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CSCI 3327 Formula Sheet

Chapter 1

Sample Mean

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Population Mean

$$\mu = \frac{1}{N}\sum_{i=1}^{N} y_i$$

Sample Variance

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (y_i - \bar{y})^2$$

Population Variance

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{N} (y_i - \mu)^2$$

Sample Standard Deviation

$$s = \sqrt{s^2}$$

Population Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

Emprical Rule

$\mu \pm \sigma$ contains approximately 68% of all the measurements

$\mu \pm 2\sigma$ contains approximately 95% of all the measurements

$\mu \pm 3\sigma$ contains almost all (99.7%) of all measurements

Chapter 2

Distributive Laws (Set Theory)

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan's Laws

$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$
$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$

Axioms for Probability of A

1. $P(A) \geq 0$
2. $P(S) = 1$
3. If $A_1, A_2, A_3,$ . . . form a sequence of pairwise mutually exclusive events in S (that is, $A_I \cap A_J = \emptyset$ if $i \neq j$), then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum_{I=1}^{\infty} P(A_i)$$

mn Rule

$$mn = m \times n$$

Permutation Formula

$$P_r^n = \frac{n!}{(n-r)!}$$

Combination Formula

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

Multinomial Coefficient Formula

$$N = \binom{n}{n_1\ n_2 \ldots\ n_k} = \frac{n!}{n_1!\,n_2! \cdots n_k!}$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

As long as $P(B) > 0$

Independence Equations

Two events A and B are considered independent if any of the following statements hold:

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \cap B) = P(A)P(B)$$

Multiplicative Law of Probability (Dependent)

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) \\ &= P(B)P(A|B) \end{aligned}$$

Multiplicative Law of Probability (Independent)

$$P(A \cap B) = P(A)P(B)$$

Additive Law of Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events, where $P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B)$$

Probability of event A

$$P(A) = 1 - P(\bar{A})$$

Partition of S

Let the sets $B_1, B_2, \dots, B_k$ be such that

1. $S = B_1 \cup B_2 \cup \dots \cup B_k$
2. $B_i \cap B_j = \emptyset$, for $i \neq j$

Then the collection of sets is said to be a partition of S.

The Law of Total Probability

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Bayes' Rule

$$P(A|B_j) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)\,P(B_i)}$$

Chapter 3

Probability Distribution statements

1. $0 \le p(y) \le 1$ for all of y
2. $\sum_y p(y) = 1$, where the summation is over all values of y with nonzero probability

Expected value of Y

$$E(Y) = \sum_{y} yp(y)$$

Expected value of g(Y)

$$E[g(Y)] = \sum_{all\ y} g(y)p(y)$$

Variance of a Random Variable

$$V(Y) = E[(Y - \mu)^2]$$

$$V(Y) = \sigma^2 = E[(Y - \mu)^2] = E(Y^2) - \mu^2$$

Standard Deviation of a Random Variable

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Theorems for Mean or Expected Value

$$E(c) = c$$

$$E[cg(Y)] = cE[g(Y)]$$

$$E[g_1(Y) + g_2(Y) + \ldots + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + \ldots + E[g_k(Y)]$$

Binomial Distribution

$$p(y) = P(Y = y) = \binom{n}{y} p^y q^{n-y}, y = 0, 1, 2, \ldots, n \text{ and } 0 \leq p \leq 1$$

Expected Value of a Binomial Distribution

$$\mu = E(Y) = np$$

Variance of a Binomial Distribution

$$\sigma^2 = V(Y) = npq$$

Standard Deviation of a Binomial Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Geometric Distribution

$$p(y) = P(Y = y) = q^{y-1}p, y = 1, 2, 3, \ldots, 0 \leq p \leq 1$$

Expected Value of a Geometric Distribution

$$\mu = E(Y) = \frac{1}{p}$$

Variance of a Geometric Distribution

$$\sigma^2 = V(Y) = \frac{1-p}{p^2}$$

Standard Deviation of a Geometric Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Hypergeometric Probability Distribution

$$p(y) = P(Y = y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}, y \leq r \text{ and } n - y \leq N - r$$

Expected Value of Hypergeometric Distribution

$$\mu = E(Y) = \frac{nr}{N}$$

Variance of Hypergeometric Distribution

$$\sigma^2 = V(Y) = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

Standard Deviation of Hypergeometric Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Negative Binomial Probability Distribution

$$p(y) = p(Y = y) = \binom{y-1}{r-1} p^r q^{y-r}, y = r, r+1, r+2, \ldots, 0 \leq p \leq 1$$

Expected Value of Negative Binomial Distribution

$$\mu = E(Y) = \frac{r}{p}$$

Variance of Negative Binomial Distribution

$$\sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

Standard Deviation of Negative Binomial Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Poisson Probability Distribution

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}, y = 0, 1, 2, \ldots, \lambda > 0$$

Mean and Variance of a Poisson Probability Distribution

$$\mu = E(Y) = \lambda \text{ and } \sigma^2 = V(Y) = \lambda$$

Lambda of Poisson Probability Distribution

$$\sigma = \sqrt{\sigma^2} = \sqrt{V(Y)}$$

Tchebysheff's Theorem

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \text{ or } P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Chapter 4

Distribution Function of Y

$$F(y) = P(Y \leq y) \text{ for } -\infty < y < \infty$$

Properties of a Distribution Function

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y. [If y_1 and y_2 are any values such that $y_1 < y_2$, then $F(y_1) \leq F(y_2)$.]

Probability Density Function for Continuous Random Variable Y

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

Properties of a Density Function

1. $f(y) \geq 0$ for all y, $-\infty < y < \infty$
2. $\int_{-\infty}^{\infty} f(y)\, dy = 1$

Probability of Continuous Random Variable Y in an Interval

$$P(a \leq Y \leq b) = \int_a^b f(y)\, dy$$

Expected Value of Continuous Random Variable Y

$$E(Y) = \int_{-\infty}^{\infty} yf(y)\ dy$$

Expected Value of g(Y)

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)\, dy$$

Theorems for Continuous Random Variable Mean or Expected Value

$$E(c) = c$$

$$E[cg(Y)] = cE[g(Y)]$$

$$E[g_1(Y) + g_2(Y) + \ldots + g_k(Y)] = E[g_1(Y)] + E[g_2(Y)] + \ldots + E[g_k(Y)]$$