CSC411 Assignment 3

Yue Guo

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1 SVM

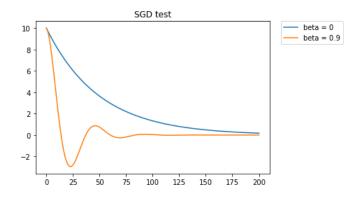


Figure 1: Plot of test SVM

2 Kernels

2.1 Positive semidefinite and quadratic form

Assume K is symmetric, we can decompose K into $U\Lambda U^T$

$$x^T K x = x^T (U \Lambda U^T) x = (x^T U) \Lambda (U^T x)$$
$$= \sum_{i=1}^d \lambda_i ([x^T U_i])^2 >= 0$$

2.2 Kernel properties

2.2.1 α

 $K_{ij} = \alpha$, the matrix K has dimension of x or y, and each element is α . Since $\alpha > 0$, and all elements are equal, K is positive semidefinite

2.2.2 f(x), f(y)

$$K_{ij} = \langle \phi(x), \phi(y) \rangle$$
, define $\phi(x) = f(x), \forall f : \mathbb{R}^d \to \mathbb{R}$ define $\phi(y) = f(y), \forall f : \mathbb{R}^d \to \mathbb{R}$ Since $f(x)$ and $f(y)$ produce a scalar, $\langle \phi(x), \phi(y) \rangle = f(x) \cdot f(y)$

2.2.3 k1 and k2

If the gram matrix, K_1 of kernel k1 and gram matrix, K_2 of kernel k2 are positive semidefinite, by scaling them and adding each element, the new gram matrix of $a \cdot k_1(x,y) + b \cdot k_2(x,y)$, call it K, each element of K is positive since a b > 0.

K is also symmetric because K_1 and K_2 are symmetric with the same dimension, and element wise addition and linear combination preserve the symmetric property.

2.2.4
$$k(x,y) = \frac{k_1(x,y)}{\sqrt{k_1(x,x)}\sqrt{k_1(y,y)}}$$

Let ϕ_1 be the mapping defined by k_1 We define a new mapping, ϕ for k(x,y) We let $\phi(x) = \frac{\phi_1(x)}{\|\phi_1(x)\|}$

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$$\phi(x) = \frac{\phi_1(x)}{\|\phi_1(x)\|}$$

$$k(x,y) = < \phi(x), \phi(y) >$$

$$= \frac{\phi_1(x)}{\|\phi_1(x)\|} \cdot \frac{\phi_1(y)}{\|\phi_1(y)\|}$$

$$= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(x)}} \cdot \frac{\phi_1(y)}{\sqrt{\phi_1(y) \cdot \phi_1(y)}}$$

$$= \frac{\phi_1(x)}{(\sqrt{\phi_1(x) \cdot \sqrt{\phi_1(y)}})} \cdot \frac{\phi_1(y)}{(\sqrt{\phi_1(x) \cdot \sqrt{\phi_1(y)}})}$$

$$= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \cdot \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}}$$

$$k(x,y) = \frac{k_1(x,y)}{\sqrt{k_1(x,x)}\sqrt{k_1(y,y)}}$$