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Power series

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_n x^n + \dots$$

a_0 → Free term, a_1 → Coeff of x, a_2 → Coeff of x^2, a_3 → Coeff of x^3, $a_n x^n$ → n - term

$$a_0 + a_1 x + a_2 x^2 + \dots$$

Convergent → $= L \neq \infty$

divergent غير متقاربة (متباعدة) → $= \infty$

Step 1 $u_n = a_n x^n$

$u_{n+1} = a_{n+1} x^{n+1}$

Step 2 $\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right| = L$

Step 3 Convergent if $|L| < 1$

Ex: $1 + \frac{1}{5}x + \frac{2}{5^2}x^2 + \dots + \frac{n}{5^n}x^n + \dots$

in Convergent if

(a) $x \in [-5, 5]$

(B) $x \in [-5, 5)$

(c) $x \in (-5, 5]$

(d) $x \in [-5, 5]$

(E) is divergent

$u_n = \frac{n}{5^n} x^n$

$u_{n+1} = \frac{n+1}{5^{n+1}} x^{n+1}$

$$\lim_{n\to\infty} \left| \frac{\left(\frac{n+1}{5^{n+1}}\right) x^{n+1}}{\left(\frac{n}{5^n}\right) x^n} \right| =$$

$$|x| \lim_{n\to\infty} \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n}$$

$$\frac{|x|}{5} \lim_{n\to\infty} \frac{n+1}{n} = \frac{|x|}{5} \lim_{n\to\infty} \frac{n}{n} = \frac{|x|}{5}$$

$$\frac{|x|}{5} < 1 \Rightarrow |x| < 5 \Rightarrow -5 < x < 5$$

ect. .... Date. ....

* Remark :- $\boxed{n+1\,! = (n+1)\, n!}$

Ex. $1 + X + 2!\,X^2 + 3!\,X^3 + \dots + n!\,X^n + \dots$ is convergent

(A) $X \in (2!, -2!)$

(B) $x \in [2, -2)$

(C) $X \in (-6, 6)$

(D) is divergent

$$U_n = n!\,X^n$$

$$U = n+1\,!\; X^{n+1}$$

$$\lim_{n\to\infty} \left| \frac{U_{n+1}}{U_n} \right| = \lim_{n\to\infty} \left| \frac{n+1!\; X^{n+1}}{n!\; X^n} \right|$$

$$= |X| \lim_{n\to\infty} \frac{n+1!}{n!} = |X| \lim_{n\to\infty} \frac{(n+1)\,n!}{n!} = \infty$$

Ex: $1 - \frac{1}{2}(X-3) + \frac{1}{3}(X-3)^2 + \dots + (-1)^n \frac{1}{n+1}(X-3)^n + \dots$

is :

(a) convergent if $X \in (2, 3)$

(b) $X \in (-2, 2)$

(c) $X \in (2, 4)$

(d) $X \in (-3, 3)$

$$U_n = \frac{1}{n+1}(X-3)^n$$

$$U_{n+1} = \frac{(X-3)^{n+1}}{n+2}$$

$$\lim_{n\to\infty} \left| \frac{\left(\frac{1}{n+2}\right)(X-3)^{n+1}}{\left(\frac{1}{n+1}\right)(X-3)^n} \right|$$

مشتقة البسط / مشتقة المقام ← لوبيتال

$$= |X-3| \lim_{n\to\infty} \frac{n+1}{n+2} = |X-3|$$

بنحلها كالمطول ← $|X-3| < 1$

$$-1 < X-3 < 1$$

$$2 < X < 4$$

Maclaurian

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots$$

$f(x) = e^x$ $\qquad$ $f(0) = e^0 = 1$

$f'(x) = e^x$ $\qquad$ $f'(0) = e^0 = 1$

$f''(x) = e^x$ $\qquad$ $f''(0) = e^0 = 1$

$$e^x \simeq 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

كود [illegible]

```
Sum = function(x)
    Input x;
    Sum = 1
for i = 1 : 1000
    Sum = Sum + x^i / i!
  end
  output sum
```

$f(x) = \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$

odd

$f(x) = \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

even

A هيجيب قيمة ال A من الامتحان

Binomial Theorem

① $C^n_r = \binom{n}{r} = \frac{n!}{r!\ n-r!}$

② $C^n_0 = C^n_n = 1$

مهمة اوي ③ $C^n_{r-1} + C^n_r = C^{n+1}_r$

Ex: $C^{100}_{10} + C^{100}_{11} =$

Ⓐ C^{101}_{10} Ⓑ C^{101}_{11} Ⓒ C^{100}_{11}

$n = 1, 2, 3, 4, \ldots \in \mathbb{N}$

$(x+y)^n = x^n + C^n_1 x^{n-1} y + C^n_2 x^{n-2} y^2 + C^n_3 x^{n-3} y^3 + \ldots + y^n$

Ex: $(1+4x)^5 = (1)^5 + C^5_1 (1)^4 (4x) + C^5_2 (1)^3 (4x)^2 + C^5_3 (1)^2 (4x)^3 + C^5_4 (1)(4x)^4 + x^5$

$n \in \mathbb{R}$

The Binomial Theorem if n is negative as fraction:

$(1+x)^n = f(0) + \frac{f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \ldots$

$= 1 + \frac{n}{1!} + \frac{n(n-1)}{2!} + \frac{n(n-1)(n-2)}{3!} \ldots + \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!} x^r$

Proof:

$f(x) = (1+x)^n \longrightarrow f(0) = 1$

$f'(x) = n(1+x)^{n-1} \longrightarrow f'(0) = n(1+0)^{0-1} = n$

$f''(x) = n(n-1)(1+x)^{n-2} \longrightarrow f''(0) = n(n-1)$

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Ex:- $(1+2x)^{-3} = 1-6x+Ex^2-80x^3+\dots$

Ⓐ $E=36$

Ⓑ $E=24$

Ⓒ $E=-24$

Ⓓ $E=-36$

$n=-3$

$(1+2x)^{-3} = 1+\frac{(-3)}{1!}(2x)+\frac{(-3)(-3-1)}{2!}(2x)^2+\dots$

$\frac{(-3)(-4)}{\cancel{2}}\overset{12\,\cancel{x}^{2}}{(4)}$

Ex: $(x+4)^{\frac{1}{2}} = 2+\frac{x}{4} - E\,x^2+\frac{x^3}{512}+\dots$

Ⓐ $E=64$

Ⓑ $E=-\frac{1}{64}$

Ⓒ $E=-64$

Ⓓ $E=\frac{1}{64}$

$(x+4)^{\frac{1}{2}} = \left[4\left(\frac{x}{4}+1\right)\right]^{\frac{1}{2}}$

$n=\frac{1}{2}$

$2\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$

$= 2\left[1+\frac{(\frac{1}{2})}{1!}\left(\frac{x}{4}\right)+\frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}\left(\frac{x}{4}\right)^2+\dots\right.$

$= \cancel{2}\,\frac{\frac{1}{2}(\frac{1}{2}-1)}{\cancel{2}}\left(\frac{1}{4}\right)^2$

$= \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(\frac{1}{16}\right) = -\frac{1}{64}$

Ex:- The coefficient of x is $(3x+4)^{-3/2}$ is

Ⓐ $-\frac{9}{63}$

Ⓑ $\frac{3}{64}$

Ⓒ $-\frac{64}{3}$

Ⓓ $-\frac{64}{3}$

$(3x+4)^{-\frac{3}{2}} = \left[4\left(1+\frac{3}{4}x\right)\right]^{-\frac{3}{2}}$

$=(4)^{-\frac{3}{2}}\left(1+\frac{3}{4}x\right)^{-\frac{3}{2}}$

$=\frac{1}{8}\left[1+\frac{(-\frac{3}{2})}{1!}\left(\frac{3}{4}x\right)^1+\dots\right]$

$=\left(\frac{1}{8}\right)\left(\frac{-3}{2}\right)\left(\frac{3}{4}\right) = \frac{-9}{64}$

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$1+x+x^2+x^3+x^4+\dots \underset{|x|<1}{\overset{x\text{ small}}{=}} \frac{1}{1-x} = (1-x)^{-1}$

$1-x+x^2-x^3+x^4+\dots = \frac{1}{1+x} = (1+x)^{-1}$

Ex:- $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots$

$1+(\frac{1}{2})+(\frac{1}{2})^2+(\frac{1}{2})^3+(\frac{1}{2})^4+\dots = (1-\frac{1}{2})^{-1}$

Ⓐ $\frac{1}{2}$

$\boxed{Ⓑ\ 2}$

Ⓒ $\frac{3}{2}$

Ⓓ $\frac{-3}{2}$

Ex:- $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\dots$

$1-(\frac{1}{2})+(\frac{1}{2})^2-(\frac{1}{2})^3+(\frac{1}{2})^4\dots = (1+\frac{1}{2})^{-1} = (\frac{3}{2})^{-1} = \frac{2}{3}$

Ⓐ $\frac{2}{3}$

Ⓑ 2

Ⓒ $\frac{3}{2}$

Ⓓ $\frac{-3}{2}$

$(1-x)^{-2} = 1+2x+3x^2+4x^3+5x^4+\dots$

$(1+x)^{-2} = 1-2x+3x^2-4x^3+5x^4\dots$

Ex:- The coefficient of x^3 is $\frac{1}{3-5x} = \dots$

Ⓐ $\frac{125}{81}$

Ⓑ $\frac{125}{27}$

Ⓒ $\frac{125}{9}$

Ⓓ $\frac{-125}{9}$

$\frac{1}{3-5x} = (3-5x)^{-1} = \frac{1}{3}(1-\frac{5}{3}x)^{-1}$

$= \frac{1}{3}[1+(\frac{5}{3}x)+(\frac{5}{3}x)^2+(\frac{5}{3}x)^3+\dots]$

$= \frac{1}{3}(\frac{5}{3})^3$

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Ex: The Cofficient of x^3 is $\left(\frac{1+x}{1-x}\right)^2 = \dots$

Ⓐ 2

$\boxed{\text{Ⓑ } 12}$

Ⓒ -12

Ⓓ -6

$$\left(\frac{1+x}{1-x}\right)^2 = \frac{(1+x)^2}{(1-x)^2} = (1+x)(1-x)^{-2}$$

$$= [1 + 2x + x^2][1 + 2x + 3x^2 + 4x^3 + \dots]$$

$$4 + 6 + 2 = 12$$

$$\sqrt{3} = (4-1)^{\frac{1}{2}}$$

$$= \left[4^{\frac{1}{2}}\left(1-\frac{1}{4}\right)^{\frac{1}{2}}\right]$$

$$= 2\left(1-\frac{1}{4}\right)^{\frac{1}{2}}$$

$$= \boxed{2\left[1 + \left(\frac{1}{2}\right)\left(\frac{-1}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)}{2!}\left(\frac{-1}{4}\right)^2 + \dots\right.}$$

Ex: $1 + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right) + \frac{(1)(3)}{(2)(4)}\left(\frac{3}{4}\right)^2 + \frac{(1)(3)(5)}{(2)4(6)}\left(\frac{3}{4}\right)^3 + \dots$

Ⓐ $\frac{1}{3}$

Ⓑ $\frac{3}{4}$

$\boxed{\text{Ⓒ } 2}$

Ⓓ $\frac{1}{4}$

$$1 + \left(\frac{-1}{2}\right)\left(\frac{-3}{4}\right)$$

$$\left(1 - \frac{3}{4}\right)^{-\frac{1}{2}}$$

$$= \left(\frac{1}{4}\right)^{-\frac{1}{2}}$$

$$= (4)^{\frac{1}{2}}$$

$$= \sqrt{4} = 2$$

* لو الاشارة كلها موجب يبقى الاس في القوس سالب
ولو سالب يبقى الاس في القوس موجب