

CSC411 Assignment 3

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1 SVM



Figure 1: Plot of test SVM

2 Kernels

2.1 Positive semidefinite and quadratic form

Assume K is symmetric, we can decompose K into $U\Lambda U^T$

$$\begin{aligned} x^T K x &= x^T (U \Lambda U^T) x = (x^T U) \Lambda (U^T x) \\ &= \sum_{i=1}^d \lambda_i ([x^T U]_i)^2 \geq 0 \end{aligned}$$

2.2 Kernel properties

2.2.1 α

$K_{ij} = \alpha$, the matrix K has dimension of x or y , and each element is α . Since $\alpha > 0$, and all elements are equal, K is positive semidefinite

2.2.2 $f(x), f(y)$

$$K_{ij} = \langle \phi(x), \phi(y) \rangle,$$

define $\phi(x) = f(x), \forall f: \mathbb{R}^d \rightarrow \mathbb{R}$

define $\phi(y) = f(y), \forall f: \mathbb{R}^d \rightarrow \mathbb{R}$

Since $f(x)$ and $f(y)$ produce a scalar, $\langle \phi(x), \phi(y) \rangle = f(x) \cdot f(y)$

2.2.3 k_1 and k_2

If the gram matrix, K_1 of kernel k_1 and gram matrix, K_2 of kernel k_2 are positive semidefinite, by scaling them and adding each element, the new gram matrix of $a \cdot k_1(x, y) + b \cdot k_2(x, y)$, call it K , each element of K is positive since $a, b > 0$.

K is also symmetric because K_1 and K_2 are symmetric with the same dimension, and element wise addition and linear combination preserve the symmetric property.

2.2.4 $k(x, y) = \frac{k_1(x, y)}{\sqrt{k_1(x, x)}\sqrt{k_1(y, y)}}$

Let ϕ_1 be the mapping defined by k_1

We define a new mapping, ϕ for $k(x, y)$

We let $\phi(x) = \frac{\phi_1(x)}{\|\phi_1(x)\|}$

$$\begin{aligned}
k(x, y) &= \langle \phi(x), \phi(y) \rangle \\
&= \frac{\phi_1(x)}{\|\phi_1(x)\|} \cdot \frac{\phi_1(y)}{\|\phi_1(y)\|} \\
&= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(x)}} \cdot \frac{\phi_1(y)}{\sqrt{\phi_1(y) \cdot \phi_1(y)}} \\
&= \frac{\phi_1(x)}{(\sqrt{\phi_1(x)} \cdot \sqrt{\phi_1(y)})} \cdot \frac{\phi_1(y)}{(\sqrt{\phi_1(x)} \cdot \sqrt{\phi_1(y)})} \\
&= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \cdot \frac{\phi_1(y)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \\
k(x, y) &= \frac{k_1(x, y)}{\sqrt{k_1(x, x)}\sqrt{k_1(y, y)}}
\end{aligned}$$