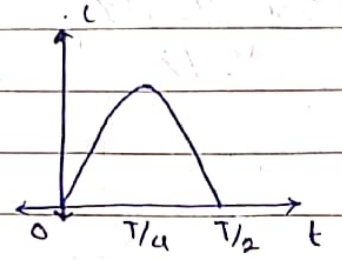
Q2) Problem Statement

Using 2-point Gauss quadrature, calculate the RMS for the following wave form.

fig,




given,

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

$$i(t) = \sin\left(\frac{2\pi t}{T}\right) \times 10^{-t/T} \quad ; \quad 0 \le t \le T/2$$

$$i(t) = 0 \quad ; \quad T/2 \le t \le T$$

$$T = 1s,$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

Sol$^n$ let $I = \int_0^T i^2(t)\,dt$

$$= \int_0^{T/2} i^2(t)\,dt + \cancel{\int_{T/2}^{T} i^2(t)\,dt}^{\,0}$$

$$= \int_0^{T/2} \left[\sin\left(\frac{2\pi t}{T}\right) \times 10e^{-t/T}\right]^2 dt$$

$$= \int_0^{1/2} \left[\sin^2(2\pi t) \times 100e^{-2t}\right] dt$$

Using gauss 2 point Quadrature,

for $a = 0$ and $b = 1/2$ ;

$$u = \frac{x - (a+b)/2}{(b-a)/2} = \frac{x - 1/2/2}{1/2/2} = \frac{(2x - 1/2)/\cancel{2}}{1/2/\cancel{2}} = \frac{(4x-1)/\cancel{2}}{1/\cancel{2}}$$

$$u = 4x - 1$$

$$x = \frac{u+1}{4}$$

$$I = \int_{-1}^{1} g(u)\,du \; ; \; g(u) = \left(\frac{b-a}{2}\right) f\left(\left(\frac{b-a}{2}\right)\cancel{x}^{\,u} + \frac{a+b}{2}\right)$$

$$g(u) = \left(\frac{1/2}{2}\right) f\left(\left(\frac{1/2}{2}\right)\left(\frac{4x+1}{\cancel{4}}\right) + \frac{1/2}{2}\right)$$

$$= \left(\frac{1}{4}\right) f\left(\frac{4x+1}{4} + \frac{1}{4}\right)$$

$$= \left(\frac{1}{4}\right) f\left(\frac{\overbrace{4x + \cancel{1}}^{u} + 1}{4}\right)$$

$$= \left(\frac{1}{4}\right) f\left(\frac{u+1}{4}\right)$$

$$= \left(\frac{1}{\cancel{4}}\right)\left(\overset{25}{\cancel{100}}\, e^{-\cancel{2}\left(\frac{u+1}{\cancel{4}\,2}\right)} \sin^2\left(\frac{\cancel{2}\pi\left(\frac{u+1}{\cancel{4}\,2}\right)}{1}\right)\right)$$

$$= 25\left(\sin^2\left(\frac{\pi(u+1)}{2}\right) \times e^{-\frac{u+1}{2}}\right)$$

$$\int_{-1}^{1} g(u)\cdot du \approx g\left(\frac{1}{\sqrt{3}}\right) + g\left(\frac{-1}{\sqrt{3}}\right)$$

$$I = \int_{0}^{1/2} i^2(t)\cdot dt = \int_{-1}^{1} g(u)\cdot du = g(1/\sqrt{3}) + g(-1/\sqrt{3})$$

$$= 25\left(\sin^2\left(\frac{\pi(u+1)}{2}\right) \times e^{-\frac{(u+1)}{2}}\right)$$

$$= 25\left(\left(\sin^2\left(\frac{\pi(1/\sqrt{3}+1)}{2}\right) \times e^{-\frac{1/\sqrt{3}+1}{2}}\right) + \left(\sin^2\left(\frac{\pi(-1/\sqrt{3}+1)}{2}\right) \times e^{\frac{1/\sqrt{3}+1}{2}}\right)\right)$$

$$= 25((0.454446 \times 0.001869) + (0.809511 \times 0.000134))$$

$$= 0.021234 + 0.002719$$

$$= 0.023953$$ —— actual integration gives 0.023949

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$$

$$= \sqrt{0.023953}$$

$$= 0.154767$$