

Population forecasting with Bayesian hierarchical models


<https://github.com/violetacln/SIPP>

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Content

- Production of population projections: problems and solutions
- Difficulties and new demands
- New methods and implementation in 
- Future plans

Problem and solution

for migration, fertility, mortality (by age, time & Z^ - characteristics)
which give, via ccpm, the population predictions*

- **Problem:**
 - *estimating and predicting* in the presence of:
 - small area/population issues and/or rare events/ shocks
 - expert assumptions
 - uncertainty of parameters and errors in data
 - while reporting *uncertainty* of estimates/predictions
- **Solution must be based on:**
 - Modeling and combining (prior) information

Types of models

Recent years: *functional models, ARDL & bsts models* $f(\text{age}, \text{time})$

- smoothing and orthogonal expansions for Fertility and Mortality components
- econometric / decay models for short / intermediate term
- time series / assumptions for long term

New: *hierarchical and/or Bayesian models of response(s)*

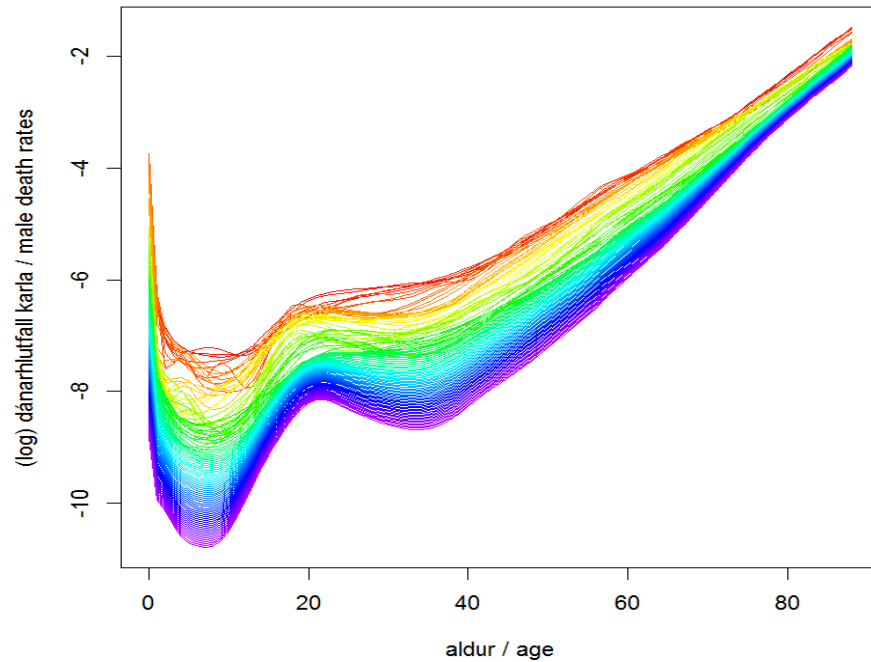
- smoothing/processes $(t, \text{space}, \text{age}, \dots)$
- complex correlation structures, spatial, demographic, social, ... characteristics
- incorporate quantitative & qualitative info: *priors*

****** *Model averaging options*

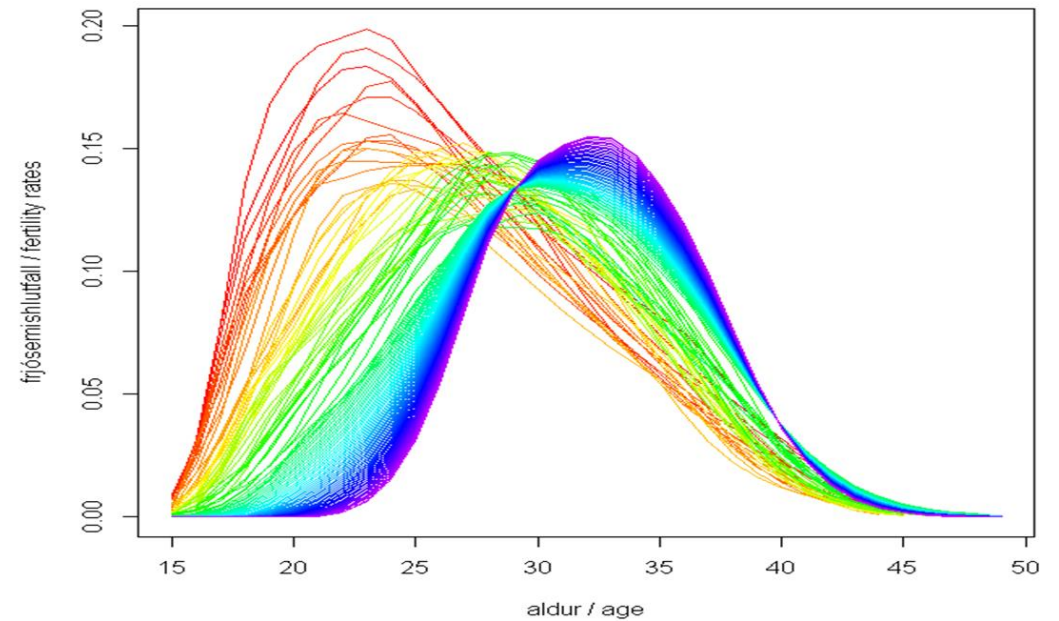
Typical result of functional models

<https://cran.r-project.org/web/packages/demography/index.html>

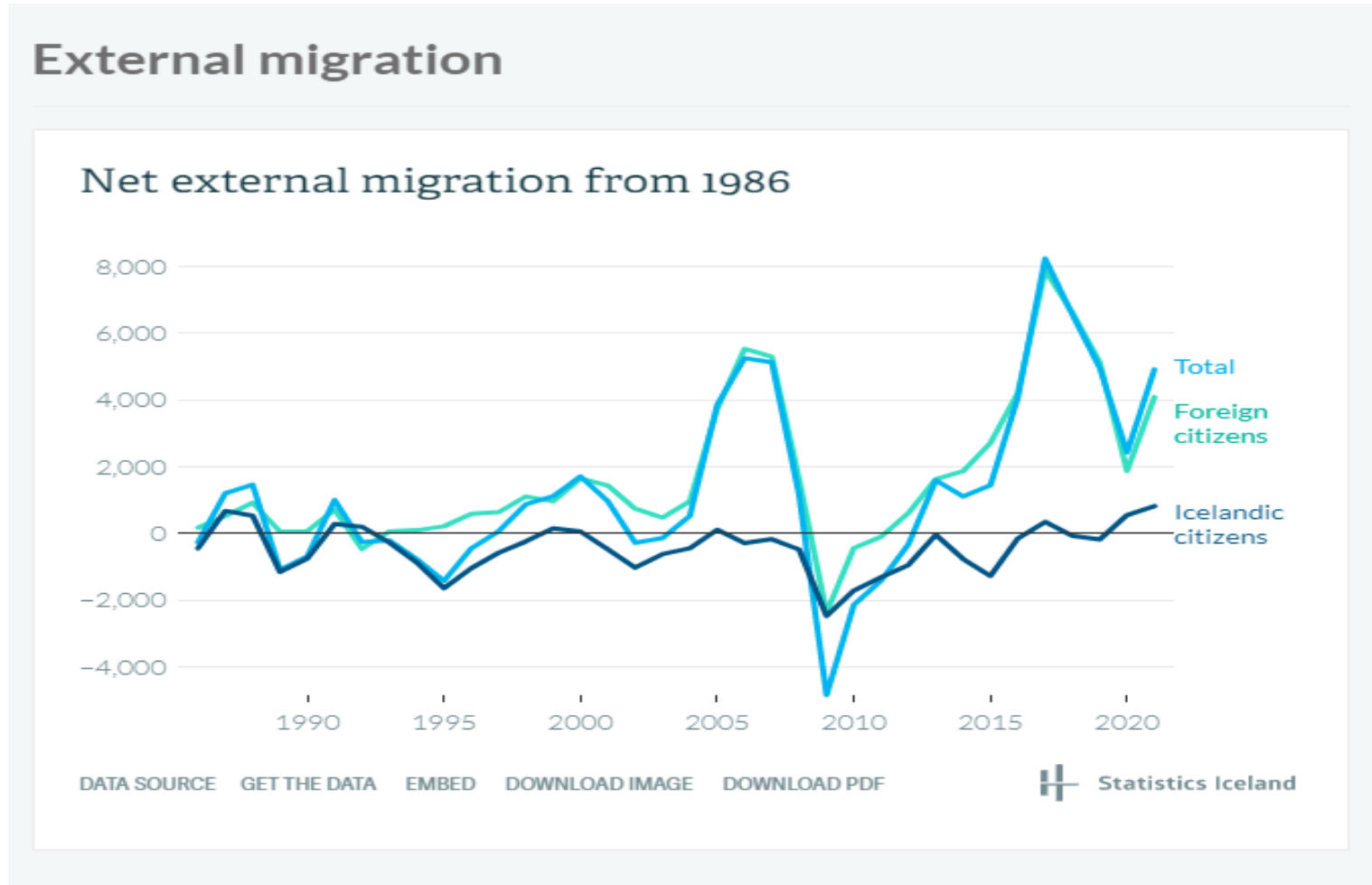
Mortality rates



Fertility rates



Migration issues and options for modeling



Predicting future values of migration/fertility/mortality:

- projections versus forecast
- advantages of Bayesian approach

Past

- Disjoint, long and short term migration predictions
- Unique models for predicting mortality and fertility, but *only* as functions of time, age (and gender)

New

*& allowing for **local** predictions*

- *Difficulties and new user demands* → Strengths of the new approach
- *Exceptions:* shocks (economical, political, natural, ...)

New Models (parametric or not!)

$P(Y/a) = F1(x)$ where $P(a/b) = F2(c, Z)$ where...

- **Type: Hierarchical / Multilevel**, Bayesian / frequentist
 - data used: *individual* or *aggregated* response (Y) versions
 - what: **processes**, since *time series* data $P(Y_i, \dots Y_n | a)$, versus $P(Y_i)$: account for (auto-) correlations
- **Why:**
 - {components/prior information/characteristics/local&total} & **uncertainty** report
- **R-Tools:**

several *Stan*-running R packages like *brms*, *arm*, *bayestestR*, ...

and the faster: *lme4*, *mgcv*, ... for frequentist / as initializations

Structure, novelty and advantages

$x=(\text{year, age, gender, citizenship, municipality, education, ...})$

- V1. Counts/rates

- $N(x, \dots) \sim \text{Poisson}(e^{f(x, \dots)} N_0(x, \dots))$ (or: neg. binom, over-disperssion , etc)
- $f_m(\{x, \dots\}) \sim \text{GP}(m(\{x, \dots\}), C(\{x, \dots\}, \{x', \dots\})) + \dots$

- V2. Microdata

- $N(x, \dots) \sim B(e^{f(x, \dots)} N_0(x, \dots))$
- $f_m(\{x, \dots\}) \sim GP(m(\{x, \dots\}), C(\{x, \dots\}, \{x', \dots\})) + \dots$
- GP – denotes here: sums of smooths/re/t2 and/or **by** {grouping factors} OR **unknown functions** with Gaussian Processes as priors (with m -> long term behaviour, C -> correl between any two response values)

Tests, Correlations, Dimensions

- Variability

- of:

fertility, mortality, migration

- through (and with auto-correl):

Time & age (*s(t)*, *gp(t, age, ...)*, *t2(t, age)*, ...)

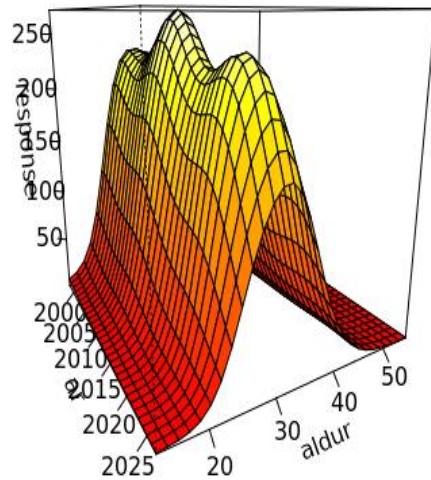
- by:

- **Municipality**, Region – significant for: migration but not/less for mortality, fertility
- Other characteristics: education, family related, municipality attributes, ...
- More grouping characteristics (ut/is, im/e, f/m)

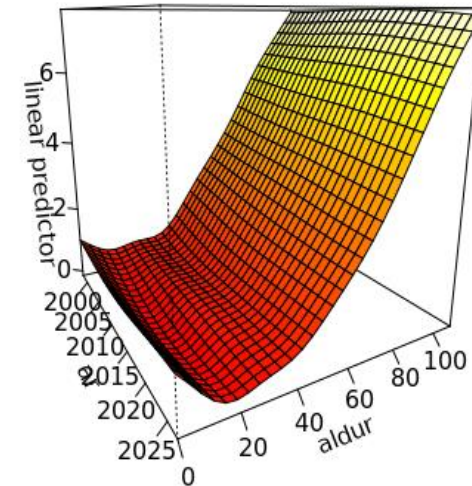
V1. Count data input

<https://cran.r-project.org/web/packages/mgcv/index.html>

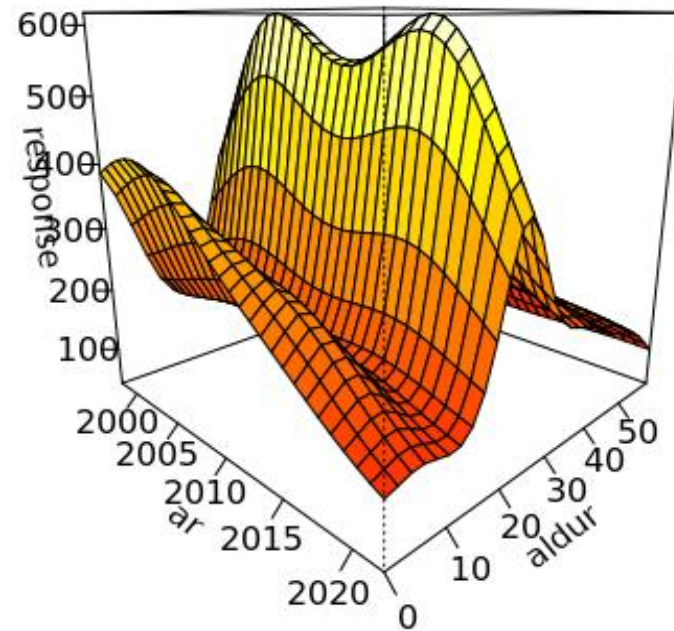
Births



Death rates (log)

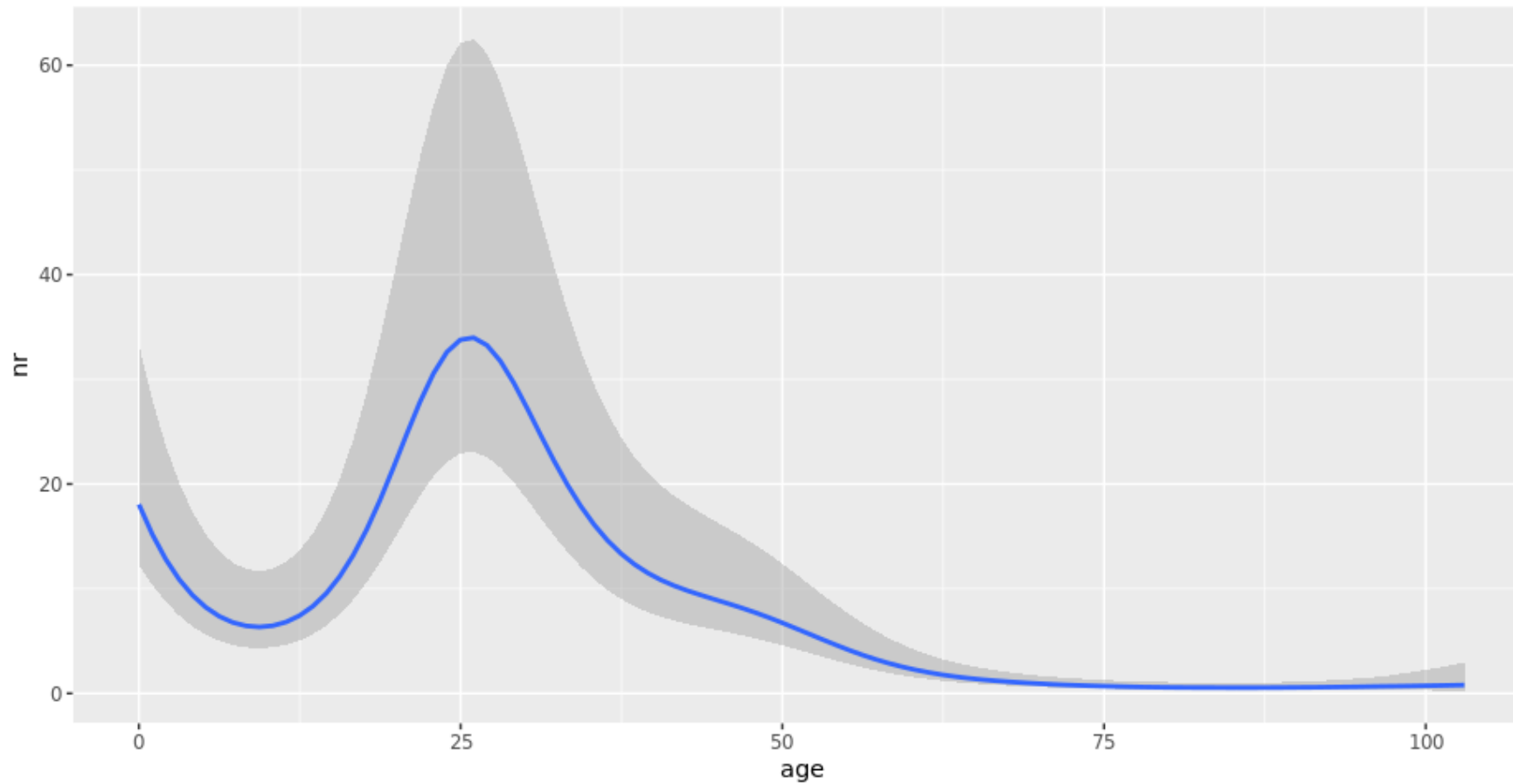


Migration

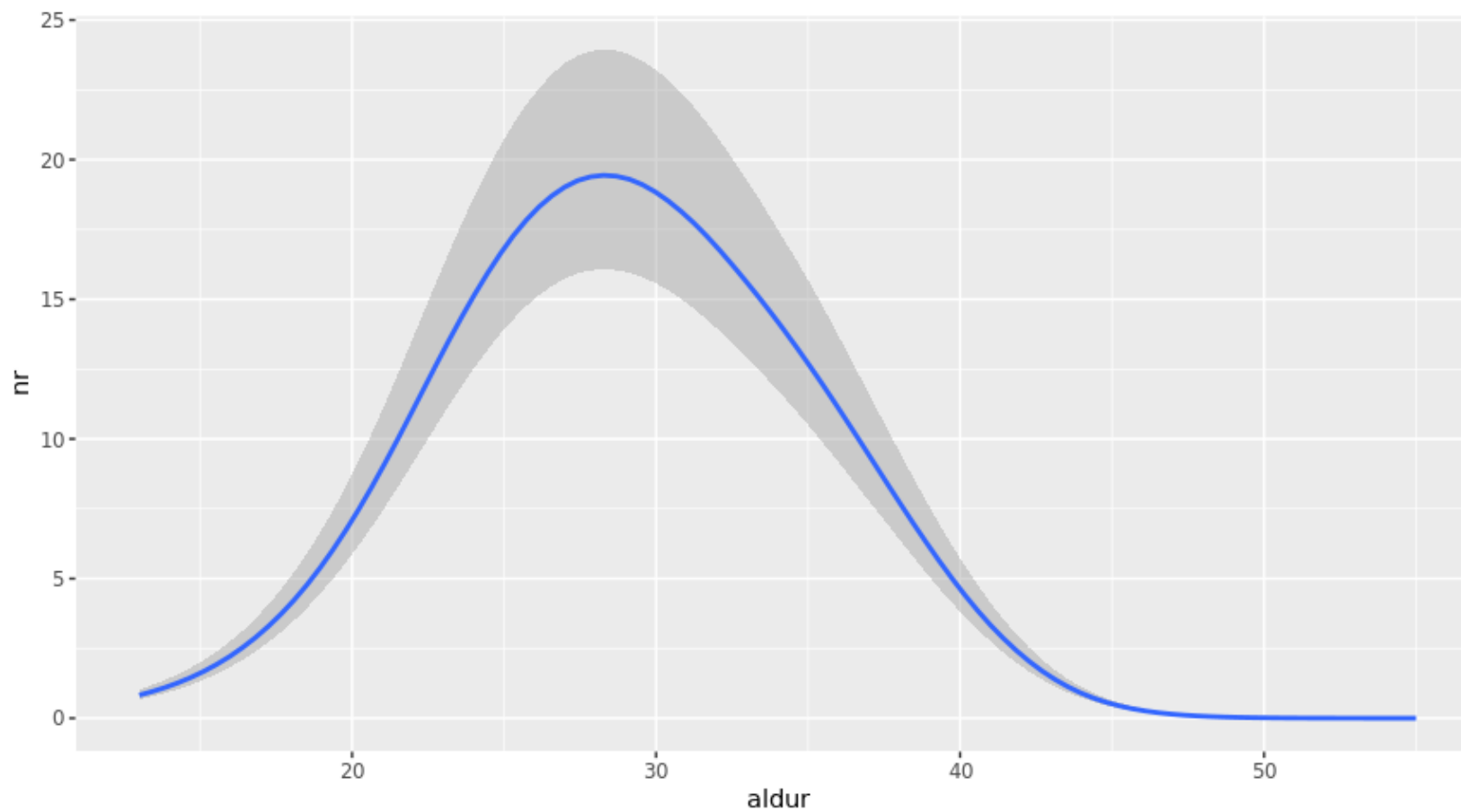


Counts/rates models: all migration categories (f_{em_ut})

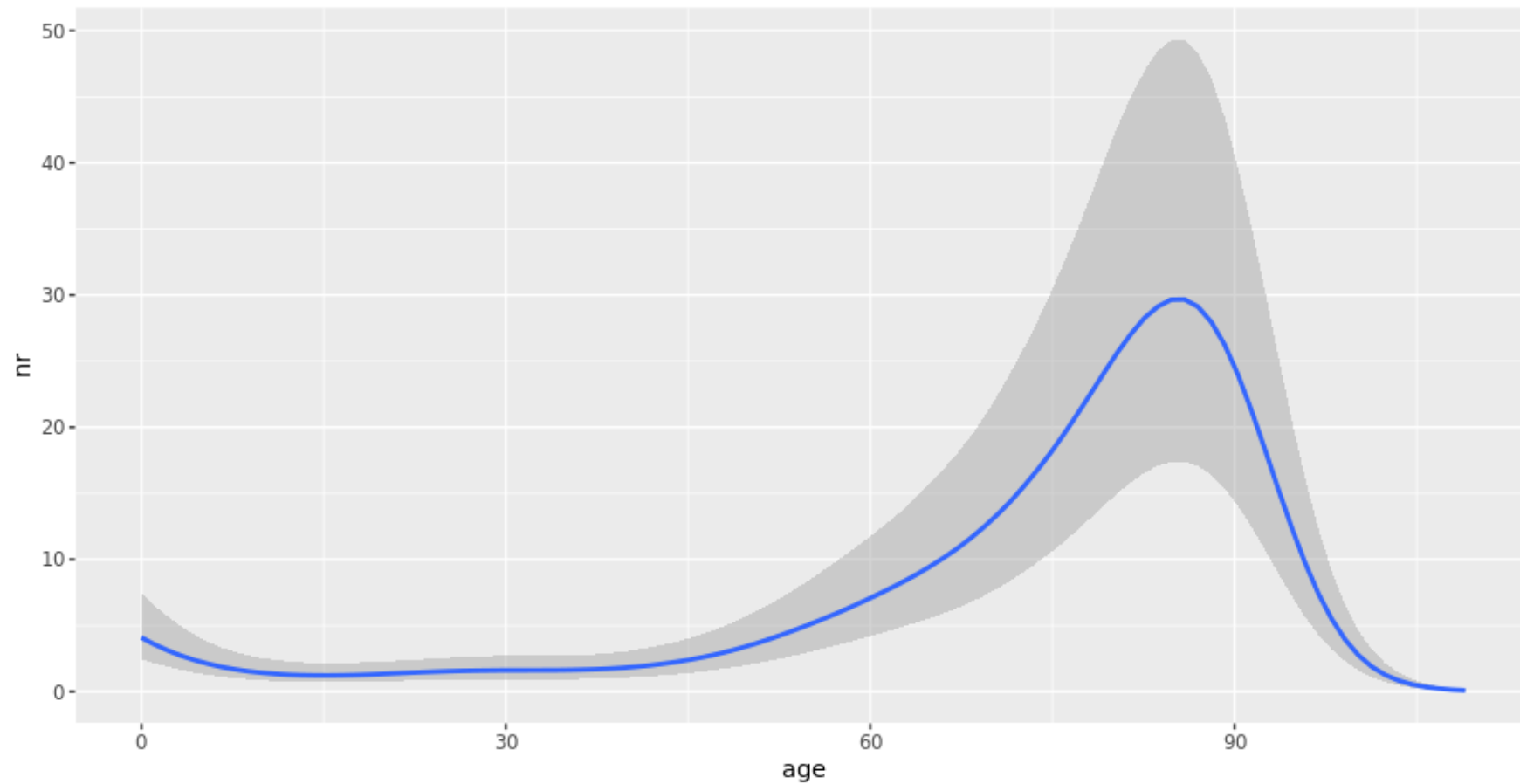
<https://cran.r-project.org/web/packages/brms/index.html>



Count data and rate models: fertility



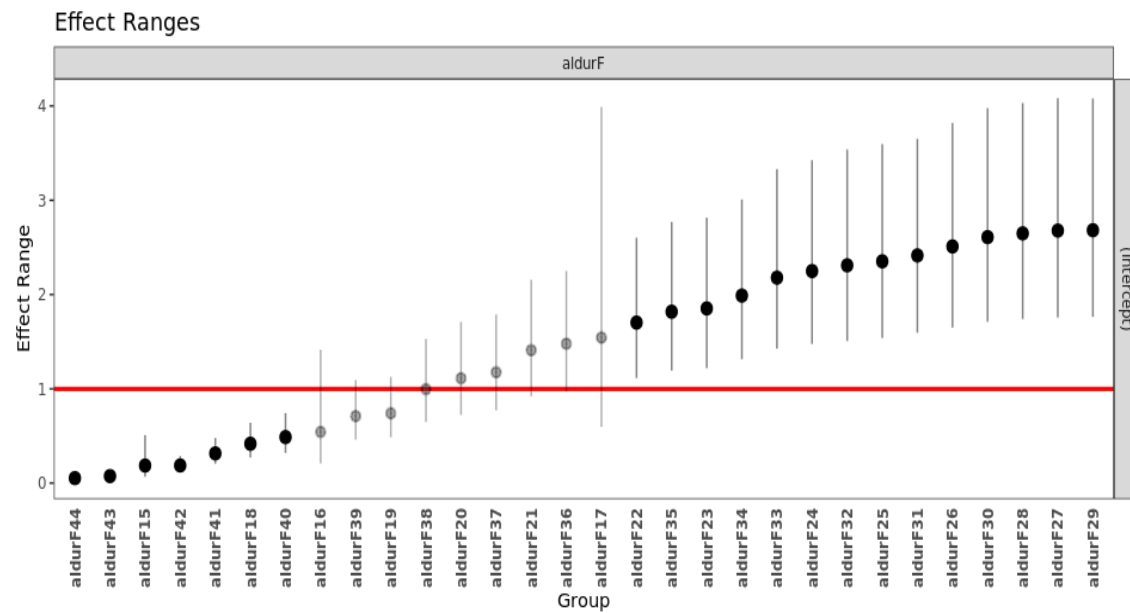
Counts/rates: mortality models (addit, s)



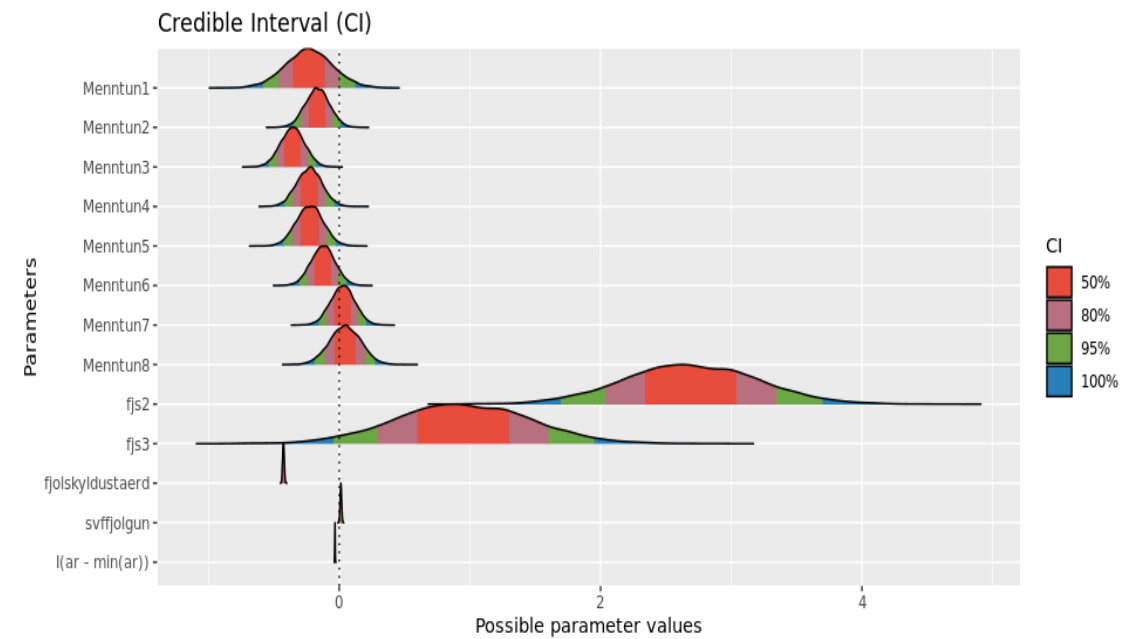
V2. Microdata input

Example 1. Significance of effects for:

Age group effect on fertility (o.r. units)



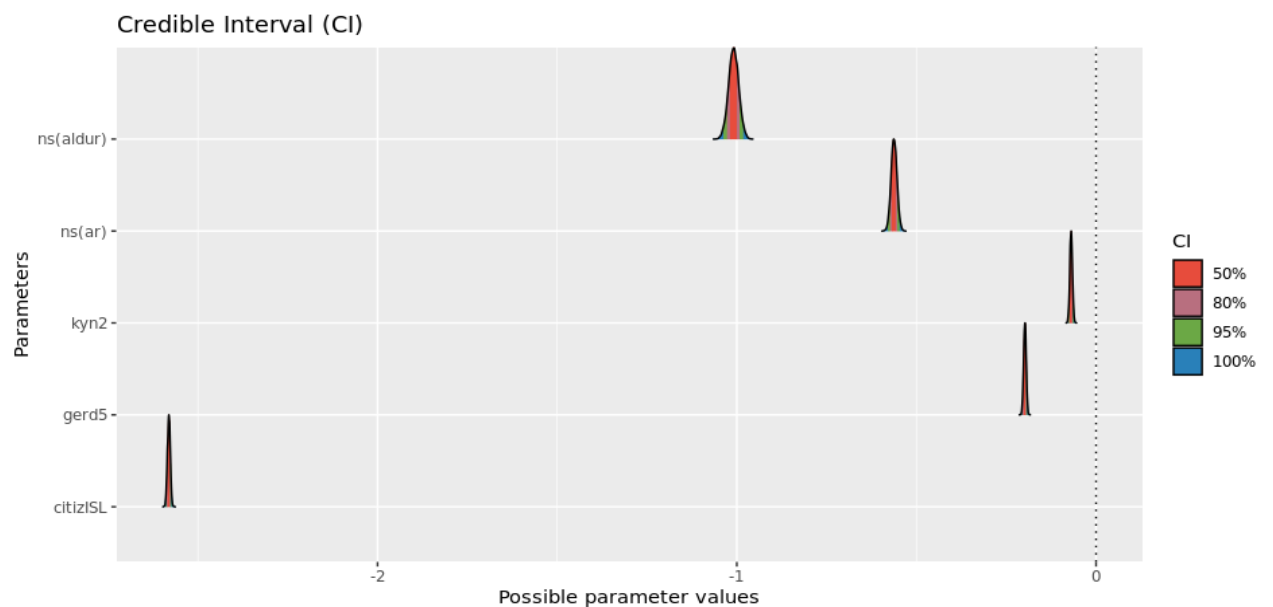
Characteristics' effects on fertility



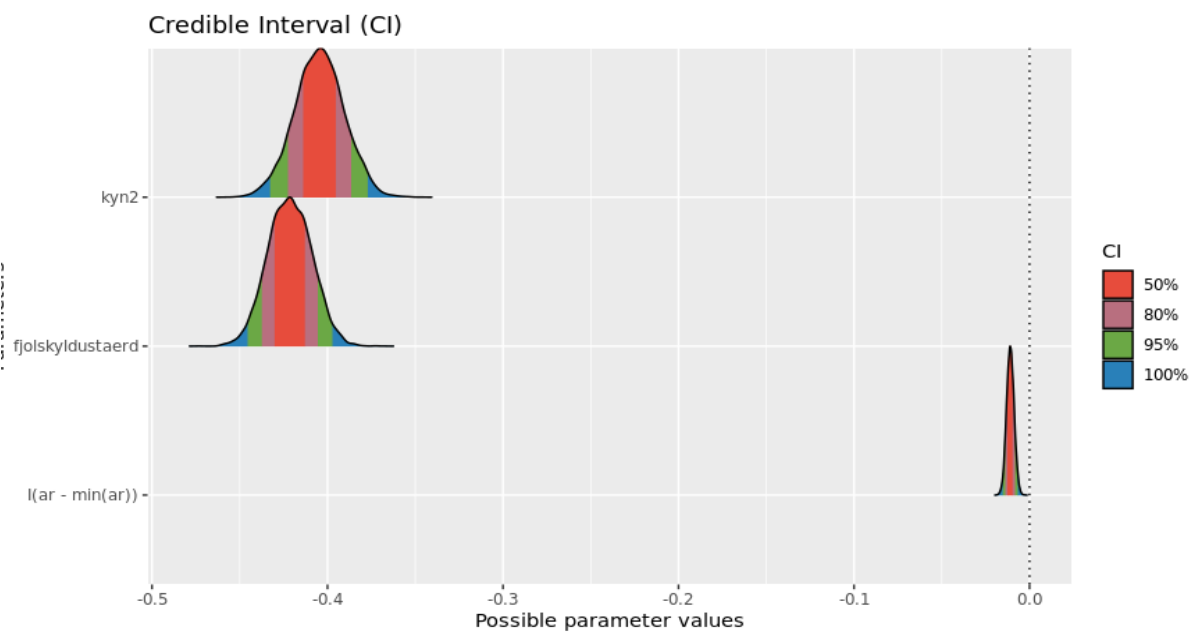
Microdata input

Example 2. Characteristics' effects on:

Migration

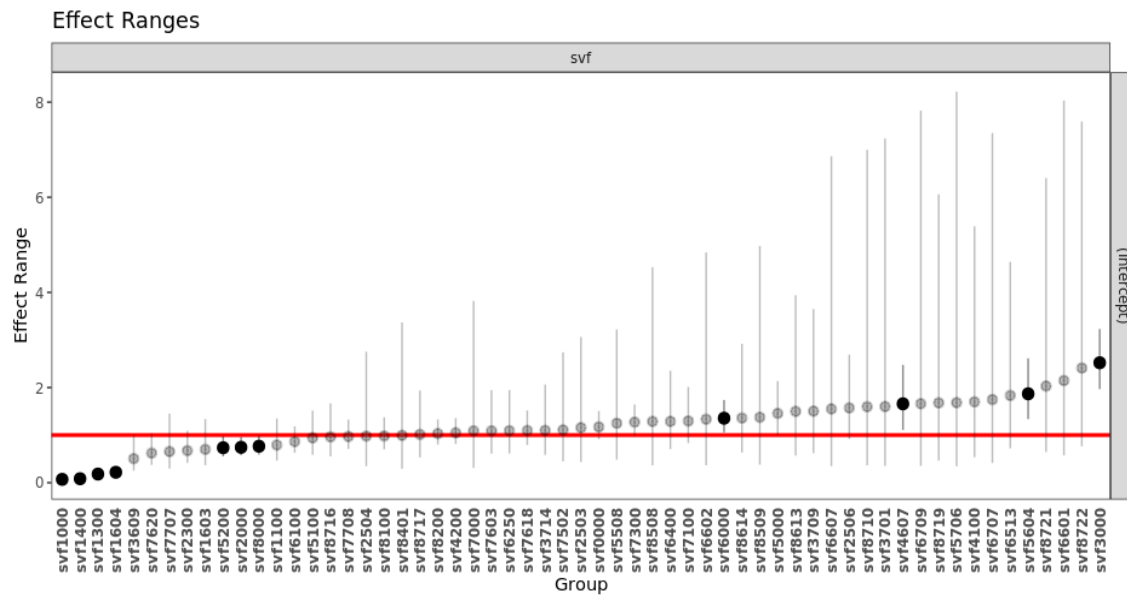


Mortality

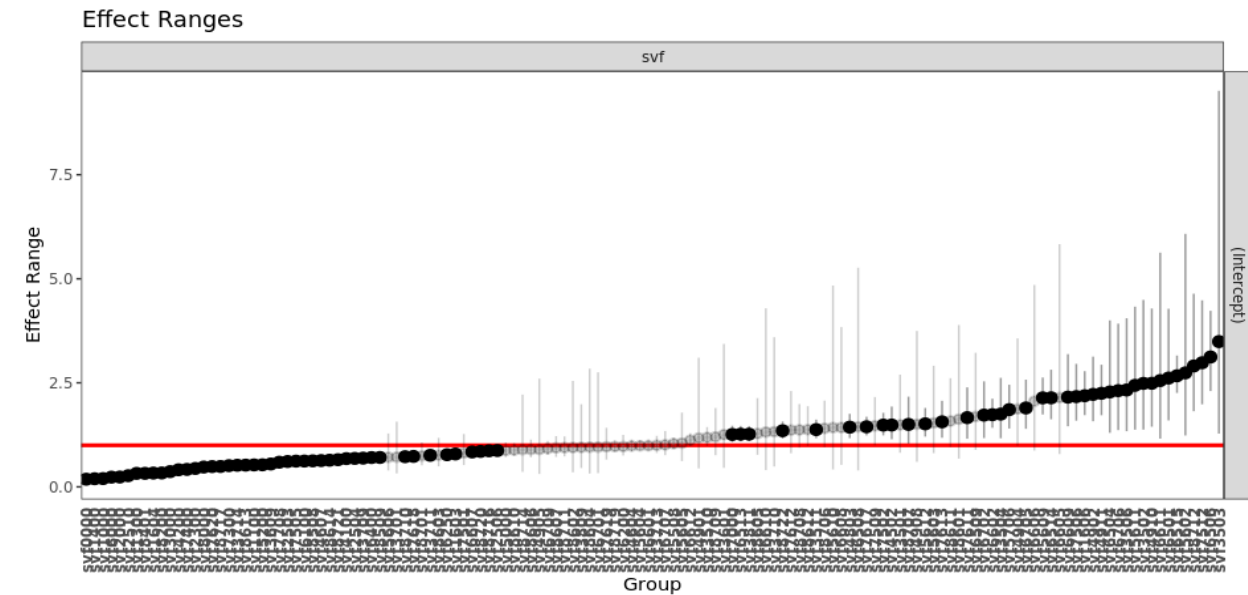


One more example: testing for location effects (although not with *mrf* yet!)

municipality - effect on fertility



Municipality – effect on migration



Future/work in progress/*

- finalise stochastic *population* predictions by combining high number of values sampled from the posterior distributions of the demographic components (migration, fertility, mortality) as in *ccpm*
- finalise the model *testing*
- forecasting with multiple *model averaging**

References

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Thank you!