MA 203 | NUMERICAL METHODS

PROBLEM SET 1

INSTRUCTIONS

- Problems marked with an asterisk (*) are tutorial problems and solutions must be submitted by the students in Google classroom by the end of the tutorial session (**6:30 pm**). Note that if the submission is not made by the end of the tutorial session or if the submission does not reflect reasonable effort and work, the student may be marked absent for the tutorial session. Late submissions will not be accepted.
- In addition, you will need to do at least one problem in PART A (Euler's Method) and one problem in PART B (Errors and Condition Numbers) as a homework assignment (excluding the tutorial problems). You must submit the assignment by **10 pm on August 17, 2023** in Google classroom.
- You should write a computer program to solve the problems in part A. You may use Matlab or any other programming language of your choice.
- The report must entail a description of the problem, the procedure used to solve it (see assignment format in the course plan for details), and results/solutions and discussions. The report must be in **PDF format**. Please upload the report and program files separately (that is, please do NOT submit all of them together as a single ZIP file) in Google classroom. **Solutions to each problem should be submitted as a separate file**. Name each file as: problemnumber_Rollnumber.***. For example, if your roll number is 22110110 and for problem A1, name your report file as A1_22110110.pdf and program file as A1_22110110.m (if you are using MATLAB).

PART A | EULER'S METHOD

Problem A1* (Tutorial problem)

Suppose that a spherical droplet of liquid evaporates at a rate that is proportional to its surface area.

$$\frac{dV}{dt} = -k\,A$$

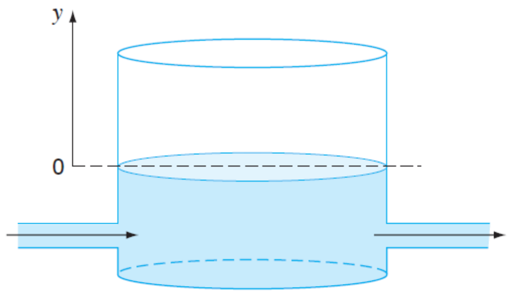
where V = volume(mm^3), t = time(min), k = the evaporation rate(mm/min), and A = surface area(mm^2). Use Euler's method to compute the volume of the droplet from t = 0 to 10 min using a step size of 0.25 min. Assume that k = 0.1 mm/min and that the droplet initially has a radius of 3 mm. Assess the validity of your results by determining the radius of your final computed volume. Calculate the average evaporation rate (change in radius/time) and verify that it is consistent with the given evaporation rate.

Problem A2

A storage tank contains a liquid at depth *y*, where *y* = 0 when the tank is half full. Liquid is withdrawn at a flow rate that depends on the depth as follows:

$Q_{out} = \alpha(1+y)^{1.5}$

The contents are resupplied at a sinusoidal rate $3Q\sin^2(t)$.



Develop a mathematical model of the process based on the principle of conservation of mass. Use Euler's method to solve for the depth y from t = 0 to 10 d with a step size of 0.5 d. The parameter values are A = 1200 m^2, where A is the surface area of the container and Q = 500 m^3/d, and α = 300. Assume that the initial condition is y = 0.

Problem A3

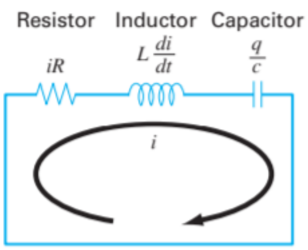
Newton's law of cooling says that the temperature of a body changes at a rate proportional to the difference between its temperature and that of surrounding medium (the ambient temperature).

$$\frac{dT}{dt} = -k(T - T_a)$$

where, T = the temperature of the body (° C), t = time (min), k = the proportionality constant (per minute), and T_o = the ambient temperature (° C). Suppose that a cup of coffee originally has a temperature of 68 °C. Use Euler's method to compute the temperature from t = 0 to 10 min using step size of 1 min if T_a = 21 ° C and k = 0.017 /min

Problem A4

An RLC circuit consists of three elements: a resistor (R), and inductor (L) and a capacitor (C).



The flow of current across each element induces a voltage drop. Kirchhoff's second voltage law states that the algebraic sum of these voltage drops around a closed circuit is zero:

$$iR + L\frac{di}{dt} + \frac{q}{C} = 0$$

where, i = current, R = resistance, L = inductance, t = time, q = charge and C = capacitance. In addition, the current is related to charge as in

$$\frac{dq}{dt} = i$$

a) If the initial values are i(0) = 0 and q(0) = 1 C, use Euler's method to solve this pair of differential equations from t = 0 to 0.1 s using a step size of Δt = 0.01 s. Employ the following parameters for your calculation: R = 200 Ω, L = 5 H and C = 10^{-4} F.
b) Develop a plot of i and q versus t.

PART B | ERRORS AND CONDITION NUMBERS

Problem B1* (Tutorial problem)

The Stefan-Boltzmann law can be employed to estimate the rate of radiation of energy *H* from a surface, as in

$$H = Ae\sigma T^4$$

where *H* is in watts, *A* = the surface area (m^2), *e* = the emissivity that characterizes the emitting properties of the surface (dimensionless), σ = a universal constant called the Stefan-Boltzmann constant (= 5.67 10^{-8} W m^{-2} K^{-4}), and *T* = absolute temperature (K). Determine the error of *H* for a steel plate with *A* = 0.15 m^2, *e* = 0.90, and *T* = 650 $\pm$ 20. Compare your results with the exact error. Repeat the computation but with *T* = 650 $\pm$ 40. Interpret your results.

Problem B2

Evaluate e^{-5} using two approaches:

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \cdots$$

and

$$e^{-x} = \frac{1}{e^x} = \frac{1}{1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots}$$

Compare with the true value of 6.737947 × 10^{-3}. Use 20 terms to evaluate each series and compute true and approximate relative errors as terms are added. Conclude which series converges faster to the true value.

Problem B3

Use zero- through third order Taylor series expansions to predict f(3) for

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

Using a base point at x=1. Compute the true percent relative error ε_t for each approximation.

Problem B4

Evaluate and interpret the condition numbers for

A. $f(x) = \sqrt{|x-1|} + 1$ for x = 1.00001
B. $f(x) = e^{-x}$ for x = 10
C. $f(x) = \sqrt{x^2+1} - x$ for x = 300
D. $f(x) = \frac{e^{-x}-1}{x}$ for x = 0.001
E. $f(x) = \frac{\sin x}{1+\cos x}$ for x = 1.0001π