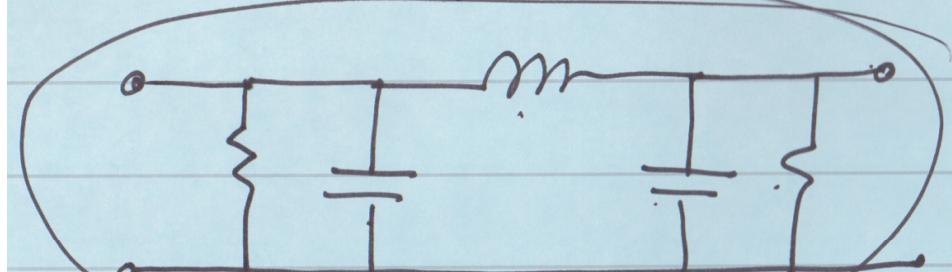


Bandstop - Notch Filter



$$n_1 = \frac{-0.10 - 1}{2 \log 0.6} = \frac{-1.10}{-0.443} = 3.678$$

Hence $n_1 = 4$

To satisfy stop band :

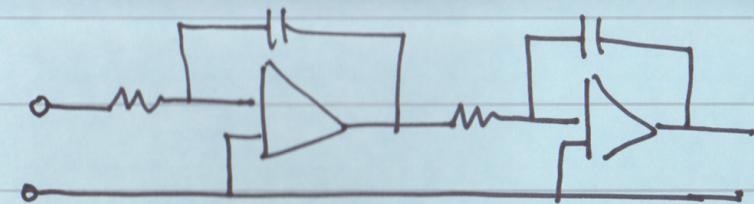
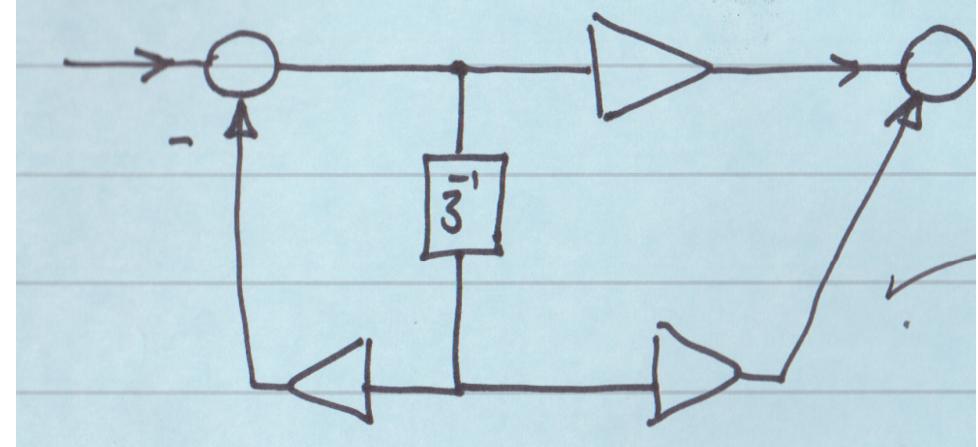
$$n_2 = \frac{\log(10^3 - 1)}{2 \log 3} = \frac{2.99}{0.954} = 3.143$$

$2.75 \Rightarrow 3$

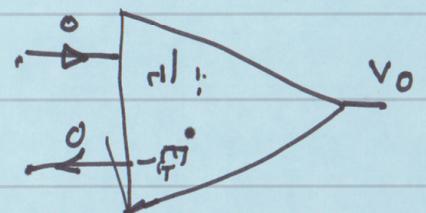
$$n_2 = 4$$

Hence n = 4

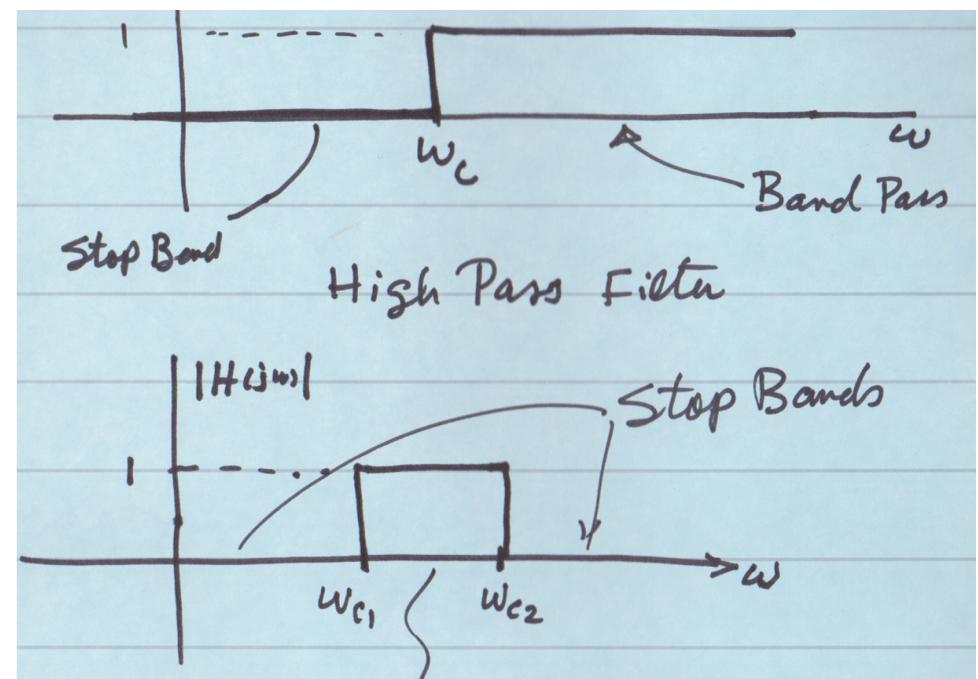
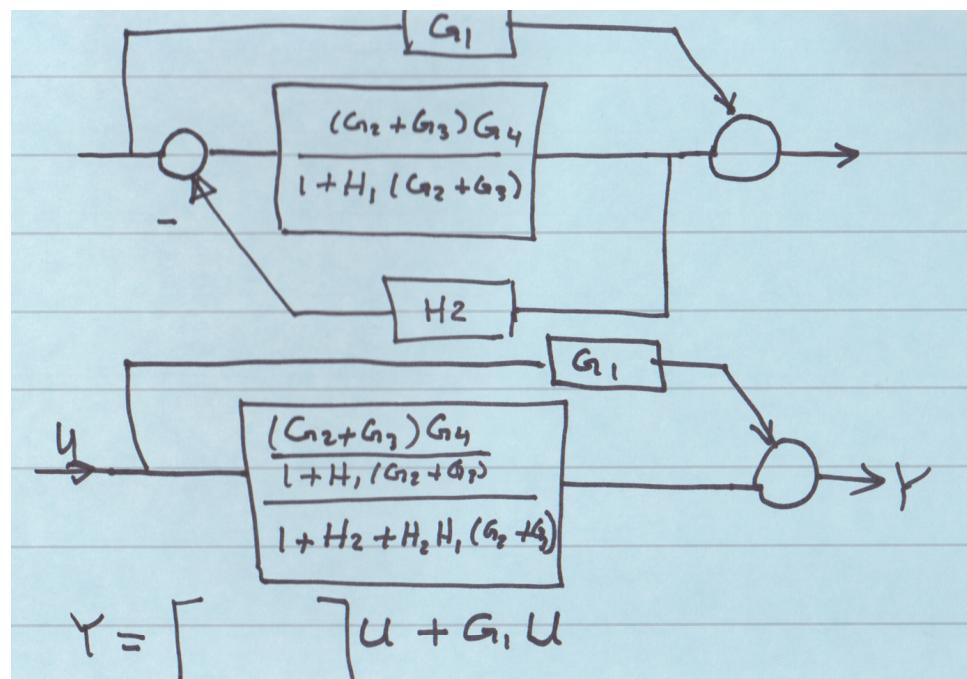
Digital Filters



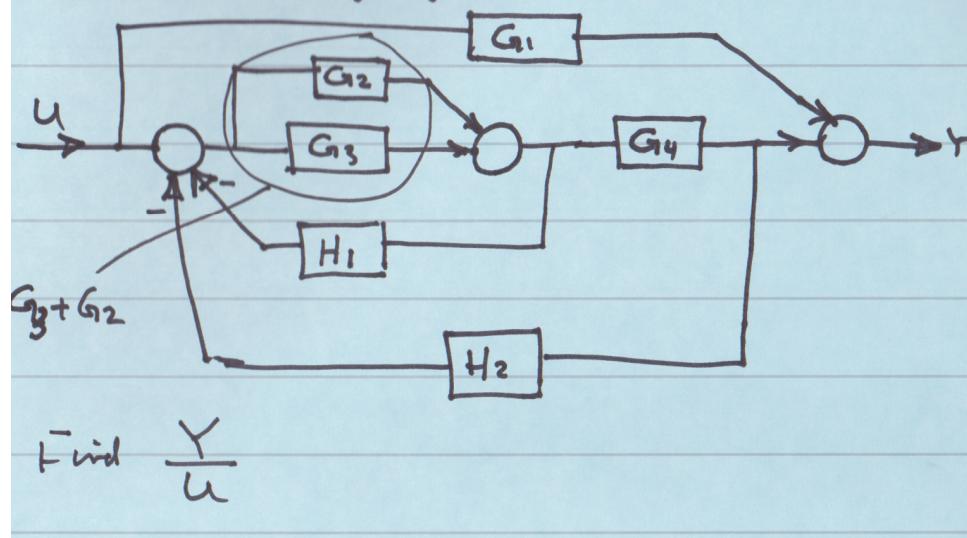
Active RC Filter



Mid 1950



Example Find The Transfer Function of the
Following system:

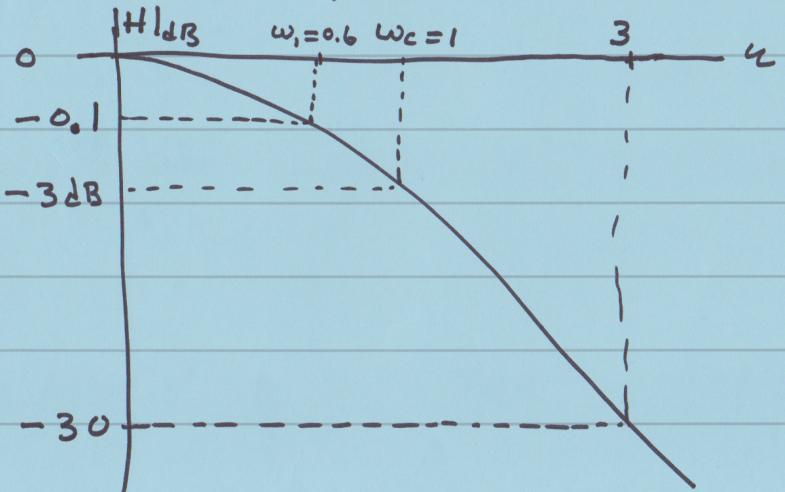


$$\frac{Y}{U} = G_1 + \frac{1 + H_1(G_2 + G_3)}{1 + H_2 + H_2H_1(G_2 + G_3)}$$

Stable iff all Poles of $H(s)$ are located in the L.H. of the s-plane

Example $H(s) = \frac{s^4 + 6s^2 + 3}{(s+6)(s+2)(s-1)(s+18)}$

Example Obtain the order and realize the low pass filter whose specs are given as:



(a) Pole Locations Order = n	P_1, s	P_2, s	P_3, s	P_4, s	P_5, s	P_6, s	P_7, s	P_8, s	P_9, s
1	-1.0000000								
2	-0.20106712 ± j0.70502168								
3	-0.50000000 ± j0.95620346								
4	-0.50000000 ± j0.95620343	-0.50000000 ± j0.95620343							
5	-0.30000000 ± j0.95620343	-0.30000000 ± j0.95620343	-0.30000000 ± j0.95620343						
6	-0.25818935 ± j0.95620343	-0.25818935 ± j0.95620343	-0.25818935 ± j0.95620343	-0.25818935 ± j0.95620343					
7	-0.22325939 ± j0.95620343								
8	-0.17544118 ± j0.95620343								
9	-0.15000000 ± j0.95620343								
(b) Denominator Polynomials $R(s) = s^9 + a_8 s^8 + a_7 s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$									
Order = n	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
1	1.00000000	1.14121356							
2	1.00000000	2.00000000	2.01312503						
3	1.00000000	2.00000000	2.52602795	3.21606798					
4	1.00000000	2.00000000	2.52602795	3.21606798	3.26703031				
5	1.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931			
6	1.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090		
7	1.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090	5.57877048	
8	1.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090	5.57877048	
9	1.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090	5.57877048	
(c) Resistor Polynomials $R(s) = R_0 + R_1 s + R_2 s^2 + R_3 s^3 + R_4 s^4 + R_5 s^5 + R_6 s^6 + R_7 s^7 + R_8 s^8$									
Order = n	R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
1	0.00000000	0.00000000							
2	0.00000000	2.00000000	2.01312503						
3	0.00000000	2.00000000	2.52602795	3.21606798					
4	0.00000000	2.00000000	2.52602795	3.21606798	3.26703031				
5	0.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931			
6	0.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090		
7	0.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090	5.57877048	
8	0.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090	5.57877048	
9	0.00000000	2.00000000	2.52602795	3.21606798	3.46410162	4.49397931	5.12303090	5.57877048	

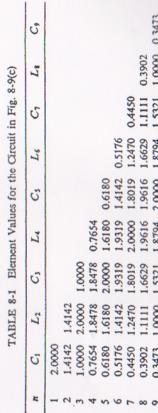
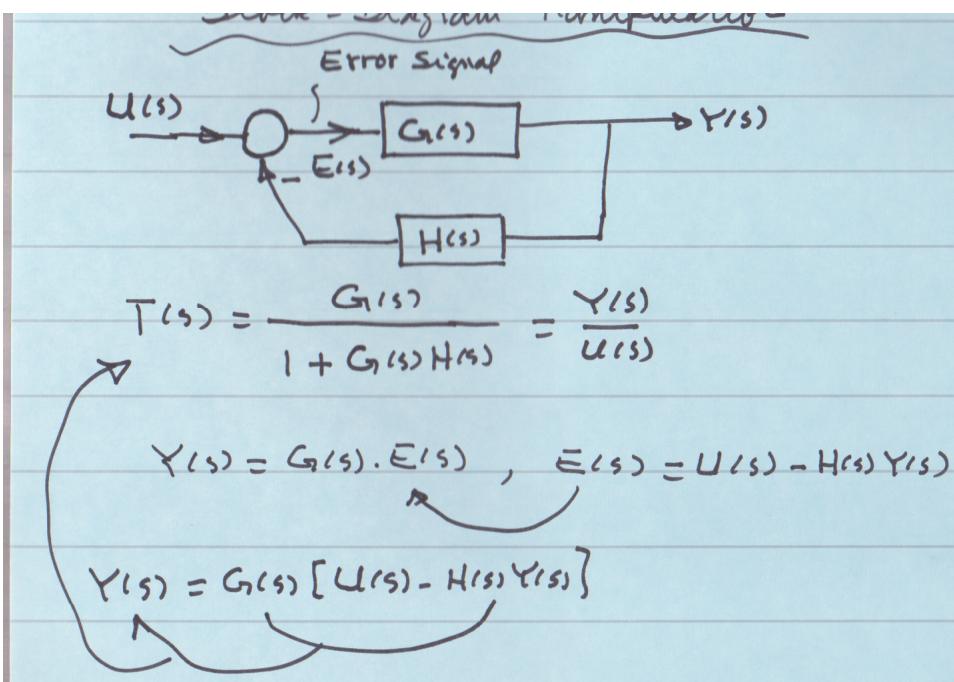
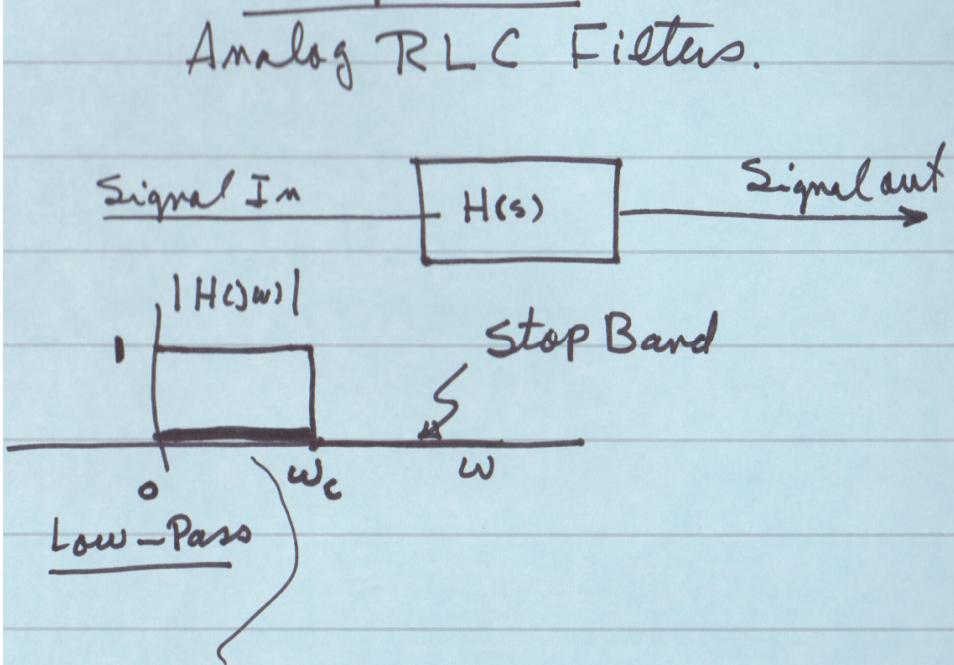


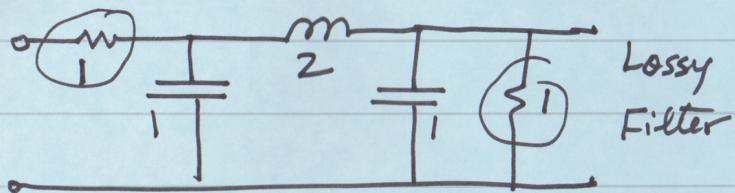
Fig. 8-9 Circuit structures of low-pass Butterworth filters.

TABLE 8-1 Element Values for the Circuit in Fig. 8-9(e)

n	C_1	L_1	C_2	L_2	C_3	L_3	C_4	L_4	C_5
1	2.0000	1.0442							
2	1.0442	2.0880	1.0000						
3	0.5221	1.8477	1.8477	0.7654					
4	0.2655	1.6180	2.0000	1.6180	0.6180				
5	0.1618	1.6180	2.0000	1.6180	1.6180	1.0716			
6	0.3176	1.2442	1.9319	1.4142	1.4142	1.2445			
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450		
8	0.3902	1.1111	1.6629	1.9616	1.6629	1.1111	0.3902		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473



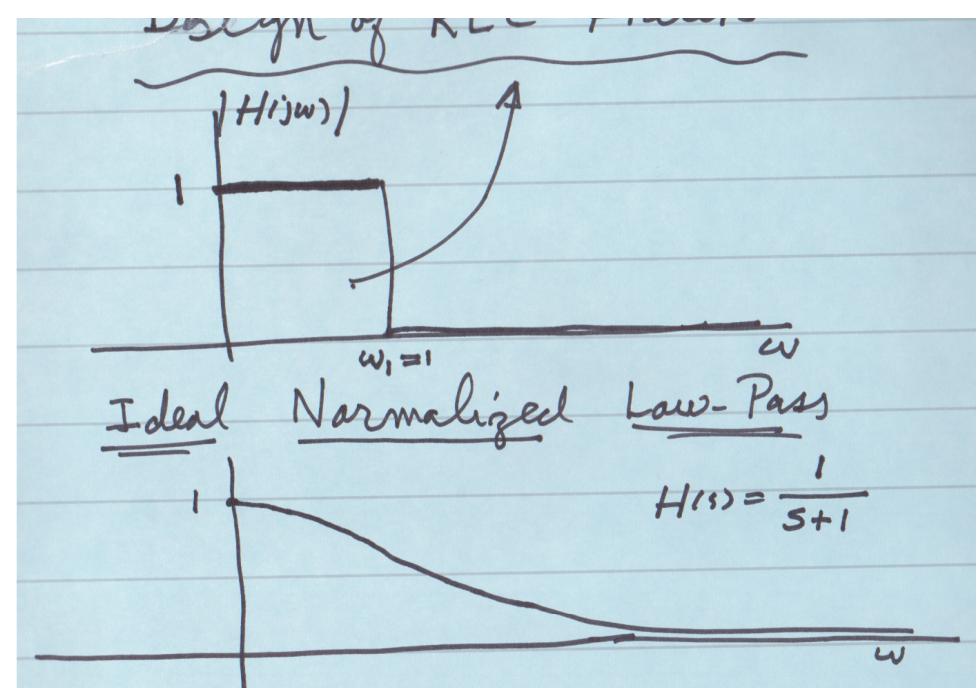
Example Give the realization of a 3rd order Normalized Butterworth Filter



Ideal Transfer Function $H(s) = \frac{1}{(s+1)(s^2+s+1)}$; $H(j\omega) = 1$

Actual Transfer Function $\hat{H}(s) = \frac{1}{2} \cdot \frac{1}{(s+1)(s^2+s+1)}$

$\hat{H}(j\omega) = \frac{1}{2}$



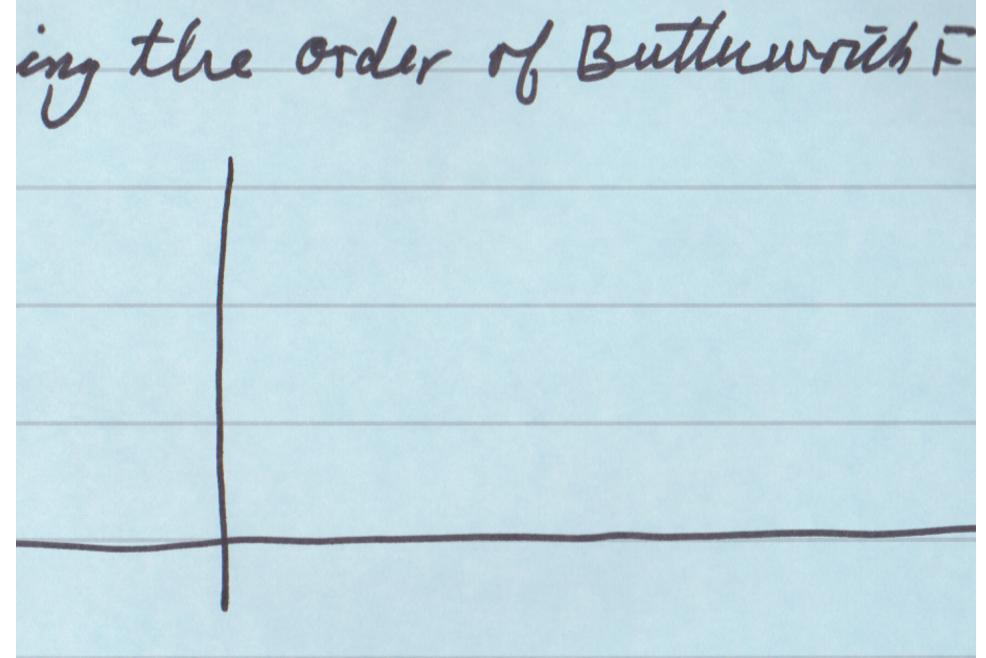
$$|H(j\omega)|^2 = \frac{1}{1+\omega^2}$$

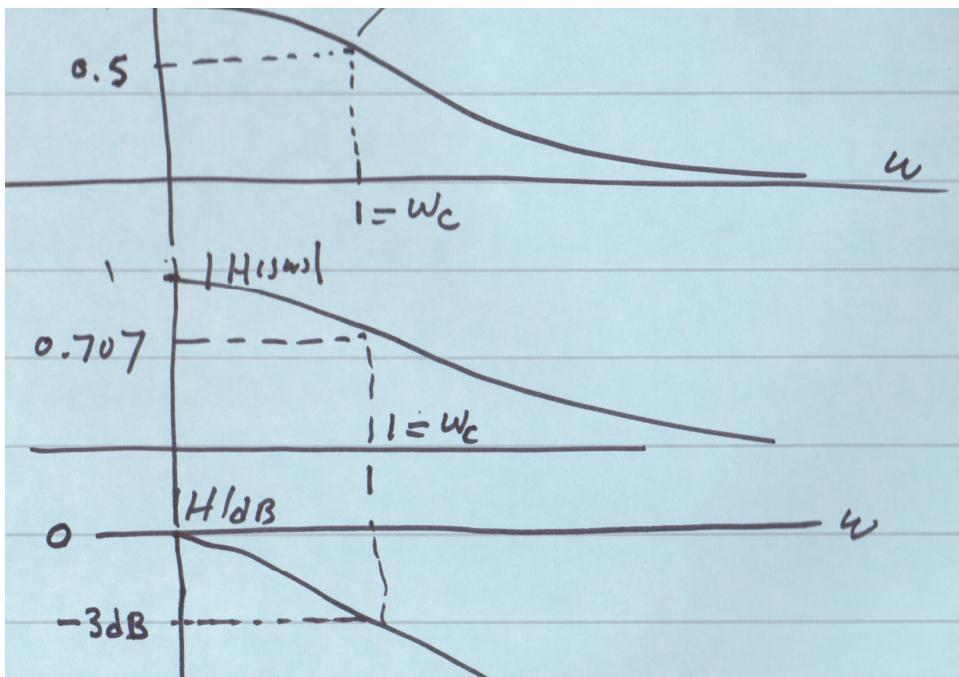
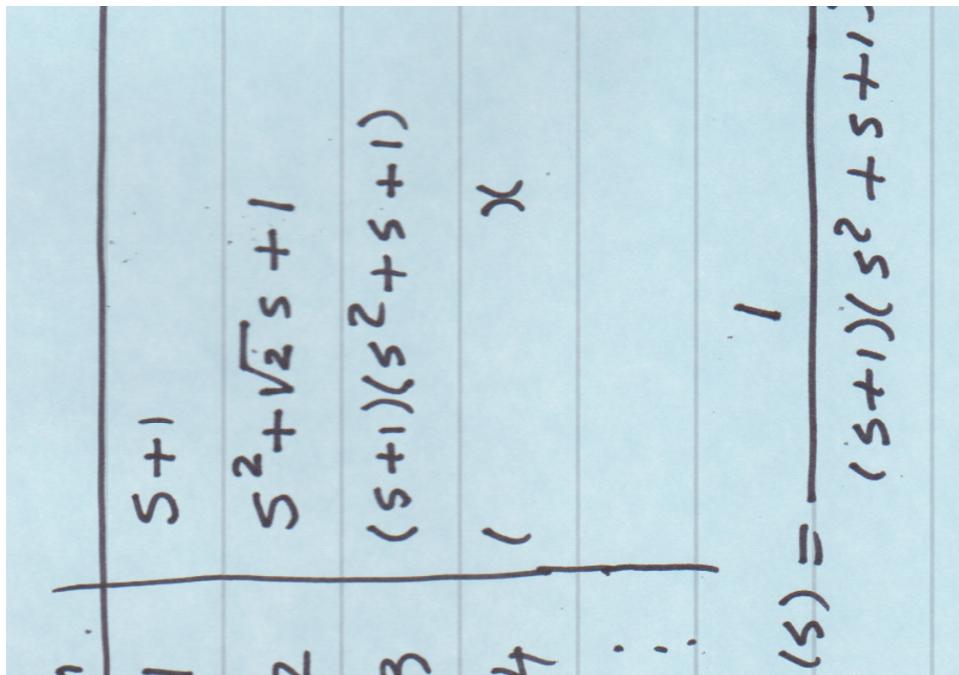
at $\omega = 1$

$$|H(j\omega)|^2 = \frac{1}{2}$$

$$|H(j\omega)| = \frac{\sqrt{2}}{2} = 0.707$$

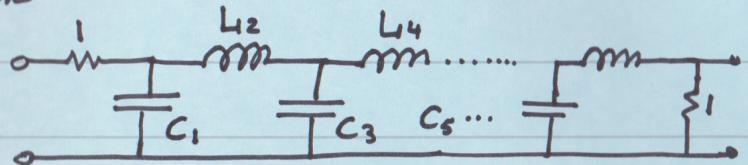
$$|H(j\omega)|_{dB} = -3 \text{ dB}$$



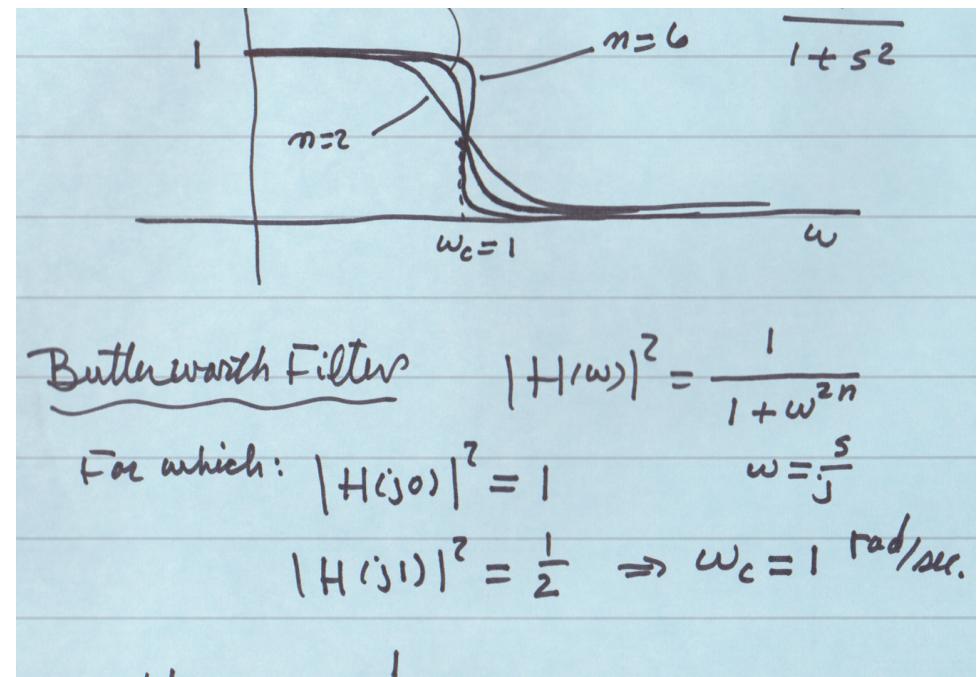


RLC Realization of $\frac{1}{D(s)}$

Darkington



n	C_1	L_2	C_3	L_4
1	2	0	0		
2	$\sqrt{2}$	$\sqrt{2}$			$C_m = 2 \sin \frac{(2m-1)\pi}{2n} C_1$
3	1	2	1		$L_m = 2 \sin \frac{(2m-1)\pi}{2n} L_2$



$$|H(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}$$

$$|H(j\omega)|_{dB} = 20 \log_{10} |H(j\omega)| = 10 \log_{10} \frac{1}{1 + \omega^{2n}}$$

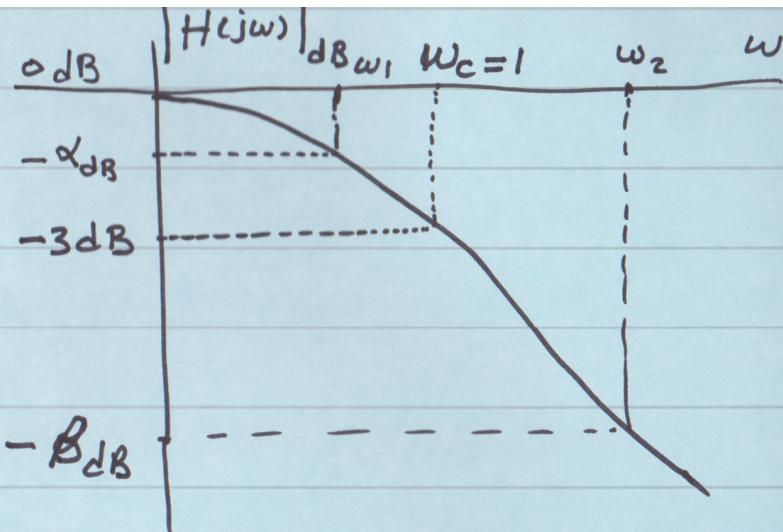
$$|H(j\omega)|_{dB} = -10 \log(1 + \omega^{2n}) = -\alpha_{dB}$$

$$10 \log(1 + \omega^{2n}) = \alpha_{dB}$$

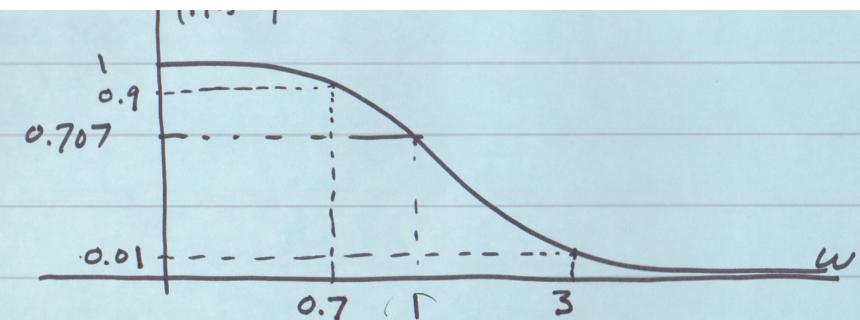
$$\therefore 1 + \omega^{2n} = 10^{0.1\alpha_{dB}}$$

$$\omega^{2n} = 10^{0.1\alpha_{dB}} - 1$$

$$\therefore \omega^{2n} = 10^{0.1\alpha_{dB}}$$



$|H(j\omega)|_{dB} \geq -\alpha_{dB}$ for $\omega \leq w_1$, Low Pass Spec.
 $|H(j\omega)|_{dB} \leq -\beta_{dB}$ for $\omega > w_2$, Band Pass Spec.



$$\alpha^2 = 0.9$$

$$\beta^2 = 0.01$$

$$n_1 = \frac{\log(\frac{1}{0.9} - 1)}{2 \log 0.7} = \frac{-0.954}{-0.309}$$

$$\boxed{n_1 = 4}$$

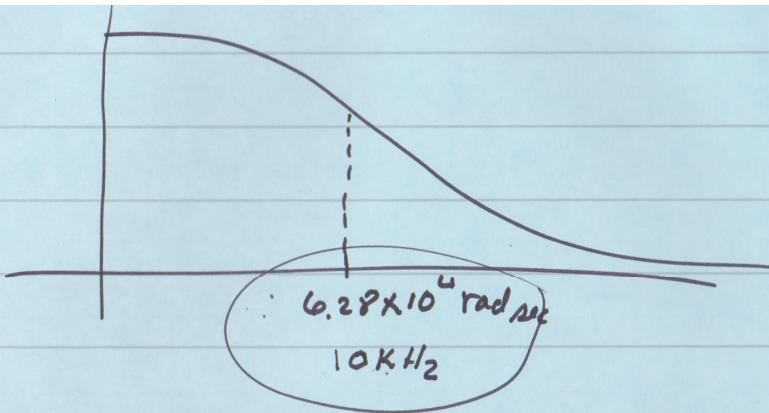
$$\boxed{n_2 = 1}$$

$$n_2 = \frac{\log(\frac{1}{0.01} - 1)}{2 \log 3} = \frac{1.99}{0.954}$$

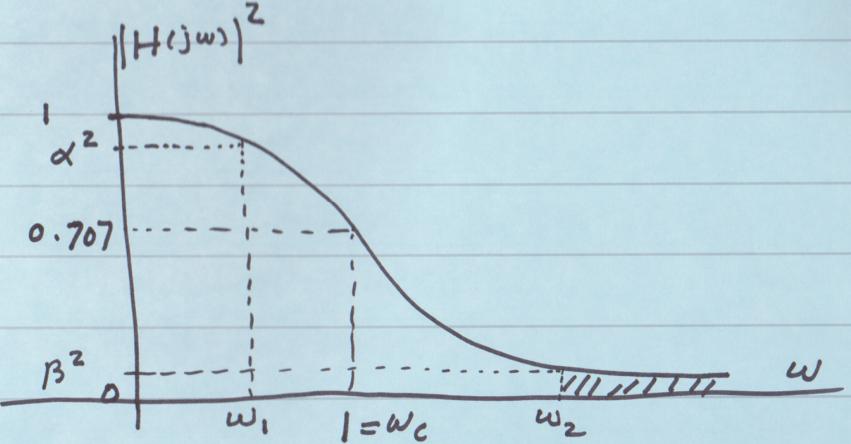
$$n_1 = \frac{\log(10^{0.1\alpha_{dB}} - 1)}{2 \log w_1} \quad \text{Pass}$$

$$n_2 = \frac{\log(10^{0.1\beta_{dB}} - 1)}{2 \log w_2} \quad \text{Stop}$$

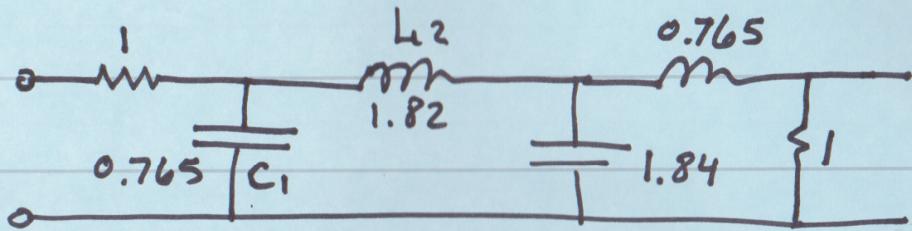
$$n = \{ \max(n_1, n_2) \}$$



If absolute mag. is given



$$H(s) = \frac{1}{(s^2 + 0.765s + 1)(s^2 + 1.847s + 1)}$$



$$C_1 = 2 \sin \frac{(2-1)\pi}{8} = 0.765 F$$

$$L_2 = 2 \sin \frac{(4-1)\pi}{8} = 1.82 H$$

D

$$|H(j\omega)| = \frac{1}{1 + \omega^{2n}}$$

$$\text{For Passband : } \frac{1}{1 + \omega_1^{2n}} \gg \alpha^2$$

$$\frac{1}{1 + \omega_1^{2n}} = \alpha^2 \Rightarrow 1 + \omega_1^{2n} = \frac{1}{\alpha^2}$$

$$\omega_1^{2n} = \frac{1}{\alpha^2} - 1$$

$$\omega_1 = \sqrt{\frac{\log(\alpha^2 - 1)}{2 \log \alpha}}$$