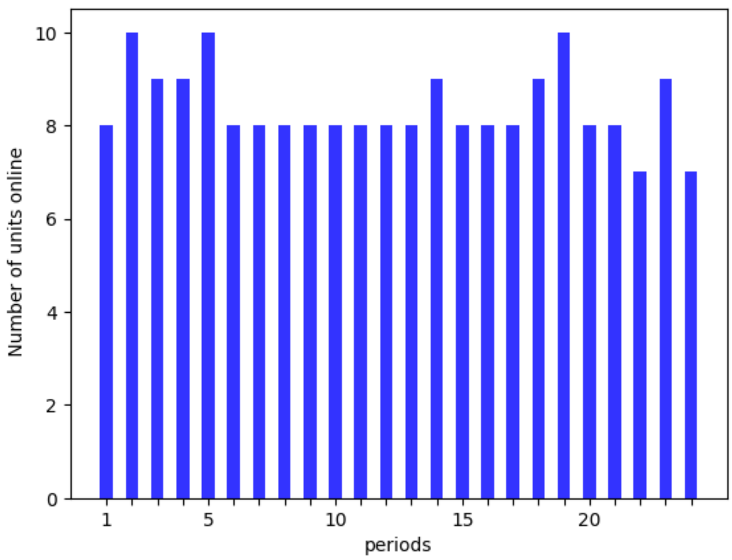


Figure 4.9: A bar graph illustrating the number of units online at each time period for the 24 hour schedule.

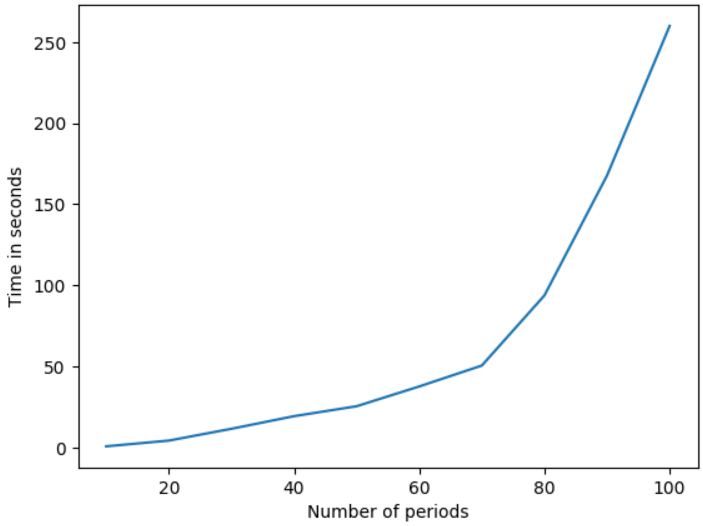


Figure 4.10: A graph illustrating the relationship between the running time of UC problem under consideration and the time periods

In checking the efficiency of an algorithm, it is not enough to only check the optimal value it generates. Power utility company will also be interested in the time complexity of the algorithm under consideration. Generally, a UC problem is considered an NP-hard problem even with $T = 1$ Bendotti et al. (2017). In our simulation, when N is fixed at 10 units and T varied, the MILP grows in polynomial time as shown in Figure 4.10. This confirms the result from (Bendotti et al., 2017) that when N is fixed and T varied, the UC problem runs with polynomial time complexity of $\mathcal{O}(T^2)$.

5. Conclusion

We started by introducing the whole concept of optimisation and different categories of optimisation problems based on the nature of objective function and the constraints. We then narrowed down to the UC optimisation problem. Its introduction and relevance in power generation is described precisely and thereafter we shifted to the main focus of this essay: reviewing optimisation models for UC problem. Three models are reviewed and a short survey on some of the solution schemes that have been extensively used in solving UC problem is presented. The survey reveals that exact algorithms adopted generally guarantees convergence to optimal solution though they are computationally inefficient especially when dealing with a large UC problem. On the other hand, meta-heuristic algorithms that have been recently adopted runs with a reasonable computational time. Nonetheless, they do not guarantee an optimal solution.

The three models reviewed reveals that different adaptations have been adopted in mathematical formulation of UC problem. These adaptations are based on the fact that different countries have different resource endowment which therefore accounts for variation in components of power generation. The three models are formulated as MILP, which further suggest that operations research techniques are gaining interest in modelling power systems. The Sage package for MILP has made it easier to solve both small scale and large scale UC problems. It provides a framework for easy adaptation through addition of an extra constraint.

In running simulations, we considered a classical UC problem with 10 units using coal, gas and diesel as a source of power and operational cost involving power generational costs, start-up costs and shut-down costs. The problem was formulated and implemented as MILP. The constraints defined were: power balance, generation limit, spanning reserve requirement,ramp up and ramp down, minimum uptime and minimum down time. A simulation is run for 100 time periods and a feasible results is obtained in 4.33 minutes. This is quite a reasonable computational time considering the dimension of the problem. The results obtained also shows that the coal-powered units have low operational costs compared to the units using gas and diesel. Units which are using diesel have higher number of start-ups and shut-downs which then increases their operational costs. The results further suggest that when the number of units is fixed and the simulation is run between 10 and 100 time periods, then the UC problem runs with a polynomial time.

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