MA 203 | NUMERICAL METHODS

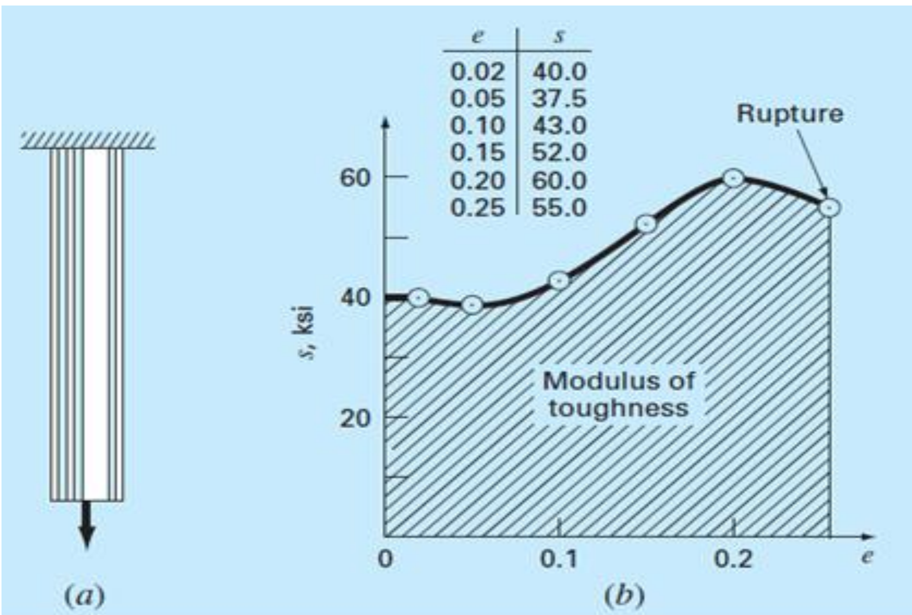
PROBLEM SET 4

INSTRUCTIONS

- Problems marked with an asterisk (*) are tutorial problems and solutions must be submitted by the students in Google classroom by the end of the tutorial session (**6:30 pm**). Note that if the submission is not made by the end of the tutorial session or if the submission does not reflect reasonable effort and work, the student may be marked absent for the tutorial session. Late submissions will not be accepted.
- In addition, you will need to do Problem 2 and any one of Problems 3 and 4. You must submit the assignment by **10 pm on September 14, 2023** in Google classroom.
- **Submission of computer code is optional for this problem sheet.**
- The report must entail a description of the problem, the procedure used to solve it (see assignment format in the course plan for details), and results/solutions and discussions. The report must be in **PDF format**. Please upload the report and program files separately (that is, please do NOT submit all of them together as a single ZIP file) in Google classroom. **Solutions to each problem should be submitted as a separate file**. Name each file as: problemnumber_Rollnumber.***. For example, if your roll number is 22110110 and for problem A1, name your report file as A1_22110110.pdf and program file (if submitted) as A1_22110110.m (if you are using MATLAB).

Problem 1* (Tutorial problem)

A rod subject to an axial load will be deformed, as shown in the stress (e) vs strain (s) curve (see figure below). The area under the curve from zero stress out to the point of rupture is called the modulus of toughness of the material. It provides a measure of the energy per unit volume required to cause the material to rupture. As such, it is representative of the material's ability to withstand an impact load. Use the Trapezoidal rule to compute the modulus of toughness for the shown stress-strain curve.



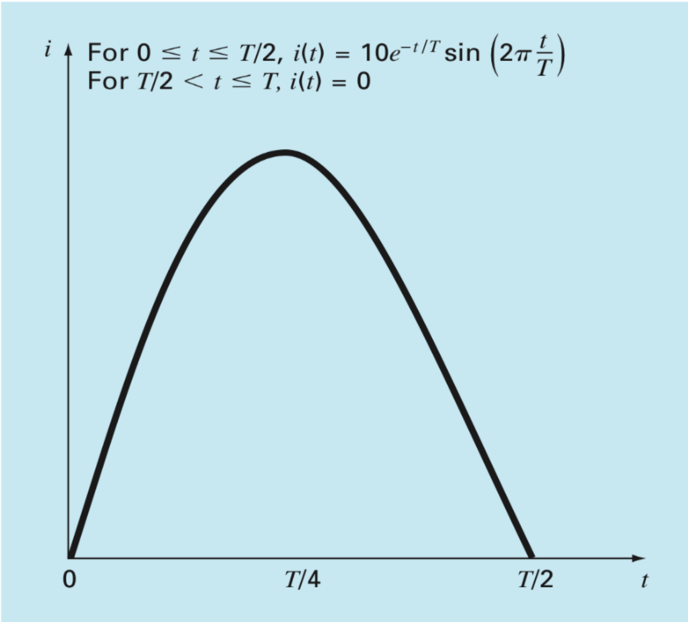
Problem 2

Consider a current described by a simple sinusoid $i(t) = \sin\left(\frac{2\pi t}{T}\right)$, as a function of time t, where T is the period. The average value of this function is zero since $\frac{1}{T}\int_0^T i(t)dt = 0$.

Even though the net result is zero, such current can perform work and generate heat. Therefore, engineers often characterize such current by its RMS or root-mean-square:

$$I_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} .$$

Using 2-point Gauss quadrature, calculate the RMS for the waveform shown in the figure below, letting $T = 1$ s.



Problem 3

Integration provides a means to compute how much mass enters or leaves a reactor over a specified time period, as in $M = \int_{t_1}^{t_2} Q\ c\,dt$, where Q is the flow rate, c is the concentration and t_1 and t_2 are the initial and final times, respectively. This formula makes intuitive sense if you recall the analogy between integration and summation. The integral represents the summation of the product of flow rate times concentration to give the total mass entering or leaving from t_1 to t_2. If the flow rate is constant, Q can be moved outside the integral to get

$$M = Q\int_{t_1}^{t_2} c\ dt.$$

Use Simpson's Rule to evaluate this equation for the data below, with $Q = 4\ \mathrm{m^3/min}$. Since the data points are unequally spaced, apply Simpson's 3/8 rule for points 1 to 4, followed by Simpson's 1/3 rule applied twice, once for points 4 to 6 and again for points 6 to 8.

t, min	0	10	20	30	35	40	45	50
c, mg/m^3	10	35	55	52	40	37	32	34

Problem 4

Water exerts pressure on the upstream face of a dam as shown in the figure below. The pressure can be characterized by the following equation

$p(z) = \rho g(D - z)$,

where $p(z)$ is pressure in Pascals exerted at an elevation z meters above the reservoir bottom; ρ is density of water, which for this problem is assumed to be a constant 10^3 $\mathrm{kg/m^3}$; $g = 9.8\ \mathrm{m/s^2}$ is the acceleration due to gravity; and D is elevation (in meters) of the water surface above the reservoir bottom. According to the given equation, pressure increases linearly with depth, as depicted in the figure.

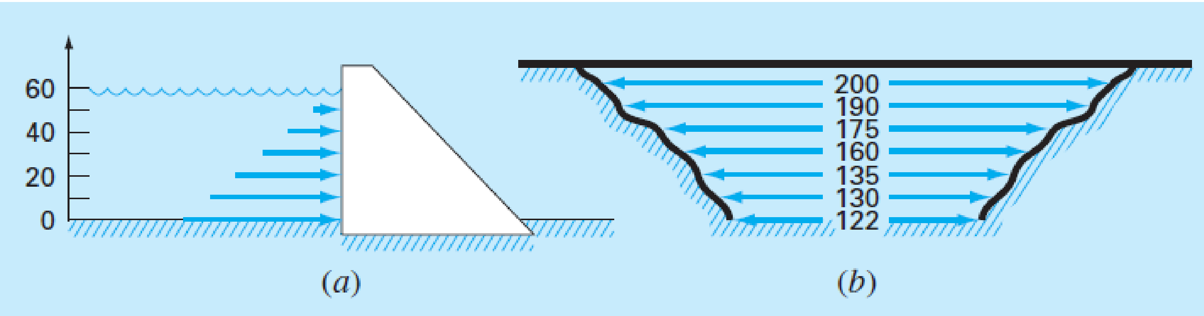


Figure (a) depicts the side view of the dam, with values of the elevation z on the y-axis. Figure (b) depicts the width of the dam at the corresponding elevation in meters. Omitting atmospheric pressure (because it works against both sides of the dam face and essentially cancels out), the total force f_t can be determined by multiplying pressure times the area of the dam face. Because both pressure and area vary with elevation, the total force is obtained by evaluating

$$f_t = \int_0^D \rho g\ w(z)(D - z)dz,$$

where $w(z) =$ width of the dam face (in meters) at elevation z . The effective height d of this force above the reservoir bottom can be calculated by the formula

$$d = \frac{\int_0^D \rho g\, z\, w(z)(D-z)dz}{\int_0^D \rho g\, w(z)(D-z)dz}.$$

Compute the values of f_t and d using Simpson's rule. Since there are an odd number of equally spaced points, apply Simpson's 1/3 rule multiple times.