#5 $$F(s) = \frac{s+2}{(s^2+2s+2)(s+1)^2}$$

$$f(0) = \lim_{s \to \infty} sF(s) = 0$$

$$f(\infty) = \lim_{s \to 0} sF(s) = 0$$

#6 $$\frac{dx}{dt} + 4x + 3\int_0^t x(\tau)\,d\tau = 5 \ ; \quad x(0) = 1$$

$$sX(s) - 1 + 4X(s) + \frac{3}{s}X(s) = \frac{5}{s}$$

$$X(s) = \frac{s+5}{s^2+4s+3} \Rightarrow \boxed{x(t) = (2e^{-t} - e^{-3t})\,u_0(t)}$$

-7

$$X(s) = \frac{s+2}{s^2+2s+3}$$

$$\mathcal{L}\left\{3x\left(\tfrac{1}{2}t\right)\right\} = 3 \cdot \frac{1}{1/2} \cdot X\left(\frac{s}{1/2}\right) = 6\,\frac{2s+2}{4s^2+4s\ldots}$$

$$\mathcal{L}\left\{t\,x(t-1)\right\} = \mathcal{L}\left\{(t-1)\,x(t-1) + x(t-1)\right\}$$

$$\mathcal{L}\left\{x(t-1)\right\} = e^{-s} \cdot \frac{s+2}{s^2+2s+3} \quad \checkmark$$

$$\mathcal{L}\left\{t\,x(t)\right\} = -\frac{1}{ds}\,X(s) = \frac{s^2+4s+1}{[s^2+2s+3]^2}$$

$$\mathcal{L}\left\{(t-1)\,x(t-1)\right\} = e^{-s}\,\frac{s^2+4s+1}{[s^2+2s+3]^2} \quad \checkmark$$

$$\left\{ \frac{dx(t)}{dt} \right\} = s\,X(s) - x(0)$$

$$x(0) = \lim_{s \to \infty} s \cdot \frac{s+2}{s^2+2s+3}$$

$$\left\{ \frac{dx}{dt} \right\} = \boxed{\frac{s(s+2)}{s^2+2s+3} - 1}$$

1

$\#4 \quad H(s) = \dfrac{s^2 + 5s + 100}{s(1 + \frac{s}{100})(1 + \frac{s}{1000})}$

$$H(s) = 100 \cdot \underbrace{\frac{(\frac{s}{10})^2 + \frac{5}{100}s + 1}{s(1 + \frac{s}{100})(1 + \frac{s}{1000})}}_{\hat{H}}$$

$\omega_n = 10$

$K = 100 \quad K_{dB} = 40\,dB$

$\hat{H}(s=1) \cong \dfrac{0 + 0 + 1}{1 \times 1 \times 1} =$

$\omega_n = 10$

$2\zeta\omega_n = 5 \qquad 20\zeta = 5$

$\zeta = 0.25$

1

#4 $H(s) = \dfrac{s^2+5s+100}{s(1+\frac{s}{100})(1+\frac{s}{1000})}$

$H(s) = 100 \cdot \dfrac{(\frac{s}{10})^2 + \frac{5}{100}s + 1}{s(1+\frac{s}{100})(1+\frac{s}{1000})}$ $\hat{H}$ $\omega_n = 10$

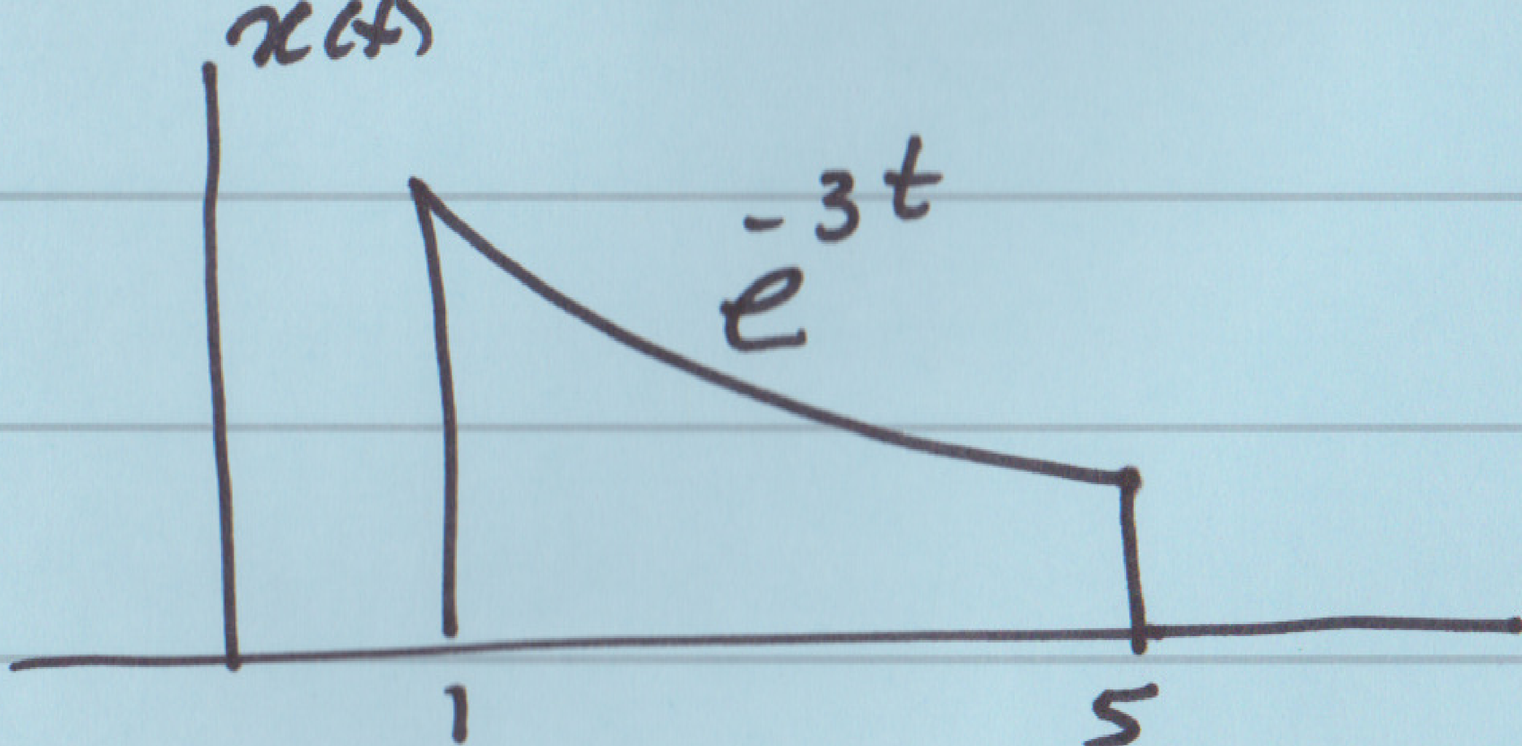
$K = 100$ $K_{dB} = 40dB$ $\hat{H}(s=1) \cong \dfrac{0+0+1}{1 \times 1 \times 1} =$

$\omega_n = 10$

$2\xi\omega_n = 5$ $20\xi = 5$

$\boxed{\xi = 0.25}$

H.W. Set #8



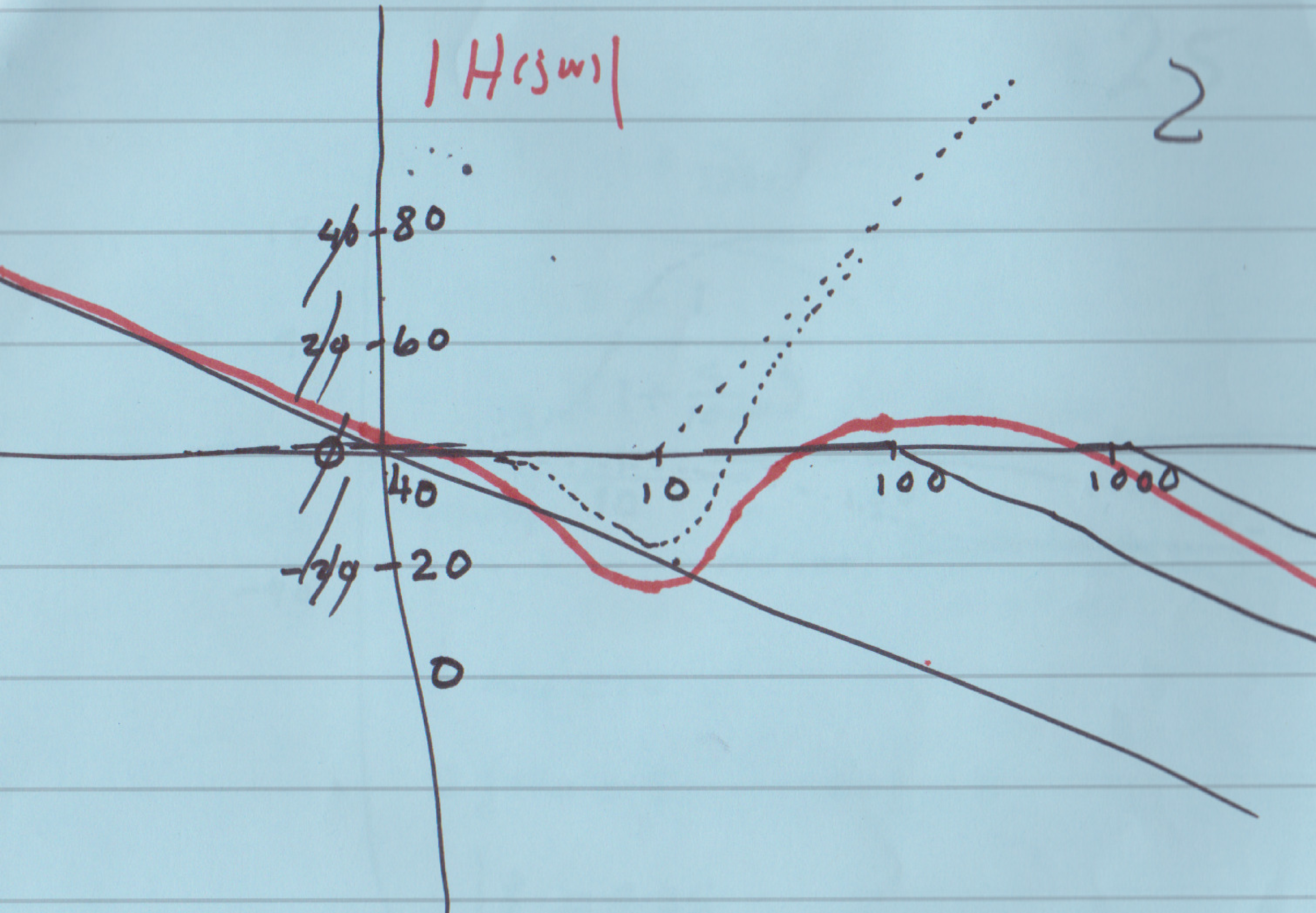
$$x(t) = e^{-3t}\left[u_0(t-1) - u_0(t-5)\right]$$

$$= e^{-3t}u_0(t-1) - e^{-3t}u_0(t-5)$$

$$= e^{-3}\cdot e^{-3(t-1)}\,u_0(t-1) - e^{-15}\cdot e^{-3(t-5)}\,u_0(t-5)$$

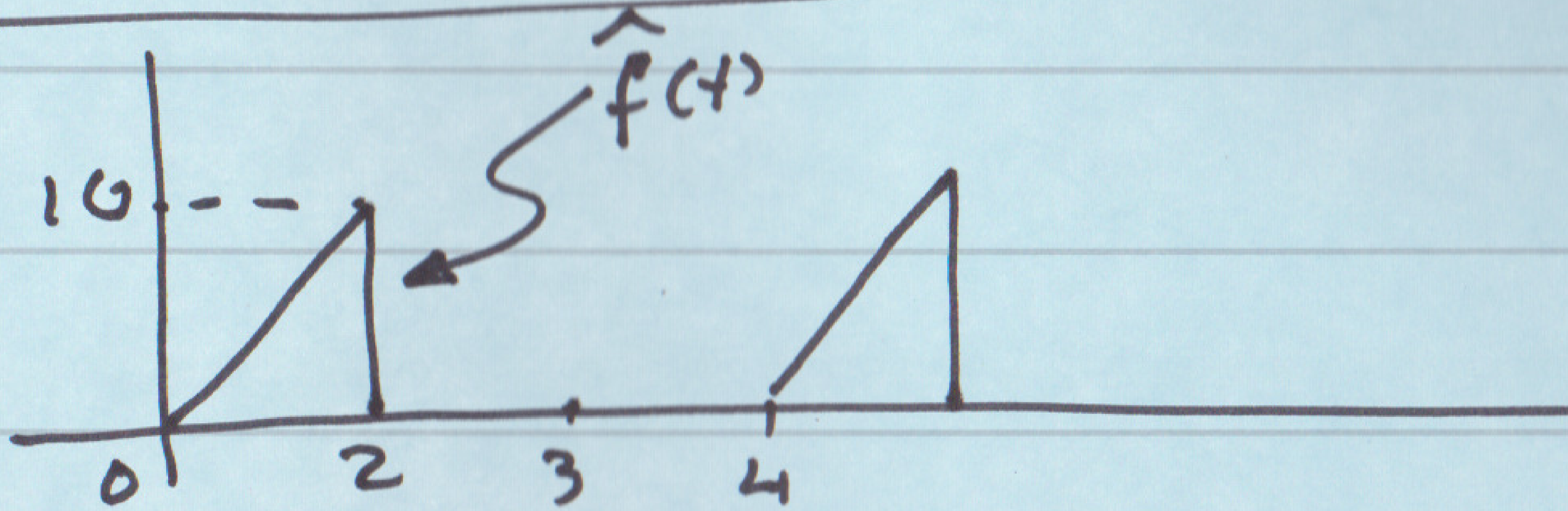
$$\downarrow$$

$$X(s) = e^{-3}\cdot\frac{e^{-s}}{s+3} - e^{-15}\cdot\frac{e^{-5s}}{s+3}$$


2
|H(jω)|
40
80
20
60
-20
20
0
40
10
100
1000

$$\bar{F}(s) = \frac{e^{-(s+3)} - e^{-s(s+3)}}{s+3}$$

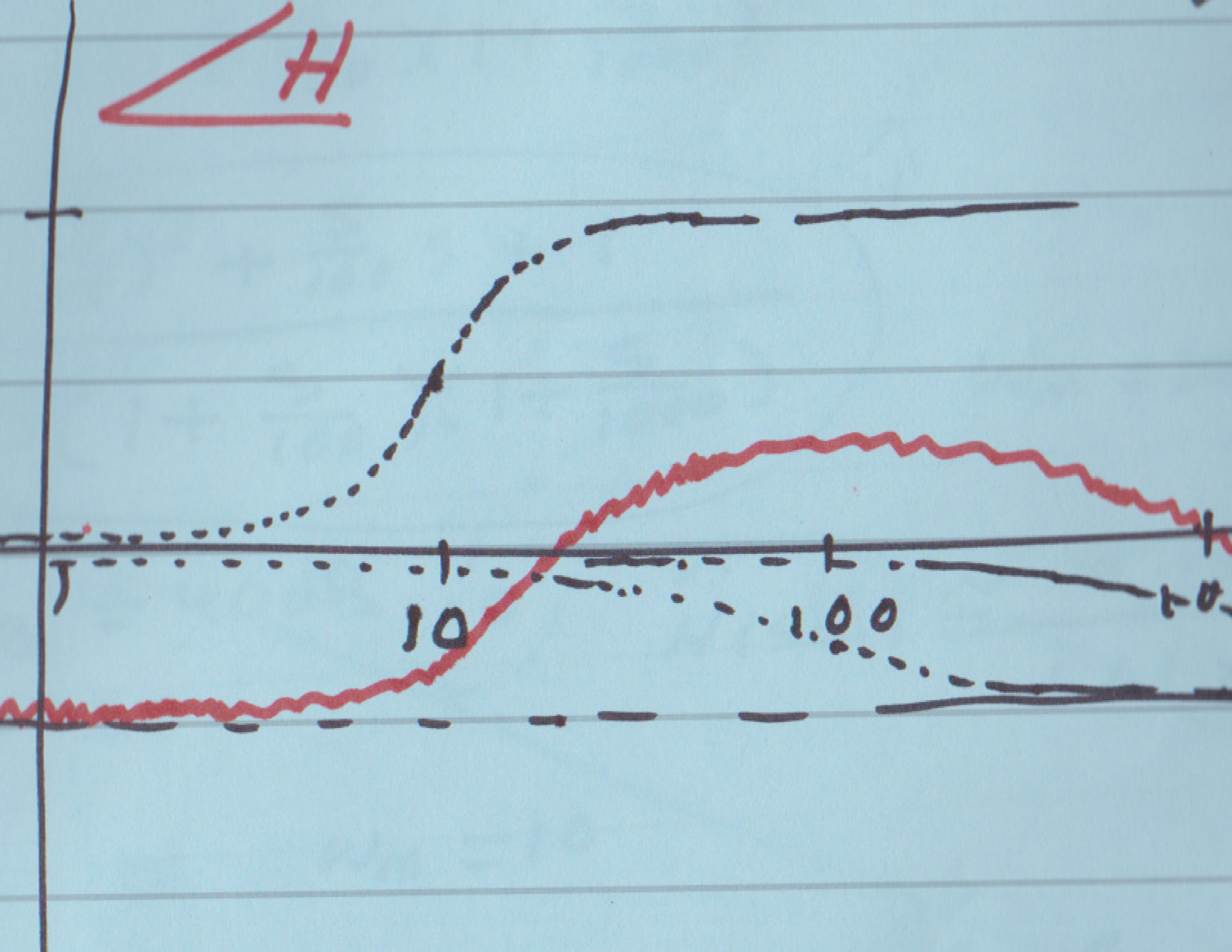
b)



$$\hat{f}(t) = 5t\,u_0(t) - 5(t-2)\,u_0(t-2) - 10\,u_0(t-2)$$

$$\hat{F}(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{10e^{-2s}}{s}$$

$$\bar{F}(s) = \frac{\hat{F}(s)}{1 - e^{-4s}}$$



$\angle H$
1
10
100
10

#2 $x(t) = t\,e\,\cos(2t+30)$

Find $X(s)$

$$f(t) = \cos(2t+30^\circ) = \cos 2t \cos 30^\circ - \sin 2t \sin 30^\circ$$

$$f(t) = \frac{\sqrt{3}}{2}\cos 2t - \frac{1}{2}\sin 2t$$

$$F(s) = \frac{\sqrt{3}}{2}\,\frac{s}{s^2+4} - \frac{1}{2}\,\frac{2}{s^2+4}$$

$$\mathcal{L}\left\{ e^{-3t}\cos(2t+30^\circ) \right\} = F(s+3)$$

$$= \frac{\sqrt{3}}{2}\,\frac{s+3}{(s+3)^2+4} - \frac{1}{2}\,\frac{2}{(s+3)^2+4}$$

$$X(s) = -\frac{\sqrt{3}}{2}\,\frac{4-(s+3)}{\left[(s+3)^2+4\right]^2}$$

$$+\frac{2(s+3)}{\left[(s+3)^2+4\right]^2}$$

#3

$$F(s) = \frac{s+2}{(s^2+2s+2)(s+1)^2}$$

$$F(s) = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{Cs+D}{(s+1)^2+1^2}$$

$A = 1$

$B = 1$

$C = -1$

$D = -2$

$$F(s) = \frac{1}{(s+1)^2} + \frac{1}{s+1} - \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}$$

$$f(t) = (t e^{-t} + e^{-t} - e^{-t}\cos t - e^{-t}\sin t)\,u_0(t)$$

#4 $$F(s) = \frac{1-e^{-3s}}{(s+2)\underbrace{(1-e^{-4s})}}$$

$$\hat{F}(s) = \frac{1}{s+2} - \frac{e^{-3s}}{s+2}$$

$$\hat{f}(t) = e^{-2t} u_0(t) - e^{-2(t-3)} u_0(t-3)$$

$$\boxed{f(t) = \sum^{\infty} \hat{f}(t-4n)}$$