Idea project LMECA2660

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Navier-stokes bulk equations on domain $\mathcal{S}$

$$\nabla \cdot \mathbf{v} = 0 \qquad \text{Mass cons.} \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + \nabla(\mathbf{v} - \mathbf{v}_{\text{mesh}}) \cdot (\mathbf{v} - \mathbf{v}_{\text{mesh}}) = -\nabla p + \frac{1}{Re}\nabla^2(\mathbf{v} - \mathbf{v}_{\text{mesh}}) - \mathbf{g}\frac{Gr}{Re^2}T \qquad \text{Momentum} \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v} - \mathbf{v}_{\text{mesh}}) \cdot \nabla T = \frac{1}{Re\,Pr}\nabla^2 T + \frac{Ec}{Re}(2\mu\,\mathbf{d}) : \mathbf{d} \qquad \text{Energy} \tag{3}$$

$$\mathbf{d} = \frac{1}{2}\Big(\nabla(\mathbf{v} - \mathbf{v}_{\text{mesh}}) + \nabla^\top(\mathbf{v} - \mathbf{v}_{\text{mesh}})\Big) \tag{4}$$

We should note that

$$\nabla \mathbf{v}_{\text{mesh}} \neq \mathbf{0} \tag{5}$$

$$\nabla \cdot \mathbf{v}_{\text{mesh}} = 0 \tag{6}$$

Boundary conditions on $\partial\mathcal{S}$

$$\mathbf{v} = \mathbf{v}_{\text{mesh}} \tag{7}$$

$$T = T_{\text{box},i}(t) \tag{8}$$

Domain rotation

$$\mathbf{v}_{\text{mesh}} = \Omega r\ \mathbf{e}_\theta \tag{9}$$

$$M\mathbf{e_z} = \int_{\partial\mathcal{S}} \mathbf{x} \times (\hat{\mathbf{n}} \cdot \boldsymbol{\sigma}) \tag{10}$$

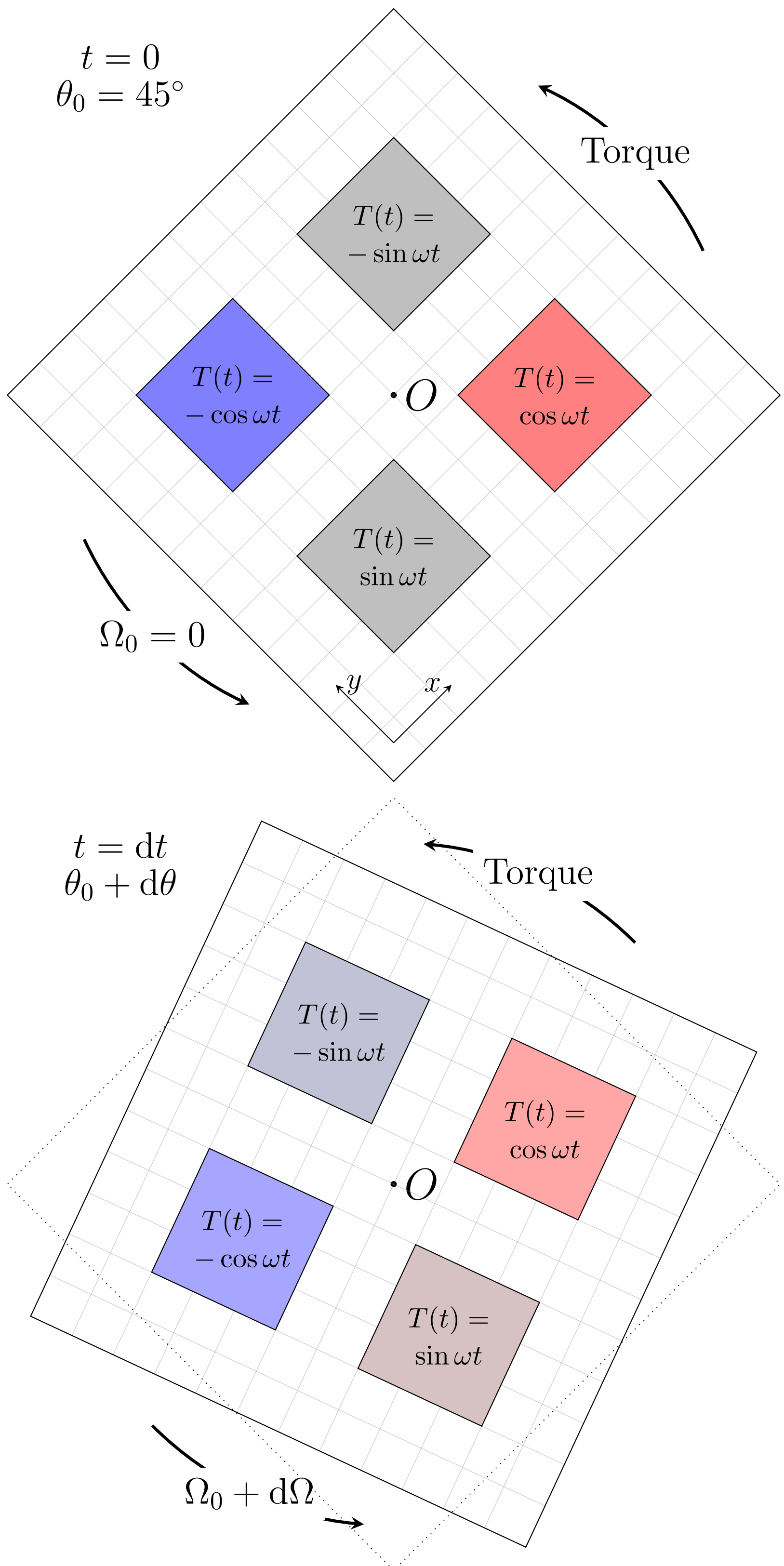
$$\boldsymbol{\sigma} = -p\mathbf{I} + \frac{1}{Re}2\mathbf{d} \tag{11}$$

$$I_{33}\frac{\mathrm{d}\Omega}{\mathrm{d}t} = M \tag{12}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \Omega \tag{13}$$

where $\mathbf{x}$ is the position vector. The inertia tensor is also assumed constant and uniform, and with only $I_{33} \neq 0$. Is it ok to neglect density variations in I_{33} ?

Which frequency ω provides the best rotation Ω ?



$t = 0$
$\theta_0 = 45^\circ$
Torque
$T(t) =$ $-\sin \omega t$
$T(t) =$ $-\cos \omega t$
$\cdot O$
$T(t) =$ $\cos \omega t$
$T(t) =$ $\sin \omega t$
$\Omega_0 = 0$
y
x
$t = \mathrm{d}t$
$\theta_0 + \mathrm{d}\theta$
Torque
$T(t) =$ $-\sin \omega t$
$T(t) =$ $\cos \omega t$
$\cdot O$
$T(t) =$ $-\cos \omega t$
$T(t) =$ $\sin \omega t$
$\Omega_0 + \mathrm{d}\Omega$