Maths Sample Paper - Answer Key

1) The HCF of two numbers is 9 and their LCM is 360. If one of the numbers is 45, what is the other number?

[Marks: 1]

(a) 60
(b) 72
(c) 80
(d) 90

Correct Answer:

(b) 72

Reason:

According to the property derived from the Fundamental Theorem of Arithmetic (page 4), for any two positive integers 'a' and 'b', HCF(a, b) × LCM(a, b) = a × b. Given HCF = 9, LCM = 360, and one number (a) = 45. Let the other number be 'b'. So, 9 × 360 = 45 × b. Solving for b, we get b = (9 × 360) / 45 = 9 × 8 = 72.

Difficulty: Medium

2) Let 'a' be a non-zero rational number and 'b' be an irrational number. Which of the following operations will always result in an irrational number?

[Marks: 1]

(a) a + b
(b) a - b
(c) a × b
(d) All of the above

Correct Answer:

(d) All of the above

Reason:

As stated on page 8 of the chapter, 'the sum or difference of a rational and an irrational number is irrational' and 'the product and quotient of a non-zero rational and irrational number is irrational'. Since 'a' is a non-zero rational number and 'b' is irrational, the sum (a+b), the difference (a-b), and the product (a × b) will all always result in an irrational number.

Difficulty: Medium

3) A quadratic polynomial has the sum of its zeroes as -2 and the product of its zeroes as -15. Which of the following could be the polynomial?

[Marks: 1]

(a) $x^2 + 2x - 15$
(b) $x^2 - 2x - 15$
(c) $x^2 + 2x + 15$
(d) $x^2 - 2x + 15$

Correct Answer:
(a) $x^2 + 2x - 15$

Reason:
For a quadratic polynomial of the form $ax^2 + bx + c$, the sum of its zeroes ($\alpha + \beta$) is given by $-b/a$, and the product of its zeroes ($\alpha\beta$) is given by c/a (page 20). Given sum of zeroes = -2 and product of zeroes = -15. If we consider a standard quadratic polynomial where a=1 (as is common for such problems unless specified otherwise), then: $-b/1 = -2 \Rightarrow b = 2$. And $c/1 = -15 \Rightarrow c = -15$. Therefore, the polynomial is $x^2 + 2x - 15$. (Refer to Example 4 on page 21 for a similar approach).

Difficulty: Medium

4) The graph of a polynomial y = p(x) intersects the x-axis at exactly three distinct points. What can be the minimum degree of the polynomial p(x)?

[Marks: 1]

(a) 1
(b) 2
(c) 3
(d) 4

Correct Answer:
(c) 3

Reason:
The zeroes of a polynomial p(x) are precisely the x-coordinates of the points where the graph of y = p(x) intersects the x-axis (page 13). If the graph intersects the x-axis at exactly three distinct points, it means the polynomial has three distinct zeroes. According to the remark on page 17, a polynomial of degree 'n' has at most 'n' zeroes. Therefore, to have three distinct zeroes, the minimum degree of the polynomial must be 3. A linear polynomial (degree 1) has at most one zero, and a quadratic polynomial (degree 2) has at most two zeroes.

Difficulty: Medium

5) What is the degree of the polynomial $p(x) = (2x^2 + 5)(x^3 - 7) - 2x$■?

[Marks: 1]

(a) 2
(b) 3
(c) 4
(d) 5

Correct Answer:

(b) 3

Reason:

To find the degree of the polynomial, we first need to simplify the expression by expanding and combining like terms. $p(x) = (2x^2 + 5)(x^3 - 7) - 2x$■. Expanding the product $(2x^2 + 5)(x^3 - 7)$: $2x^2(x^3) + 2x^2(-7) + 5(x^3) + 5(-7) = 2x$■ $- 14x^2 + 5x^3 - 35$. Now substitute this back into the original expression: $p(x) = (2x$■ $- 14x^2 + 5x^3 - 35) - 2x$■. $p(x) = 2x$■ $- 2x$■ $+ 5x^3 - 14x^2 - 35$. $p(x) = 5x^3 - 14x^2 - 35$. The highest power of the variable x in the simplified polynomial is 3. Therefore, the degree of the polynomial is 3. (Refer to Section 2.1 Introduction on page 10 for the definition of the degree of a polynomial).

Difficulty: Medium

6) Using the Fundamental Theorem of Arithmetic, explain why 6■ can never end with the digit zero for any natural number n.

[Marks: 2]

Solution:

A number ending with the digit zero must have 5 as a prime factor. The prime factorization of 6■ is (2 × 3)■ = 2■ × 3■. According to the Fundamental Theorem of Arithmetic, the prime factorization of any natural number is unique. Since 5 is not a prime factor in the unique factorization of 6■, it cannot end with the digit zero.

Difficulty: Medium

Source: Chapter: Real Numbers, Page: 1 | Chapter: Real Numbers, Page: 3 | Chapter: Real Numbers, Page: 4

7) Find a quadratic polynomial whose sum and product of zeroes are -1/2 and -3/2 respectively.

[Marks: 2]

Solution:

Let the quadratic polynomial be $ax^2 + bx + c$. The sum of zeroes $(\alpha+\beta) = -b/a$ and product of zeroes $(\alpha\beta) = c/a$. Given $\alpha+\beta = -1/2$ and $\alpha\beta = -3/2$. We can consider a simplified polynomial of the form x^2 - (sum of zeroes)x + (product of zeroes). Substituting the given values: $x^2 - (-1/2)x + (-3/2) = x^2 + 1/2x - 3/2$. To get integer coefficients, we can multiply the entire polynomial by 2, resulting in $2x^2 + x - 3$. This is one possible quadratic polynomial.

Difficulty: Medium

Source: Chapter: Polynomials, Page: 19 | Chapter: Polynomials, Page: 23

8) The HCF of two numbers is 9 and their product is 2700. Find their LCM.

[Marks: 2]

Solution:

For any two positive integers 'a' and 'b', the product of their HCF and LCM is equal to the product of the numbers. This can be written as: HCF(a, b) × LCM(a, b) = a × b. Given HCF = 9 and the product of the numbers (a × b) = 2700. Substituting these values into the formula: 9 × LCM = 2700. Therefore, LCM = 2700 / 9 = 300.

Difficulty: Medium

Source: Chapter: Real Numbers, Page: 4