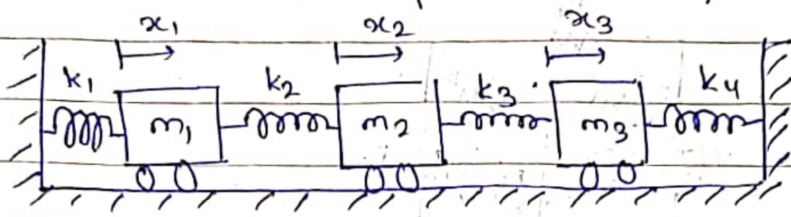
Q5 Problem Statement,

considering 3 masses and 4 springs as a system in the following form, find equation of motion for each mass, (write eq^n in matrix form)

digram,



given, $k_1 = k_4 = 10 N/m$

$k_2 = k_3 = 30 N/m$

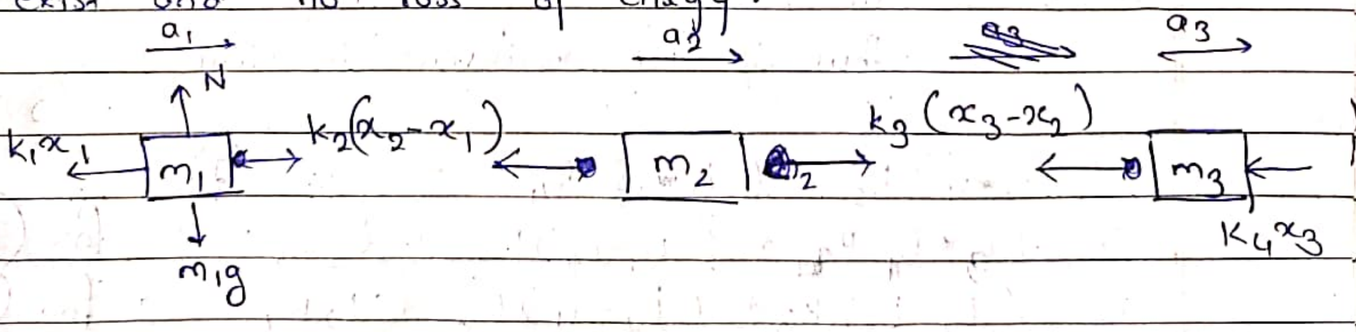
$m_1 = m_2 = m_3 = 2kg$

matrix form [Displacement x] [k/m matrx] + [acc^n a] = 0
matrix matrix

at, time when $a_1 = -0.4 \, m/s^2$, $a_2 = a_3 = 0$ this forms tridiagonal matrix, solve this matrix with (TDMA)

assumptions,

the system is ideal, friction is not exist and no loss of energy.



$F_1 = k_1 x_1 - k_2 (x_2 - x_1)$ — for mass 1

$F_2 = k_2 (x_2 - x_1) - k_3 (x_3 - x_2)$ — for mass 2

$F_3 = k_3 (x_3 - x_2) - k_4 x_3$ — for mass 3

$$F_1 = k_2(x_2 - x_1) - k_1 x_1 \quad \text{--- (I)}$$

$$F_2 = k_3(x_3 - x_2) - k_2(x_2 - x_1) \quad \text{--- (II)}$$

$$F_3 = -k_3(x_3 - x_2) - k_4(x_3) \quad \text{--- (III)}$$

from (1)
$$30x_2 - 30x_1 - 10x_1 = m_1 a_1$$
$$30x_2 - 40x_1 = 2a_1$$
$$15x_2 - 20x_1 = a_1$$

from (II)
$$30x_3 - 30x_2 - 30x_2 + 30x_1 = m_2 a_2$$
$$30x_1 - 60x_2 + 30x_3 = 2a_2$$
$$15x_1 - 30x_2 + 15x_1 = a_2$$

from (III)
$$-30x_3 + 30x_2 - 10x_3 = m_3 a_3$$
$$-40x_3 + 30x_2 = 2a_3$$
$$-20x_3 + 15x_2 = a_3$$

$$\underbrace{\begin{bmatrix} -20 & 15 & 0 \\ 15 & -30 & 15 \\ 0 & 15 & -20 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}}_{B}$$

Now as per que $a_1 = -0.4$; $a_2 = 0$; $a_3 = 0$

So converting A to LU

$R_2 \longrightarrow R_2 - \left(\frac{15}{-20}\right) R_1$ multiple$_1$ = $\frac{\text{coff } \cancel{20}\ A[0][0]}{\text{coff } A[0][1]}$

$R_3 \longrightarrow R_3 - \left(\frac{15}{-30}\right) R_2$ multiple$_2$ = $\frac{\text{coff } A[\text{[illegible]}][\text{[illegible]}]}{\text{coff } A[1][2]}$

$$U = \begin{bmatrix} -20 & 15 & 0 \\ 0 & -75/4 & 15 \\ 0 & 0 & 25/2 \end{bmatrix} \quad \text{and} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ \text{multiple}_1 & 1 & 0 \\ 0 & \text{mulipl}_2 & 1 \end{bmatrix}$$

Now, $LD = B$ find such D.

$$\begin{bmatrix} 1 & 0 & 0 \\ 15/-20 & 1 & 0 \\ 0 & 15/-30 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

Now back substitution.

$UX = D$

$$\begin{bmatrix} -20 & 15 & 0 \\ 0 & -75/4 & 15 \\ 0 & 0 & 25/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

$x_1 = \cancel{0.09} \quad 0.05$

$x_2 = \cancel{0.12} \quad 0.04$

$x_3 = \cancel{0.09} \quad 0.03$