LMECA2300 - Advanced numerical methods - Assignment 4

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Friday 13 May 2022

1 Analytical and numerical solution comparison

We consider a test case without mean flow, and with a speed of sound $c = 1$ m/s. The initial pressure condition has a centered Gaussian distribution:

$$p(x, y, 0) = e^{-20(x^2+y^2)}$$

The initial conditions $u(x, y, t = 0)$ and $v(x, y, t = 0)$ are both zero everywhere.

Using the formula from [1], the analytical solution is then given by

$$p(x, y, t) = \frac{1}{40} \int_0^{\infty} e^{-\frac{\xi^2}{80}} \cos(\xi t) J_0(\xi \eta) d\xi$$

where J_0 is the Bessel function of the first kind, with order 0.

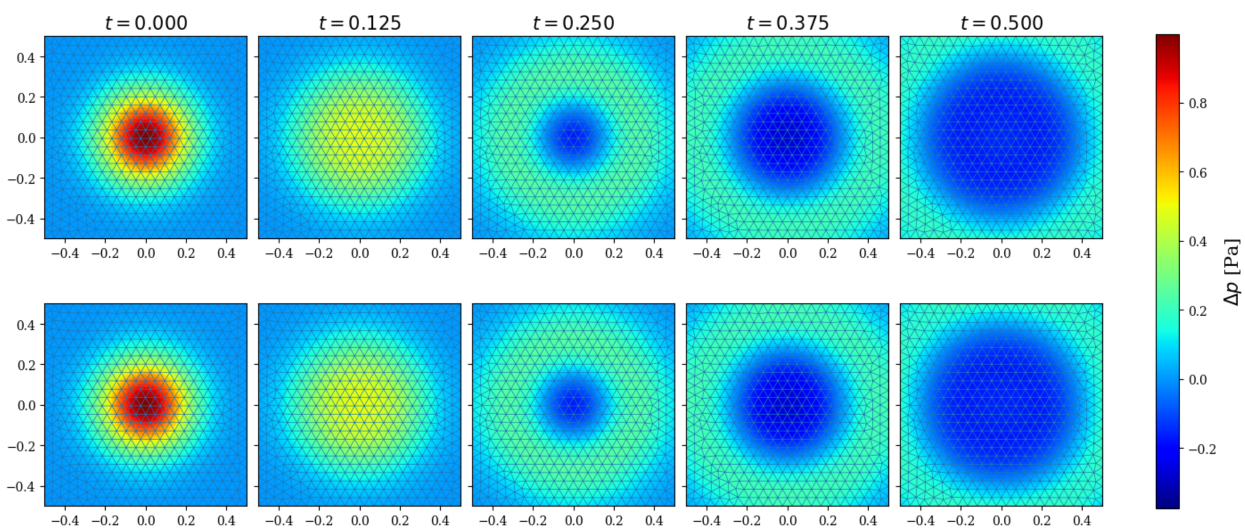

Figure 1. Comparison between the analytical (above) and the numerical (below) solutions. The x and y axes are both expressed in meters, and the time units are seconds.

The result of the implementation is really close to the solution predicted by the theory. We also observe that the simplified non-reflective boundary condition works really well in this test case. For the record, we implemented the true Riemann solver for the numerical flux.

2 Slip wall condition

In this simulation, we implemented a slip wall condition for the lateral walls of a channel. The initial condition is still a Gaussian, centered this time at $(175, 125)$. We then solved the Euler equations with 3 different mean flows: subsonic $M = 0.5$, transonic $M = 1.$ and supersonic $M = 1.5$.

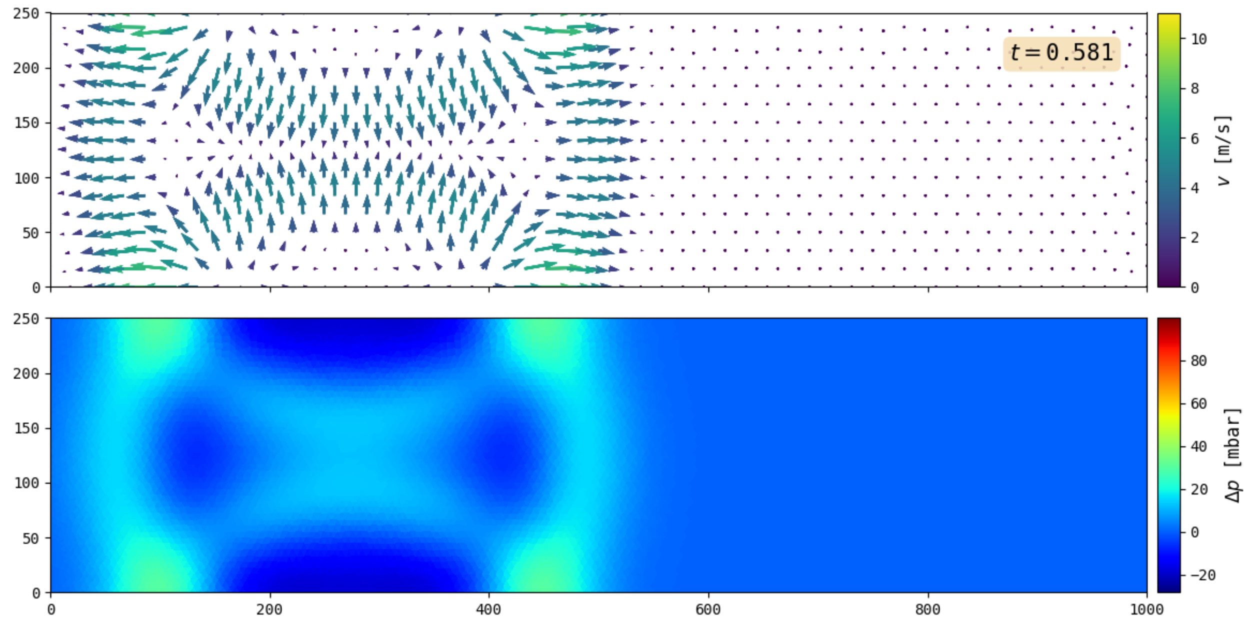

Figure 2. Simulation in a channel at $M = 0.5$.

The results in the other cases are quite similar: they are simply being shifted to the right since the mean flow u_0 is increased. Therefore, we will not show more images of these simulations. However, we made videos that are available in the directory `Animations` that show the differences between the 3 cases.

3 Doppler effect with a periodic source

The source $f(x, y, t)$ is modeled as a Gaussian peak, repeated periodically every 0.25 second. The period is 25 times longer than the duration of the peak, which is thus $\Delta t = 0.01$ second.

$$f(x, y, t) = \epsilon_1 \exp\left[-\alpha_1(x_c^2 + y_c^2)\right] g(t)$$
$$g(t) = \exp\left[-\frac{t^2}{2\left(\frac{0.025}{25}\right)^2}\right]$$

where $\epsilon_1 = 10^4$, $\alpha_1 = 5$, $x_c = x - 250$, $y_c = y - 125$, $g(t)$ is repeated periodically, and the channel keeps the same dimensions $L = 1000$ m and $H = 250$ m.

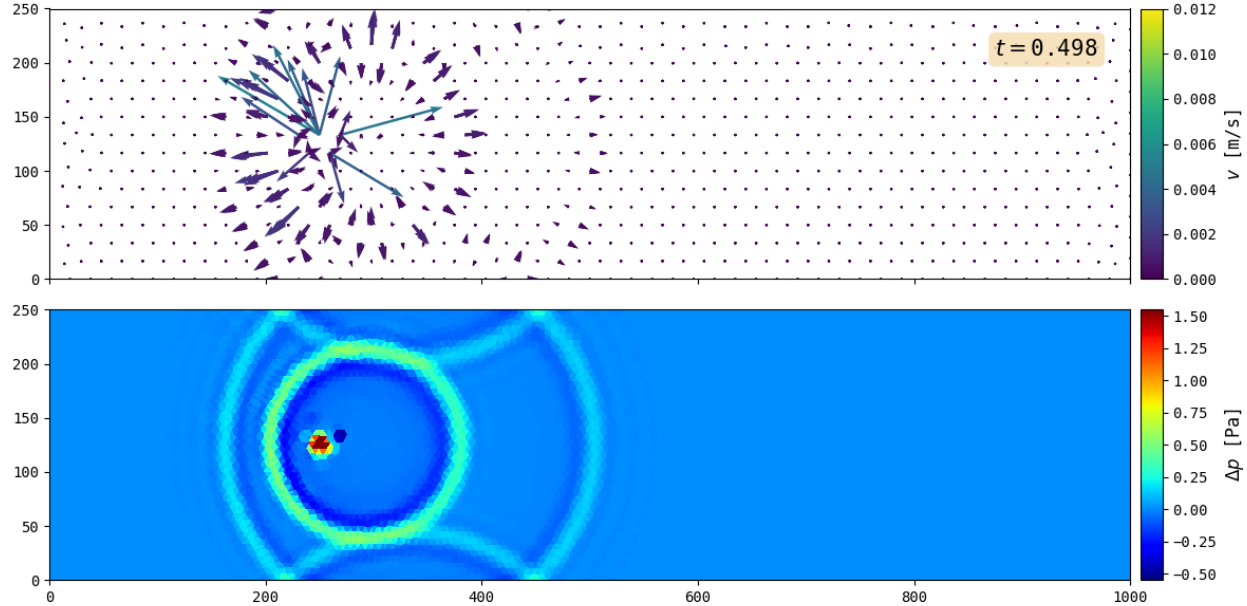

Figure 3. Periodic source in a channel at $M = 0.5$.

In figure 3, we can clearly see how the wave is distorted due to the mean flow. The wave traveling to the left moves at a speed $c - u_0 = 170$ m/s, while the wave traveling to the right moves at a speed $c + u_0 = 510$ m/s.

Once again, videos of the Doppler effect are available in the directory `Animations`. There are 3 videos: subsonic $M = 0.5$ as in figure 3, transonic $M = 1.$ and supersonic $M = 1.5$. In the transonic case, we can observe a vertical shock-wave created at the location of the source. In the supersonic case, as expected, we can observe that the pattern of the waves forms a cone.

References

[1] Christopher K.W. Tam and Jay C. Webb. "Dispersion-Relation-Preserving Finite Difference Schemes for Computational Acoustics." In: *Journal of Computational Physics* 107.2 (1993), pp. 262–281. ISSN: 0021-9991. DOI: `https://doi.org/10.1006/jcph.1993.1142`.

[2] Jan S. Hesthaven and Tim Warburton. *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis, and Applications*. 1st. Springer Publishing Company, 2007. ISBN: 0387720650.