

# CSC411 Assignment 3

Yue Guo

November 30, 2017

## 1 20 News Group

### 1.1 Algorithms

#### 1.1.1 KNN

$k = 20$

Feature selection:

=====  $f = 180$  KNN train accuracy = 0.510783100583 KNN test accuracy = 0.438263409453 =====  $k = 320$  KNN train accuracy = 0.56222379353 KNN test accuracy = 0.456850770048 =====  $k = 350$  KNN train accuracy = 0.565847622415 KNN test accuracy = 0.458311205523 =====  $k = 500$  KNN train accuracy = 0.562047021389 KNN test accuracy = 0.445698353691

#### 1.1.2 Decision Tree

feature, depth

#### 1.1.3 Random forest

sfgdf

## 2 SVM

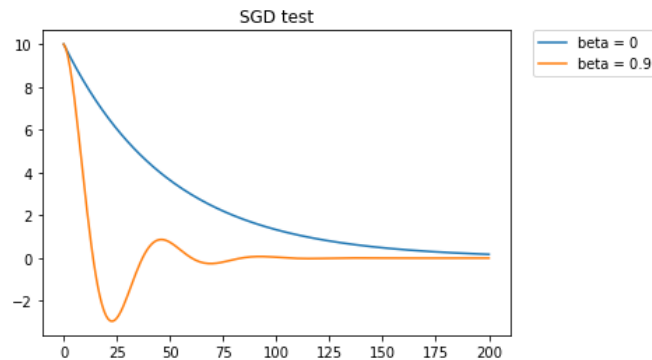


Figure 1: Plot of test SVM

### 3 Kernels

#### 3.1 Positive semidefinite and quadratic form

Assume  $K$  is symmetric, we can decompose  $K$  into  $U\Lambda U^T$

$$\begin{aligned}x^T K x &= x^T (U\Lambda U^T) x = (x^T U) \Lambda (U^T x) \\&= \sum_{i=1}^d \lambda_i ([x^T U]_i)^2 \geq 0\end{aligned}$$

#### 3.2 Kernel properties

##### 3.2.1 $\alpha$

$K_{ij} = \alpha$ , the matrix  $K$  has dimension of  $x$  or  $y$ , and each element is  $\alpha$ . Since  $\alpha > 0$ , and all elements are equal,  $K$  is positive semidefinite

##### 3.2.2 $f(x), f(y)$

$$K_{ij} = \langle \phi(x), \phi(y) \rangle,$$

define  $\phi(x) = f(x), \forall f : \mathbb{R}^d \rightarrow \mathbb{R}$

define  $\phi(y) = f(y), \forall f : \mathbb{R}^d \rightarrow \mathbb{R}$

Since  $f(x)$  and  $f(y)$  produce a scalar,  $\langle \phi(x), \phi(y) \rangle = f(x) \cdot f(y)$

##### 3.2.3 $k_1$ and $k_2$

If the gram matrix,  $K_1$  of kernel  $k_1$  and gram matrix,  $K_2$  of kernel  $k_2$  are positive semidefinite, by scaling them and adding each element, the new gram matrix of  $a \cdot k_1(x, y) + b \cdot k_2(x, y)$ , call it  $K$ , each element of  $K$  is positive since  $a, b > 0$ .

$K$  is also symmetric because  $K_1$  and  $K_2$  are symmetric with the same dimension, and element wise addition and linear combination preserve the symmetric property.

##### 3.2.4 $k(x, y) = \frac{k_1(x, y)}{\sqrt{k_1(x, x)} \sqrt{k_1(y, y)}}$

Let  $\phi_1$  be the mapping defined by  $k_1$

We define a new mapping,  $\phi$  for  $k(x, y)$

We let  $\phi(x) = \frac{\phi_1(x)}{\|\phi_1(x)\|}$

$$\begin{aligned}k(x, y) &= \langle \phi(x), \phi(y) \rangle \\&= \frac{\phi_1(x)}{\|\phi_1(x)\|} \cdot \frac{\phi_1(y)}{\|\phi_1(y)\|} \\&= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(x)}} \cdot \frac{\phi_1(y)}{\sqrt{\phi_1(y) \cdot \phi_1(y)}} \\&= \frac{\phi_1(x)}{(\sqrt{\phi_1(x)} \cdot \sqrt{\phi_1(y)})} \cdot \frac{\phi_1(y)}{(\sqrt{\phi_1(x)} \cdot \sqrt{\phi_1(y)})} \\&= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \cdot \frac{\phi_1(y)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \\k(x, y) &= \frac{k_1(x, y)}{\sqrt{k_1(x, x)} \sqrt{k_1(y, y)}}\end{aligned}$$