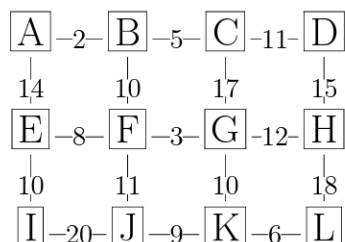**Problem 1.** [10] The graph below represents a group of computers in a dorm, and the distance between them. We would like to network them, using the least amount of wire possible.

$$
\begin{array}{cccc}
\boxed{A} -2- \boxed{B} -5- \boxed{C} -11- \boxed{D} \\
14 \qquad 10 \qquad 17 \qquad 15 \\
\boxed{E} -8- \boxed{F} -3- \boxed{G} -12- \boxed{H} \\
10 \qquad 11 \qquad 10 \qquad 18 \\
\boxed{I} -20- \boxed{J} -9- \boxed{K} -6- \boxed{L}
\end{array}
$$

Use any Algorithm to find a minimal spanning tree on the graph shown above. Find the amount of wire used in the resulting network.

Problem 2. [10] Find an Eulerian circuit on the graph below, indicating your path by drawing arrows and numbering the edges in the order that you traverse them.
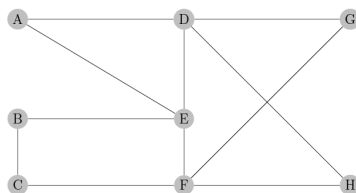
Write down the adjacency matrix for the graph above. (Please order the vertices in alphabetical order.)

Problem 3. [10] Let S be the power set of $\{1, 2, 3\}$.

(a) Determine which one of the following two relations is **not** an equivalence relation. (You only need to explain why your selection is not an equivalence relation. You are not required to explain why the other one is an equivalence relation.)

    i. $AR_1B \iff A \cap B = \emptyset$.

    ii. $AR_2B \iff |A| = |B|$.

(b) For the relation above that **is** an equivalence relation, find the associated partition of S into equivalence classes.

Problem 4. [5] Show that the product of a nonzero rational number and an irrational number is irrational.

Problem 5. [10] Define a function $f : \mathbb{R} \to \mathbb{R}$ by:

$$f(x) = \begin{cases} \frac{x-2}{x-4}, & x \neq 4 \\ 1, & x = 4 \end{cases}$$

Show that f is a bijection.

Problem 6. [10] In a standard 52-card deck, there are 4 suits of 13 cards each.

(a) How many cards do you need to draw to be guaranteed of drawing at least 2 cards of the same suit?

(b) How many cards do you need to draw to be guaranteed of drawing at least 3 cards of the same suit?

Problem 7. [5, Bonus] Give a formula for the coefficient of x^k in the expansion of $\left(x - \frac{1}{x}\right)^{100}$, where k is an integer.