7

$$X(s) = \frac{s+2}{s^2+2s+3}$$

a) $\mathcal{L}\{3x(\tfrac{1}{2}t)\} = 3 \cdot \frac{1}{1/2} \cdot X\left(\frac{s}{1/2}\right) = 6\,\frac{2s+2}{4s^2+4s+3}$

b) $\mathcal{L}\{t\,x(t-1)\} = \mathcal{L}\{(t-1)x(t-1) + x(t-1)\}$

$\mathcal{L}\{x(t-1)\} = e^{-s} \cdot \frac{s+2}{s^2+2s+3}$ ✓

$\mathcal{L}\{t\,x(t)\} = -\frac{d}{ds}X(s) = \frac{s^2+4s+1}{[s^2+2s+3]^2}$

$\mathcal{L}\{(t-1)x(t-1)\} = e^{-s}\,\frac{s^2+4s+1}{[s^2+2s+3]^2}$ ✓

c) $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0)$

But $x(0) = \lim_{s\to\infty} s \cdot \frac{s+2}{s^2+2s+3} = 1$

$$\boxed{\mathcal{L}\left\{\frac{dx}{dt}\right\} = \frac{s(s+2)}{s^2+2s+3} - 1}$$ ✓

1

4

$$H(s) = \frac{s^2+5s+100}{s(1+\frac{s}{100})(1+\frac{s}{1000})}$$

$$H(s) = 100 \cdot \frac{(\frac{s}{10})^2 + \frac{5}{100}s + 1}{s(1+\frac{s}{100})(1+\frac{s}{1000})} \quad \hat{H}$$

$\omega_n = 10$

$K = 100 \quad K_{dB} = 40\,dB \quad \hat{H}(s=1) \cong \frac{0+0+1}{1\times1\times1} = 1$

$\omega_n = 10$

$2\zeta\omega_n = 5 \quad 20\zeta = 5$

$\zeta = 0.25$

5

$$F(s) = \frac{s+2}{(s^2+2s+2)(s+1)^2}$$

$f(0) = \lim_{s\to\infty} sF(s) = 0$

$f(\infty) = \lim_{s\to 0} sF(s) = 0$

6

$$\frac{dx}{dt} + 4x + 3\int_0^t x(\sigma)\,d\sigma = 5 \;;\; x(0) = 1$$

$sX(s) - 1 + 4X(s) + \frac{3}{s}X(s) = \frac{5}{s}$

$X(s) = \frac{s+5}{s^2+4s+3} \Rightarrow \boxed{x(t) = (2e^{-t} - e^{-3t})u_0(t)}$

#2 $x(t) = t\,e^{-3t}\cos(2t+30^\circ)$

Find $X(s)$

$$f(t) = \cos(2t+30^\circ) = \cos 2t \cos 30^\circ - \sin 2t \sin 30^\circ$$

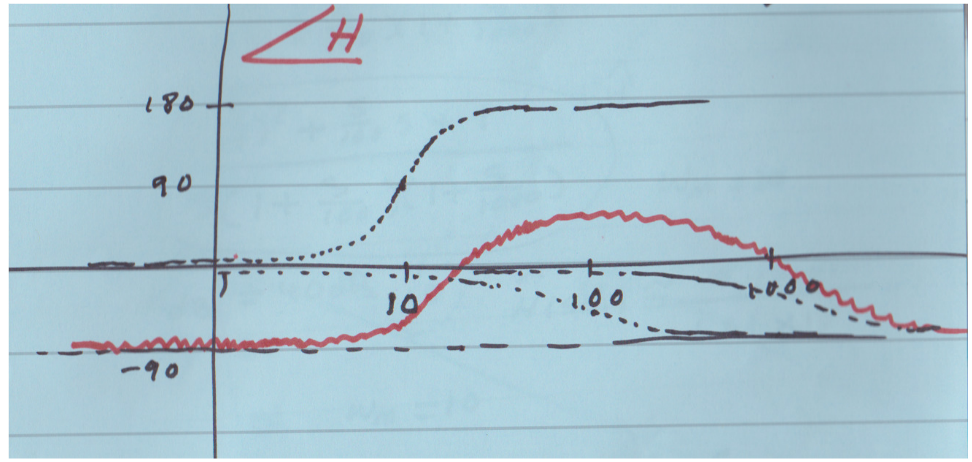
$$f(t) = \frac{\sqrt{3}}{2}\cos 2t - \frac{1}{2}\sin 2t$$

$$F(s) = \frac{\sqrt{3}}{2}\,\frac{s}{s^2+4} - \frac{1}{2}\,\frac{2}{s^2+4}$$

$$\mathcal{L}\{e^{-3t}\cos(2t+30^\circ)\} = F(s+3)$$

$$= \frac{\sqrt{3}}{2}\,\frac{s+3}{(s+3)^2+4} - \frac{1}{2}\,\frac{2}{(s+3)^2+4}$$

$$\mathcal{L}\{t\,e^{-3t}\cos(2t+30^\circ)\} = -\frac{d}{ds}\{\cdots\}$$



$$X(s) = -\frac{\sqrt{3}}{2}\,\frac{4-(s+3)^2}{[(s+3)^2+4]^2} + \frac{2(s+3)}{[(s+3)^2+4]^2}$$

#3 $F(s) = \dfrac{s+2}{(s^2+2s+2)(s+1)^2}$

$A = 1$

$B = 1$

$C = -1$

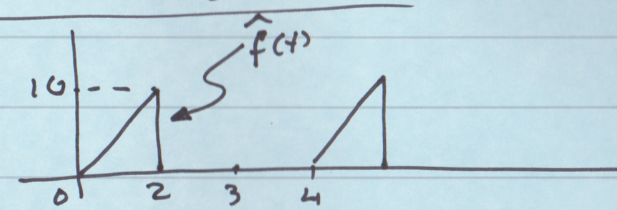
$D = -2$

$$F(s) = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{Cs+D}{(s+1)^2+1^2}$$

$$F(s) = \frac{1}{(s+1)^2} + \frac{1}{s+1} - \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}$$

$$f(t) = (t e^{-t} + e^{-t} - e^{-t}\cos t - e^{-t}\sin t)\,u_0(t)$$

$$\boxed{F(s) = \frac{e^{-(s+3)} - e^{-5(s+3)}}{s+3}}$$



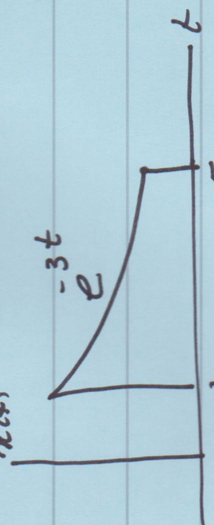
$$\hat{f}(t) = 5t\,u_0(t) - 5(t-2)\,u_0(t-2) - 10\,u_0(t-2)$$

$$\hat{F}(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{10e^{-2s}}{s}$$

$$F(s) = \frac{\hat{F}(s)}{1-e^{-4s}}$$

H.W. Set #8

#1



$x(t) = e^{-3t}\left[u_0(t-1) - u_0(t-5)\right]$

$= e^{-3t}u_0(t-1) - e^{-3t}u_0(t-5)$

$= e^{-3}\cdot e^{-3(t-1)}u_0(t-1) - e^{-15}\cdot e^{-3(t-5)}u_0(t-5)$

$\downarrow$

$F(s) = e^{-3}\cdot\dfrac{e^{-s}}{s+3} - e^{-15}\cdot\dfrac{e^{-5s}}{s+3}$

#4 $F(s) = \dfrac{1-e^{-3s}}{(s+2)\underbrace{(1-e^{-4s})}}$

$\hat{F}(s) = \dfrac{1}{s+2} - \dfrac{e^{-3s}}{s+2}$

$\hat{f}(t) = e^{-2t}u_0(t) - e^{-2(t-3)}u_0(t-3)$

$\boxed{f(t) = \sum_{n=0}^{\infty} \hat{f}(t-4n)}$

1

#4 $H(s) = \dfrac{s^2+5s+100}{s(1+\frac{s}{100})(1+\frac{s}{1000})}$

$H(s) = 100\cdot\dfrac{(\frac{s}{10})^2 + \frac{5}{100}s + 1}{s(1+\frac{s}{100})(1+\frac{s}{1000})}$ $\hat{H}$

$\omega_n = 10$

$K = 100$ $K_{dB} = 40\text{dB}$ $\hat{H}(s=1) \cong \dfrac{0+0+1}{1\times1\times1} = 1$

$\omega_n = 10$

$2\zeta\omega_n = 5$ $20\zeta = 5$

$\zeta = 0.25$