Консултация КЧ

14.12

Зад. 1 Намерете X

$$(X+B)^{-1}XA = C$$

където $A = \begin{pmatrix} 2 & 0 & -1 \\ -1 & 4 & 1 \\ -1 & -2 & 6 \end{pmatrix}$

$B = \begin{pmatrix} -4 & 0 & 16 \\ -1 & 1 & 5 \\ 0 & 4 & 4 \end{pmatrix}$ $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Реш:

$(X+B)^{-1}XA = C \quad / \cdot (X+B)$ отляво

$XA = (X+B)C$

$XA = XC + BC$

$XA - XC = BC$

$X(A-C) = BC$

$\Rightarrow$ имаме ур-е от вида
$XD = M$

$$A - C = D = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 3 & 1 \\ -1 & -2 & 5 \end{pmatrix}$$

$$BC = B = \begin{pmatrix} -4 & 0 & 16 \\ -1 & 1 & 5 \\ 0 & 4 & 4 \end{pmatrix} = M$$

$$(XD)^t = M^t = \begin{pmatrix} -4 & -1 & 0 \\ 0 & 1 & 4 \\ 16 & 5 & 4 \end{pmatrix} = D^t X^t$$

$$(D^t \mid M^t) \sim \ldots \sim (E \mid X^t)$$

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & -4 & -1 & 0 \\ 0 & 3 & -2 & 0 & 1 & 4 \\ -1 & 1 & 5 & 16 & 5 & 4 \end{array}\right) (1) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & -4 & -1 & 0 \\ 0 & 3 & -2 & 0 & 1 & 4 \\ 0 & 0 & 4 & 12 & 4 & 4 \end{array}\right) \begin{array}{l} \\ \\ /:4 \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & -4 & -1 & 0 \\ 0 & 3 & -2 & 0 & 1 & 4 \\ 0 & 0 & 1 & 3 & 1 & 1 \end{array}\right) \begin{matrix} \\ (2) \quad (1) \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 0 & 1 \\ 0 & 3 & 0 & 6 & 3 & 6 \\ 0 & 0 & 1 & 3 & 1 & 1 \end{array}\right) \ 1{:}3 \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 & 1 \end{array}\right) \begin{matrix} \\ (1) \\ \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1 & 1 \end{array}\right)$$

$$\Rightarrow X = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

Зад. 2 Да се пресметне дет. от n-ти ред $(a \neq 0)$

$$\Delta_n = \begin{array}{c} \begin{vmatrix} x+1 & x & \cdots & x \\ x & x+a & \ddots & \vdots \\ \vdots & x & x+a^2 & \ddots \\ \vdots & \vdots & \ddots & x \\ x & x & \cdots\ x & x+a^n \end{vmatrix} \end{array} \begin{array}{l} (-1) \\ \\ \\ \\ \\ \end{array} =$$

$$= \begin{vmatrix} x+1 & x & \cdots & \cdots & x \\ -1 & a & 0 & \cdots & 0 \\ \vdots & 0 & a^2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -1 & 0 & \cdots & 0 & a^n \end{vmatrix} =$$

$$\begin{array}{ccc} & \left(\frac{1}{a}\right) & \cdots\ \left(\frac{1}{a^n}\right) \end{array}$$

$$= \begin{vmatrix} x+1+\sum_{i=1}^{n} \frac{x}{a^i} & x & \cdots & x \\ 0 & a & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & a^n \end{vmatrix} =$$

$$= a^{\frac{n(n+1)}{2}} \left(x + 1 + \sum_{i=1}^{n} \frac{x}{a^i} \right) =$$

$$= a^{\frac{n(n+1)}{2}} \left(1 + x \frac{a^{n+1} - 1}{a^n (a-1)} \right)$$

Зад. 3 Нека $a_1 = (2, 3, 5)$, $a_2 = (0, 1, 2)$ $a_3 = (1, 0, 0)$ $b_1 = (1, 1, 1)$ $b_2 = (1, 1, 0)$ $b_3 = (2, 1, 2)$

Док, че a_1, a_2, a_3 обр. базис на $\mathbb{R}^3$ и ? матр. на φ, опр. чрез $\varphi(a_i) = b_i$, $i = 1, 2, 3$ спр. станд. базис (e_1, e_2, e_3)

Реш: $\begin{pmatrix} 2 & 3 & 5 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{pmatrix}$ – ЛНЗ $\Rightarrow$ базис

Нека ψ, σ – л. оп.

$$\begin{tikzcd} & e \arrow[dl, "\psi"'] \arrow[dr, "\sigma"] & \\ a \arrow[rr, "\varphi"'] & & b \end{tikzcd}$$

$\psi(e_i) = a_i$

$\sigma(e_i) = b_i$

$\sigma(e_i) = b_i = \varphi(a_i)$

$= \varphi(\psi(e_i)) =$

$= (\varphi \circ \psi)(e_i)$

$\sigma = \varphi \cdot \psi$

$$\left.\begin{array}{l} A \text{ - матр. на } \psi \\ B \text{ - матр. на } \sigma \\ C \text{ - матр. на } \varphi \end{array}\right\} \begin{array}{l} \text{спр. базиса} \\ e_1, e_2, e_3 \end{array}$$

$\Rightarrow B = CA$

$B = C \cdot A \Rightarrow B^t = (CA)^t = A^t \cdot C^t$

$\Rightarrow (A^t \mid B^t) \sim \dots \sim (E \mid C^t)$

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 5 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 & 1 & 2 \end{array}\right) (-2) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 0 & 3 & 5 & -3 & -1 & -3 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 & 1 & 2 \end{array}\right) (-3) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 0 & 0 & -1 & -6 & -4 & -3 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 & 1 & 2 \end{array}\right) (2)$$

$$\sim \left(\begin{array}{ccc|ccc} 0 & 0 & -1 & -6 & -4 & -3 \\ 0 & 1 & 0 & -11 & -7 & -6 \\ 1 & 0 & 0 & 2 & 1 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & -11 & -7 & -6 \\ 0 & 0 & 1 & 6 & 4 & 3 \end{array}\right)$$

$$\Rightarrow C = \begin{pmatrix} 2 & -11 & 6 \\ 1 & -7 & 4 \\ 2 & -6 & 3 \end{pmatrix}$$