HW Solutions Chapter 3 & 24 Feb Notes #4

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Chapter 3 H.W.

#7

$$x(t) = 2 + \frac{1}{2}\cos(t+45^\circ) + 2\cos 3t - 2\sin(4t+30^\circ)$$

a) $$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

1, 3, 4

$$\Rightarrow \omega_0 = 1$$

$$x(t) = 2 + \frac{1}{4}\left[e^{j1(\omega_0 t+45^\circ)} + e^{-j(\omega_0 t+45^\circ)}\right] + \left[e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right]$$

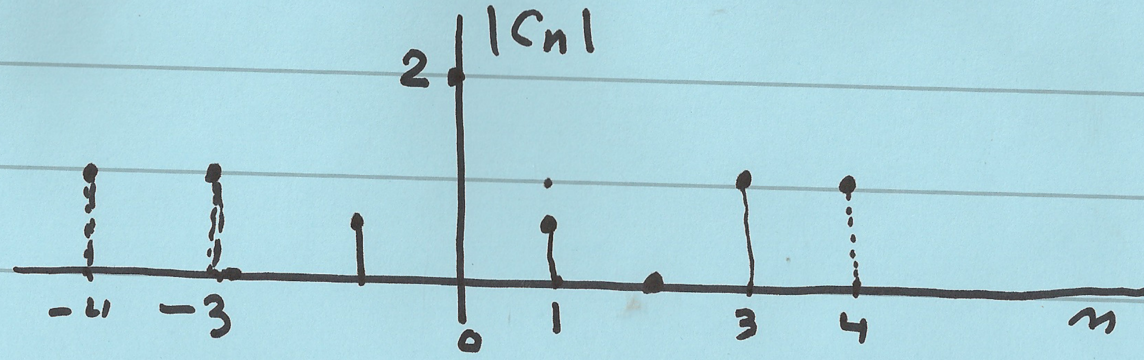
$$+j\left[e^{j(4\omega_0 t+30^\circ)} - e^{-j(4\omega_0 t+30^\circ)}\right]$$

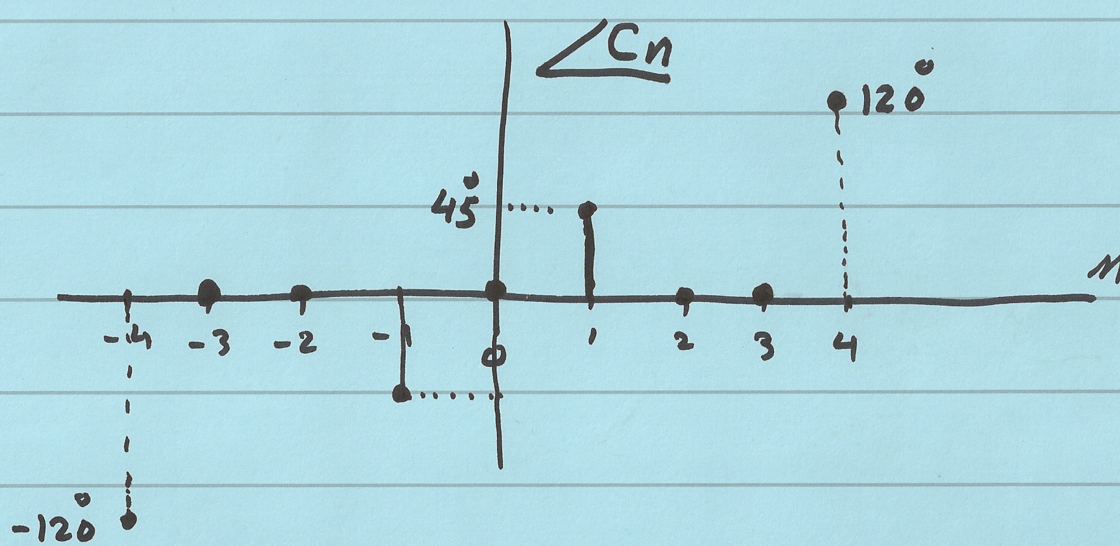
$C_0 = 2$, $C_1 = \frac{1}{4}\cdot e^{j45^\circ}$, $C_{-1} = \frac{1}{4} e^{-j45^\circ}$

$C_2 = C_{-2} = 0$,

$C_3 = 1$, $C_{-3} = 1$, $C_4 = j e^{j30^\circ}$, $C_{-4} = -j e^{-j30^\circ}$

$C_n = 0$ otherwise





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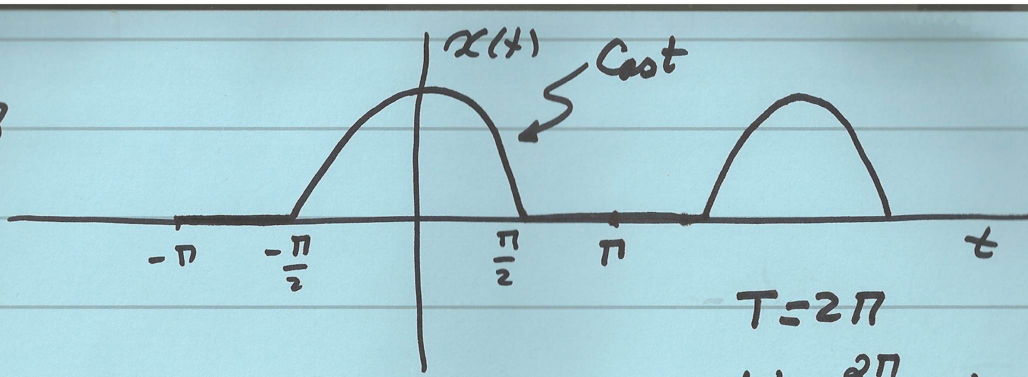


$T = 2\pi$

$\omega_0 = \frac{2\pi}{T} = 1$

$$C_n = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \, e^{-jnt}\,dt$$

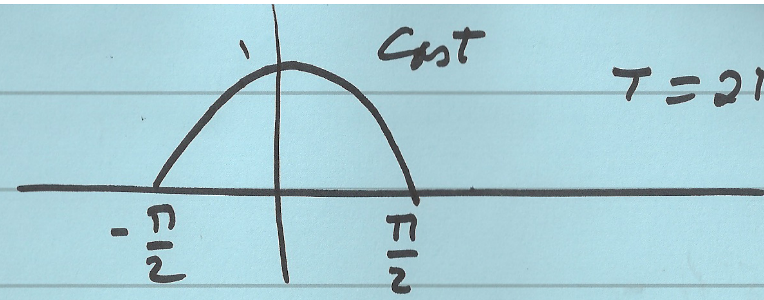
$$C_n = \frac{1}{2\pi}\int \frac{e^{jt}+e^{-jt}}{2}\cdot e^{-jnt}\,dt$$

$$C_n = \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[e^{j(1-n)t} + e^{-j(1+n)t}\right]dt$$

$$\frac{e^{jx} + e^{-jx}}{2j} = \sin x$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

3.9



$T = 2\pi, \ \omega_0 = 1$

$$a_n = 2\,\mathrm{Re}\{C_n\} \quad , \quad b_n = -2\,\mathrm{Im}\{C_n\}$$

$$\Downarrow$$

$$b_n = 0$$

But $C_n = \dfrac{1}{\pi(1-n^2)} \cos n\dfrac{\pi}{2}$

$$\therefore \ a_n = \frac{2}{\pi(1-n^2)} \cos n\frac{\pi}{2} \quad , \quad a_1 = \frac{1}{2}$$

$$x(t) = \underset{\text{D.C. Level}}{\underset{\downarrow}{a_0}} + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

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$$C_n = \frac{1}{4\pi}\left[\frac{1}{j(1-n)}\left(e^{j(1-n)\frac{\pi}{2}} - e^{-j(1-n)\frac{\pi}{2}}\right)\right.$$

$$\left. + \frac{1}{j(1+n)}\left(e^{j(1+n)\frac{\pi}{2}} - e^{-j(1+n)\frac{\pi}{2}}\right)\right]$$

$$C_n = \frac{1}{4\pi}\left[\frac{2\sin(1-n)\frac{\pi}{2}}{1-n} + \frac{2\sin(1+n)\frac{\pi}{2}}{1+n}\right]$$

$$C_n = \frac{1}{2\pi}\left[\frac{\cancelto{1}{\sin\frac{\pi}{2}}\cos n\frac{\pi}{2} - \cancelto{0}{\cos\frac{\pi}{2}}\sin n\frac{\pi}{2}}{1-n}\right.$$

$$\left. + \frac{\cancel{\sin\frac{\pi}{2}}\cos n\frac{\pi}{2} + \cancelto{0}{\cos\frac{\pi}{2}}\sin\frac{n\pi}{2}}{1+n}\right]$$

$$c_n = \frac{1}{2\pi}\left[\frac{1}{1-n} + \frac{1}{1+n}\right]\cos n\frac{\pi}{2}$$

$$C_n = \frac{1}{2\pi}\,\frac{1+n+1-n}{1-n^2}\cos n\frac{\pi}{2}$$

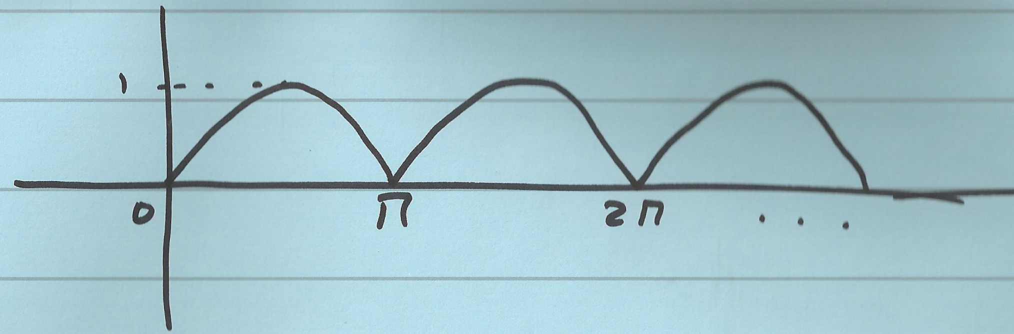
$$\boxed{C_n = \frac{\cos n\frac{\pi}{2}}{\pi(1-n^2)} \qquad n \neq 1}$$

$$\boxed{C_1 = \frac{1}{4}}$$

$$C_1 = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^{jt}+e^{-jt}}{2}\cdot e^{-jt}\,dt$$

$$= \frac{1}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(e^{j(1-1)t}+e^{-j(2)t}\right)dt = \frac{1}{4}$$

3.10



$T = \pi$

$\omega_0 = \frac{2\pi}{T} = 2$

$$C_n = \frac{1}{\pi}\int_0^{\pi} \sin t\, e^{-j2nt}\, dt$$

$$C_n = \frac{1}{2\pi j}\int_0^{\pi} \frac{e^{j(1-2n)t} - e^{-j(1+2n)t}}{1}\, dt$$

$$= \frac{1}{2\pi j}\left[\frac{e^{j(1-2n)\pi} - 1}{j(1-2n)} - \frac{e^{-j(1+2n)\pi} - 1}{j(1+2n)}\right]$$

$$= \frac{1}{-2\pi}\left[\frac{\cos(1-2n)\pi - 1}{1-2n} + \frac{\cos(1+2n)\pi - 1}{1+2n}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{1-2n} + \frac{1}{1+2n}\right]$$

$$\boxed{C_n = \frac{2}{\pi(1-4n^2)}}$$

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$$C_n = \frac{1}{2}\left[\int_0^1 \frac{1}{2} e^{-jn\omega_0 t} - \int_1^2 \frac{1}{2} e^{-j\omega_0 t}\, dt\right]$$

$$= \frac{1}{4}\left[\frac{1}{-j\omega_0 n} e^{-jn\omega_0 t}\Big|_0^1 - \frac{1}{-j\omega_0 n} e^{-jn\omega_0 t}\Big|_1^2\right]$$

$$C_n = \frac{1}{j4\pi n}\left[-e^{-j\pi n} + \underset{1}{\cancel{e^{0}}} + \underset{1}{\cancel{e^{-j2\pi n}}} - e^{-j\pi n}\right]$$

$$\boxed{C_n = \frac{1}{j2\pi n}\left[1 - e^{-jn\pi}\right]} \Rightarrow C_n = \frac{1}{jn\pi}\ ;\ n = 1, 3, 5, 7, \dots$$

$$= 0\ ;\ n = 2, 4, 6, \dots$$

$$|C_n| = \frac{1}{n\pi}\ ,\ \angle C_n =$$

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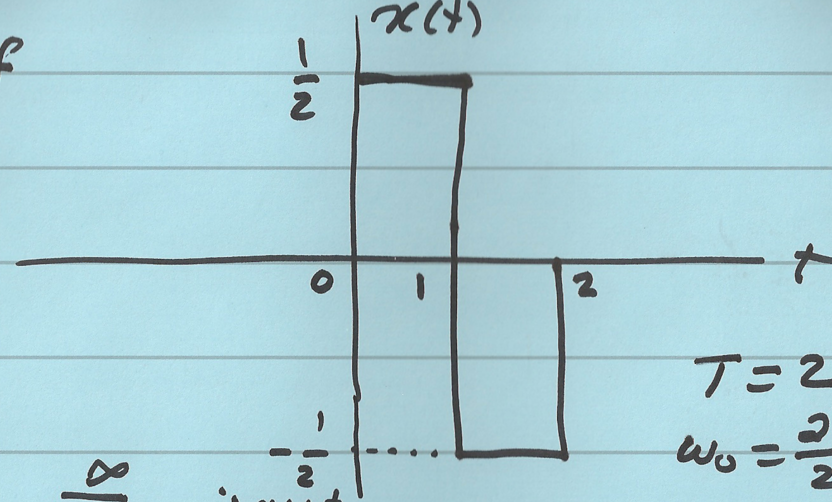
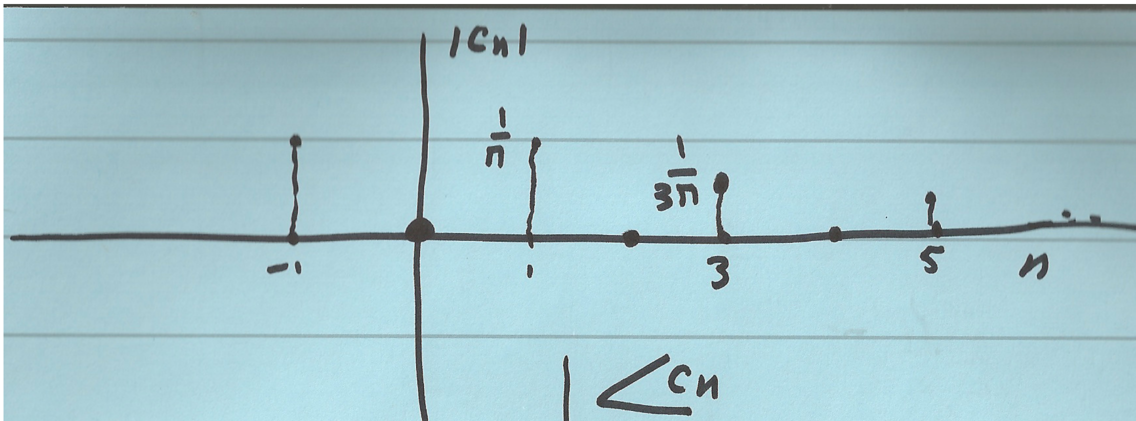


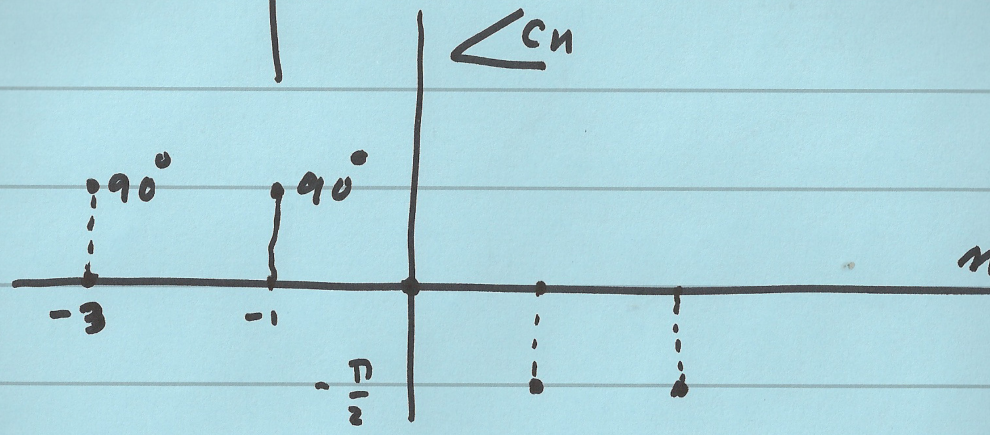
$T = 2$

$\omega_0 = \frac{2\pi}{2} = \pi$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T}\int_0^T x(t) e^{-j\omega_0 n t}\, dt$$





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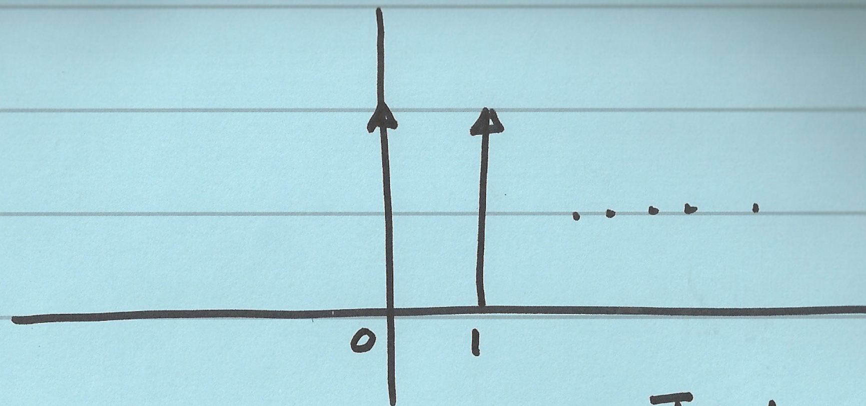


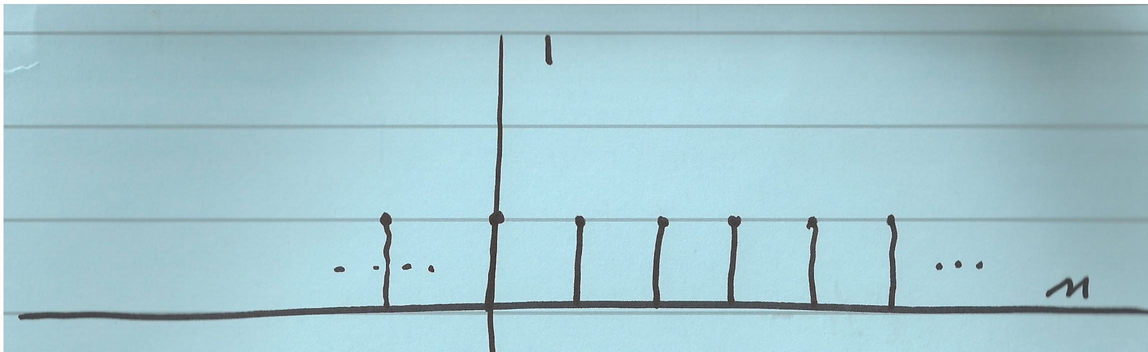
$T = 1$

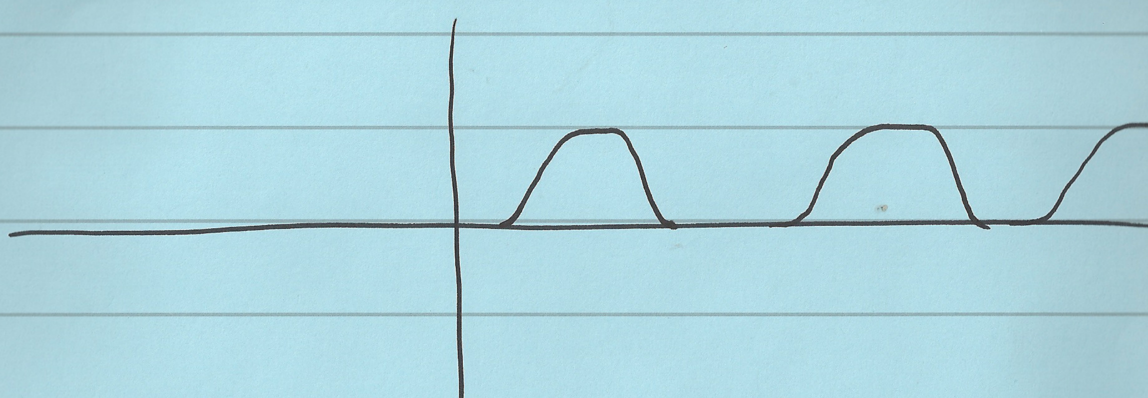
$\omega_0 = 2\pi$

$$x(t) = \sum c_n e^{j2\pi nt}$$

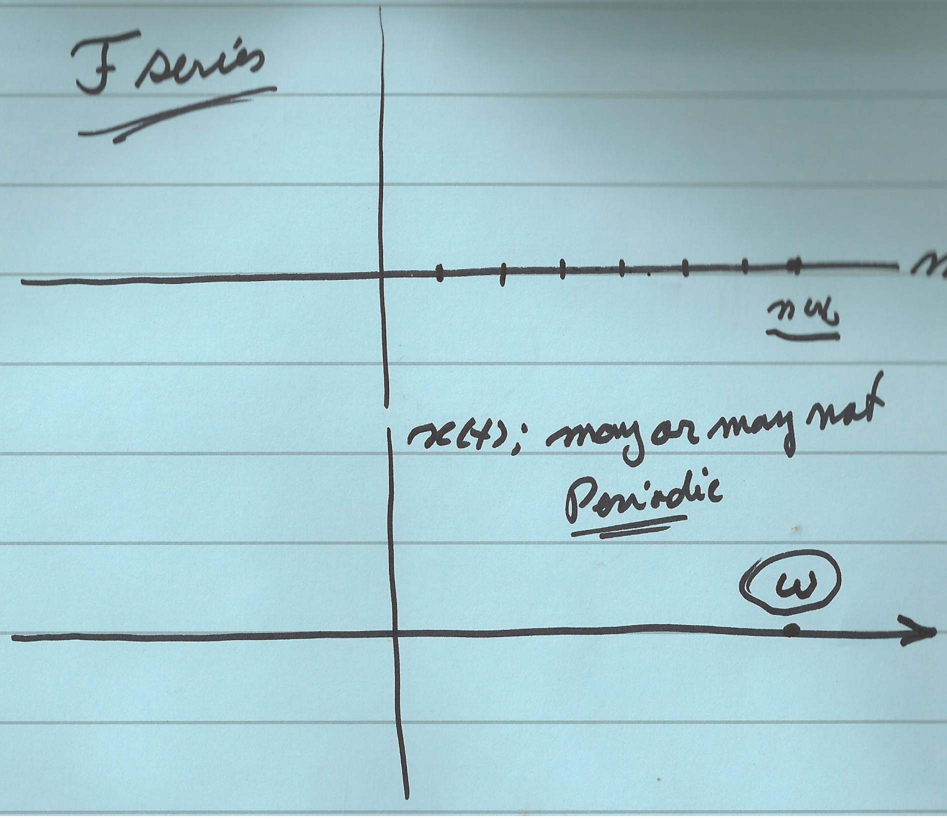
$$c_n = \frac{1}{1}\int_{-\frac{1}{2}}^{\frac{1}{2}} \delta(t)\, e^{j2\pi nt}\, dt = 1$$

$c_n = 1 \quad ; \; n = 0 \longrightarrow \pm\infty$





F series



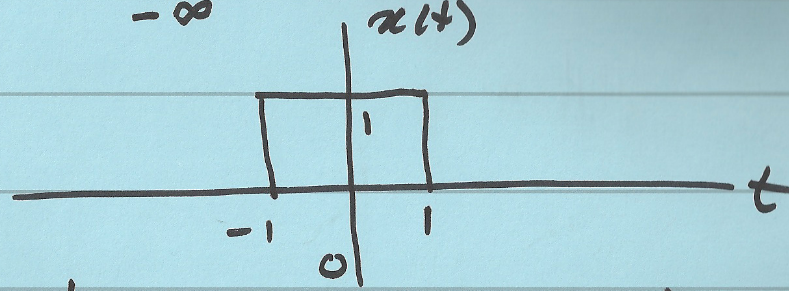
Chapter 4

Fourier Transform

$$= \underbrace{X(\omega)}_{\text{Freq. Domain}} = \int_{-\infty}^{\infty} \underbrace{x(t) \cdot e^{-j\omega t}}_{\text{Time Domain}} \, dt \triangleq \mathcal{F}\{x(t)\}$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

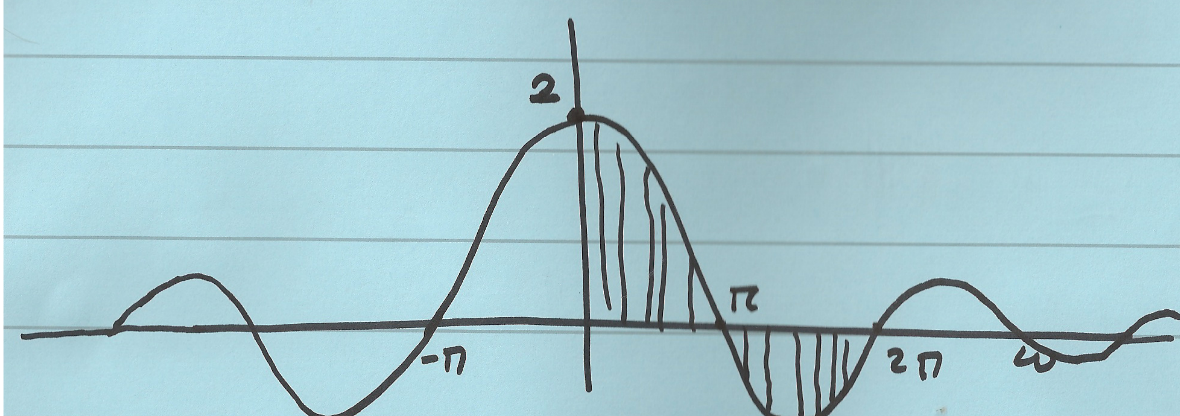
<u>Example</u>

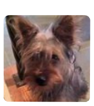


$$X(\omega) = \int_{-1}^{1} 1\cdot e^{-j\omega t}\, dt = \frac{1}{-j\omega}\cdot e^{-j\omega t}\Big|_{-1}^{1}$$

$$\boxed{X(\omega) = 2\,\frac{\sin\omega}{\omega}}$$

$|X(\omega)| \Rightarrow$ Spectrum of $x(t)$





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