Time: 30 minutes   Max. Marks: 10

Name and Roll No.: _____

> **Instructions:**
> - Do not plagiarize. Do not assist your classmates in plagiarism.
> - Show your full solution for the questions to get full credit.
> - Attempt all questions that you can.
> - True / False questions will get full credit only if the justification and answer are both correct.
> - A multiple choice question may have one or more correct answers. Credit will only be awarded if all correct answers are marked and none of the incorrect answers are marked.
> - In the unlikely case that you find a question ambiguous, discuss it with an invigilating TA/invigilator. Please ensure that you clearly write any assumptions you make, even after clarification from the invigilator.
> **V. Imp.**: If you do not write your **Name and Roll No.**, you will get a zero.

1. (3 points) If either Jolie or Cruz are in the film, then Brad is in the film. However, Brad is not in the film. Prove by resolution that Jolie is not in the film.

   Solution:
   Let $J \equiv$ Jolie is in the film, $C \equiv$ Cruz is in the film and $B \equiv$ Brad is in the film. Then $\alpha = \neg J$, which means our assumption for proof by contradiction is $\neg \alpha = J$ (i.e., we assume that Jolie is in the film). The KB is:

   $$(J \vee C) \Rightarrow B$$
   $$\neg B$$

   Writing the first sentence of KB in the CNF form, we have

   $$\neg(J \vee C) \vee B$$
   $$\equiv (\neg J \wedge \neg C) \vee B$$
   $$\equiv (\neg J \vee B) \wedge (\neg C \vee B)$$

   Using the CNF form, we have KB as

   $$(\neg J \vee B) \wedge (\neg C \vee B) \wedge (\neg B)$$

   and $KB \wedge \neg \alpha$ is

   $$(\neg J \vee B) \wedge (\neg C \vee B) \wedge (\neg B) \wedge (J)$$

   Writing this as a set of clauses, we have: $\{(\neg J \vee B), (\neg C \vee B), (\neg B), (J)\} \equiv \{C_1, C_2, C_3, C_4\}$
   Starting the resolution steps:

   1. Pick C_1 and C_3 and resolve B with the resolvent as $C_5 \equiv \neg J$

   2. Pick C_5 and C_4 and resolve J leading to an empty clause.

   3. This step confirms the contradiction, implying that J has to be False, proving that Jolie is not in the film.

2. (2 points) Show that a resolution step on a pair of Horn clauses will result in another Horn clause.

   Solution:
   Horn clauses are disjunctions of literals with at most one positive literal. Therefore, for a pair of Horn clauses, we have the following three cases:

1. **Case-1: Zero positive literals across the two clauses**. In this case, resolution is not possible as it needs *complementary literals*.

2. **Case-2: One positive literal across the two clauses**. In this case, if there are complimentary literals for the resolution to occur, the resolvent will end up with zero positive literals, thus making the resolvent clause a Horn Clause with zero positive literals.

3. **Case-3: Two positive literals, one in each clause**. In this case, if there are complimentary literals for the resolution to occur, the resolvent will end up with one positive literal, thus making the resolvent clause a Horn Clause with one positive literal.

3. $(1 \times 3 = 3$ points) State True or False with justification. In case a statement is False, provide a counter-example.

   (a) Entailment can be established by solving some satisfiability (SAT) problem.
   Solution:
   True. To show $A \vDash B$, we can solve the satisfiability problem of $A \wedge \neg B$. If it is unsatisfiable, then $A \vDash B$, else $A \nvDash B$.

   (b) Forward Chaining is *sound* for propositional logic.
   Solution:
   True. Since forward chaining uses modus ponens, i.e., it builds upon affirmations from what it knows (from the knowledge-base for example), every deduction is a true deduction. Therefore it is sound.

   (c) Forward Chaining is *complete* for propositional logic.
   Solution:
   False. Consider the following example where you have two sentences in the $KB \equiv \{(P \vee Q), (\neg P)\} \equiv \{C_1, C_2\}$ and we want to query Q.
   Here, we see that Q can be inferred by resolution, but not by forward chaining. We can start with the initial fact $\neg P$ but we cannot use forward chaining (modus ponens) to infer anything from the C_1 clause because it is not a Horn clause (more than one positive literal). Therefore, forward chaining can not infer all sentences. It is complete *only if* the KB comprises of Horn clauses and if it has to infer a single literal.

4. (2 points) Draw the `AND-OR` graph for the following set of sentences. The house has a front and a back door and both are open. When both doors are open, wind blows through the house. The wind also blows through the house when both the front door and the rear window are open. The wind blowing through the house carries the smell of bananas. A monkey enters the house if the rear door is open and the wind is blowing. If there is a monkey in the house and it is windy, then the rear window is open.

   Solution:
   Interpretation:

   1. $A \equiv$ Front door is open.
   2. $B \equiv$ Rear/Back door is open.
   3. $L \equiv$ Wind blows through the house.
   4. $P \equiv$ Rear/Back window is open.
   5. $M \equiv$ Monkey enters the house.

   The Knowledge-Base:

   1. A
   2. B
   3. $A \wedge B$
   4. $A \wedge B \Rightarrow L$

5. $A \wedge P \Rightarrow L$

6. $B \wedge L \Rightarrow M$
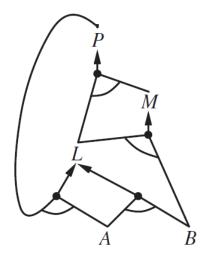
7. $L \wedge M \Rightarrow P$

The And-Or graph is shown in Fig. 1.

Figure 1: And-Or Graph for Q4