#2 $x(t) = t e^{-3t} \cos(2t + 30^\circ)$

Find $X(s)$

$$f(t) = \cos(2t + 30^\circ) = \cos 2t \cos 30^\circ - \sin 2t \sin 30^\circ$$

$$f(t) = \frac{\sqrt{3}}{2}\cos 2t - \frac{1}{2}\sin 2t$$

$$F(s) = \frac{\sqrt{3}}{2}\frac{s}{s^2+4} - \frac{1}{2}\frac{2}{s^2+4}$$

$$\mathcal{L}\{e^{-3t}\cos(2t+30^\circ)\} = F(s+3)$$

$$= \frac{\sqrt{3}}{2}\frac{s+3}{(s+3)^2+4} - \frac{1}{2}\frac{2}{(s+3)^2+4}$$

$$\mathcal{L}\{t e^{-3t}\cos(2t+30^\circ)\} = -\frac{d}{ds}\{\cdots\}$$

$$X(s) = -\frac{\sqrt{3}}{2}\,\frac{4-(s+3)^2}{[(s+3)^2+4]^2} + \frac{2(s+3)}{[(s+3)^2+4]^2}$$

#3 $\quad F(s) = \dfrac{s+2}{(s^2+2s+2)(s+1)^2}$

$A = 1$

$B = 1$

$C = -1$

$D = -2$

$$F(s) = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{Cs+D}{(s+1)^2+1^2}$$

$$F(s) = \frac{1}{(s+1)^2} + \frac{1}{s+1} - \frac{s+1}{(s+1)^2+1} - \frac{1}{(s+1)^2+1}$$

$$f(t) = \left(te^{-t} + e^{-t} - e^{-t}\cos t - e^{-t}\sin t\right)u_0(t)$$

#4 $$F(s) = \frac{1 - e^{-3s}}{(s+2)(1 - e^{-4s})}$$

$$\hat{F}(s) = \frac{1}{s+2} - \frac{e^{-3s}}{s+2}$$

$$\hat{f}(t) = e^{-2t} u_0(t) - e^{-2(t-3)} u_0(t-3)$$

$$f(t) = \sum_{n=0}^{\infty} \hat{f}(t - 4n)$$

#5 $$F(s) = \frac{s+2}{(s^2+2s+2)(s+1)^2}$$

$$f(0) = \lim_{s\to\infty} sF(s) = 0$$

$$f(\infty) = \lim_{s\to 0} sF(s) = 0$$

#6 $$\frac{dx}{dt} + 4x + 3\int_0^t x(\sigma)\,d\sigma = 5 \; ; \; x(0) = -1$$

$$sX(s) - 1 + 4X(s) + \frac{3}{s}X(s) = \frac{5}{s}$$

$$X(s) = \frac{s+5}{s^2+4s+3} \Rightarrow \boxed{x(t) = (2e^{-t} - e^{-3t})u_0(t)}$$

#7

$$X(s) = \frac{s+2}{s^2+2s+3}$$

a) $\mathcal{L}\{3x(\frac{1}{2}t)\} = 3 \cdot \frac{1}{1/2} \cdot X(\frac{s}{1/2}) = 6\,\frac{2s+2}{4s^2+4s+3}$

b) $\mathcal{L}\{t\,x(t-1)\} = \mathcal{L}\{(t-1)x(t-1) + x(t-1)\}$

$$\mathcal{L}\{x(t-1)\} = e^{-s} \cdot \frac{s+2}{s^2+2s+3}$$ ✓

$$\mathcal{L}\{t\,x(t)\} = -\frac{1}{ds} X(s) = \frac{s^2+4s+1}{[s^2+2s+3]^2}$$

$$\mathcal{L}\{(t-1)x(t-1)\} = e^{-s}\,\frac{s^2+4s+1}{[s^2+2s+3]^2}$$ ✓

c) $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = s\,X(s) - x(0)$

But $x(0) = \lim_{s\to\infty} s \cdot \frac{s+2}{s^2+2s+3} = 1$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = \frac{s(s+2)}{s^2+2s+3} - 1$$ ✓

#4 $$H(s) = \frac{s^2 + 5s + 100}{s\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{1000}\right)}$$

$$H(s) = 100 \cdot \frac{\left(\frac{s}{10}\right)^2 + \frac{s}{100}s + 1}{s\left(1 + \frac{s}{100}\right)\left(1 + \frac{s}{1000}\right)} \quad \hat{H}$$

$\omega_n = 10$

$K = 100 \qquad K_{dB} = 40dB$

$$\hat{H}(s=1) \cong \frac{0 + 0 + 1}{1 \times 1 \times 1} = 1$$

$\omega_n = 10$

$2\xi\omega_n = 5 \qquad 20\xi = 5$

$$\boxed{\xi = 0.25}$$

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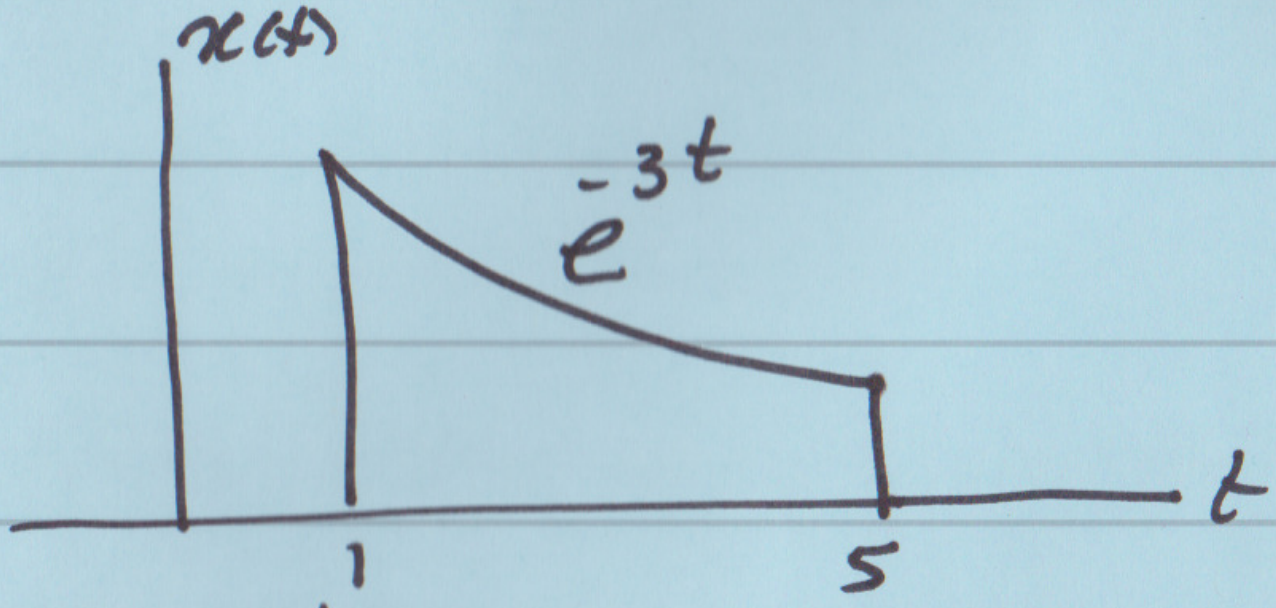
$\omega_n = 10$

$2\xi\omega_n = 5 \qquad 20\xi = 5$

$$\boxed{\xi = 0.25}$$

H.W. Set #8

#1



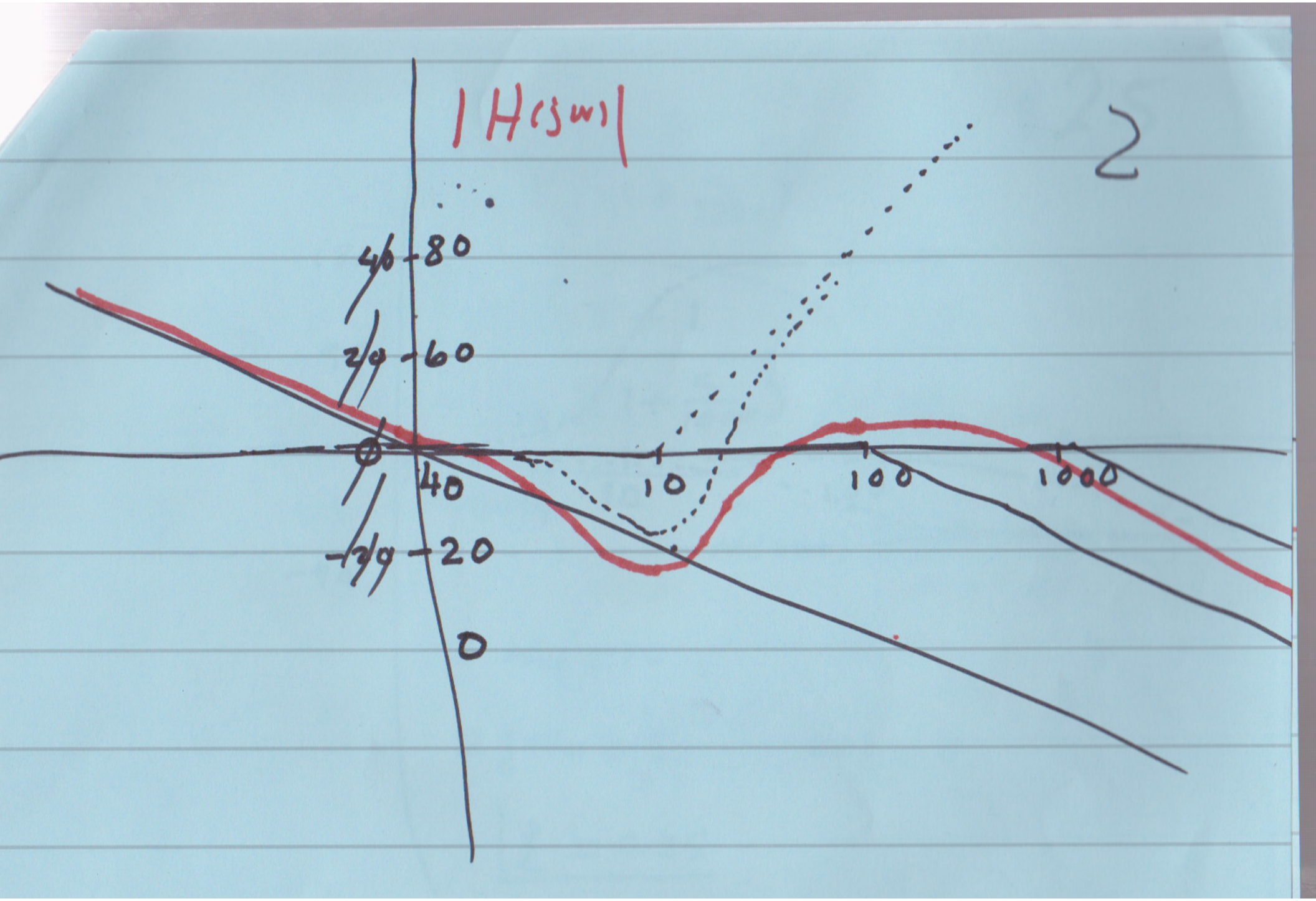
$$x(t) = e^{-3t}\left[u_0(t-1) - u_0(t-5)\right]$$

$$= e^{-3t}u_0(t-1) - e^{-3t}u_0(t-5)$$

$$= e^{-3}\cdot e^{-3(t-1)}u_0(t-1) - e^{-15}\cdot e^{-3(t-5)}u_0(t-5)$$

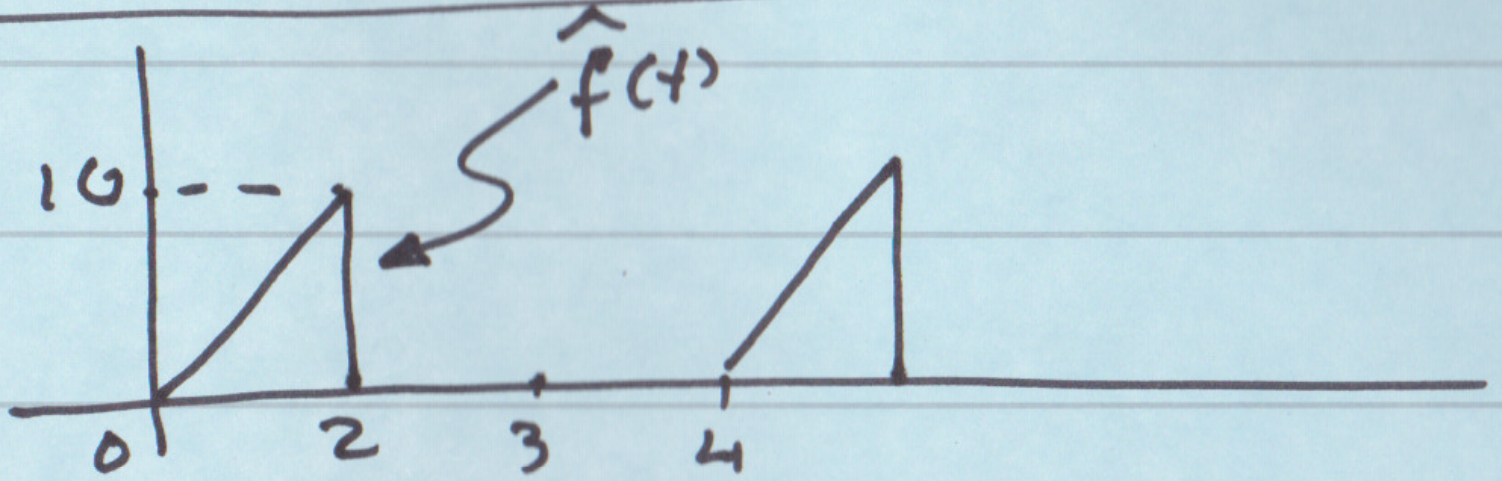
$$\downarrow$$

$$F(s) = e^{-3}\cdot\frac{e^{-s}}{s+3} - e^{-15}\cdot\frac{e^{-5s}}{s+3}$$


|H(jw)|
2
80
60
40
20
0
4/9
2/9
-2/9
10
100
1000

$$F(s) = \frac{e^{-(s+3)} - e^{-5(s+3)}}{s+3}$$

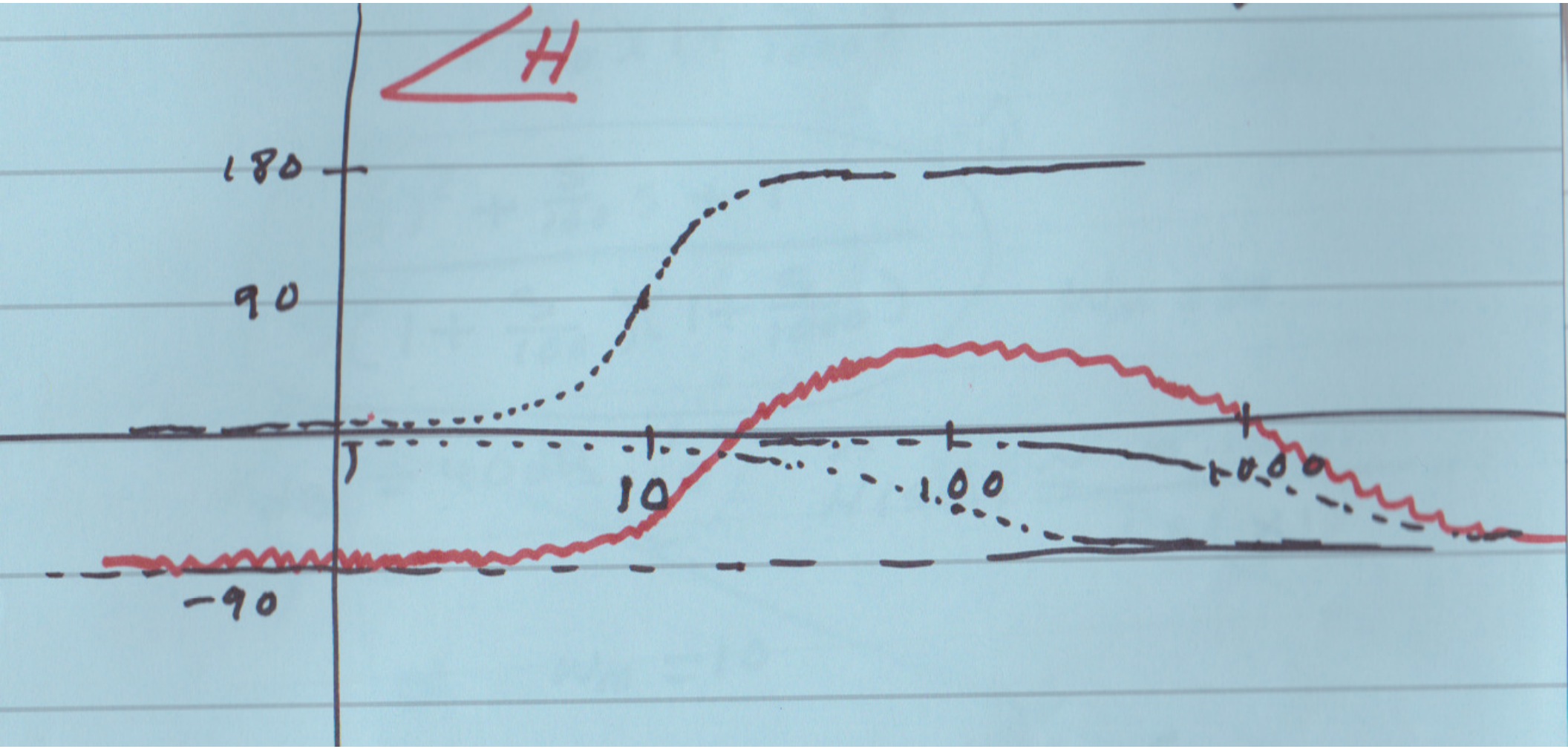
b)



$$\hat{f}(t) = 5t\,U_0(t) - 5(t-2)U_0(t-2) - 10\,U_0(t-2)$$

$$\hat{F}(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{10e^{-2s}}{s}$$

$$F(s) = \frac{\hat{F}(s)}{1 - e^{-4s}}$$ ✓


∠H
180
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