MA 203 | NUMERICAL METHODS

PROBLEM SET 2

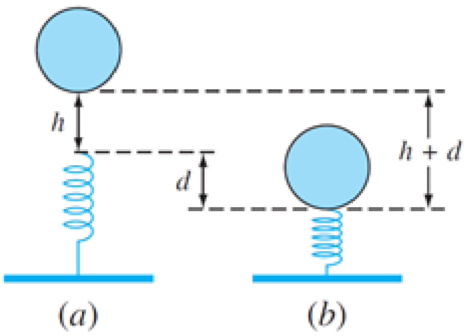
INSTRUCTIONS

- Problems marked with an asterisk (*) are tutorial problems and solutions must be submitted by the students in Google classroom by the end of the tutorial session (**6:30 pm**). Note that if the submission is not made by the end of the tutorial session or if the submission does not reflect reasonable effort and work, the student may be marked absent for the tutorial session. Late submissions will not be accepted.
- In addition, you will need to do Problem 2 and any one of the remaining problems as a homework assignment (excluding the tutorial problem). You will need to use the false position method for Problem 2 and any of the open methods for the other problem. You must submit the assignment by **10 pm on August 24, 2023** in Google classroom.
- You should write a computer program to solve the problems. You may use Matlab or any other programming language of your choice.
- The report must entail a description of the problem, the procedure used to solve it (see assignment format in the course plan for details), and results/solutions and discussions. The report must be in **<u>PDF format</u>**. Please upload the report and program files separately (that is, please do NOT submit all of them together as a single ZIP file) in Google classroom. **Solutions to each problem should be submitted as a separate file**. Name each file as: problemnumber_Rollnumber.***. For example, if your roll number is 22110110 and for problem A1, name your report file as A1_22110110.pdf and program file as A1_22110110.m (if you are using MATLAB).

Problem 1* (Tutorial problem)

Real mechanical systems may involve the deflections of non-linear springs. In Figure below, a mass *m* is released a distance *h* above a non-linear spring. The resistance force *F* of the spring is given by

$$F = -\left(k_1 d + k_2 d^{\frac{3}{2}}\right)$$



Conservation of energy can be used to show that

$$0 = \frac{2k_2 d^{\frac{5}{2}}}{5} + \frac{1}{2}k_1 d^2 - mgd - mgh$$

Solve for d using the Bisection method, given the following parameter values. $k_1 = 50{,}000\ \frac{\mathrm{g}}{\mathrm{s}^2}, k_2 = 40\ \frac{\mathrm{g}}{\mathrm{s}^2\mathrm{m}^{0.5}},\ m = 90\ \mathrm{g},\ g = 9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}, h = 0.45\ \mathrm{m}.$

Problem 2

The ideal gas equation of state is valid only for a limited range of pressures and temperatures. An alternative equation of state for gases is the van der Waals equation:

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

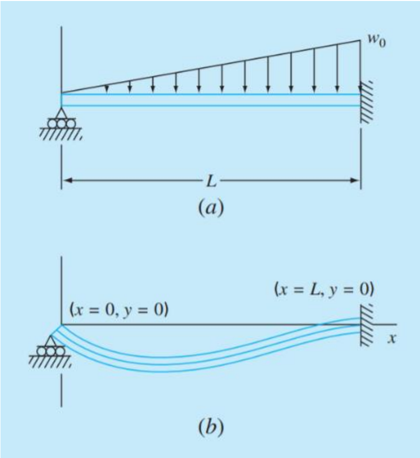
where *v* is the molar volume, and *a* and *b* are the empirical constants for a gas. A chemical engineering design project requires you to accurately estimate the molar volume of ethyl alcohol (*a* = 12.02 and *b* = 0.08407) at a temperature of 400 K and pressure of 2.5 atm. Use the false position method. Compare your results with the ideal gas law.

Problem 3

Figure below shows a uniform beam subject to a linearly increasing distributed load. The equation for the resulting elastic curve is (see figure below)

$$y = \frac{w_0}{120EIL}(-x^5 + 2L^2x^3 - L^4x)$$

Determine the point of maximum deflection. Then substitute this value into the above equation to determine the value of the maximum deflection. Use the following parameter values in your computation: $L = 600\ \mathrm{cm}$, $E = 50{,}000\,\frac{\mathrm{kN}}{\mathrm{cm}^2}$, $I = 30{,}000\ \mathrm{cm}^4$, $w_0 = 2.5\,\frac{\mathrm{kN}}{\mathrm{cm}}$. Use any open method for root-finding.

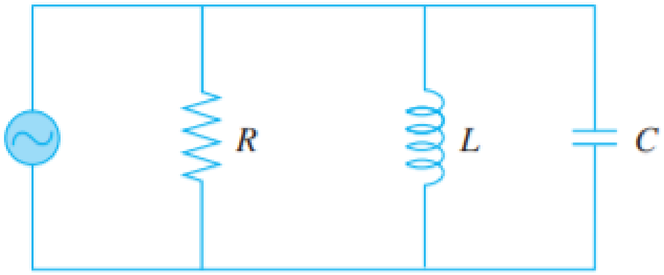


Problem 4

Figure below shows a circuit with a resistor, an inductor, and a capacitor in parallel. Kirchhoff's rules can be used to express the impedance of the system as

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

where $Z = \text{Impedance}\ (\Omega)$ and $\omega = \text{angular frequency.}$ Find the ω that results in an impedance of $75\,\Omega$ for the following parameters: $R = 225\ \Omega, C = 0.6 \times 10^{-6}\ \text{F},\ L = 0.5\ \text{H}.$ Use any open method for root-finding.



Problem 5

For fluid flow in pipes, friction is described by a dimensionless number, the *Fanning friction factor* f. The Fanning friction factor is dependent on a number of parameters related to the size of the pipe and the fluid, which can all be represented by another dimensionless quantity, the *Reynolds number* Re. A formula that predicts the value of f given Re is the *von Karman equation,*

$$\frac{1}{\sqrt{f}} = 4log_{10}\left(Re\sqrt{f}\right) - 0.4$$

Typical values for the Reynolds number for turbulent flows are $10{,}000$ to $500{,}000$ and for Fanning friction factor are 0.001 to 0.01. Develop a function that solves for f given a user-supplied value of Re between 2500 and $1{,}000{,}000$. Design the function in such a way that the relative error tolerance is sufficiently low to enable accurate computation of friction factors for all values of Reynolds numbers. Use any open method for root-finding.