

CS3050: Exercise Sheet — Graph Theory and Induction I

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1. Can you draw a graph with 7 vertices, each of degree 5?
2. Let G be a graph with v vertices and e edges. Prove that if M is the maximum degree among the vertices of G , then $2e/v \leq M$.
3. Prove that among 100 people, there are at least 9 who were born in the same month.
4. Can six houses be connected to two utilities without any of the connections crossing?
5. Prove that a graph G is bipartite if and only if we can colour the vertices of G using one of two colours, such that each vertex gets a colour and no two vertices of the same colour are neighbours.
6. Prove that for all positive integers n , $1 \times 1! + 2 \times 2! + \cdots + n \times n! = (n+1)! - 1$
7. Prove that for all positive integers n and $r \neq 1$, $1 + r + r^2 + \cdots + r^n = \frac{r^{n+1}-1}{r-1}$
8. Prove that for all positive integers n , 3 divides $(5^{2n} - 1)$.
9. Prove that for all positive integers n , 43 divides $6^{n+1} + 7^{2n-1}$.