

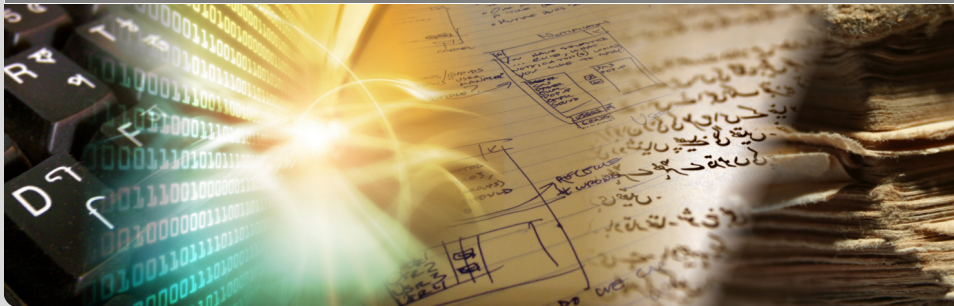
# Bachelor's Thesis

## Flexible User-Friendly Trip Planning Queries

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- *Sequenced Route Queries* (SRQ) - finding routes passing through multiple *Points of Interest* (Pols)
- Advances in *Location Based Services* (LBS) and *Geographic Information System* (GIS) applications (e.g. logistics and supply chain management)
- Aim: Designing a language to enable the user to express his query requirements in a flexible manner

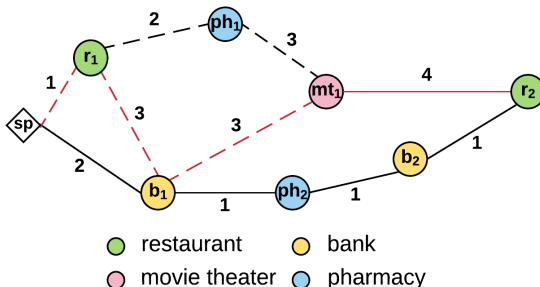


- Vector vs. metric spaces
- *Trip Planning Queries* (TPQ) [1]
- The *Optimal Sequenced Route* (OSR) Query [2]
- The Skylyne concept applied to SRQ [3]
- Considering multiple factors of a route – rating of Pols, distance and category weights, dynamic factors (e.g. traffic information) [4]
- SRQ issued by users moving along a route [5]
- *Multi-rule Partial Sequenced Route* (MRPSR) Query [6]

- **Category sequence:** (restaurant, bank, movie theater, restaurant)  
**Condition:** equal restaurants
- *Optimal Sequenced Route (OSR):*  $(r_1, b_1, mt_1, r_2)$ , length: 11 (shown with red lines)
- Optimal route with equal restaurants:  $(r_1, b_1, mt_1, r_1)$ , length: 12 (shown with dashed lines)

# Example

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- **Problem:** Need for flexibility in route finding queries
- **Solution:** Developing a query language operators to fulfill the essential user's requirements:
  - Relationships among the Pols
  - Order and priority of the Pols
  - Expressing multiple travel variations
- Proposing four essential operators: "equality" operator, "inequality" operator, "or" operator, "order" operator
- Making use of existing approaches (PNE (*Progressive Neighbor Exploration*) [2]) to transform the complex user query

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- *Input*: A category sequence  $M = (c_1, c_2, \dots, c_l)$ , a starting point  $sp$  in  $\mathbb{R}^2$  and indices of the equal Pols  $i$  and  $j$ , where  $c_i = c_j$
- *Output*: Optimal route  $R = (r_1, r_2, \dots, r_l)$ , where  $r_i = r_j$
- **Proposed approach**: uses the Progressive Neighbor Explorator (PNE) as its base to upgrade on and extends it with a heuristic approach to shrink the search space
- **Baseline/trivial approach**: extends the PNE with forcing the Pols  $r_i$  and  $r_j$  to be equal; does not use optimization techniques

# Heuristic

Given a sequence of categories  $M = (c_1, c_2, \dots, c_l)$  and a PSR  $R' = (r_1, r_2, \dots, r_k)$  the **heuristic** for this route is defined as:

$$h(R') = \max_{i \in [k+1, l]} \text{nearestNeighbor}(r_k, C_{M_i}) \quad (1)$$

- Informal: The heuristic of a certain PSR is the maximum distance out of the distances to the nearest Pols from the set of categories that are yet to be expanded.

The **lower bound** of a PSR  $R'$  represents the sum of its length and its heuristic:

$$LB(R') = \text{length}(R') + h(R') \quad (2)$$

- The proposed algorithm uses a heap, sorted by the lower bound of the routes

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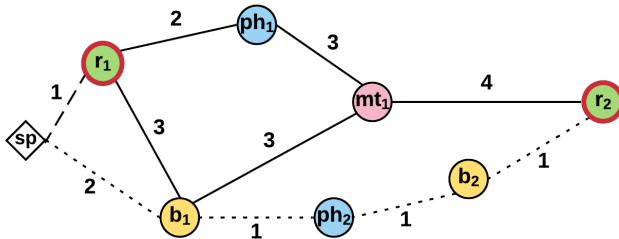
$$LB(R') = \text{length}(R') + h(R') \quad (2)$$

- The proposed algorithm uses a heap, sorted by the lower bound of the routes

# Example: Step 1

$sp, M = (r, b, mt, r), EQUAL(0, 3)$

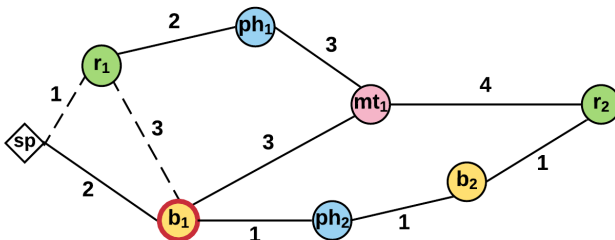
- Optimal route found with PNE:  $(r_1, b_1, mt_1, r_2)$
- Dummy SR:  $(r_1, b_1, mt_1, r_1)$ ; Upper Bound:  $length(dummySR) = 12$ , Candidate SR:  $dummySR$



Step	Heap contents (PSR $R$ : $length(R), heuristic(R)$ )
1	$(r_1 : 1, 5), (r_2 : 5, 4)$

## Example: Step 2

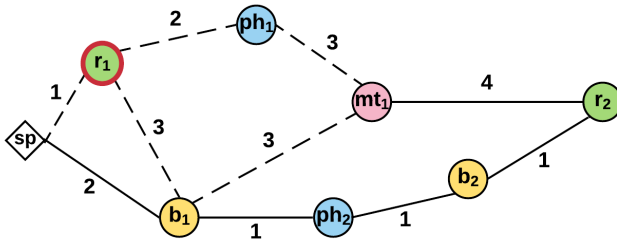
$sp, M = (r, b, mt, r), EQUAL(0, 3)$



Step	Heap contents (PSR $R$ : $length(R)$ , $heuristic(R)$ )
1	$(r_1 : 1, 5), (r_2 : 5, 4)$
2	$(r_1, b_1 : 4, 3), (r_2 : 5, 4)$

# Example: Step 8

$sp, M = (r, b, mt, r), EQUAL(0, 3)$

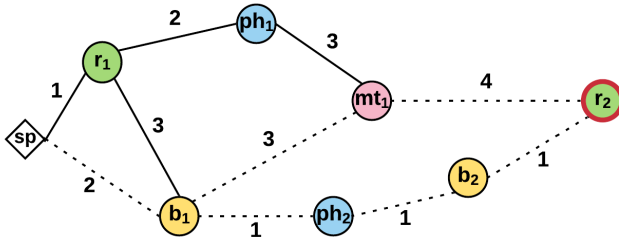


Candidate SR:  $(r_1, b_1, mt_1, r_1 : 12, 0)$

Step	Heap contents (PSR $R : length(R), heuristic(R)$ )
7	$(r_1, b_1, mt_1 : 7, 5), (r_2, b_1, mt_1 : 11, 4), (r_2, b_2, mt_1 : 11, 4), (r_1, b_2, mt_1 : 11, 5)$
8	$(r_2, b_1, mt_1 : 11, 4), (r_2, b_2, mt_1 : 11, 4), (r_1, b_2, mt_1 : 11, 5)$

# Example: Step 9

$sp, M = (r, b, mt, r), EQUAL(0, 3)$

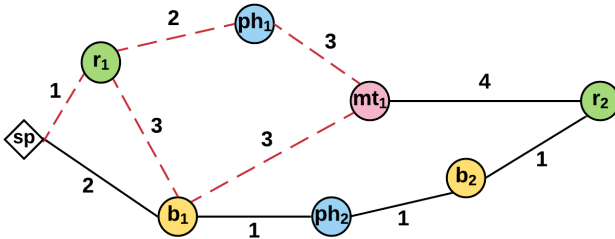


Candidate SR:  ~~$(r_2, b_1, mt_1, r_2 : 15, 0)$~~

Step	Heap contents (PSR $R : length(R), heuristic(R)$ )
8	$(r_2, b_1, mt_1 : 11, 4), (r_2, b_2, mt_1 : 11, 4), (r_1, b_2, mt_1 : 11, 5)$
9	$(r_2, b_2, mt_1 : 11, 4), (r_1, b_2, mt_1 : 11, 5)$

# Example: Result

$sp, M = (r, b, mt, r), EQUAL(0, 3)$



Found SR:  $(r_1, b_1, mt_1, r_1 : 12, 0)$



- *Input*: A category sequence  $M = (c_1, c_2, \dots, c_l)$ , a starting point  $sp$  in  $\mathbb{R}^2$  and indices of the unequal Pols  $i$  and  $j$ , where  $c_i = c_j$
- *Output*: Optimal route  $R = (r_1, r_2, \dots, r_l)$ , where  $r_i \neq r_j$
- **Proposed approach**: generates routes based on the Progressive Neighbor Explorer (PNE) and inspects partial routes for satisfying the requirement of unequal Pols and modifies them accordingly

- **OR sequence:** An OR sequence  $OR = (M_1, M_2, \dots, M_m)$  represents the disjunction of category sequences, such as  $M_1 = (c_1, c_2, \dots, c_l)$ .
- *Input:* A sequence of OR sequences  $S = (OR_1, OR_2, \dots, OR_n)$  and a starting point  $sp$  in  $\mathbb{R}^2$
- *Output:* Optimal route  $R = (r_1, r_2, \dots, r_l)$
- **Proposed approach:** progressively inspects each option  $M_i$  from the OR sequences  $OR_i$  in  $S = (OR_1, OR_2, \dots, OR_n)$ , compares them and continues with the best one, based on length, until it reaches a full sequenced route
- **Proposed approach:** runs the PNE algorithm on all possible combinations of the query to find the shortest route out of them

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# "Order" operator

- **ORDER sequence:** An order sequence  $ORDER = (i_1, i_2, \dots, i_k)$ , is a sequence of indices in a category sequence  $M = (c_1, c_2, \dots, c_l)$ , which indicate that the categories at the given indices should remain in the fixed positions in this category sequence.
- **Input:** A sequence of categories  $M = (c_1, c_2, \dots, c_l)$ , a starting point  $sp$  in  $\mathbb{R}^2$  and an ORDER sequence  $ORDER = (i_1, i_2, \dots, i_k)$
- **Output:** Optimal route  $R = (r_1, r_2, \dots, r_l)$
- **Proposed approach:** inspects progressively each category option for the indices out of the NOTORDERED sequence  $NOTORDERED = \overline{ORDER}$ , compares them and continues with the best one, based on length, until it reaches a full sequenced route.
- **Proposed approach:** runs the PNE algorithm on all possible permutations of the query to find the shortest route out of them

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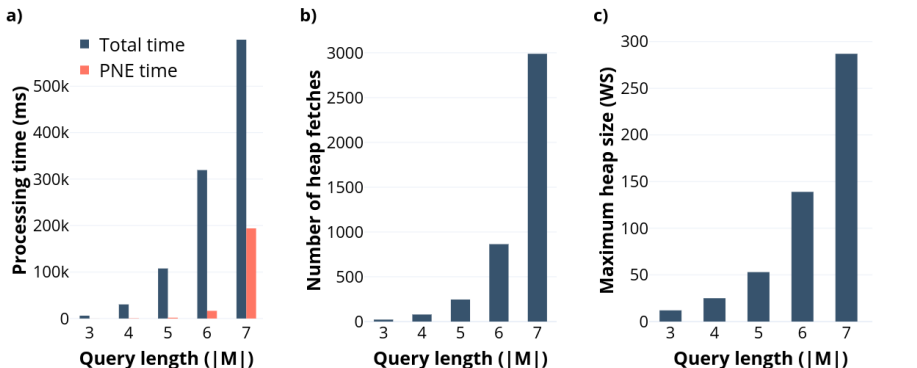
- Road network of Berlin, with 428769 crossroad nodes, 504229 road edges, 5548 Pols and 7 category types
- 1000 queries with randomly selected starting points

<i>Categories</i>	<i>Size</i>	<i>Frequency</i>
Restaurants	2081	High
Coffee shops	1002	
Pubs and bars	958	
Atms/Banks	597	Middle
Pharmacies	589	
Gas stations	180	Low
Movie theaters	141	

- **Quantitative evaluation criteria:** run time, number of heap fetches, maximum heap size (work space)
- **Qualitative evaluation:** compare against baseline approaches

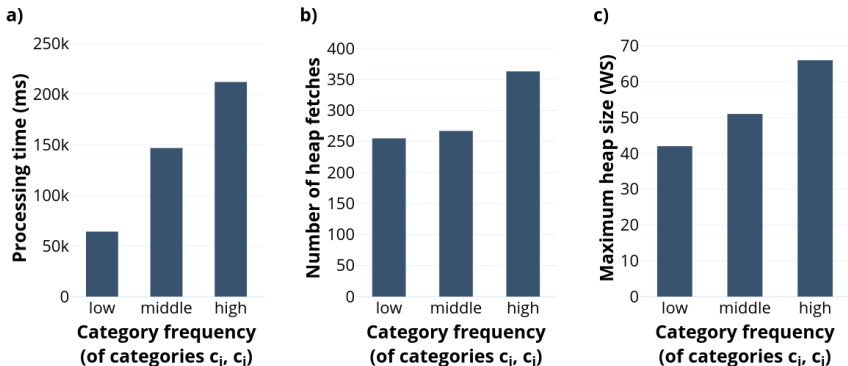
# "Equality" operator: Query length

- **Evaluation parameters:** (1) query length, (2) frequency of the categories  $c_i$ ,  $c_j$  in the category sequence and (3) distance between the categories  $c_i$ ,  $c_j$



# "Equality" operator: Category frequency

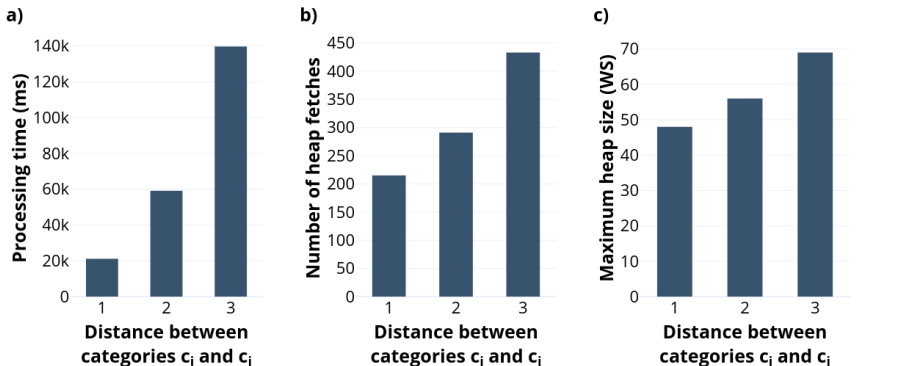
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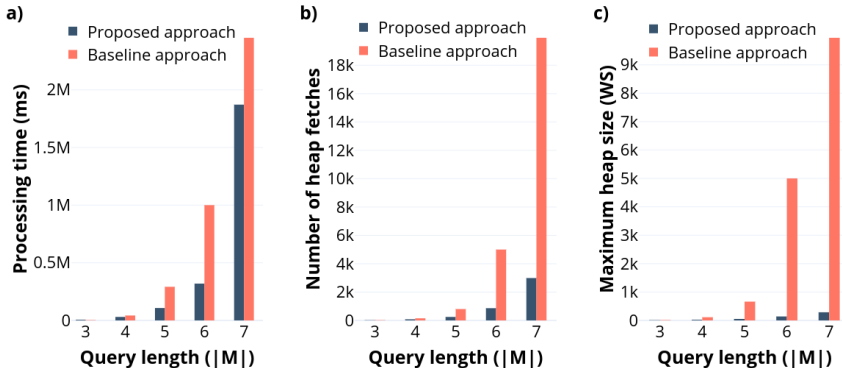


# "Equality" operator: Category distance

- **Evaluation parameters:** (1) query length, (2) frequency of the categories  $c_i$ ,  $c_j$  in the category sequence and (3) distance between the categories  $c_i$ ,  $c_j$

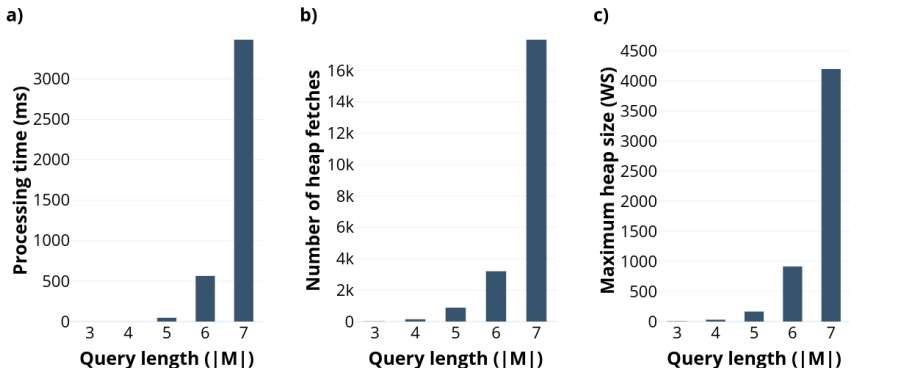


# "Equality" operator: Baseline approach



# "Inequality" operator: Query length

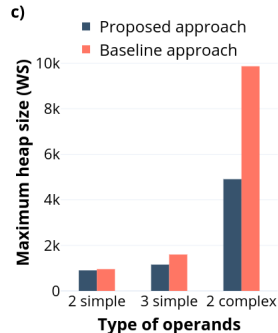
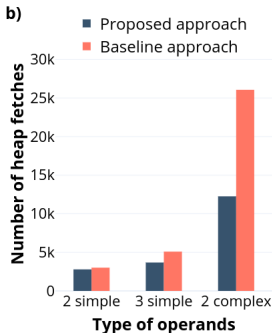
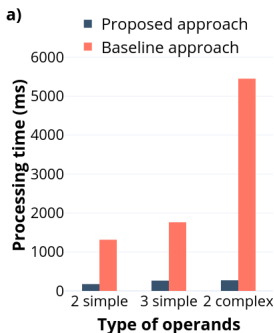
- **Evaluation parameters:** (1) query length, (2) frequency of the categories  $c_i$ ,  $c_j$  in the category sequence and (3) distance between the categories  $c_i$ ,  $c_j$



# "Or" operator

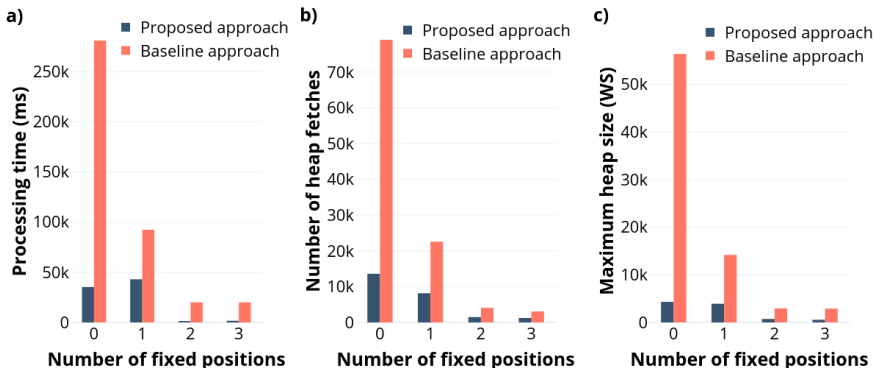
## ■ Evaluation parameter: type and number of operands

- simple OR operand:  $|M| = 1$
- complex OR operand:  $|M| > 1$



# "Order" operator

■ **Evaluation parameter:** number of fixed positions in  $M$







- Proposed four essential route query language operators
- Developed algorithms that deliver an optimal result in metric spaces
- Evaluated the approaches qualitatively and quantitatively and proved the efficiency of the algorithms




- Easily modified: using a different quantifying parameter such as travel duration, finding  $k$  optimal routes
- Additional operators are possible: "hop", "and", "not", "necessity"
- A different approach to implementing the operators
  - Skyline concept [3]
- Extending the proposed algorithms to also support dynamic road networks, in which traffic information is provided (e.g., travel time, traffic congestion, etc.). [7]

# The End

*Thank you for the attention!*

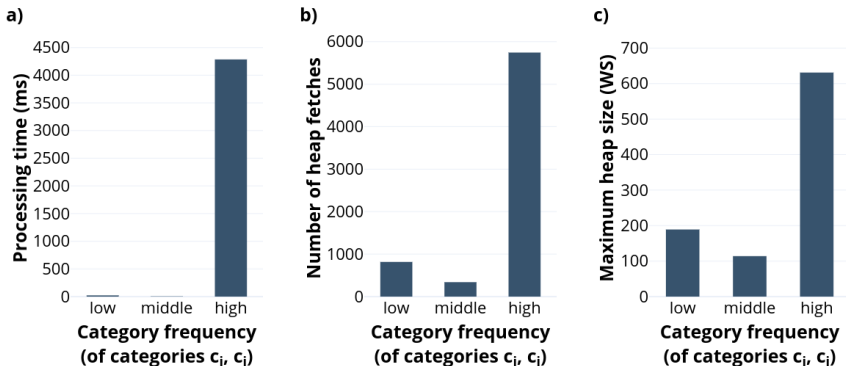


-  Feifei Li, Dihan Cheng, Marios Hadjieleftheriou, George Kollios and Shang-Hua Teng. *On trip planning queries in spatial databases*. In Bauzer Medeiros C., Egenhofer M.J., Bertino E. (eds) *Advances in Spatial and Temporal Databases (SSTD)*, pages 273–290, 2005.
-  Mehdi Sharifzadeh, Mohammad Kolahdouzan and Cyrus Shahabi. *The optimal sequenced route query*. *The VLDB Journal*, 17:765–787, 2008.
-  Yuya Sasaki, Yoshiharu Ishikawa, Yasuhiro Fujiwara and Makoto Onizuka. *Sequenced route query with semantic hierarchy*. In *EDBT*, pages 37–48, 2018.
-  Jian Dai, Chengfei Liu, Jiajie Xu<sup>4</sup> and Zhiming Ding. *On personalized and sequenced route planning*. *World Wide Web*, 19:679–705, 2016.

-  Jochen Eisner and Stefan Funke. *Sequenced route queries: Getting things done on the way back home*. In SIGSPATIAL '12 Proceedings of the 20th International Conference on Advances in Geographic Information Systems, pages 502–505, 2012.
-  Haiquan Chen, Wei-Shinn Ku, Min-Te Sun and Roger Zimmermann. *The partial sequenced route query with traveling rules in road networks*. Geoinformatica, 15:541–569, 2011.
-  Nirmesh Malviya, Samuel Madden and Arnab Bhattacharya. A continuous query system for dynamic route planning. *A continuous query system for dynamic route planning*. In ICDE '11 Proceedings of the 2011 IEEE 27th International Conference on Data Engineering, pages 792–803, 2011.

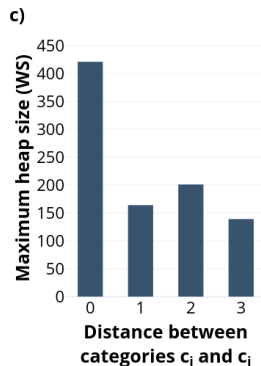
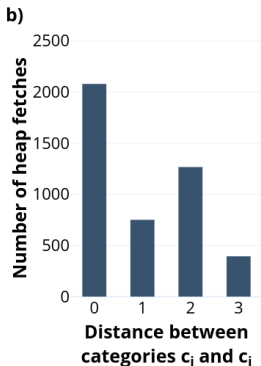
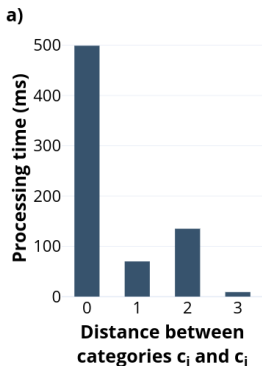
# Inequality operator - category frequency

- **Evaluation parameters:** (1) query length, (2) frequency of the categories  $c_i, c_j$  in the category sequence and (3) distance between the categories  $c_i, c_j$



# Inequality operator - category distance

- **Evaluation parameters:** (1) query length, (2) frequency of the categories  $c_i$ ,  $c_j$  in the category sequence and (3) distance between the categories  $c_i$ ,  $c_j$



---

## Algorithm 1: PNE

---

```
1 fetch a PSR from the heap;  
2 switch  $s = \text{size}(PSR)$  do  
3   case  $s == l$  do  
4     PSR is the optimal route;  
5     return PSR;  
6   case  $s \neq l$  do  
7     a)  
8        $NN(r_{|PSR|}, C_{M_{|PSR|+1}})$ ;  
9       update PSR and perform trimming in case it is a candidate  
10      SR ;  
11      put PSR back on the heap ;  
12     b)  
13       $kNN(r_{|PSR|-1}, C_{M_{|PSR|}})$ ;  
14      generate a new PSR and place it on the heap;
```

---

<i>Symbol</i>	<i>Meaning</i>
$C_i$	a point set for a category in $\mathbb{R}^2$
$ C_i $	cardinality of the set $C_i$
$n$	number of point sets $C_i$
$dist(.,.)$	distance function in $\mathbb{R}^2$
$M$	category sequence, $= (c_1, c_2, \dots, c_l)$
$ M $	$l$ , size of sequence $M$ = number of items in $M$
$c_i$	$i$ th member of $M$
$R$	route, $= (r_1, r_2, \dots, r_r)$
$ R $	$r$ , size of route $R$ = number of points in $R$
$r_i$	$i$ th point in $R$
$length(R)$	length of $R$
$length(sp, R)$	length of $R_{sp} = (sp, r_1, r_2, \dots, r_r)$ , $= length(R_{sp})$
$Q(sp, M)$	sequenced route query
$NN(r_q, c_p)$	nearest neighbor from the category point set $C_p$ to a route point $r_q$
$KNN(r_q, c'_p)$	next nearest neighbor $r'_p$ from the category point set $C_p$ to a route point $r_q$

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**Algorithm 2:** modifiedPNE()

---

```
1 foreach  $r_1$  in  $C_{M_1}$  do Checking the upper bound for every  $r_1$  neighbor  
   of  $sp$  in the category set  $C_{M_1}$   
2   build a new  $PSR$  with  $r_1$ ;  
3   if  $LB(PSR) \leq UB$  then  
4   | place the new  $PSR(r_1)$  on the heap;  
5 while heap is not empty do  
6    $current \leftarrow$  fetch a  $PSR$  from the heap;  
7   switch  $s = size(current)$  do  
8   | case  $s \leq j - 1$  do Finding  $PSRs$  before  $r_j$   
9   | | caseBefore();  
10  | case  $s = j$  do Finding  $PSR$  containing  $r_j$   
11  | | caseContaining();  
12  | case  $s = j + 1$  do Finding  $PSR$  after/containing  $r_j$   
13  | | caseAfterOrContaining();  
14  | case  $s \geq j + 2$  do Finding  $PSRs$  after  $r_j$   
15  | | caseAfter();  
16 return foundSR
```

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## Procedure trim( $PSR$ )

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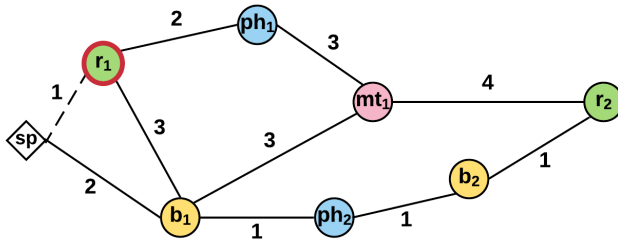
```
1 if size( $PSR$ ) = 1 then
2   if  $PSR[i] \neq PSR[j]$  then
3     // Optimization: length check
4     if length( $PSR$ )  $\leq$  length(candidate) then
5       update candidate;
6       place  $PSR$  on the heap;
7   else
8     // In case  $j$  is the last index in the route, we
      find the  $k$ th neighbor of the previous PoI to
      the last one
9     if size( $PSR$ ) =  $j + 1$  then
10      kNN( $r_{|PSR|-1}, C_{M_{|PSR|}}$ );
11      generate a new  $PSR$ ;
12      trim( $PSR$ );
13 else
14   // Optimization: length check
15   if length( $PSR$ )  $\leq$  length(candidate) then
16     place  $PSR$  on the heap;
```



# Example: Step 1

$M = (r, ph, r), UNEQUAL(0, 2)$

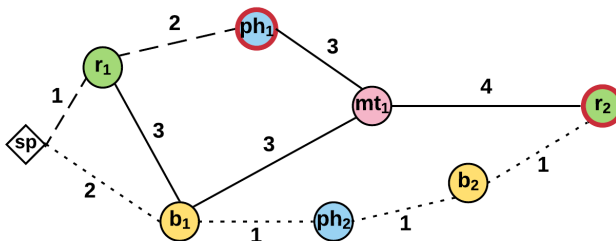
- Optimal route found with PNE:  $(r_1, ph_1, r_1)$



Step	Heap contents (PSR $R$ : $length(R)$ )
1	$(r_1 : 1)$

## Example: Step 2

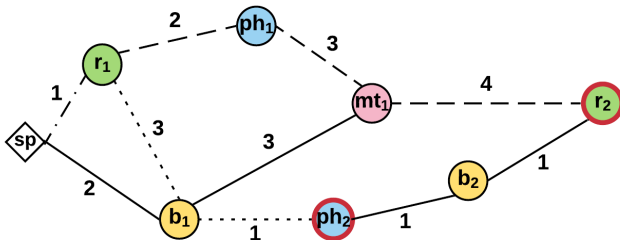
$M = (r, ph, r), UNEQUAL(0, 2)$



Step	Heap contents (PSR $R$ : $length(R)$ )
1	$(r_1 : 1)$
2	$(r_1, ph_1 : 3), (r_2 : 5)$

# Example: Step 3

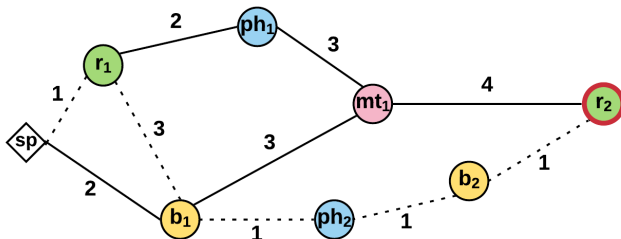
$M = (r, ph, r), UNEQUAL(0, 2)$



Step	Heap contents (PSR $R$ : $length(R)$ )
2	$(r_1, ph_1 : 3), (r_2 : 5)$
3	$(r_2 : 5), (r_1, ph_2 : 5), (r_1, ph_1, r_2 : 10)$

# Example: Step 5

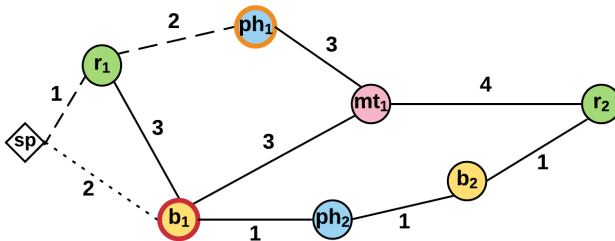
$M = (r, ph, r), UNEQUAL(0, 2)$



Step	Heap contents (PSR $R : length(R)$ )
4	$(r_1, ph_2 : 5), (r_2, ph_2 : 7), (r_1, ph_1, r_2 : 10)$
5	$(r_1, ph_2, r_2 : 7), (r_2, ph_2 : 7), (\cancel{r_1, ph_1, r_2 : 10})$

# Example: Step 1

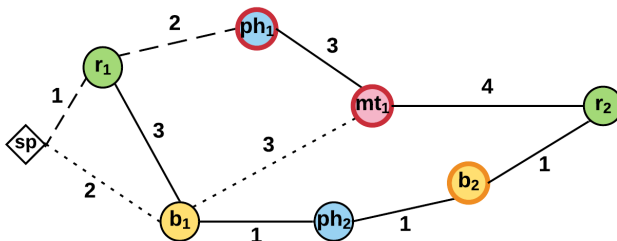
$S = (OR_1, OR_2, OR_3)$ ,  $OR_1 = ((b), (ph))$ ,  $OR_2 = ((mt))$ ,  $OR_3 = ((r))$



Step	Heap contents (PSR $R$ : $length(r)$ , $index(R)$ )
1	$(b_1 : 2, 1)$

## Example: Step 2

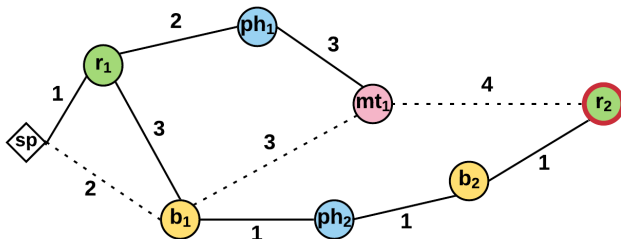
$S = (OR_1, OR_2, OR_3)$ ,  $OR_1 = ((b), (ph))$ ,  $OR_2 = ((mt))$ ,  $OR_3 = ((r))$



Step	Heap contents (PSR $R$ : $length(r)$ , $index(R)$ )
1	$(b_1 : 2, 1)$
2	$(ph_1 : 3, 1), (b_1, mt_1 : 5, 2)$

## Example: Step 6

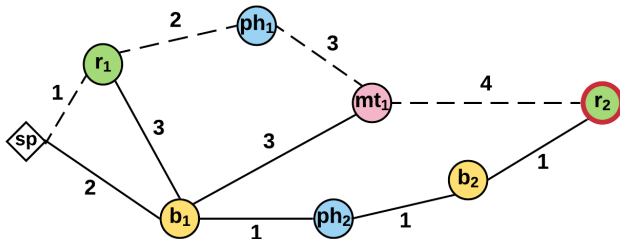
$S = (OR_1, OR_2, OR_3)$ ,  $OR_1 = ((b), (ph))$ ,  $OR_2 = ((mt))$ ,  $OR_3 = ((r))$



Step	Heap contents (PSR $R$ : $length(r)$ , $index(R)$ )
5	$(b_1, mt_1 : 5, 2), (ph_1, mt_1 : 6, 2), (ph_2, mt_1 : 7, 2), (b_2, mt_1 : 9, 2)$
6	$(ph_1, mt_1 : 6, 2), (ph_2, mt_1 : 7, 2), (b_1, mt_1, r_2 : 9, 3), (b_2, mt_1 : 9, 2)$

## Example: Step 7

$S = (OR_1, OR_2, OR_3)$ ,  $OR_1 = ((b), (ph))$ ,  $OR_2 = ((mt))$ ,  $OR_3 = ((r))$

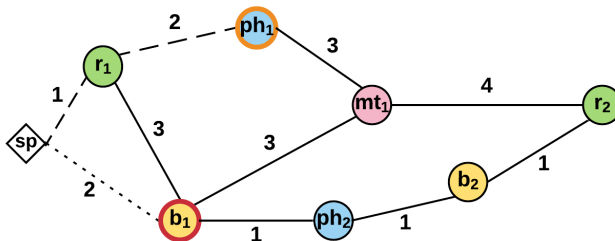


Step	Heap contents (PSR $R$ : $length(r)$ , $index(R)$ )
6	$(ph_1, mt_1 : 6, 2)$ , $(ph_2, mt_1 : 7, 2)$ , $(b_1, mt_1, r_2 : 9, 3)$ , $(b_2, mt_1 : 9, 2)$
7	$(ph_2, mt_1 : 7, 2)$ , $(b_1, mt_1, r_2 : 9, 3)$ , $(b_2, mt_1 : 9, 2)$ , <del><math>(ph_1, mt_1, r_2 : 10, 3)</math></del>
8	$(b_1, mt_1, r_2 : 9, 3)$ , $(b_2, mt_1 : 9, 2)$ , <del><math>(ph_2, mt_1, r_2 : 11, 3)</math></del>



# Example: Step 1

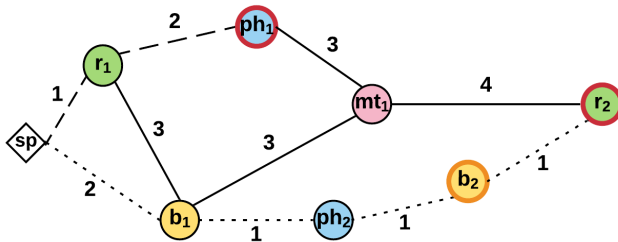
$M = (b, r, ph)$ ,  $ORDER = (1)$



Step	Heap contents (PSR $R : length(r), r.notordered$ )
1	$(b_1 : 2, [ph])$

# Example: Step 1

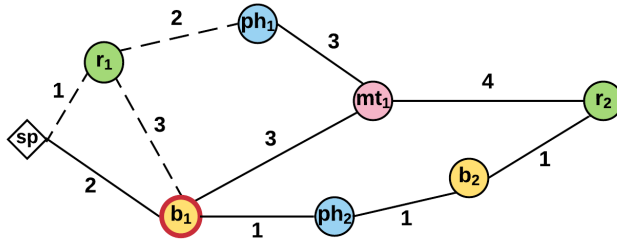
$M = (b, r, ph)$ ,  $ORDER = (1)$



Step	Heap contents (PSR $R : length(r), r.notordered$ )
1	$(b_1 : 2, [ph])$
2	$(ph_1 : 3, [b]), (b_1, r_2 : 5, [ph])$

# Example: Step 6

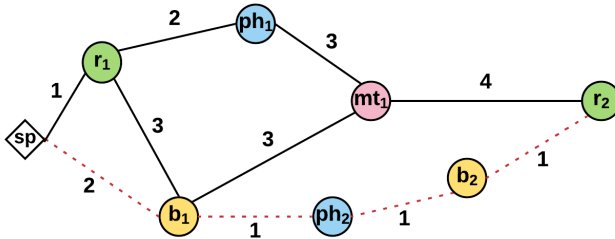
$M = (b, r, ph)$ ,  $ORDER = (1)$



Step	Heap contents (PSR $R : length(r), r.notordered$ )
5	$(ph_1, r_1 : 5, [b]), (b_2, r_2 : 5, [ph]), (ph_2, r_2 : 5, [b]), (b_1, r_2 : 5, [ph])$
6	$(b_2, r_2 : 5, [ph]), (ph_2, r_2 : 5, [b]), (b_1, r_2 : 5, [ph]), (ph_1, r_1, b_1 : 8, [])$

# Example: Step 12

$M = (b, r, ph)$ ,  $ORDER = (1)$



Step	Heap contents (PSR $R$ : $length(r)$ , $r.notordered$ )
10	$(b_2, r_2 : 5, [ph]), (ph_2, r_2, b_2 : 6, []), (b_1, r_1, ph_1 : 7, []), (ph_2, r_1 : 7, [b]), (b_2, r_1 : 9, [ph]), (ph_1, r_2 : 10, [b])$
11	$(ph_2, r_2, b_2 : 6, []), (ph_2, r_1 : 7, [b]), (b_2, r_1 : 9, [ph]), (ph_1, r_2 : 10, [b]), (\cancel{b_2, r_2, ph_1 : 12, []})$