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CSCI 3327 Formula Sheet

Chapter 1

Sample Mean

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Population Mean

$$\mu = \frac{1}{N}\sum_{i=1}^{N} y_i$$

Sample Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (y_i - \bar{y})^2$$

Population Variance

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (y_i - \mu)^2$$

Sample Standard Deviation

$$s = \sqrt{s^2}$$

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Emprical Rule

$\mu \pm \sigma$ contains approximately 68% of all the measurements

$\mu \pm 2\sigma$ contains approximately 95% of all the measurements

$\mu \pm 3\sigma$ contains almost all (99.7%) of all measurements

Chapter 2

Distributive Laws (Set Theory)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan's Laws

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$
$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

Axioms for Probability of A

1. $P(A) \geq 0$
2. $P(S) = 1$
3. If $A_1, A_2, A_3, \ldots$ form a sequence of pairwise mutually exclusive events in S (that is, $A_I \cap A_J = \emptyset$ if $i \neq j$), then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum_{I=1}^{\infty} P(A_i)$$

mn Rule

$$mn = m \times n$$

Permutation Formula

$$P_r^n = \frac{n!}{(n-r)!}$$

Combination Formula

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

Multinomial Coefficient Formula

$$N = \binom{n}{n_1\; n_2 \ldots\; n_k} = \frac{n!}{n_1!\, n_2! \cdots n_k!}$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

As long as $P(B) > 0$

Independence Equations

Two events A and B are considered independent if any of the following statements hold:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \cap B) = P(A)P(B)$$

Multiplicative Law of Probability (Dependent)

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) \\ &= P(B)P(A|B) \end{aligned}$$

Multiplicative Law of Probability (Independent)

$$P(A \cap B) = P(A)P(B)$$

Additive Law of Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events, where $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

Probability of event A

$$P(A) = 1 - P(\bar{A})$$

Partition of S

Let the sets $B_1, B_2, \ldots, B_k$ be such that

1. $S = B_1 \cup B_2 \cup \ldots \cup B_k$
2. $B_i \cap B_j = \emptyset$, for $i \neq j$

Then the collection of sets is said to be a partition of S.

The Law of Total Probability

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Bayes' Rule

$$P(A|B_j) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)\,P(B_i)}$$

<u>Chapter 3</u>

Probability Distribution statements

1. $0 \leq p(y) \leq 1$ for all of y
2. $\sum_y p(y) = 1$, where the summation is over all values of y with nonzero probability

Expected value of Y

$$E(Y) = \sum_{y} yp(y)$$

Expected value of g(Y)

$$E[g(Y)] = \sum_{all\ y} g(y)p(y)$$

Variance of a Random Variable

$$V(Y) = E[(Y - \mu)^2]$$

$$V(Y) = \sigma^2 = E[(Y - \mu)^2] = E(Y^2) - \mu^2$$

Standard Deviation of a Random Variable

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Theorems for Mean or Expected Value

$$E(c) = c$$

$$E[cg(Y)] = cE[g(Y)]$$

$$E[g_1(Y) + g_2(Y) + \ldots + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + \ldots + E[g_k(Y)]$$

Binomial Distribution

$$p(y) = P(Y = y) = \binom{n}{y} p^y q^{n-y}, y = 0, 1, 2, \ldots, n \text{ and } 0 \le p \le 1$$

Expected Value of a Binomial Distribution

$$\mu = E(Y) = np$$

Variance of a Binomial Distribution

$$\sigma^2 = V(Y) = npq$$

Standard Deviation of a Binomial Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Geometric Distribution

$$p(y) = P(Y = y) = q^{y-1}p, y = 1, 2, 3, \ldots, 0 \le p \le 1$$

Expected Value of a Geometric Distribution

$$\mu = E(Y) = \frac{1}{p}$$

Variance of a Geometric Distribution

$$\sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Standard Deviation of a Geometric Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Hypergeometric Probability Distribution

$$p(y) = P(Y = y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}, y \leq r \text{ and } n - y \leq N - r$$

Expected Value of Hypergeometric Distribution

$$\mu = E(Y) = \frac{nr}{N}$$

Variance of Hypergeometric Distribution

$$\sigma^2 = V(Y) = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Standard Deviation of Hypergeometric Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Negative Binomial Probability Distribution

$$p(y) = p(Y = y) = \binom{y-1}{r-1} p^r q^{y-r}, y = r, r+1, r+2, \ldots, 0 \leq p \leq 1$$

Expected Value of Negative Binomial Distribution

$$\mu = E(Y) = \frac{r}{p}$$

Variance of Negative Binomial Distribution

$$\sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

Standard Deviation of Negative Binomial Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Poisson Probability Distribution

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}, y = 0, 1, 2, \ldots, \lambda > 0$$

Mean and Variance of a Poisson Probability Distribution

$$\mu = E(Y) = \lambda \text{ and } \sigma^2 = V(Y) = \lambda$$

Lambda of Poisson Probability Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Tchebysheff's Theorem

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \text{ or } P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Chapter 4

Distribution Function of Y

$$F(y) = P(Y \leq y) \text{ for } -\infty < y < \infty$$

Properties of a Distribution Function

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y. [If y_1 and y_2 are any values such that $y_1 < y_2$, then $F(y_1) \leq F(y_2)$.]

Probability Density Function for Continuous Random Variable Y

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

Properties of a Density Function

1. $f(y) \geq 0$ for all y, $-\infty < y < \infty$
2. $\int_{-\infty}^{\infty} f(y)\, dy = 1$

Probability of Continuous Random Variable Y in an Interval

$$P(a \leq Y \leq b) = \int_a^b f(y)\, dy$$

Expected Value of Continuous Random Variable Y

$$E(Y) = \int_{-\infty}^{\infty} y f(y)\, dy$$

Variance of a Continuous Random Variable Y

$$\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2$$

Expected Value of g(Y)

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y) f(y)\, dy$$

Theorems for Continuous Random Variable Mean or Expected Value

$$E(c) = c$$

$$E[cg(Y)] = cE[g(Y)]$$

$$E[g_1(Y) + \ g_2(Y) + \ldots + g_k(Y)] = \ E[g_1(Y)] + \ E[g_2(Y)] + \ldots + E[g_k(Y)]$$

Uniform Probability Distribution

$$f(y) = \begin{cases} \dfrac{1}{\theta_2 - \theta_1}, & \theta_1 \le y \le \theta_2 \\ 0, & elsewhere \end{cases} = \begin{cases} \dfrac{1}{b-a}, a \le y \le b \\ 0, elsewhere \end{cases}$$

Finding the Probability using Uniform Probability Distribution

$$P(c \le y \le d) = \int_c^d f(y)dy = \int_c^d \frac{1}{b-a} dy = \frac{d-c}{b-a}$$

Expected Value for Uniform Probability Distribution

$$\mu = E(Y) = \frac{b-a}{2}$$

Variance for Uniform Probability Distribution

$$\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2 = \frac{(b-a)^2}{12}$$

Standard Deviation for Uniform Probability Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Normal Probability Distribution

If $\sigma > 0$ and $-\infty < y < \infty$, the density function of Y is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}, \qquad -\infty < y < \infty$$

Expected Value for Normal Probability Distribution

$$E(Y) = \mu$$

Variance for Normal Probability Distribution

$$V(Y) = \sigma^2$$

Standard Deviation for Normal Probability Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Finding a probability from a to b using a Normal Density Function

$$\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/(2\sigma^2)}\ dy$$

Transforming a normal random variable to a standard normal variable

$$Z = \frac{Y - \mu}{\sigma}$$

Gamma Probability Distribution

$$f(y) = \begin{cases} \dfrac{y^{\alpha-1}e^{-y/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, & 0 \leq y < \infty \\ 0, & elsewhere \end{cases}$$

where

$$\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1}e^{-y}\, dy$$

Expected Value for Gamma Probability Distribution

$$\mu = E(Y) = \alpha\beta$$

Variance for Gamma Probability Distribution

$$\sigma^2 = V(Y) = \alpha\beta^2$$

Standard Deviation for Gamma Probability Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Chi-square Distribution

Let v be a positive integer. A random variable Y is said to have a chi-square distribution with v degrees of freedom if and only if Y is a gamma-distribution random variable with parameters of $\alpha = \frac{v}{2}$ and $\beta = 2$.

Expected Value for Chi-Square Distribution

$$\mu = E(Y) = v$$

Variance for Chi-Square Distribution

$$\sigma^2 = V(Y) = 2v$$

Standard Deviation for Chi-Square Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Exponential Distribution

If $\alpha = 1$ and $\beta > 0$, then

$$f(y) = \begin{cases} \frac{1}{\beta} e^{-y/\beta}, & 0 \leq y < \infty \\ 0, & elsewhere \end{cases}$$

Expected Value for Exponential Distribution

$$\mu = E(Y) = \beta$$

Variance for Exponential Distribution

$$\sigma^2 = V(Y) = \beta^2$$

Standard Deviation for Exponential Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Beta Probability Distribution

$$f(y) = \begin{cases} \dfrac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)}, & 0 \leq y \leq 1 \\ 0, & elsewhere \end{cases}$$

where

$$B(\alpha,\beta) = \int_0^1 y^{\alpha-1}(1-y)^{\beta-1}\ dy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

Expected Value for Beta Probability Distribution

$$\mu = E(Y) = \frac{\alpha}{\alpha+\beta}$$

Variance for Beta Probability Distribution

$$\sigma^2 = V(Y) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Standard Deviation for Beta Probability Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Chapter 5

Joint (Bivariate) Probability Function

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2), \qquad -\infty < y_1 < \infty, -\infty < y_2 < \infty$$

Theorems for Joint (Bivariate) Probability Functions

If Y_1 and Y_2 are discrete random variables with joint probability function $p(y_1, y_2)$, then

1. $p(y_1, y_2) \geq 0$ for all y_1, y_2
2. $\sum_{y_1, y_2} p(y_1, y_2) = 1$, where the sum is over all values (y_1, y_2) That are assigned nonzero probabilities

Joint (Bivariate) Distribution Function

$$F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2), \qquad -\infty < y_1 < \infty, -\infty < y_2 < \infty$$

Joint Distribution Function from a Joint Probability Density Function

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 dt_1, \quad -\infty < y_1 < \infty, -\infty < y_2 < \infty$$

Theorems for Joint Distribution Functions

If Y_1 and Y_2 are random variables with joint distribution function $F(y_1, y_2)$, then

1. $F(-\infty, -\infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$
2. $F(\infty, \infty) = 1$
3. If $y_1^* \geq y_1$ and $y_2^* \geq y_2$, then

$$F(y_1^*, y_2^*) - F(y_1^*, y_2) - F(y_1, y_2^*) + F(y_1, y_2) \geq 0$$

Theorems for Joint Density Functions

If Y_1 and Y_2 are jointly continuous random variables with a joint density function given by $f(y_1, y_2)$, then

1. $f(y_1, y_2) \geq 0$ for all y_1, y_2
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1 dy_2 = 1$

Marginal Probability Functions

$p_1(y_1) = \sum_{all\ y_2} p(y_1, y_2)$ and $p_2(y_2) = \sum_{all\ y_1} p(y_1, y_2)$

Marginal Density Functions

$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_2$ and $f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2)\, dy_1$

Conditional Discrete Probability Function

$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1=y_1, Y_2=y_2)}{P(Y_2=y_2)} = \frac{p(y_1, y_2)}{p_2(y_2)}$, as long as $p_2(y_2) > 0$

Conditional Distribution Function

$$F(y_1|y_2) = P(Y_1 \leq y_1|Y_2 = y_2)$$

Conditional Density Functions

$f(y_1|y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$, where $f_2(y_2) > 0$

$f(y_2|y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$, where $f_1(y_1) > 0$

Independence Definition for Joint Random Variables using Distribution Functions

$F(y_1, y_2) = F_1(y_1)F_2(y_2)$ for every pair of real numbers (y_1, y_2)

If Y_1 and Y_2 are not independent, then they are said to be dependent

Independence Definition for Joint Discrete Random Variable using Probability Functions

$p(y_1, y_2) = p_1(y_1)p_2(y_2)$ for every pair of real numbers (y_1, y_2)

Independence Definition for Joint Continuous Random Variable using Probability Functions

$f(y_1, y_2) = f_1(y_1)f_2(y_2)$ for every pair of real numbers (y_1, y_2)

Independence Definition for Joint Continuous Random Variable if the Limits of Integration are Constants

Let Y_1 and Y_2 have a joint density $f(y_1, y_2)$ that is positive if and only if $a \leq y_1 \leq b$ and $c \leq y_2 \leq d$, for constants a, b, c and d; and $f(y_1, y_2) = 0$ otherwise. Then Y_1 and Y_2 are independent random variables if and only if

$$f(y_1, y_2) = \mathrm{g}(y_1)h(y_2)$$

where $\mathrm{g}(y_1)$ is a nonnegative function of y_1 alone and $h(y_2)$ is a nonnegative function of y_2 alone.