Problem Statement,

Using Simpson's Rule evaluate this equations for the data table, with $Q = 4 m^3/min$. apply Simpson's 3/8 rule and 1/3 rule respectively accordingly.

Given,

$$M = \int_{t_1}^{t_2} Q\,c\,dt$$

$$Q = 4 m^3/min$$

t, min	0	10	20	30	35	40	45	50
C, mg/m3	10	35	55	32	40	37	82	34
	1	2	3	4	5	6	7	8

fo

for points 1 to 4, apply Simpson's 3/8 rule
for points 4 to 6, apply Simpson's 1/3 rule
for points 6 to 8, apply Simpson's 1/3 rule

formula,

Rule $\frac{1}{3}$: $J = \int_{x_0}^{x_2} f(x)\,dx \approx \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2) \right)$

Rule $\frac{3}{8}$: $J = \int_{x_0}^{x_3} f(x)\,dx \approx \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right]$

Soln

$$M = \int_{t_1}^{t_2} Q\,c\,dt = Q\int_{t_1}^{t_2} c\,dt = Q\left[(x_0 \to x_4) + (x_4 \to x_6) + (x_6 \to x_8) \right]$$

$$= Q\left[J_1 + J_2 + J_3 \right] \quad \text{—}\textcircled{I}$$

for: $I_1 \Rightarrow x_1 \to x_4 \Rightarrow$

x_1	0	10	20	30	$\Rightarrow \therefore h = 10$
x_4					$x_0 = 0; x_1 = 10; x_2 = 20; x_3 = 30$

$$J_1 = \frac{3h}{8} \left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right)$$

$$= \frac{3(10)}{8} \left(f(0) + 3f(10) + 3f(20) + f(30) \right)$$

$$= \frac{30}{8} \left[10 + 3 \times 35 + 3 \times 55 + 52 \right]$$

$$J_1 = 1245$$

$$J_2 = \frac{h}{3} \left(f(x_0) + 4f(x_1) + f(x_2) \right)$$

$$= \frac{5}{3}$$

for: $J_2 \Rightarrow x_4 \rightarrow x_6 \Rightarrow \quad \begin{array}{ccc} 30 & 35 & 40 \\ x_4 & & x_6 \end{array} \Rightarrow \quad \therefore h = 5$

$x_0 = 30 ; x_1 = 35 ; x_2 \, 40$

$$J_2 = \frac{h}{3} \left(f(x_0) + 4(f(x_1)) + f(x_2) \right)$$

$$= \frac{5}{3} \left(f(30) + 4 f(35) + f(40) \right)$$

$$= \frac{5}{3} (40 + 37 + 82) = \frac{5}{3} (52 + 4 \times 40 + 37)$$

$$= 415$$

$$J_2 = 415$$

for: $I_3 \Rightarrow x_6 \rightarrow x_8 \Rightarrow \quad \begin{array}{ccc} 40 & 45 & 50 \\ x_6 & & x_8 \end{array} \Rightarrow \quad \therefore h = 5$

$x_0 = 40 ; x_1 = 45 ; x_2 = 50$

$$I_3 = \frac{h}{3} \left(f(x_0) + 4(f(x_1)) + f(x_2) \right)$$

$$= \frac{5}{3} \left(f(40) + 4 f(45) + f(50) \right)$$

$$= \frac{5}{3} \ [\ 37 + 4 + 32 + 34\ ]$$

$$J_3 = 331.67$$

as& above eq ①

$$M = Q\ [\ I_1 + I_2 + J_3\ ]$$
$$= 4 + [\ 1245 + 415 + 331.67\ ]$$
$$= 7966.68 \ mg$$