CSC411 Assignment 3

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1 Kernels

1.1 Positive semidefinite and quadratic form

Assume K is symmetric, we can decompose K into $U\Lambda U^T$

$$\begin{aligned} x^T K x &= x^T(U\Lambda U^T)x = (x^T U)\Lambda(U^T x) \\ &= \Sigma_{i=1}^d \lambda_i([x^T U_i])^2 >= 0 \end{aligned}$$

1.2 Kernel properties

1.2.1 α

$K_{ij} = \alpha$, the matrix K has dimension of x or y, and each element is α. Since $\alpha > 0$, and all elements are eqaul, K is positive semidefinite

1.2.2 $f(x), f(y)$

$K_{ij} =< \phi(x), \phi(y) >$,
define $\phi(x) = f(x), \forall f : \mathbb{R}^d \rightarrow \mathbb{R}$
define $\phi(y) = f(y), \forall f : \mathbb{R}^d \rightarrow \mathbb{R}$
Since f(x) and f(y) produce a scalar, $< \phi(x), \phi(y) >= f(x) \cdot f(y)$

1.2.3 k1 and k2