Mid-semester Exam Maximum Marks: 50, Maximum Time: 60 mins
Date : 07/10/2024
MTH210: Discrete Structures **Name:** ____________________
Semester: Monsoon 2024 **Tutorial Section:** ____________________

Problem 1. [4] Let

$$X = \{0, 1, 2, 3, 4\}$$

and define an equivalence relation R on X by

$$nRm \text{ if } n^2 - m^2 \text{ is divisible by 5}$$

How many equivalence classes are there? List all the elements in each equivalence class of the relation R.

Problem 2. [8] Solve the recurrence relation

$$a_n - 4a_{n-1} + 5a_{n-2} - 2a_{n-3} = 0$$

subject to the initial conditions $a_0 = 0$, $a_1 = 1$, and $a_2 = 2$.

Problem 3. [4] Let $P(x)$, $Q(x)$, and $R(x)$ be abbreviations for the following predicates:

- $P(x)$: x is perfect.
- $Q(x)$: x failed the quiz.
- $R(x)$: x read a lot of books.

Write these propositions using $P(x)$, $Q(x)$, $R(x)$, logical connectives, and quantifiers.

(a) Everyone who is perfect read a lot of books.

(b) No one who read a lot of books failed the quiz.

(c) Someone who failed the quiz isn't perfect.

(d) If everyone reads a lot of books, then no one will fail the quiz.

Problem 4. [3] Use a truth table to show that $(p \to q) \wedge (p \to r)$ is logically equivalent to $p \to (q \wedge r)$. Explain in a sentence why your truth table shows that they are logically equivalent.

Problem 5. [4] Prove that at a party where there are at least two people, there are at least two people who know the same number of people. Assume that if person A knows person B, then person B also knows person A. Furthermore, assume that each person A knows him/herself.

Problem 6. [4] Determine the truth value of each of the following statements if the universe of discourse for each variable consists of all real numbers. Simply write “True" or “False" for each; no need to explain why.

(a) $\forall x\, \exists y\, (y = 3x + 2)$

(b) $\exists y\, \forall x\, (y = 3x + 2)$

(c) $\exists x\, \exists y\, (x^2 + y^2 = 1)$

(d) $\forall y\, \forall x\, ((x + y)^2 = x^2 + y^2)$

Problem 7. [3] Given statements 1 and 2, find an argument to conclude statement n. For each intermediate statement you make, state what rule of inference you use and the number(s) of the previous lines that rule uses. **Statements:**

1. Samay, a student in this class, has a Coldplay concert ticket.
2. Everyone who has a Coldplay concert ticket is traveling to Mumbai in January.

n. Someone in this class is traveling to Mumbai in January.

(You don’t have to use symbolic notation for this problem, but it may help. Also, if you don’t remember the name for a rule of inference, then just state the whole rule symbolically.)

Problem 8. [12] For each statement, mention if it is “True" or “False". Justify your answer in each case.

(a) There is no one-to-one correspondence between the set of all positive integers and the set of all odd positive integers because the second set is a proper subset of the first.

(b) If f is a function $A \to B$, and S and T are subsets of A, then $f(S \cap T) = f(S) \cap f(T)$.

(c) $\lfloor 14.85 \rfloor + \lceil 14.85 \rceil = 30$.

(d) $\sum_{k=0}^{10} 7 = 70$.

(e) If $f(n) = (3n^6 + 5n^2 - 6)(x + \log x)$, then f is $O(x^6 \log x)$.

(f) If the product $A \times B$ of two sets A and B is the empty set $\emptyset$, then both A and B must be $\emptyset$.

Problem 9. [8] Solve the recurrence relation

$$a_n - 4a_{n-1} + 4a_{n-2} = 2^n$$

with initial conditions $a_0 = 1$ and $a_1 = 6$.