26 Feb & Chapter 4 Starting HW #5

Open **vipulkohli** opened this issue an hour ago · 8 comments

vipulkohli commented an hour ago Owner

Start with HW problems 4.5 and 4.6 all parts

Chapter 4 – Fourier Transform

Given $x(t)$

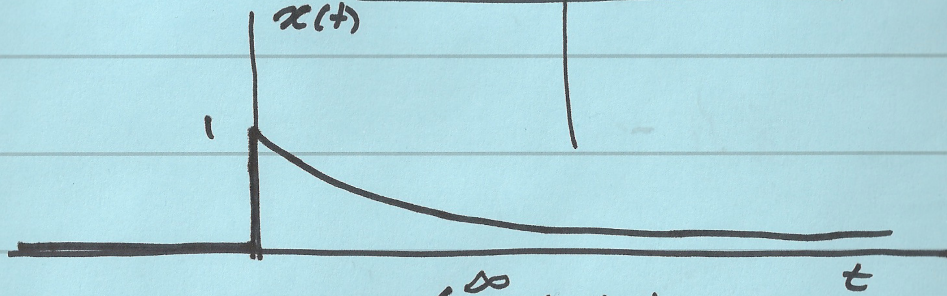
Define $$\mathcal{F}\{x(t)\} = X(j\omega) = \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt$$

$$\mathcal{F}^{-1}\{X(j\omega)\} = x(t) = \frac{1}{2\pi}\int X(j\omega)\cdot e^{j\omega t}\,d\omega$$

vipulkohli commented an hour ago Owner

Example Given $x(t)$ as:

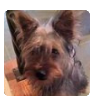
$$x(t) = e^{-\alpha t} u_0(t) \quad \Big| \quad \frac{1}{\alpha + j\omega}$$



Find $X(j\omega)$, $\quad X(j\omega) = \int_0^\infty e^{-\alpha t} \cdot e^{-j\omega t} \cdot dt$

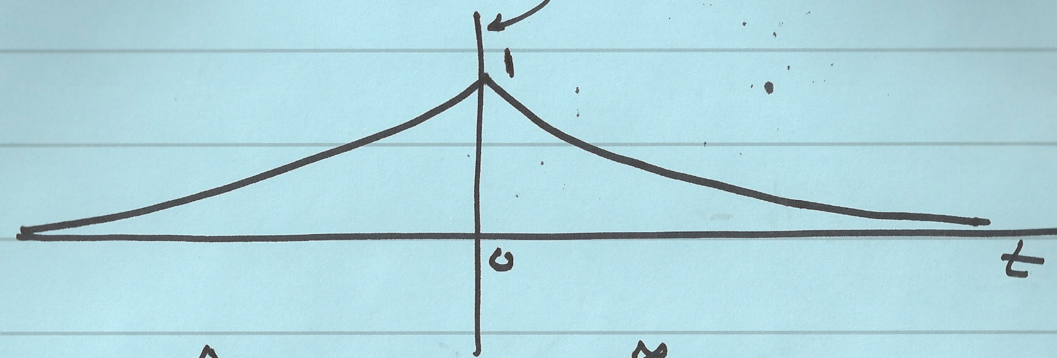
$$X(j\omega) = \int_0^\infty e^{-(\alpha + j\omega)t} \, dt = \frac{1}{-(\alpha + j\omega)} e^{-(\alpha + j\omega)t} \Big|_0^\infty$$

$$\boxed{X(j\omega) = \frac{1}{\alpha + j\omega}}$$



vipulkohli commented 40 minutes ago Owner

Example $x(t) = e^{-\alpha|t|}$ $\quad -\infty < t < \infty$

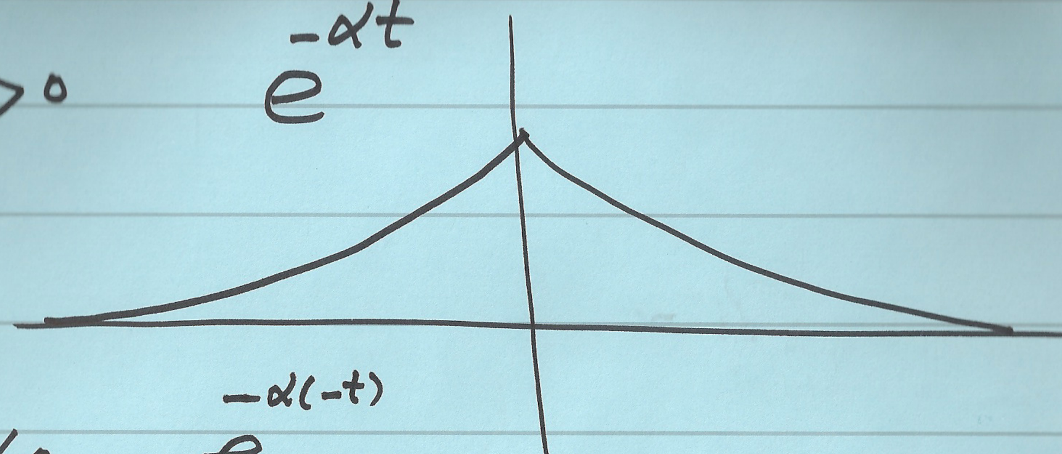


$$X(j\omega) = \int_{-\infty}^{0} e^{\alpha t} \cdot e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-\alpha t} \cdot e^{-j\omega t}\, dt$$

$$X(j\omega) = \frac{1}{\alpha - j\omega} + \frac{1}{\alpha + j\omega} = \frac{\alpha + j\omega + \alpha - j\omega}{\alpha^2 + \omega^2}$$

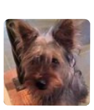
$$\boxed{X(\omega) = \frac{2\alpha}{\alpha^2 + \omega^2}}$$

$t > 0 \quad e^{-\alpha t}$



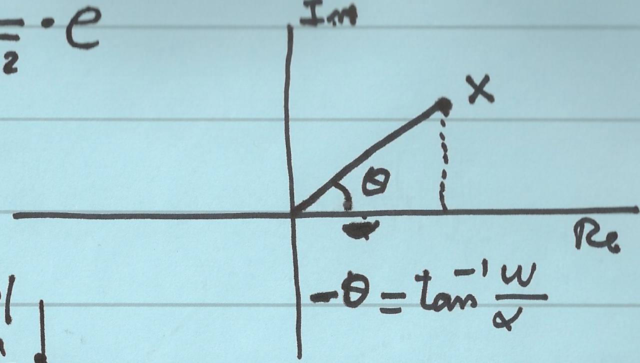
$t < 0 \quad e^{-\alpha(-t)}$

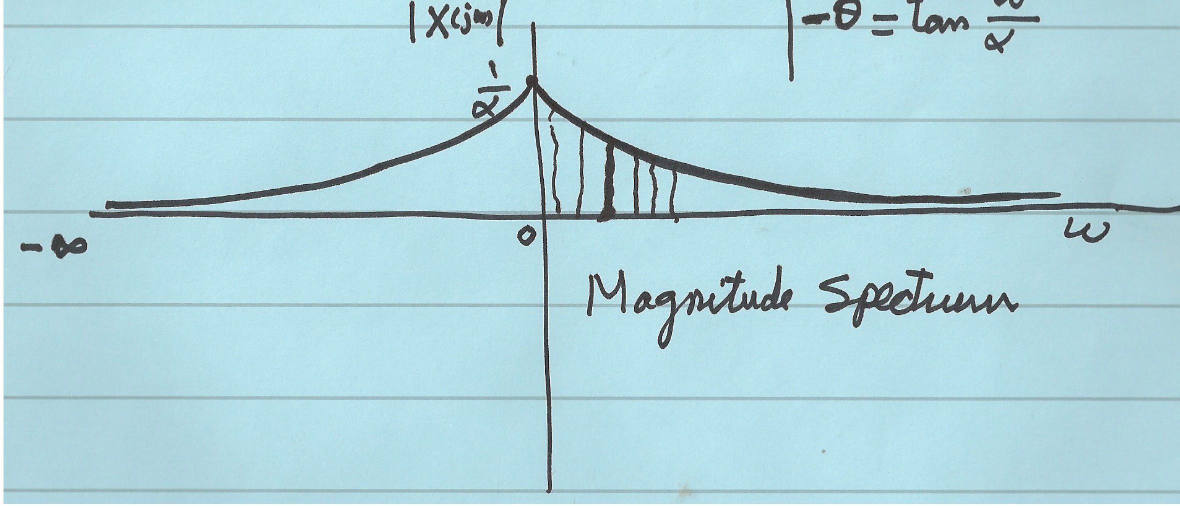
$= e^{\alpha t}$



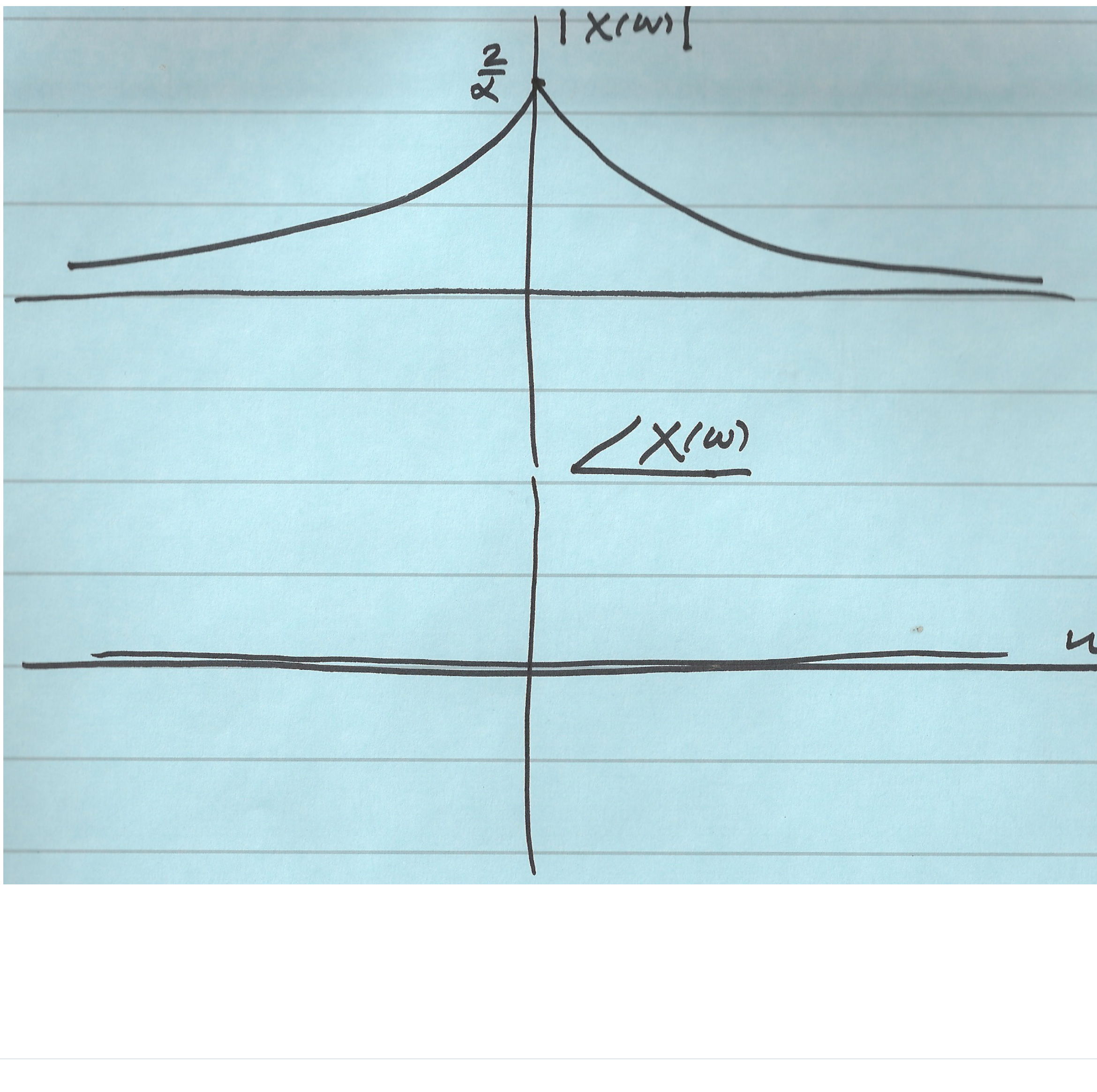
vipulkohli commented 37 minutes ago Owner

$$X(j\omega) = \frac{1}{\sqrt{\alpha^2+\omega^2}} \cdot e^{-j\left(\tan^{-1}\frac{\omega}{\alpha}\right)}$$





Magnitude Spectrum



$$X(j\omega) = \frac{1}{\alpha + j\omega} = \frac{1}{\sqrt{\alpha^2+\omega^2}\cdot e^{+j\tan^{-1}\frac{\omega}{\alpha}}}$$

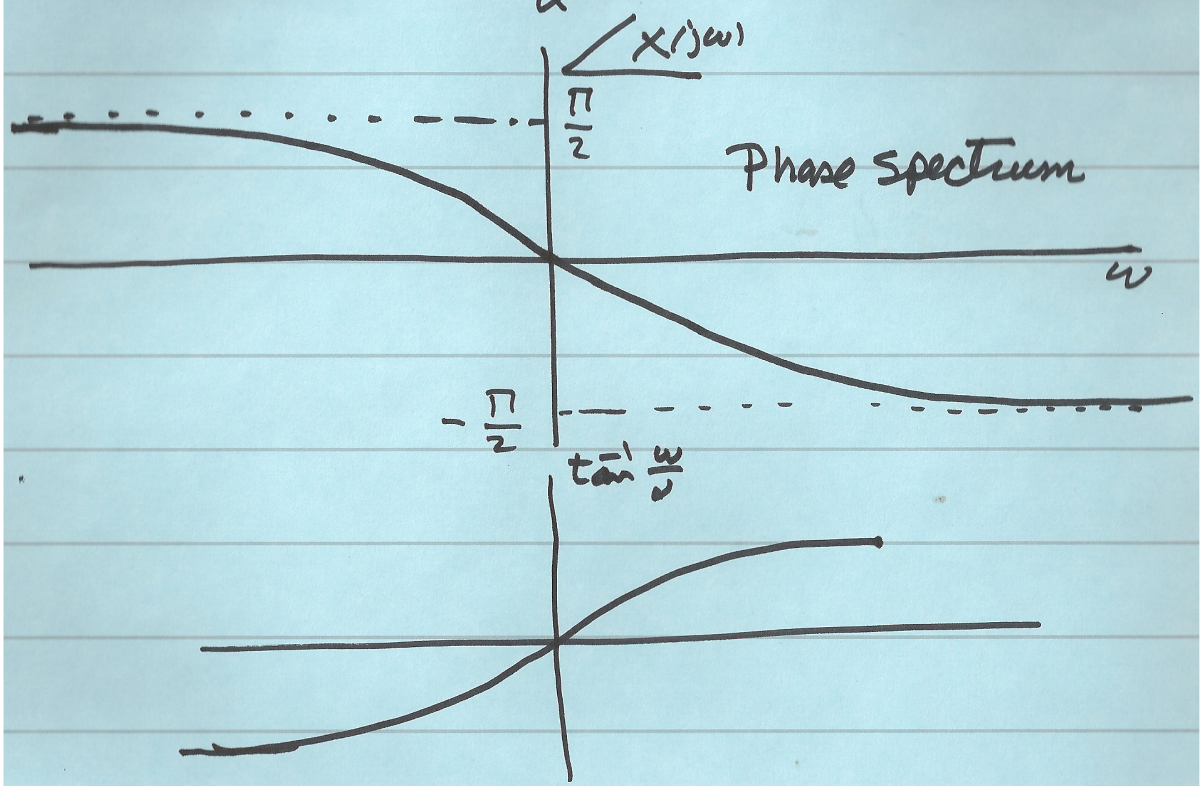
$$= \frac{1}{\sqrt{\alpha^2+\omega^2}}\cdot e^{-\tan^{-1}\frac{\omega}{\alpha}}$$

$$\frac{1}{e^x} = e^{-x}$$

vipulkohli commented 34 minutes ago

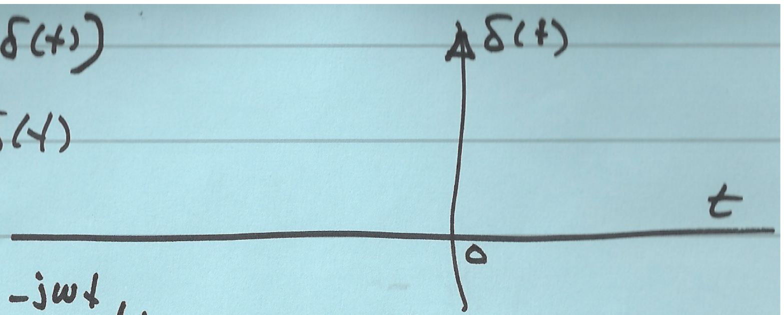
Owner

$$\angle X(j\omega) = -\tan^{-1}\frac{\omega}{\alpha}$$



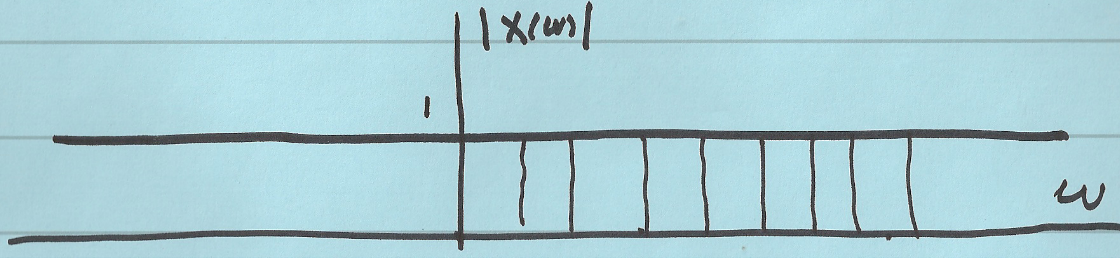
Example $\mathcal{F}(\delta(t))$

$\therefore \quad x(t) = \delta(t)$

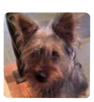


$$X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} . dt$$

$$X(\omega) \Rightarrow \boxed{X(\omega) = 1}$$



$$\angle X(\omega) = 0$$



vipulkohli commented 32 minutes ago

Owner

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 1 \cdot e^{j\omega t}\, d\omega$$

$$\delta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 1 \cdot e^{j\omega t} \cdot dt$$

$$2\pi\,\delta(\omega) = \int_{-\infty}^{\infty} 1 \cdot e^{j\omega t}\, dt$$

$$2\pi\,\delta(\omega) = -\int_{\infty}^{-\infty} e^{-j\omega t}\, d(-t)$$

$$2\pi\,\delta(\omega) = \int 1 \cdot e^{-j\omega t}\, dt = \mathcal{F}\{1\}$$

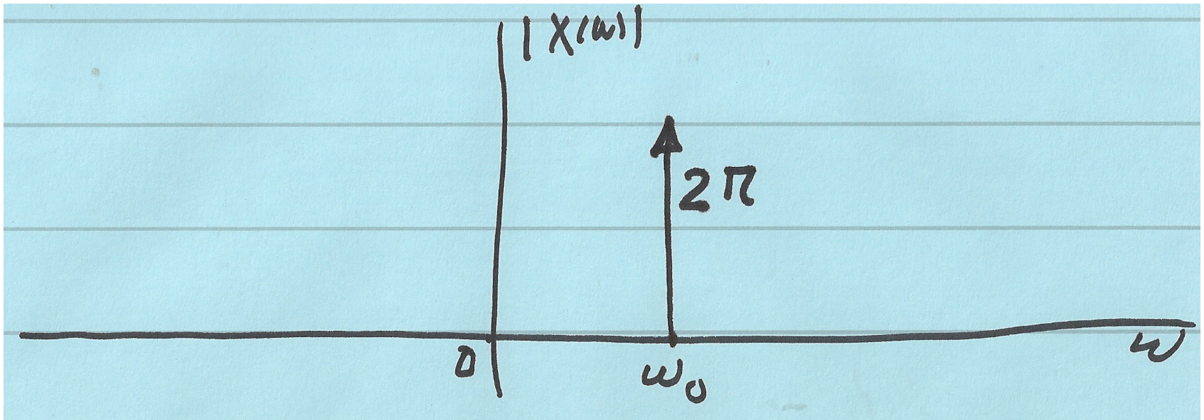
$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 \cdot e^{j\omega t} \, d\omega$$

$$\delta(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 1 \cdot e^{j\omega t} \cdot dt$$

$$2\pi \delta(\omega) = \int_{-\infty}^{\infty} 1 \cdot e^{j\omega t} \, dt$$

$$2\pi \delta(\omega) = - \int_{\infty}^{-\infty} e^{-j\omega t} \, d(-t)$$

$$2\pi \delta(\omega) = \int 1 \cdot e^{-j\omega t} \, dt = \mathcal{F}\{1\}$$



$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \int \delta(t)\,dt = 1 & \end{cases}$$

$$2\pi\,\delta(t - t_0) = \begin{cases} 0 & t \neq t_0 \\ \int \quad = 2\pi & \end{cases}$$

vipulkohli commented 26 minutes ago

Owner

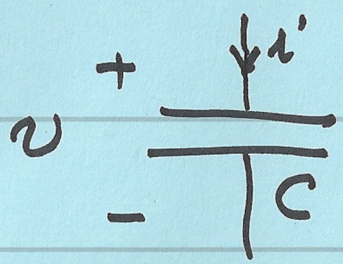
Example $x(t) = e^{j\omega_0 t}$

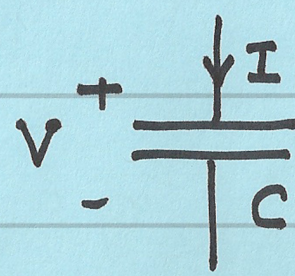
$$X(\omega) = \int_{-\infty}^{\infty} e^{j\omega_0 t} \cdot e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} 1 \cdot e^{-j(\omega-\omega_0)t}\, dt$$

$$X(\omega) = 2\pi\, \delta(\omega - \omega_0)$$

$$\mathcal{F}\{e^{j\omega_0 t}\} = 2\pi\, \delta(\omega-\omega_0)$$

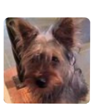




$$i = C\frac{dv}{dt}$$

$$I = j\omega C V$$

$$\frac{V}{I} = \frac{1}{j\omega C}$$



vipulkohli commented 25 minutes ago Owner

Properties of Fourier Transforms

1) $\mathcal{F}$-transform is a linear operation

$$\mathcal{F}\{\alpha x_1(t) + \beta x_2(t)\} = \alpha X_1(\omega) + \beta X_2(\omega)$$

2) $$\mathcal{F}\left\{\frac{d}{dt}x(t)\right\} = j\omega X(\omega)$$

Proof: $$x(t) = \frac{1}{2\pi}\int X(j\omega)\cdot e^{j\omega t}\,d\omega$$

$$\frac{d}{dt}x(t) = \underbrace{\frac{1}{2\pi}\int j\omega\cdot X(\omega)}\cdot e^{j\omega t}\,d\omega$$

$$\therefore \quad j\omega X(\omega) = \mathcal{F}\left\{\frac{d}{dt}x(t)\right\}$$

vipulkohli commented 23 minutes ago Owner

$\hat{2})$ $$\mathcal{F}\left\{\int_{-\infty}^{t} x(\tau)\,d\tau\right\} = \frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$$

3) $$\mathcal{F}\{x(t-t_0)\} = e^{-j\omega t_0}X(\omega)$$

4) $$\mathcal{F}\{e^{j\omega_0 t}\cdot x(t)\} = X(\omega-\omega_0)$$

5) $$\mathcal{F}\{x(\alpha t)\} = \frac{1}{|\alpha|}X\left(\frac{\omega}{\alpha}\right)$$

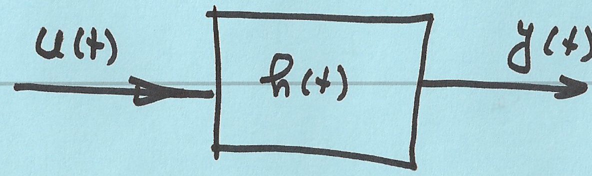
6) Parsaval's Theorem

$$E\{x(t)\} = \int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega$$

7) Energy in a Frequency band $(\omega_2-\omega_1)$

$$\Delta E = \int_{\omega_1}^{\omega_2}\frac{1}{2\pi}|X(\omega)|^2\,d\omega$$

8) $$\mathcal{F}\{x_1(t) * x_2(t)\} = X_1(\omega)\cdot X_2(\omega)$$



$$\therefore\ y(t) = h(t) * u(t)$$

$$\boxed{Y(j\omega) = H(j\omega)\cdot U(j\omega)}$$

If Periodic

Let $x(t)$ be Periodic

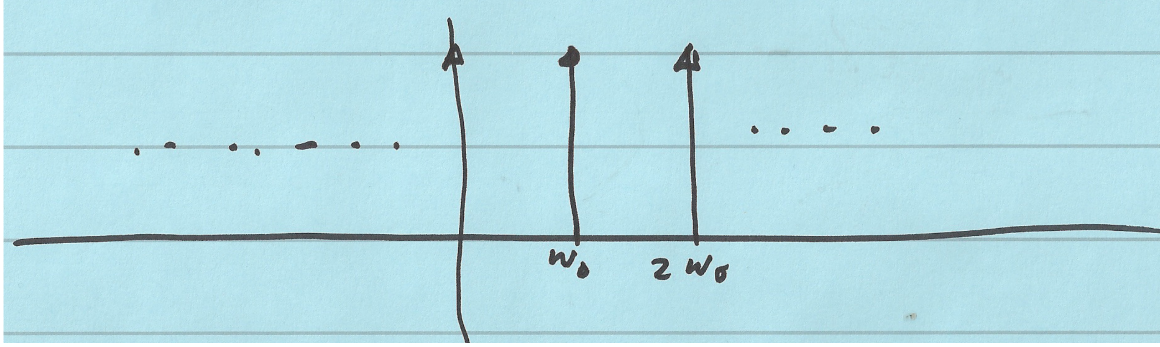
Then by $\mathcal{F}$-series;

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{-jn\omega_0 t}$$

Hence $$\mathcal{F}\{x(t)\} = \sum \mathcal{F}\left[c_n e^{-j\omega_0 n t}\right]$$

$$\mathcal{F}\{x(t)\} = \sum_{n=-\infty}^{\infty} c_n \cdot 2\pi\, \delta(\omega - n\omega_0)$$

$$\boxed{\mathcal{F}\{x(t)\} = 2\pi \sum_{n=-\infty}^{\infty} c_n\, \delta(\omega - n\omega_0)}$$

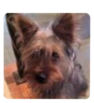


$$\int_{-\infty}^{\infty} x(t)\cdot x^{*}\,dt$$

$$\mathcal{F}\{t\,e^{-\alpha t}\} = \frac{1}{(\alpha+j\omega)^2}$$

H.W. 4.5, 4.6,

Due Thurs. March 5th



Write Preview

Markdown supported Edit in fullscreen

Leave a comment

Attach images by dragging & dropping or selecting them.

Close issue Comment