CSC411 Assignment 3

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1 Kernels

1.1 Positive semidefinite and quadratic form

Assume K is symmetric, we can decompose K into $U\Lambda U^T$

$$\begin{aligned} x^T K x &= x^T(U\Lambda U^T)x = (x^T U)\Lambda(U^T x) \\ &= \Sigma_{i=1}^d \lambda_i([x^T U_i])^2 >= 0 \end{aligned}$$

1.2 Kernel properties

1.2.1 α

$K_{ij} = \alpha$, the matrix K has dimension of x or y, and each element is α. Since $\alpha > 0$, and all elements are eqaul, K is positive semidefinite

1.2.2 $f(x), f(y)$

$K_{ij} =< \phi(x), \phi(y) >$,
define $\phi(x) = f(x), \forall f : \mathbb{R}^d \to \mathbb{R}$
define $\phi(y) = f(y), \forall f : \mathbb{R}^d \to \mathbb{R}$
Since f(x) and f(y) produce a scalar, $< \phi(x), \phi(y) >= f(x) \cdot f(y)$

1.2.3 k1 and k2

If the gram matrix, K_1 of kernel k1 and gram matrix, K_2 of kernel k2 are positive semidefinite, by scaling them and adding each element, the new gram matrix of $a \cdot k_1(x, y) + b \cdot k_2(x, y)$, call it K, each element of K is positive since a ,b > 0.
K is also symmetric because K_1 and K_2 are symmetric with the same dimension, and element wise addition and linear combination preserve the symmetric property.

1.2.4 $k(x, y) = \frac{k_1(x,y)}{\sqrt{k_1(x,x)}\sqrt{k_1(y,y)}}$

Let ϕ_1 be the mapping defined by k_1
We define a new mapping, ϕ for $k(x, y)$
We let $\phi(x) = \frac{\phi_1(x)}{\|\phi_1(x)\|}$

$$
\begin{aligned}
k(x,y) &=< \phi(x), \phi(y) > \\
&= \frac{\phi_1(x)}{\|\phi_1(x)\|} \cdot \frac{\phi_1(y)}{\|\phi_1(y)\|} \\
&= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(x)}} \cdot \frac{\phi_1(y)}{\sqrt{\phi_1(y) \cdot \phi_1(y)}} \\
&= \frac{\phi_1(x)}{(\sqrt{\phi_1(x)} \cdot \sqrt{\phi_1(y)})} \cdot \frac{\phi_1(y)}{(\sqrt{\phi_1(x)} \cdot \sqrt{\phi_1(y)})} \\
&= \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \cdot \frac{\phi_1(x)}{\sqrt{\phi_1(x) \cdot \phi_1(y)}} \\
k(x,y) &= \frac{k_1(x,y)}{\sqrt{k_1(x,x)}\sqrt{k_1(y,y)}}
\end{aligned}
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