1 1

(a)

 $\hat{I}^{+} = \text{DFGI}$

 $H^+ = ACEH$

 $BI^+ = ABCDEFGHIJK$

 $B^+ = BHCEA$

 $CI^+ = CDFGIK$

 $I \!\!\to DGF, \, H \!\!\to CEA, \, G \!\!\to H, \, CI \!\!\to K$ violate BCNF because the left sides are not super keys.

(b)

Firstly, we split R using I^+ . Then, we get $R_1(DFGI)$, $R_2(ABCEHIJK)$.

No FD in S violates BCNF for $R_1(DFGI)$.

For $R_2(ABCEHIJK)$, $H \rightarrow CEA$ violates BCNF. We split $R_2(ABCEHIJK)$ into $R_3(ACEH)$, $R_4(BHIJK)$.

No FD violates BCNF in $R_3(ACEH)$.

For $R_4(BHIJK)$, $B \rightarrow H$ violates BCNF. We split $R_4(BHIJK)$ into $R_5(BH)$, $R_6(BIJK)$.

No FDs violates BCNF in $R_5(BH)$ and $R_6(BIJK)$. Therefore, we have $R_1(DFGI)$, $R_3(ACEH)$, $R_5(BH)$ and $R_6(BIJK)$.

2 2

(a)

Step 1: Split the RHSs to get our initial set of FDs,S1:

- (a) $ACDE \rightarrow B$
- (b) $BF \rightarrow A$
- (c) BF \rightarrow D
- (d) $B \rightarrow C$
- (e) $B \rightarrow F$
- (f) CD→A
- (g) $CD \rightarrow F$
- (h) ABF \rightarrow C
- (i) $ABF \rightarrow D$
- (j) $ABF \rightarrow H$

Step 2: For each FD, try to reduce the LHS: (a) No singleton LHS yields anything, we need only consider LHSs with two or more attributes. $CD^+ = \text{CDAF}$, do we can reduce it to CDE \rightarrow B.

- (b) $B \rightarrow CF$, we can reduce it to $B \rightarrow A$.
- (c) Same as (b), $B \rightarrow D$.
- (d) Only B at LHS, cannot reduce.
- (e) Only B at LHS, cannot reduce.
- (f) No singleton LHS yields anything, cannot reduce.
- (g) No singleton LHS yields anything, cannot reduce.

- (h) BF \rightarrow A, and B \rightarrow CF, we can reduce as B \rightarrow C.
- (i) Smae as h, we can reduce as $B\rightarrow D$.
- (j) Smae as h, we can reduce as $B \rightarrow H$.

Now we have:

- (a) $CDE \rightarrow B$
- (b) $B \rightarrow A$
- (c) $B \rightarrow D$
- (d) $B \rightarrow C$
- (e) $B \rightarrow F$
- (f) $CD \rightarrow A$
- (g) $CD \rightarrow F$
- (h) $B \rightarrow C$
- (i) $B \rightarrow D$
- $(j) B \rightarrow H$

Step 3: Try to eliminate each FD. We have:

- ${\rm B}{\rightarrow}{\rm C}$
- $B{
 ightarrow} D$
- $\mathrm{B}{\to}\mathrm{H}$
- $\text{CD} {
 ightarrow} \text{A}$
- $CD \rightarrow F$
- $CDE \rightarrow B$

(b)

Combining minimal basis FDs:

- $B \rightarrow CDH$
- $CD \rightarrow AF$
- ${\rm CDE} {\rightarrow} {\rm B}$

The keys are b, CD, CDE

(c)

The relations:

 $R_1(ACDF), R_2(BCDE), R_3(BCDH)$

Since G never appears, so we need to add a new relation $R_4(\text{CDEG})$

 $CDEG^+ = ABCDEFGH.$

The relations are $R_1(ACDF)$, $R_2(BCDE)$, $R_3(BCDH)$, $R_4(CDEG)$

(d)

B is not the superkey. It violate BCNF. So R_3 allow redundancy. So the schema allows redundancy.