CS3050: Logic and Reasoning

Exercise 2: Propositional Logic II

Adam D. Barwell

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Exercises taken from Huth and Ryan §1.7.

1. Prove the validity of the following sequents.

    (a) $Q \to R \vdash (P \to Q) \to (P \to R)$

    (b) $P \to Q, R \to S \vdash P \vee R \to Q \vee S$

    (c) $(P \vee (Q \to P)) \wedge Q \vdash P$

    (d) $P \to Q, R \to S \vdash P \wedge R \to Q \wedge S$

    (e) $P \to Q \wedge R \vdash (P \to Q) \wedge (P \to R)$

    (f) $\vdash \neg P \to P \to P \to Q$

    (g) $P \wedge Q \vdash \neg(\neg P \vee \neg Q)$

    (h) $P \to Q \vdash \neg P \vee Q$

2. For the sequents below, show which are valid and which are not.

    (a) $\neg P \vee \neg Q \vdash \neg(P \wedge Q)$

    (b) $\neg P \wedge \neg Q \vdash \neg(P \vee Q)$

    (c) $P \vee Q, \neg Q \vee R \vdash P \vee R$

    (d) $\neg(\neg P \vee Q) \vdash P$