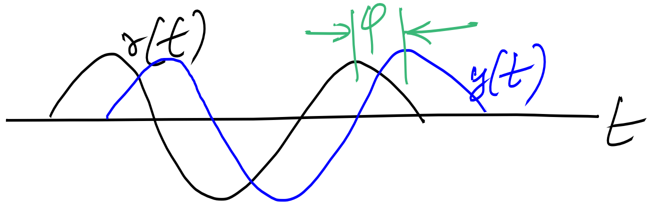
15-Feb-2018

Lecture 19: Bode Plots (End), Minimum-Phase, Stability

Brihaspati > TA-assignments > L18-HW.pdf.

Minimum-phase and non-minimum-phase TFs.



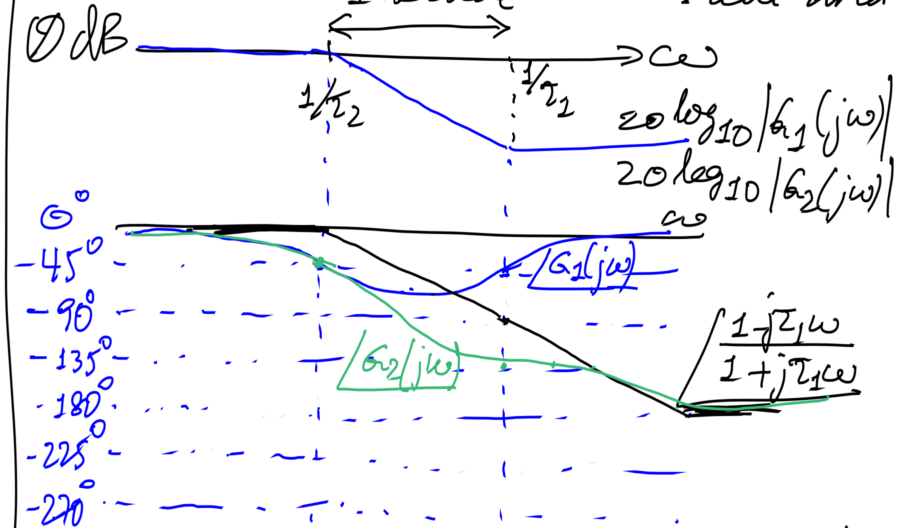
φ - phase shift of the output y(t) w.r.t. the input r(t).

→ phase shift of output w.r.t. input.

$$G_1(s) = \frac{1+\tau_1 s}{1+\tau_2 s}, \quad G_2(s) = \frac{1-\tau_1 s}{1+\tau_2 s},$$

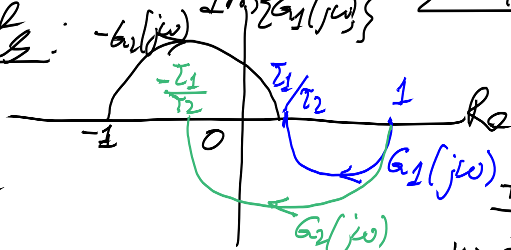
$$1/\tau_1 = \frac{10}{\tau_2} \Rightarrow \tau_2 = 10\tau_1$$

$\tau_1, \tau_2 > 0$. (Time constants are always real and positive.)

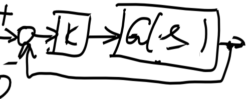


$$\angle G_2(j\omega) = \angle G_1(j\omega) + \angle \frac{1-j\tau_1\omega}{1+j\tau_1\omega}.$$

polar plots:



HW: sketch the polar plot of $-G_2(j\omega)$.



$K > 0$

Among all the stable TFs with the same BMP the one whose polar plot spans the fewest number of quadrants is called minimum-phase shift TF or minimum-phase TF. Both these terms were introduced by Bode in his ~~book~~ Network Analysis and Feedback Amplifier Design, published in 1945.

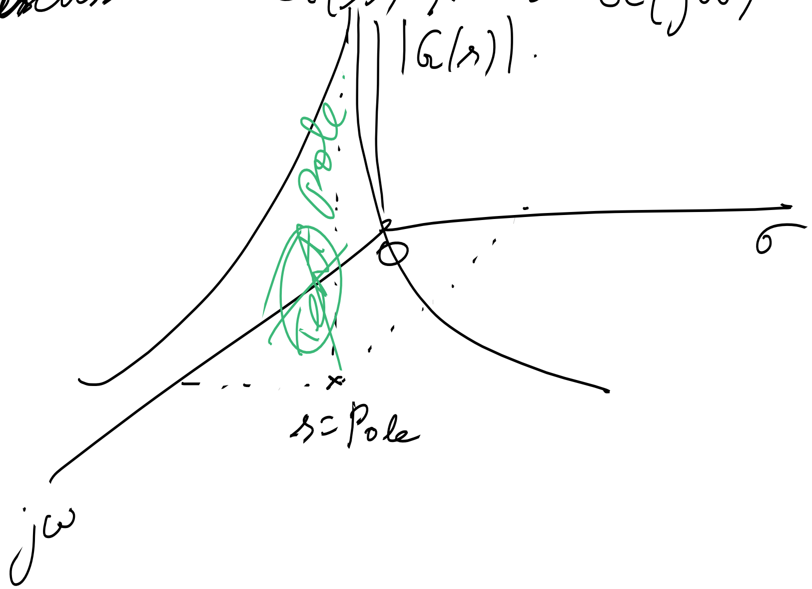
What is the importance of minimum- and non-minimum-phase TFs? The control system built around the NMP version of a TF could be unstable under the same controller that stabilizes the closed loop system built around the corresponding MP version. In the figure, the control system is stable for all $K > 0$ when $G(s) = G_1(s)$, but is unstable for K larger than some positive value for $G(s) = G_2(s)$. Note that the polar plots of $G_1(j\omega)$ and $G_2(j\omega)$ are nearly identical at low frequencies, and diverge at higher frequencies. Such a similarity does not exist between $G_1(j\omega)$ and $-G_2(j\omega)$ at any ω. So, it is hard to see that $-G_2(s)$ is the NMP version of $G_1(s)$ until we see that $G_2(s)$ is the NMP version of $G_1(s)$.

Most books define NMP TFs as those that have RHP zeros, or as those that respond "the wrong way".

Response y(t) goes the right way. Response y(t) starts out the wrong way.

poles & zeros:

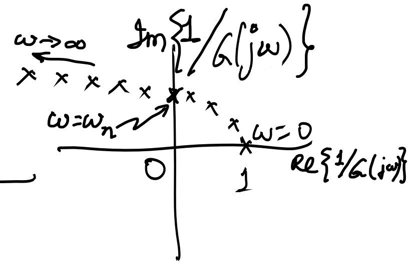
discussed w.r.t $G(s)$, not $G(j\omega)$.



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega} = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right] + j2\zeta\frac{\omega}{\omega_n}}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$



$$\angle G(j\omega) = -\angle\left(\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + j2\zeta\frac{\omega}{\omega_n}\right)$$

$0° \longrightarrow -180°$.