Solution Q1:

Small Step Sizes (e.g., 0.0001, 0.001, 0.01)

- **Smooth Descent & Convergence**
  For small learning rates, the spiral-like function appears to be minimized steadily. The plots show a slow but stable decrease in the objective y, often reaching values near 10−^3 or lower.
- **Longer Time to Converge**
  The algorithm takes many iterations to get close to the minimum, but it does so without blowing up.

Medium Step Size (e.g., 0.1)

- **Mixed Behavior**
  Some seeds show a partial descent and then the function value skyrockets, while others might converge slowly or in a somewhat erratic pattern.
- **Occasional Divergence**
  The plots where x1 and x2 blow up to very large magnitudes and the function value grows extremely large indicate divergence.
- **Sensitivity to Initial Conditions**
  The difference in behavior across seeds suggests that with λ=0.1, some initial points still allow partial descent, while others cause large jumps and overshooting.

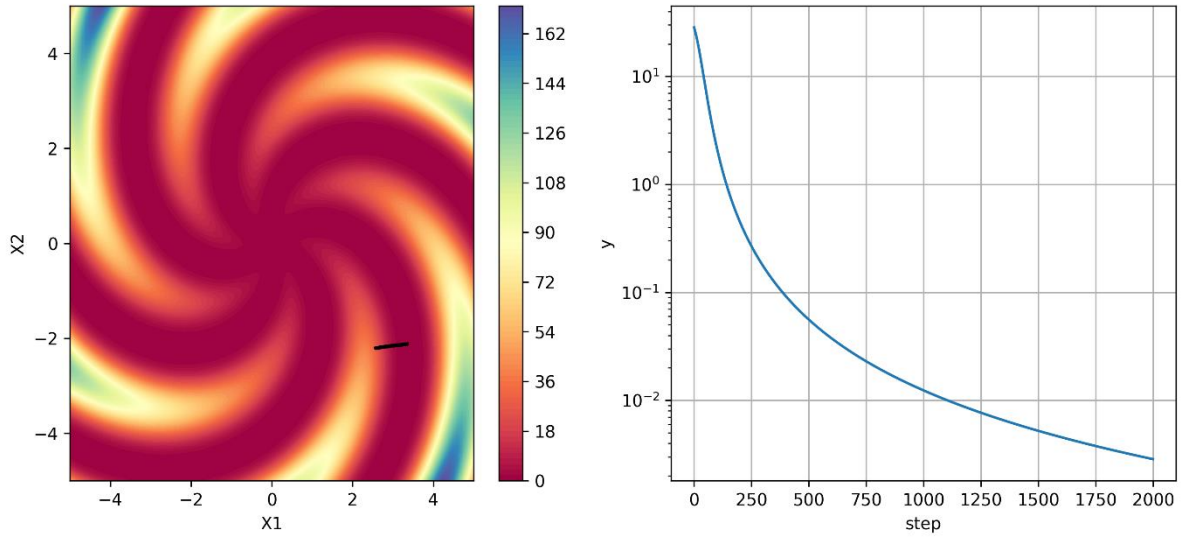
Large Step Size (e.g., 1.0)

- **Frequent or Immediate Divergence**
  Most seeds cause the updates to overshoot drastically. x1 and x2 move rapidly to very large absolute values, and the objective y can jump orders of magnitude upward.
- **Minimal or No Convergence**
  The few seeds that don't immediately diverge tend to hover around large values or eventually blow up. You rarely see stable decreases in y.

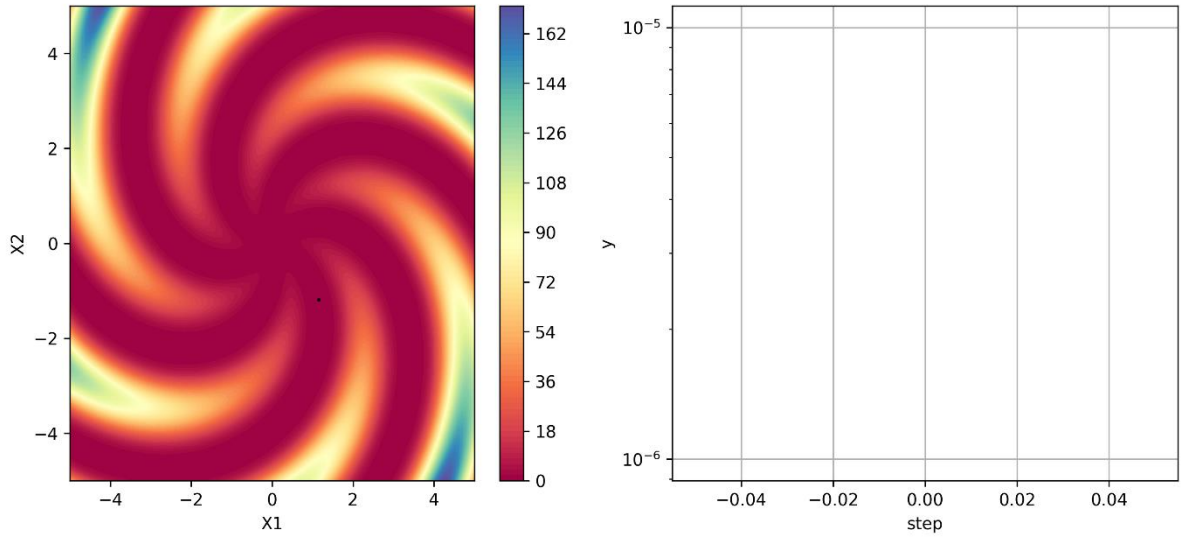
Conclusion:

- **Small λ**: Stable, slow, eventually convergent.
- **Moderate λ**: Can be effective but sometimes diverges, depends heavily on the starting point.
- **Large λ**: High likelihood of divergence—parameters explode to large values, and y becomes huge.

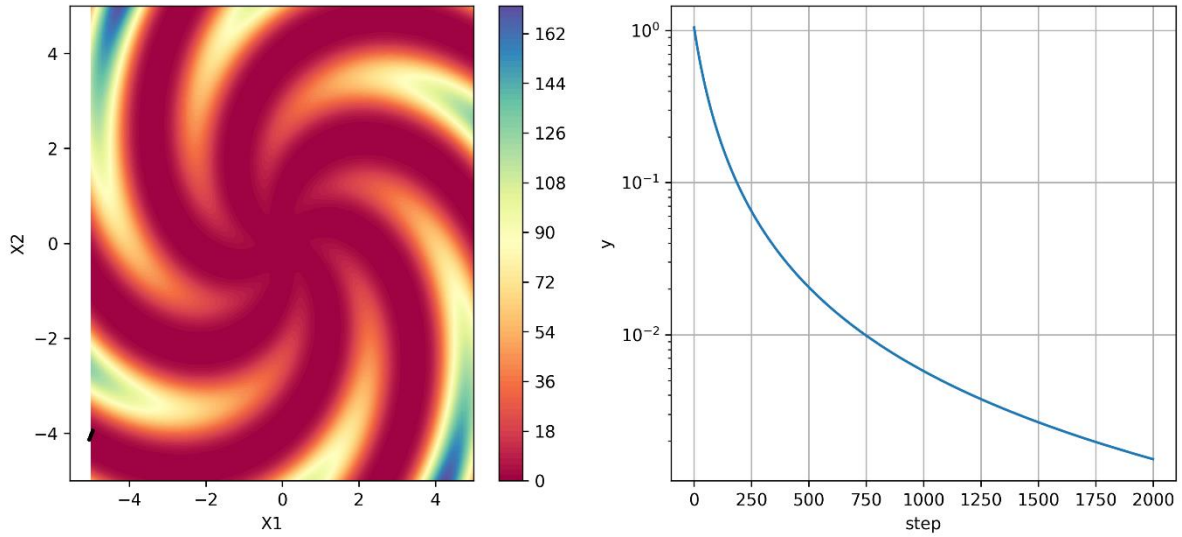
Step size: 0.0001, seed: 1



Step size: 0.0001, seed: 2



Step size: 0.0001, seed: 3



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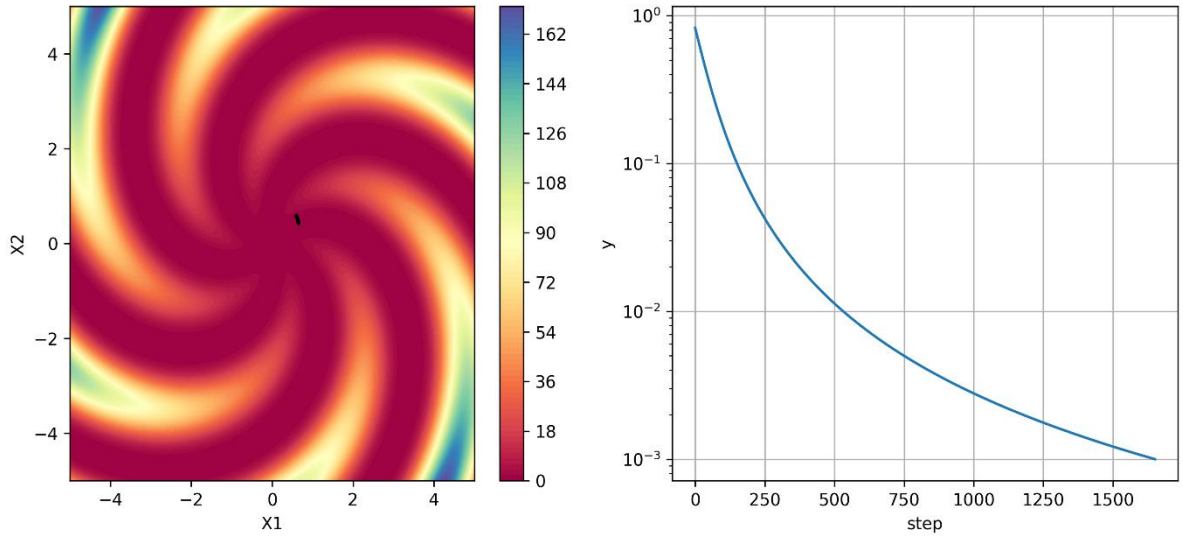
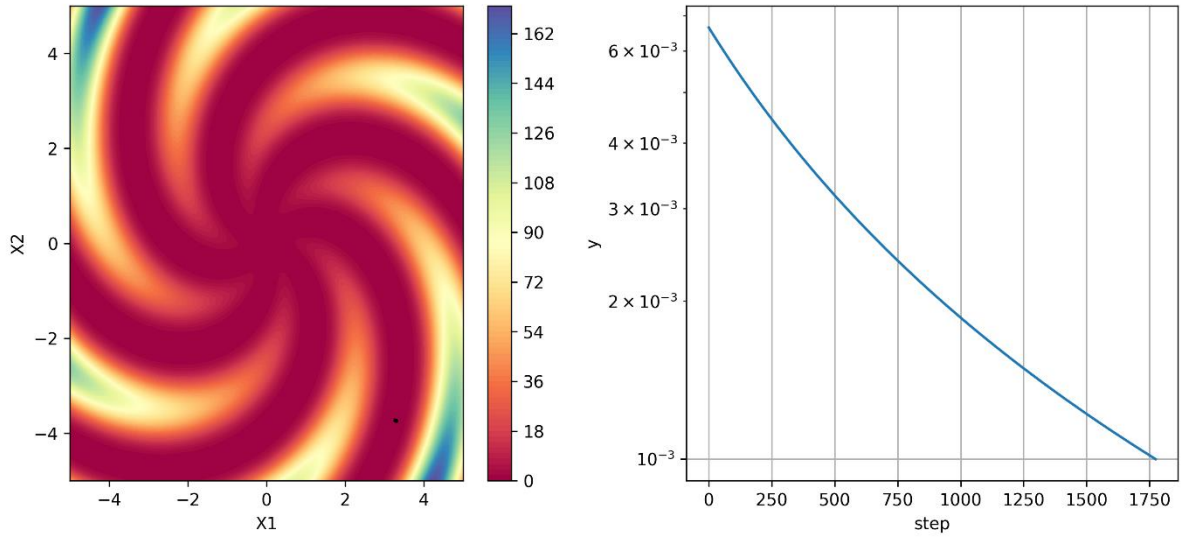


Step size: 0.0001, seed: 5



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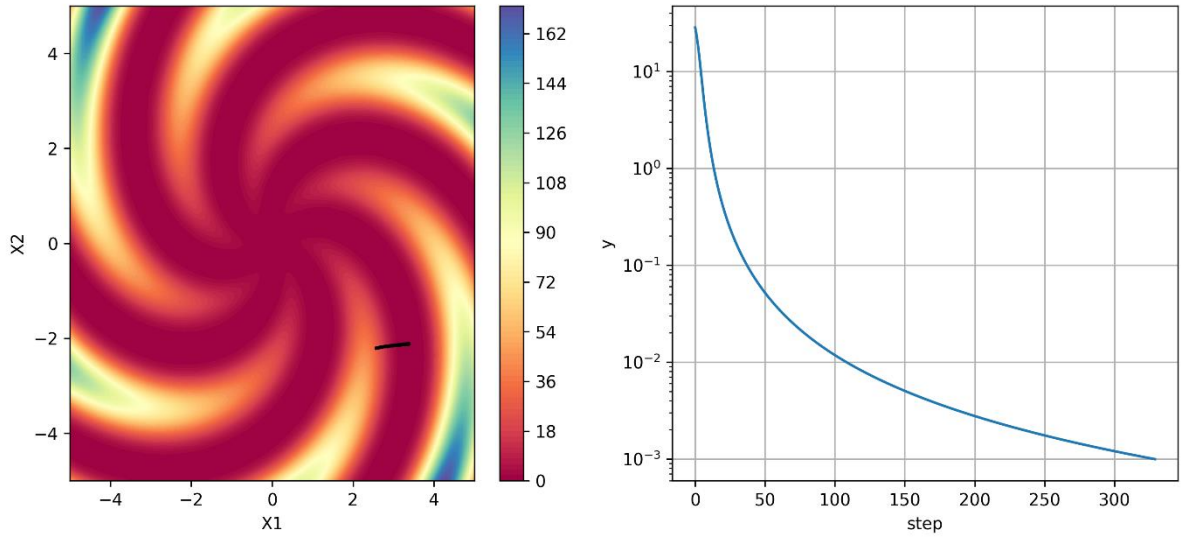

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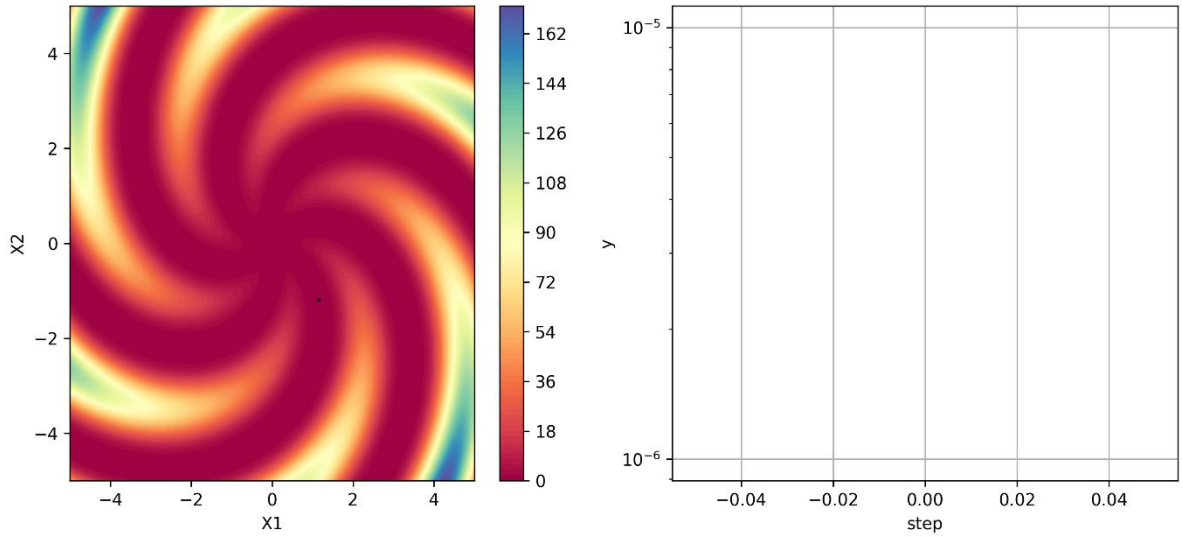

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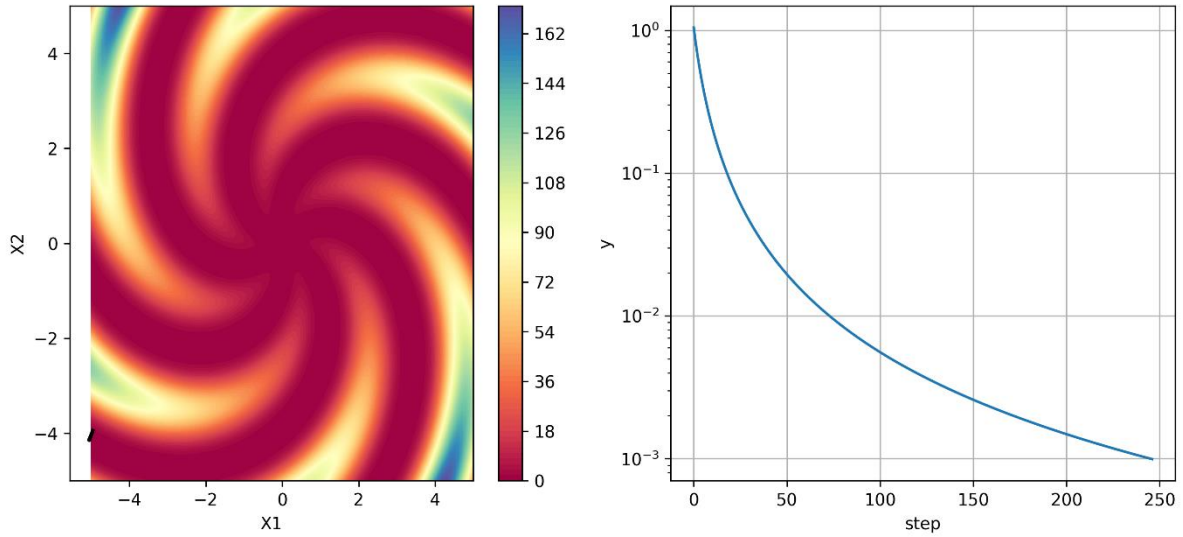

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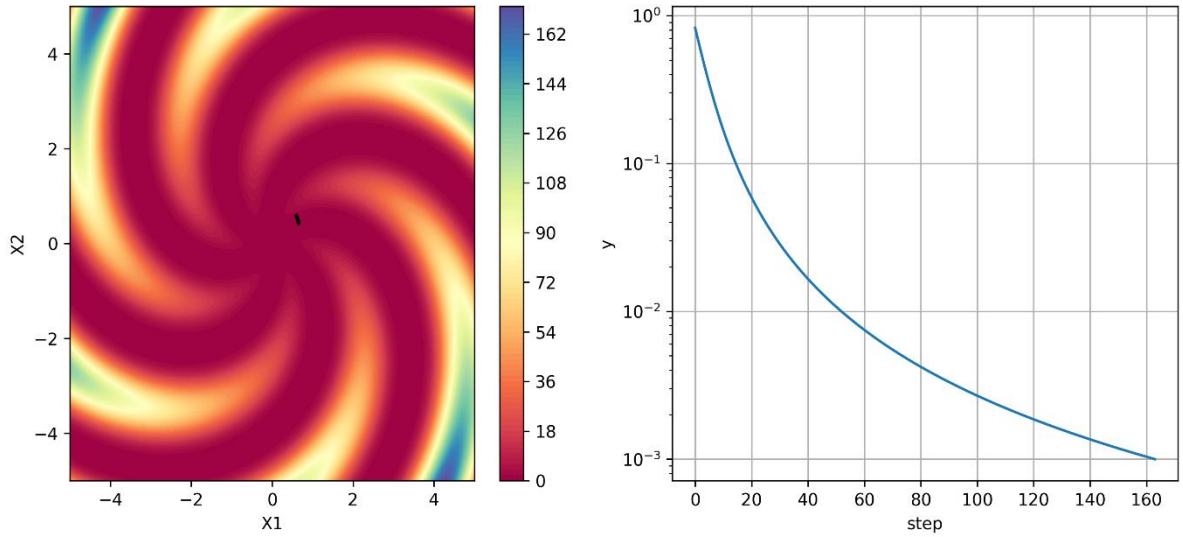

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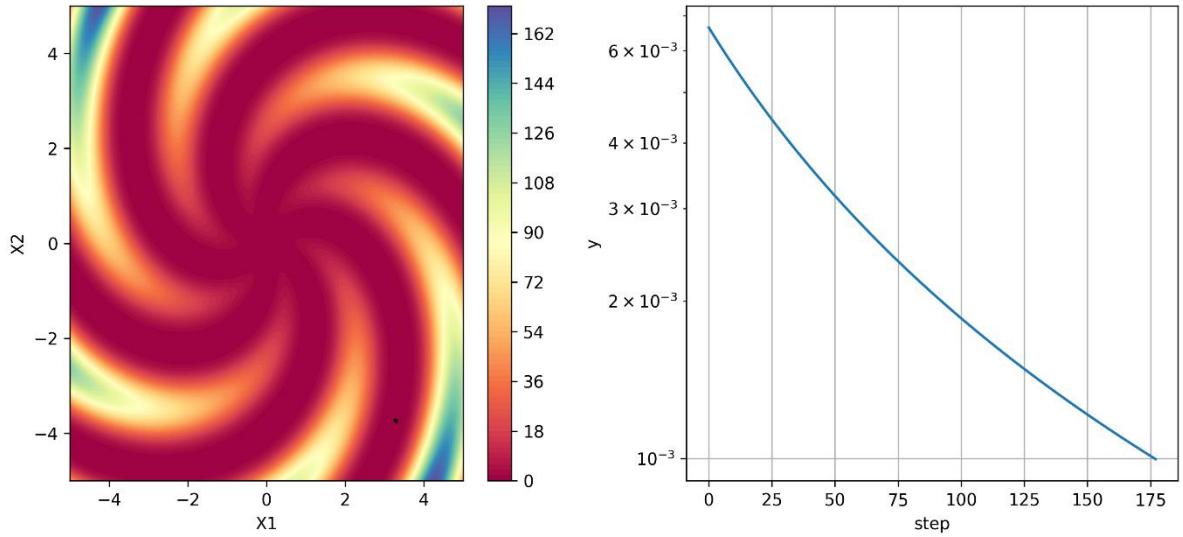

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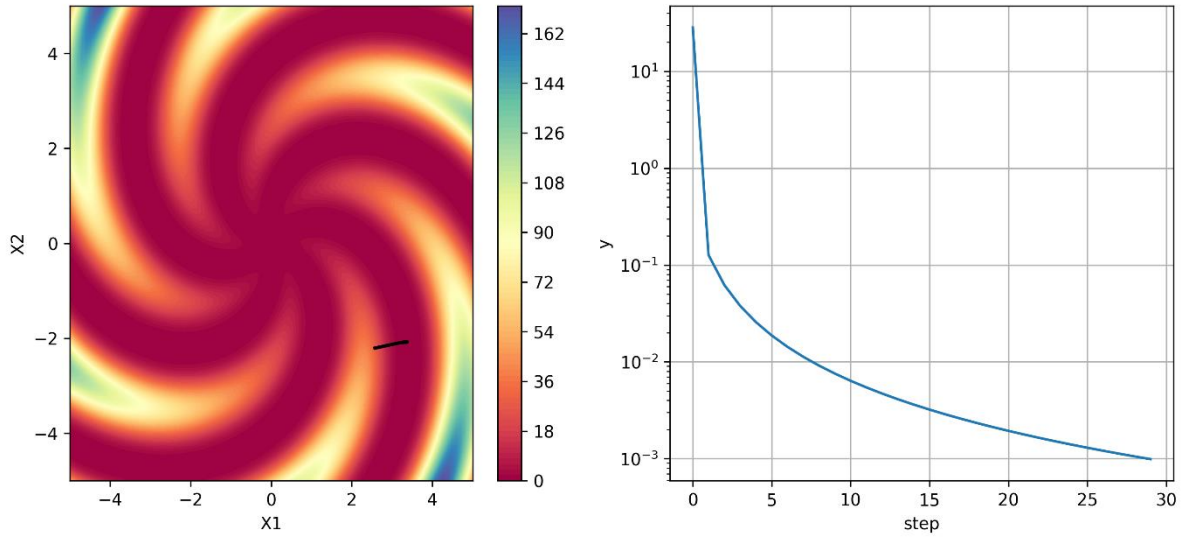

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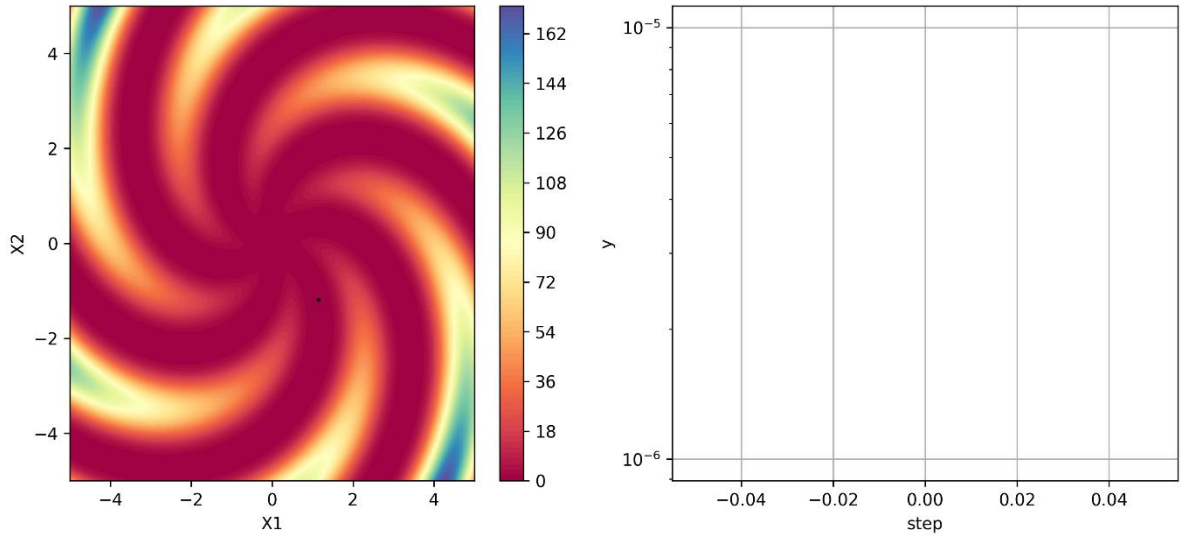

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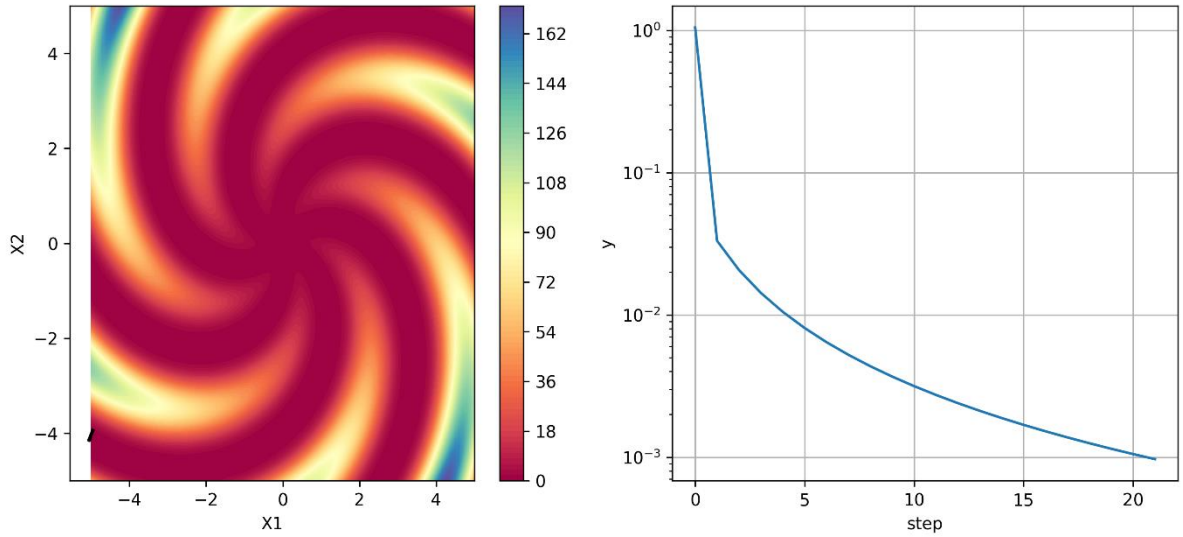

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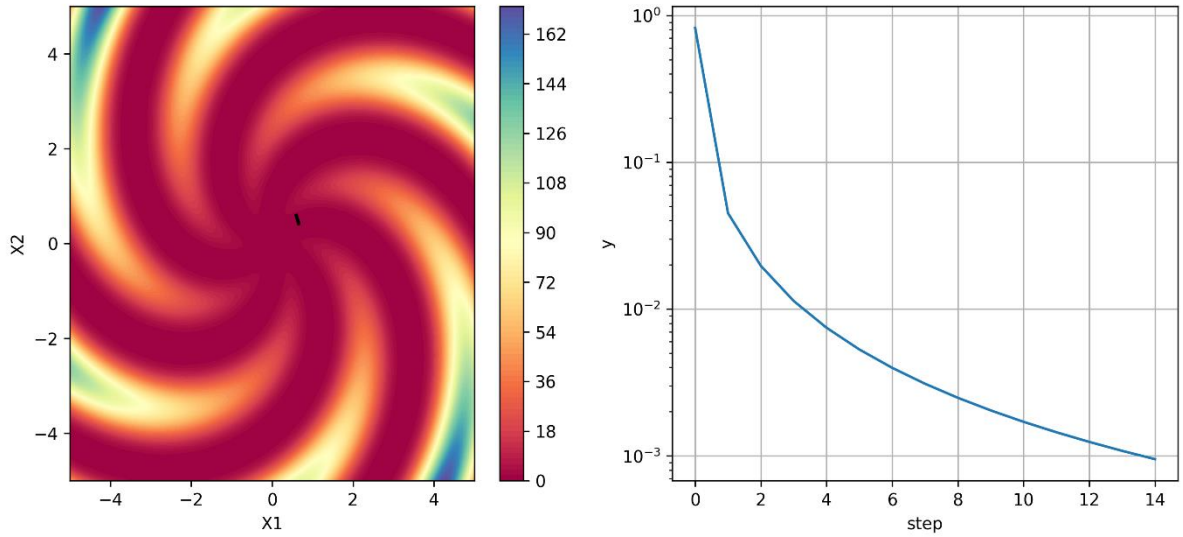

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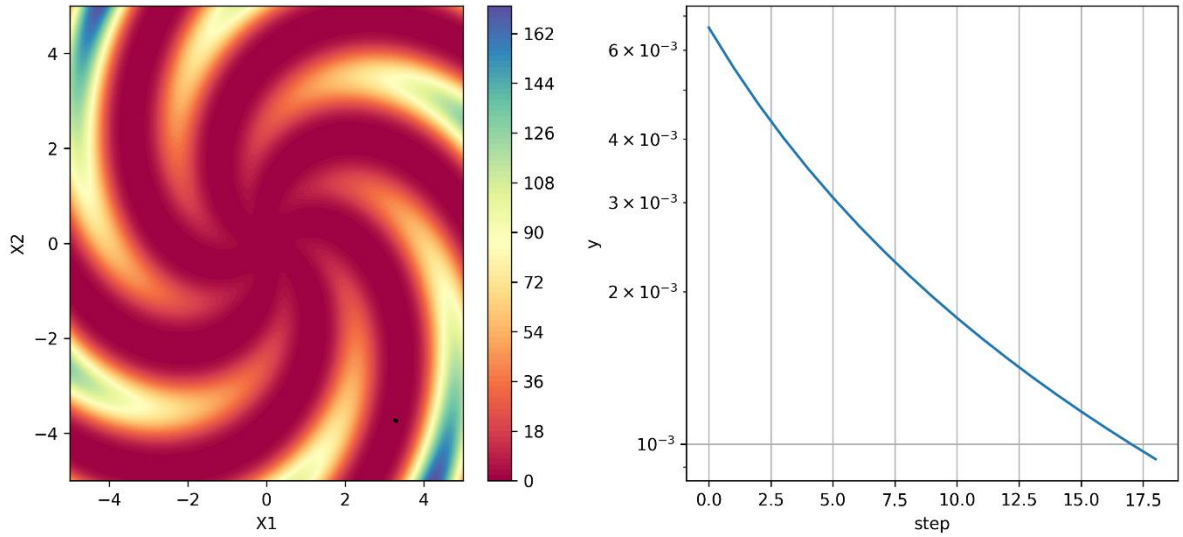

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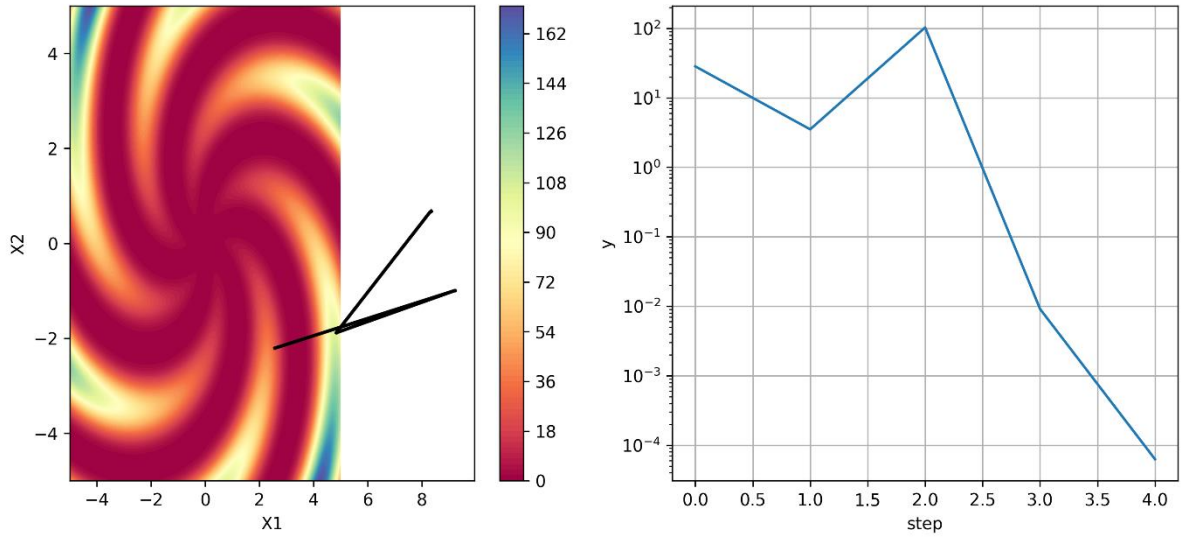

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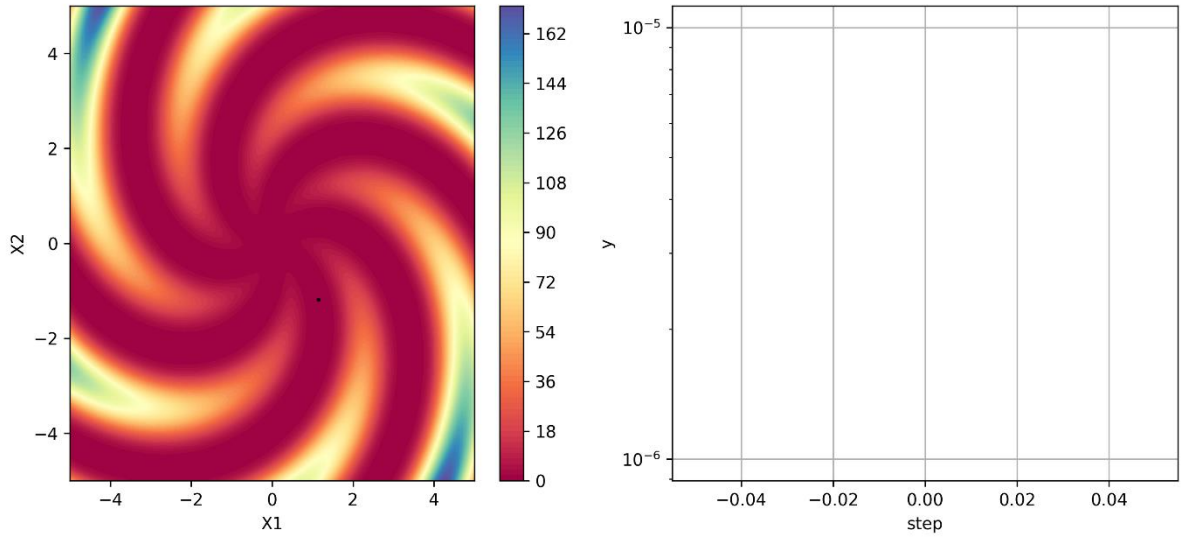

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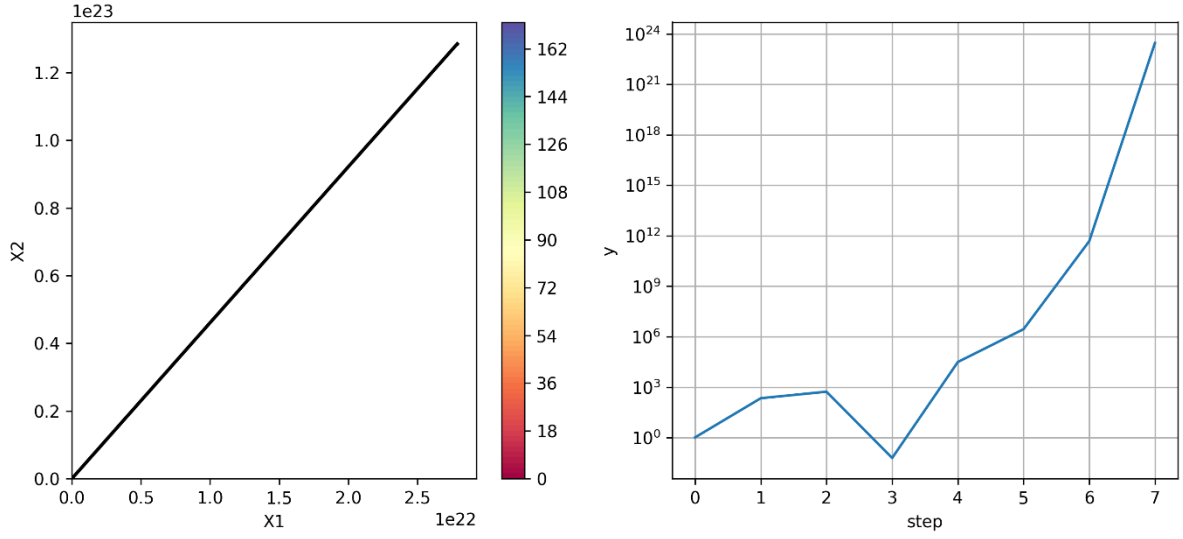

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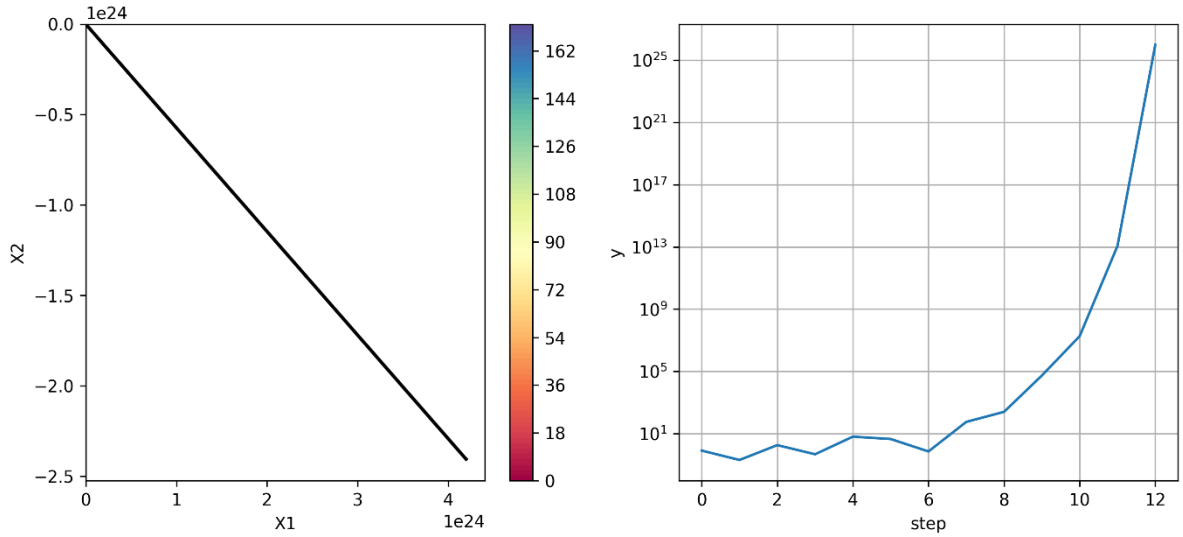

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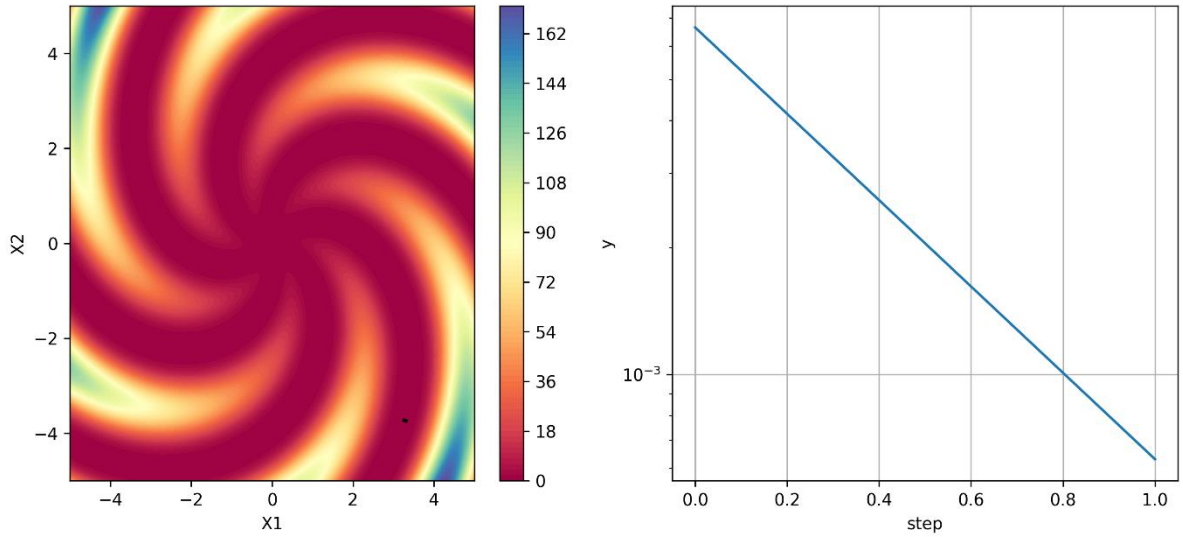

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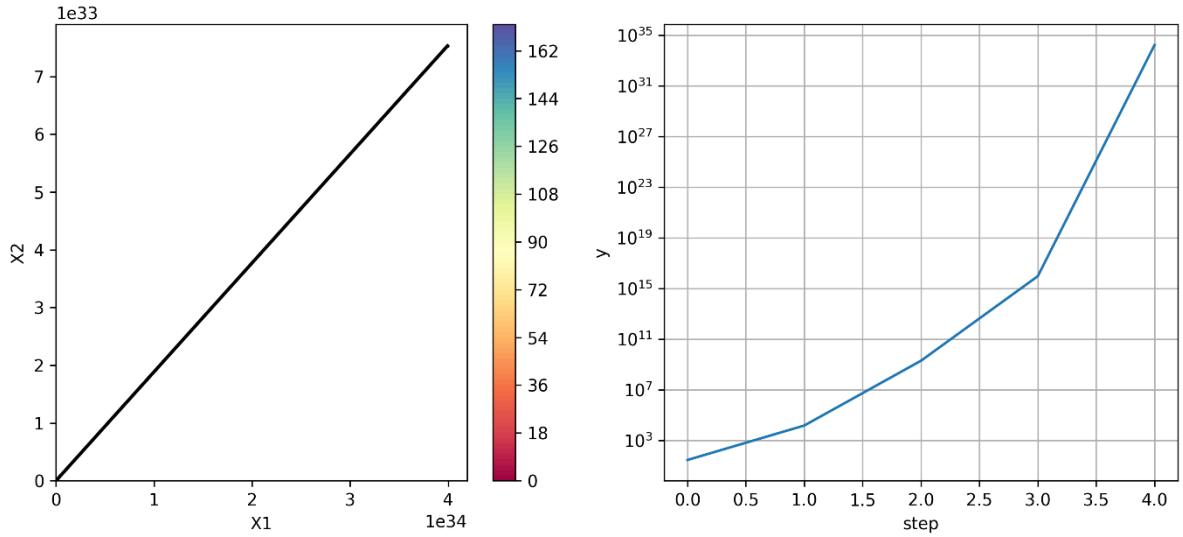

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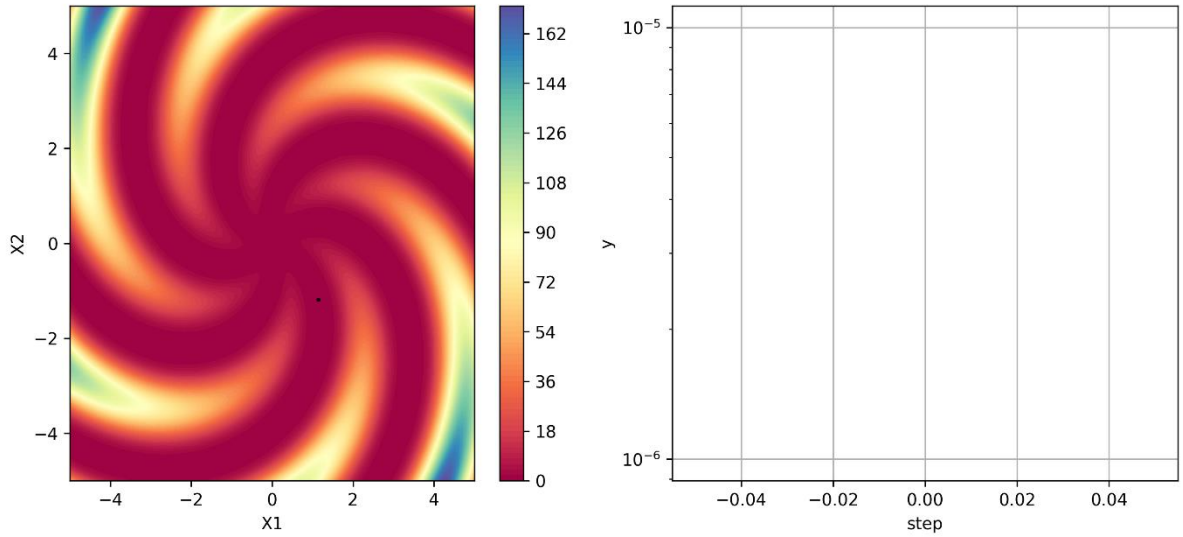

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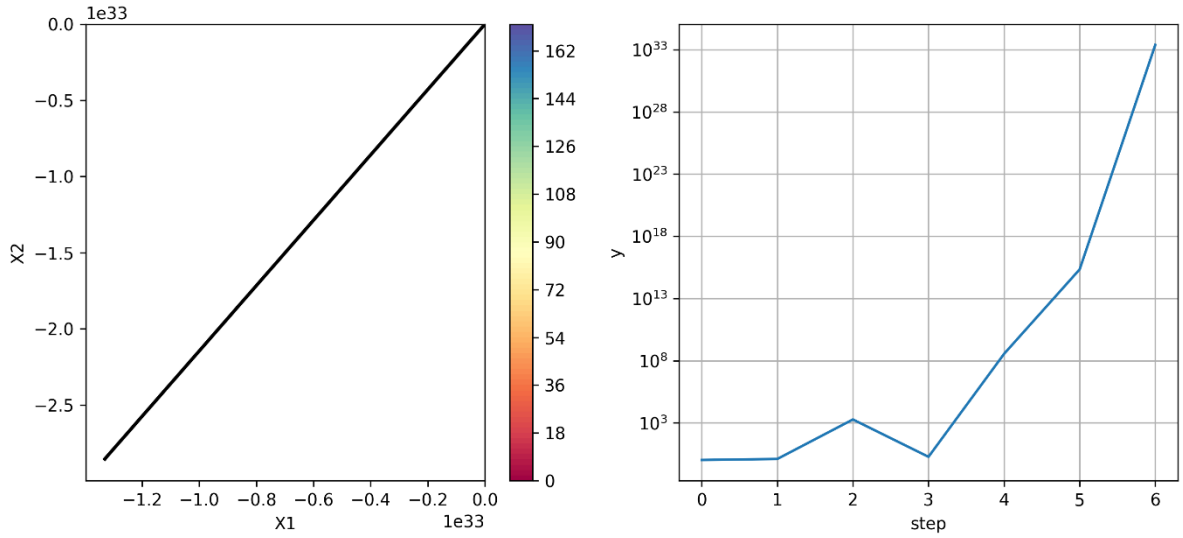

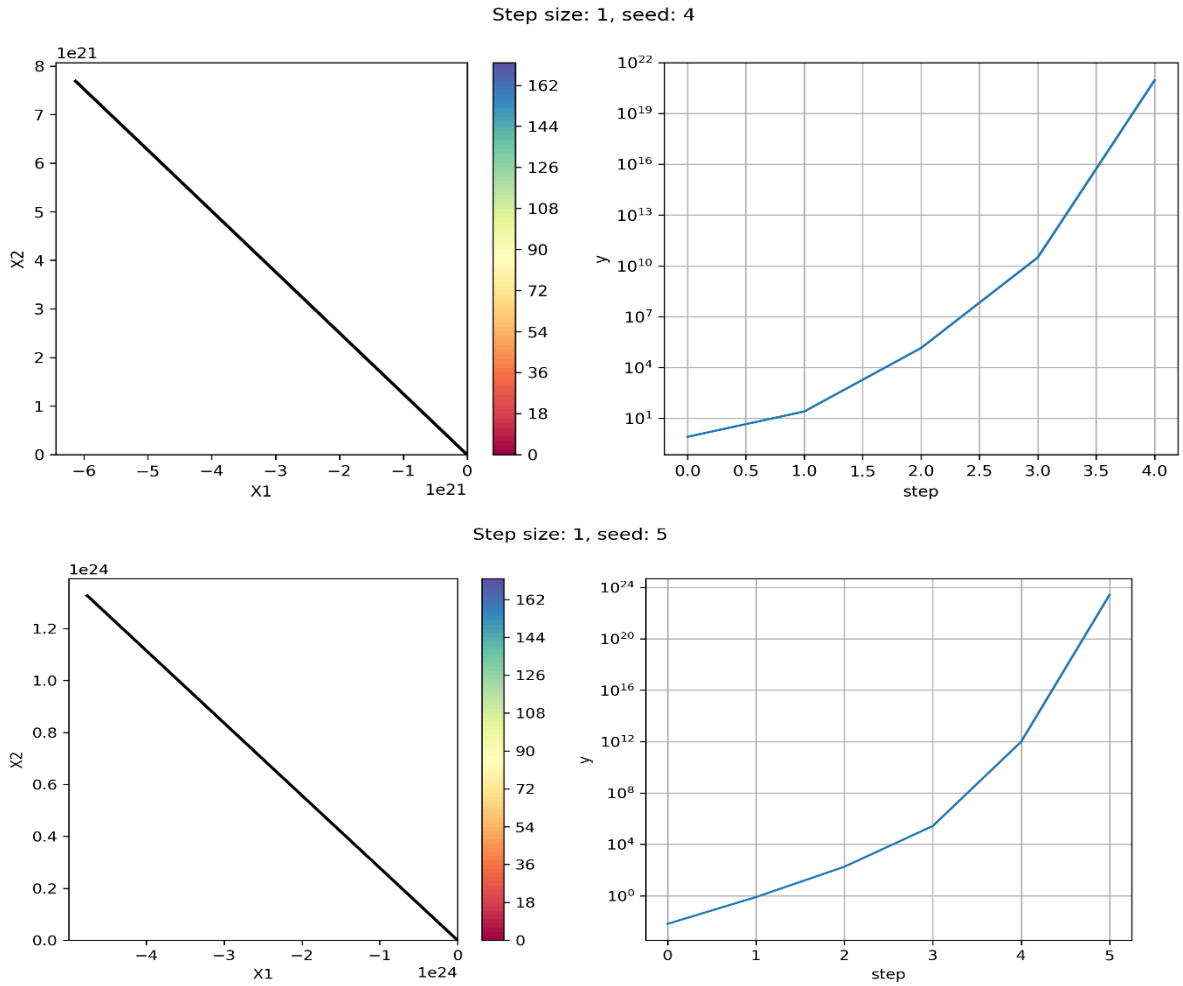
Step size: 1, seed: 4
1e21
X2
X1
1e21
y
step
Step size: 1, seed: 5
1e24
X2
X1
1e24
y
step

Solution Q2: