Discrete mathematics

- **Discrete mathematics**
  - study of mathematical structures and objects that are fundamentally **discrete** rather than **continuous**.
- **Examples of objects** with discrete values are
  - **integers**, **graphs**, or **statements in logic**.
- Discrete mathematics and **computer science**.
  - Concepts from discrete mathematics are useful for describing **objects and problems in computer algorithms and programming languages**. These have applications in cryptography, automated theorem proving, and software development.

Logic

Logic:

- defines a formal language for representing knowledge and for making logical inferences
- It helps us to understand how to construct a valid argument

Logic defines:

- Syntax of statements
- The meaning of statements
- The rules of logical inference (manipulation)

Propositional logic

- **The simplest logic**

- **Definition:**
  - **A proposition is a statement that is either true or false.**

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh.
    - **(T)**
  - 5 + 2 = 8.
    - **(F)**
  - It is raining today.
    - **(either T or F)**

Propositional logic

- **Examples (cont.):**
  - How are you?
    - **a question is not a proposition**
  - x + 5 = 3
    - **since x is not specified, neither true nor false**
  - 2 is a prime number.
    - **(T)**
  - She is very talented.
    - **since she is not specified, neither true nor false**
  - There are other life forms on other planets in the universe.
    - **either T or F**

Composite statements

- More complex propositional statements can be build from elementary statements using **logical connectives.**

Example:

- Proposition A: It rains outside
- Proposition B: We will see a movie
- A new (combined) proposition:

  If it rains outside then we will see a movie

Composite statements

- More complex propositional statements can be build from elementary statements using **logical connectives.**

- **Logical connectives:**
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive or
  - Implication
  - Biconditional

Negation

Definition: Let p be a proposition. The statement "It is not the case that p." is another proposition, called the **negation of p**. The negation of p is denoted by ¬ p and read as "not p."

Example:

- Pitt is located in the Oakland section of Pittsburgh.

  →

- It is not the case that Pitt is located in the Oakland section of Pittsburgh.

Other examples:

- $5 + 2 \neq 8$.
- 10 is not a prime number.
- It is not the case that buses stop running at 9:00pm.

Negation

- **Negate the following propositions:**
  - It is raining today.
    - It is not raining today.
  - 2 is a prime number.
    - 2 is not a prime number
  - There are other life forms on other planets in the universe.
    - It is not the case that there are other life forms on other planets in the universe.

Negation

- A **truth table** displays **the relationships between truth values** (T or F) of different propositions.

p	¬p
T	F
F	T

Rows: all possible values of elementary propositions:

Conjunction

- **Definition**: Let p and q be propositions. The proposition "**p and q**" denoted by $p \wedge q$, is true when both p and q are true and is false otherwise. The proposition $p \wedge q$ is called the **conjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh and 5 + 2 = 8
  - It is raining today and 2 is a prime number.
  - 2 is a prime number and $5 + 2 \neq 8$.
  - 13 is a perfect square and 9 is a prime.

Disjunction

- **Definition**: Let p and q be propositions. The proposition "**p or q**" denoted by $p \vee q$, is false when both p and q are false and is true otherwise. The proposition $p \vee q$ is called the **disjunction** of p and q.

- **Examples:**
  - Pitt is located in the Oakland section of Pittsburgh or 5 + 2 = 8.
  - It is raining today or 2 is a prime number.
  - 2 is a prime number or $5 + 2 \neq 8$.
  - 13 is a perfect square or 9 is a prime.

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T		
T	F		
F	T		
F	F		

Rows: all possible combinations of values for elementary propositions: 2^n values

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T	T	
T	F	F	
F	T	F	
F	F	F	

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

Truth tables

- **Conjunction and disjunction**
- Four different combinations of values for p and q

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

- NB: $p \vee q$ (the or is used inclusively, i.e., $p \vee q$ is true when either p or q or both are true).

Exclusive or

- **Definition:** Let p and q be propositions. The proposition "p exclusive or q" denoted by $p \oplus q$, is true when exactly one of p and q is true and it is false otherwise.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F