Exercise 3: Predicate Logic 1

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1. Convert these statements to first order logic formulae, making clear what making clear what the domain is, and what each predicate or function you use represents:
   (a) Every student in this class knows Java.
   (b) Somebody in this class knows a language that is not English.
   (c) Some person cannot swim.
   (d) Everybody has a friend who can swim.
   (e) There is a person whose friends can all swim.
2. If the domain is the integers, with $\times$ being the multiplication function, say in English what the following formulæ in First Order Logic represent:
   (a) $\forall x \, \forall y \, (x \times y) = (y \times x)$
   (b) $\forall x \, \exists y \, x = (y \times y)$
   (c) $\exists y \, \forall x \, x = (y \times y)$
3. Which of the formulæ of part 2 are true when the domain is integers?
4. Which of the formulæ of part 2 are true when the domain is real numbers?
5. Use the rules of natural deduction to prove the following inferences are valid:
   (a) $\forall x \, (P(x) \wedge Q(x)) \vdash \forall y \, P(y) \wedge \forall z \, Q(z)$
   (b) $\forall x \, \forall y \, R(x, y) \vdash \forall z \, R(z, z)$
   (c) $\exists x \, (P(x) \wedge Q(x)) \vdash \exists y \, P(y) \wedge \exists z \, Q(z)$
   (d) $\forall x \, P(x) \vdash \neg(\exists y \, (\neg P(y)))$