Time: 60 minutes Max. Marks: 15

Instructions:
- Do not plagiarize. Do not assist your classmates in plagiarism.
- Show your full solution for the questions to get full credit.
- Attempt all questions that you can.
- True/False questions will get full credit only if the justification and answer are both correct.
- In the unlikely case that you find a question ambiguous, discuss it with an invigilating TA/invigilator. Please ensure that you clearly write any assumptions you make, even after clarification from the invigilator.

1. (5 points) (a) (2 points) Show that A* search is cost-optimal when the heuristic used is admissible.
   (b) (3 points) State True or False. Justify your answer or provide a counter example.
      i. (1 point) A hill-climbing algorithm that never visits states with lower value (or higher cost) is guaranteed to find the optimal solution if given enough time to find a solution.
      ii. (1 point) Assume that a rook can move on a chessboard (8×8 grid) any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.
      iii. (1 point) For any local-search problem, hill-climbing will return a global optimum if the algorithm is run starting at any state that is a neighbor of a neighbor of a globally optimal state.

Solution:

(a) An admissible heuristic is one that never overestimates the cost to goal. We approach the proof by contradiction and assume that A^* with $f(n) = g(n) + h(n)$ returns a path with cost $C > C^*$, where $g(n)$ is the cost to node n and $h(n)$ is the heuristic function returning the estimated cost to goal from node n. We assume that $h(\cdot)$ is an admissible heuristic. We define $g^*(n)$ as the cost of the optimal path from start to n and $h^*(n)$ as the cost of the optimal path from n to a goal node. Then for any admissible heuristic $h(n) \leq h^*(n)$.

$$
\begin{aligned}
&f(n) > C^* && \text{(otherwise } n \text{ would have been expanded)}\\
&f(n) = g(n) + h(n) && \text{(by definition)}\\
&f(n) = g^*(n) + h(n) && \text{(because } n \text{ is on an optimal path)}\\
&f(n) \leq g^*(n) + h^*(n) && \text{(because of admissibility, } h(n) \leq h^*(n))\\
&f(n) \leq C^* && \text{(by definition, } C^* = g^*(n) + h^*(n))
\end{aligned}
$$

See a more detailed explanation in [RN-Ch-3.5.2: A^* Search] in the 4th edition pdf.

(b) True/False with justification or a counter-example.
   i. **False**. Such an algorithm will reach a local optimum and stop (or wander on a plateau).
   ii. **False**. A rook can move across the board in one move, although the Manhattan distance from start to finish is 8.
   iii. **False**. The intervening neighbor could be a local minimum and the current state is on another slope leading to a local maximum. Consider, for example, starting at the state valued 5 in the linear sequence 7,6,5,0,9,0,0.

Rubric : 1 (a)
0.5 marks – If admissible heuristic is properly defined.
0.5 marks – If A* criteria of expansion is explained correctly.
1 mark – If the correct proof is provided, by contradiction or otherwise.

Rubric : 1 (b)
Full marks only if both answer (True/False) and justification is correct.
0.5 marks if only True/False is correct.

2. (2 points) Describe a common challenge with respect to state spaces in environments that have non-deterministic actions or partial observability or both. Explain briefly the {`Predict, Possible Percepts, Update`} stages often used in such settings.

Solution: An environment that has non-deterministic actions or partial observability may lead to a non-deterministic state transition as a consequence of an agents action or inaction. This may result in uncertainty around state estimates, which may need the agent to keep track of multiple states as belief states.

Refer to Textbook [RN-Ch-4.4.2; 4th ed pdf] for a description of the three stages used in partially observable environments.

Rubric:
0.5 mark for the reasoning.
0.5 mark each for brief explanations of Predict, Possible Percept and Update.

3. (5 points) We often build knowledge bases from English sentences.
   (a) (3 points) Consider the problem of map coloring and the predicates $In(x, y)$, $Borders(x, y)$, and $State(x)$. For each of the logical expressions listed below, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid; or (3) is syntactically valid but does not express the meaning of the English sentence. Explain your choice in one sentence.
      i. (1.5 points) Patna and Nalanda are both in Bihar.
         α) $In(Patna \wedge Nalanda, Bihar)$
         β) $In(Patna, Bihar) \wedge In(Nalanda, Bihar)$
         γ) $In(Patna, Bihar) \vee In(Nalanda, Bihar)$
      ii. (1.5 points) There is a state that borders both China and Pakistan.
         α) $\exists s\ State(s) \wedge Border(s, China) \wedge Border(c, Pakistan)$
         β) $\exists s\ State(s) \Rightarrow [Border(s, China) \wedge Border(\not{c}\ s, Pakistan)]$
         γ) $\exists s\ Border(State(s), China \wedge Pakistan)$
   (b) (2 points) Consider a vocabulary with the following symbols:
      - Occupation$(p, o) \equiv O(p, o)$: Predicate. Person p has occupation o.
      - Customer$(p_1, p_2) \equiv C(p_1, p_2)$: Predicate. Person p_1 is a customer of person p_2.
      - Boss$(p_1, p_2) \equiv B(p_1, p_2)$: Predicate. Person p_1 is a boss of person p_2.
      - (D)octor, (S)urgeon, (L)awyer, (A)ctor: Constants (symbols in parentheses) for occupations.
      - (E)mily, (J)oe: Constants (symbols in parentheses) for people.

      Use these symbols to write the following assertions in first-order logic

i. All surgeons are doctors
ii. Emily has a boss who is a lawyer.
iii. Joe is an actor, but he also holds another job.
iv. Every surgeon has a lawyer.

Solution:

(a) i. (1.5 points) Patna and Nalanda are both in Bihar.

α) (0.5 points) $In(Patna \wedge Nalanda, Bihar)$
(2) Syntactically invalid. Cannot use conjunction inside a term.

β) (0.5 points) $In(Patna, Bihar) \wedge In(Nalanda, Bihar)$
(1) Correct. Both predicates are independently true, confirming that Patna is in Bihar and Nalanda is in Bihar are *simultaneously* true.

γ) (0.5 points) $In(Patna, Bihar) \vee In(Nalanda, Bihar)$
(3) Incorrect. Disjunction does not express "both."

ii. (1.5 points) There is a state that borders both China and Pakistan.

α) (0.5 points) $\exists s\ State(s) \wedge Border(s, China) \wedge Border(s, Pakistan)$
(1) Correct. There exists a state which shares a border with China and simultaneously shares a border with Pakistan.

β) (0.5 points) $\exists s\ State(s) \Rightarrow [Border(s, China) \wedge Border(s, Pakistan)]$
(3) Incorrect. Use of implication in existential. Whenever $State(s)$ is False, the statement on the right of the implication becomes vacuously true, which does not represent the English sentence given to us.

γ) (0.5 points) $\exists s\ Border(State(s), China \wedge Pakistan)$
(2) Syntactically invalid. Cannot use conjunction inside a term.

(b) (i) $\forall p\ O(p; S) \Rightarrow O(p, D)$.
(ii) $\exists p B(p, E) \wedge O(p, L)$.
(iii) $O(J, A) \wedge \exists p, (p \neq A) \wedge O(J, p)$.
(iv) $\forall p O(p, S)) \Rightarrow \exists q O(q, L) \wedge C(p, q)$.

Rubric : 3 (a)
Straightforward, 0.5 mark for each correct answer. 0 mark if no explanation provided along with the choice of (1), (2), or (3).

Rubric : 3 (b)
0.5 mark for each logically correct representation. Evaluate the logic of the answer if it doesn't match the rubric exactly.

4. (3 points) Given the following sentences in the KB:
$\{S_1 : A \Leftrightarrow (B \vee E); S_2 : E \Rightarrow D; S_3 : C \wedge F \Rightarrow \neg B; S_4 : E \Rightarrow B; S_5 : B \Rightarrow F; S_6 : B \Rightarrow C\}$.
Here S_i are used only for indicating the sentence number. Use resolution to prove the query $Q : \neg A \wedge \neg B$

Solution: To prove the conjunction, it suffices to prove each literal separately. To prove $\neg B$, we follow the steps below:

- Add the negated goal $S_7 : B$.

- Resolve S_7 with $S_5 \equiv (\neg B \lor F)$, giving $S_8 : F$.
- Resolve S_7 with $S_6 \equiv (\neg B \lor C)$, giving $S_9 : C$.
- Resolve S_8 with S_3, giving $S_{10} : (\neg C \lor \neg B)$.
- Resolve S_9 with S_{10}, giving $S_{11} : \neg B$.
- Resolve S_7 with S_{11} giving the empty clause.

To prove $\neg A$,

- Add the negated goal $S_7 : A$.
- Resolve S_7 with the first clause of S_1, giving $S_8 : (B \lor E)$.
- Resolve S_8 with S_4, giving $S9 : B$.
- Proceed as above to derive the empty clause.

Note: The solution should mention that the empty clause signifies a contradiction (i.e. unsatisfiability) and hence the KB entails the query.

Rubric:

- 1 mark for arriving at the resolution of B when resolving A. This means 1 mark for the 3 resolution steps needed.
- 2 marks for resolving B and arriving at the empty clause, 1 mark for every 3 correct steps in the correct order as per solution.
- 0.5 mark deduction if the proof is complete, but does not mention that the KB entails the query as the resolution resulted in the empty clause (or that it was a contradiction). This deduction is only when the rest of the proof is complete, but for partial proofs marks should only be given according to steps specified above.

5. (3 points) (*Bonus*) We know that Horn clauses can be written in an implication form.
   (a) (1 point) Show that every clause (regardless of the number of positive literals) can be written in the form $(P_1 \land \ldots \land P_m) \Rightarrow (Q_1 \lor \ldots \lor Q_n)$, where the Ps and Qs are proposition symbols. This form is also called as the *implicative normal form*.
   (b) (2 points) Write the full resolution rule for sentences in the implicative normal form. You may use intermediate representations in any form, but your initial statements and your resolvent must be presented in the implicative normal form.

Solution:

(a) A clause can have positive and negative literals; let the negative literals have the form $\neg P_1, \ldots, \neg P_m$ and let the positive literals have the form $Q_1, \ldots, Q_n$, where the P_is and Q_js are propositional symbols. Then the clause can be written as $(\neg P_1 \lor \ldots \lor \neg P_m \lor Q_1 \lor \ldots \lor Q_n)$. By the previous argument, with Q = $Q_1 \lor \ldots \lor Q_n$, it is immediate that the clause is equivalent to

$$(P_1 \land \ldots \land P_m) \Rightarrow (Q_1 \lor \ldots \lor Q_n).$$

(b) Let two clauses in the implicative normal form be:

$$(p_1 \wedge \ldots \wedge p_j \wedge \ldots \wedge p_{n1}) \Rightarrow (r_1 \vee \ldots \vee r_{n2})$$

$$(s_1 \wedge \ldots \wedge s_{n3}) \Rightarrow (q_1 \vee \ldots \vee q_k \vee \ldots \vee q_{n4})$$

Let p_j and q_k be complementary literals, then the full resolution step in implicative normal form will be:

$$\frac{(p_1 \wedge \ldots \wedge p_j \wedge \ldots \wedge p_{n1}) \Rightarrow (r_1 \vee \ldots \vee r_{n2}) \qquad (s_1 \wedge \ldots \wedge s_{n3}) \Rightarrow (q_1 \vee \ldots \vee q_k \vee \ldots \vee q_{n4})}{\text{The resolvent:}}$$

$$(p_1 \wedge \ldots \wedge p_{j-1} \wedge p_{j+1} \wedge \ldots \wedge p_{n1} \wedge s_1 \wedge \ldots \wedge s_{n3}) \Rightarrow (r_1 \vee \ldots \vee r_{n2} \vee q_1 \vee \ldots \vee q_{k-1} \vee q_{k+1} \vee \ldots \vee q_{n4})$$

The intermediate steps can be derived by employing implication elimination and DeMorgan's Law.

Rubric: No part marking. Full marks if correct proof, else 0.