

## Exercise 4: Predicate Logic 2

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1. Use the rules of natural deduction to prove the following inferences are valid:

- (a)  $\forall x (P(x) \rightarrow Q(x)), \forall y (Q(y) \rightarrow R(y)) \vdash \forall z (P(z) \rightarrow R(z))$
- (b)  $\exists x (P(x) \wedge Q(x)), \forall y (Q(y) \rightarrow R(y)) \vdash \exists z (P(z) \wedge R(z))$
- (c)  $\exists x P(x) \vdash \neg(\forall y (\neg P(y)))$
- (d)  $P(b) \vdash \forall x (b = x \rightarrow P(x))$
- (e)  $x = y, y = z \vdash x = z$

2. Let  $m$  be a constant, let  $f$  be a function symbol of arity 1, and let  $S$  and  $B$  be two predicate symbols of arity 2. Which of the following strings are formulas in predicate logic? Specify a reason for failure for strings which aren't.

- (a)  $S(m, x)$
- (b)  $B(m, f(m))$
- (c)  $f(m)$
- (d)  $B(B(m, x), y)$
- (e)  $(B(x, y) \rightarrow (\exists z S(z, y)))$
- (f)  $(S(x, y) \rightarrow S(y, f(f(x))))$
- (g)  $B(x) \rightarrow B(B(x))$

3. Consider the domain  $D = \{a, b, c, d, e\}$ , and the predicates  $P$  (arity 1) and  $Q$  (arity 2). For each of the following formulas, define a model under which the formula is true.

- (a)  $\exists x P(x)$
- (b)  $\forall x P(x)$
- (c)  $\forall x (\neg P(y) \rightarrow \exists y Q(x, y))$
- (d)  $\forall x \exists y (Q(x, y) \vee Q(y, x))$
- (e)  $\exists x (\neg P(x) \wedge \forall y (Q(x, y)))$