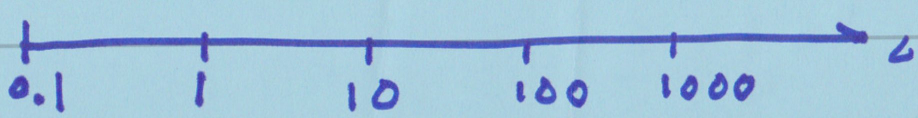
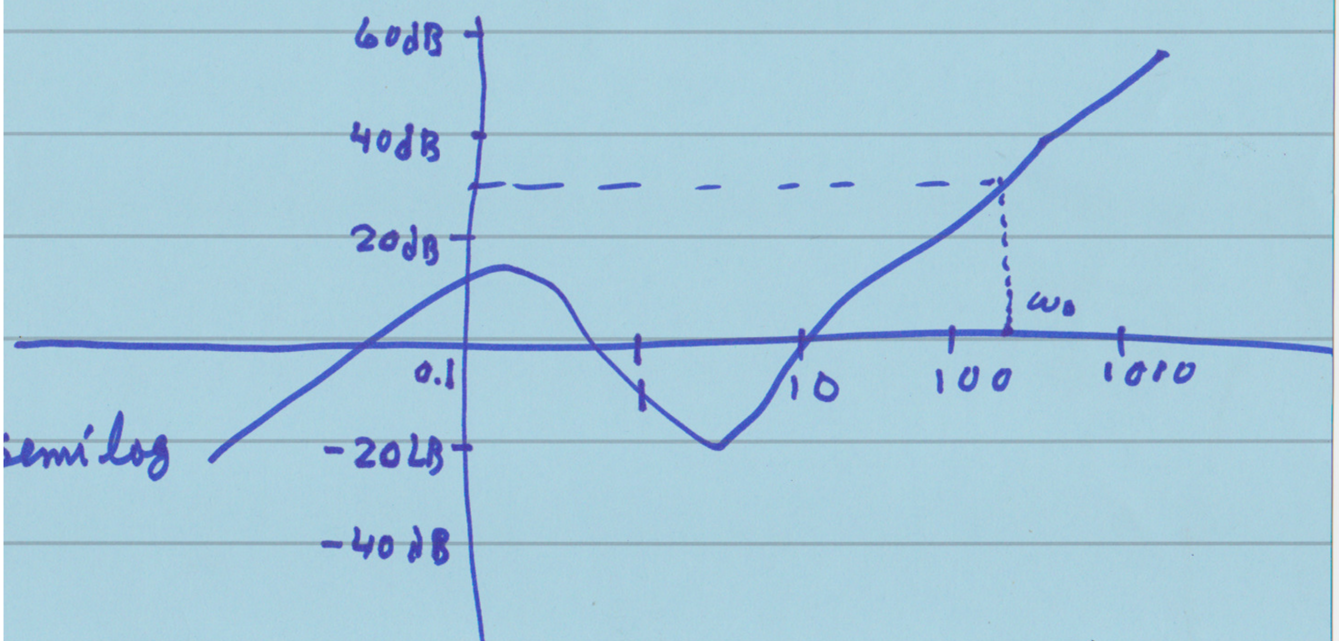
Bode Plots

$$H(s) = \frac{N(s)}{D(s)}\bigg|_{s=j\omega} = \frac{N(j\omega)}{D(j\omega)}$$

$$|H(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|} \qquad \omega : 0 \longrightarrow \infty$$



$$|H(j\omega)| \longrightarrow |H(j\omega)|_{dB} = 20\log_{10}|H(j\omega)|$$



semi log

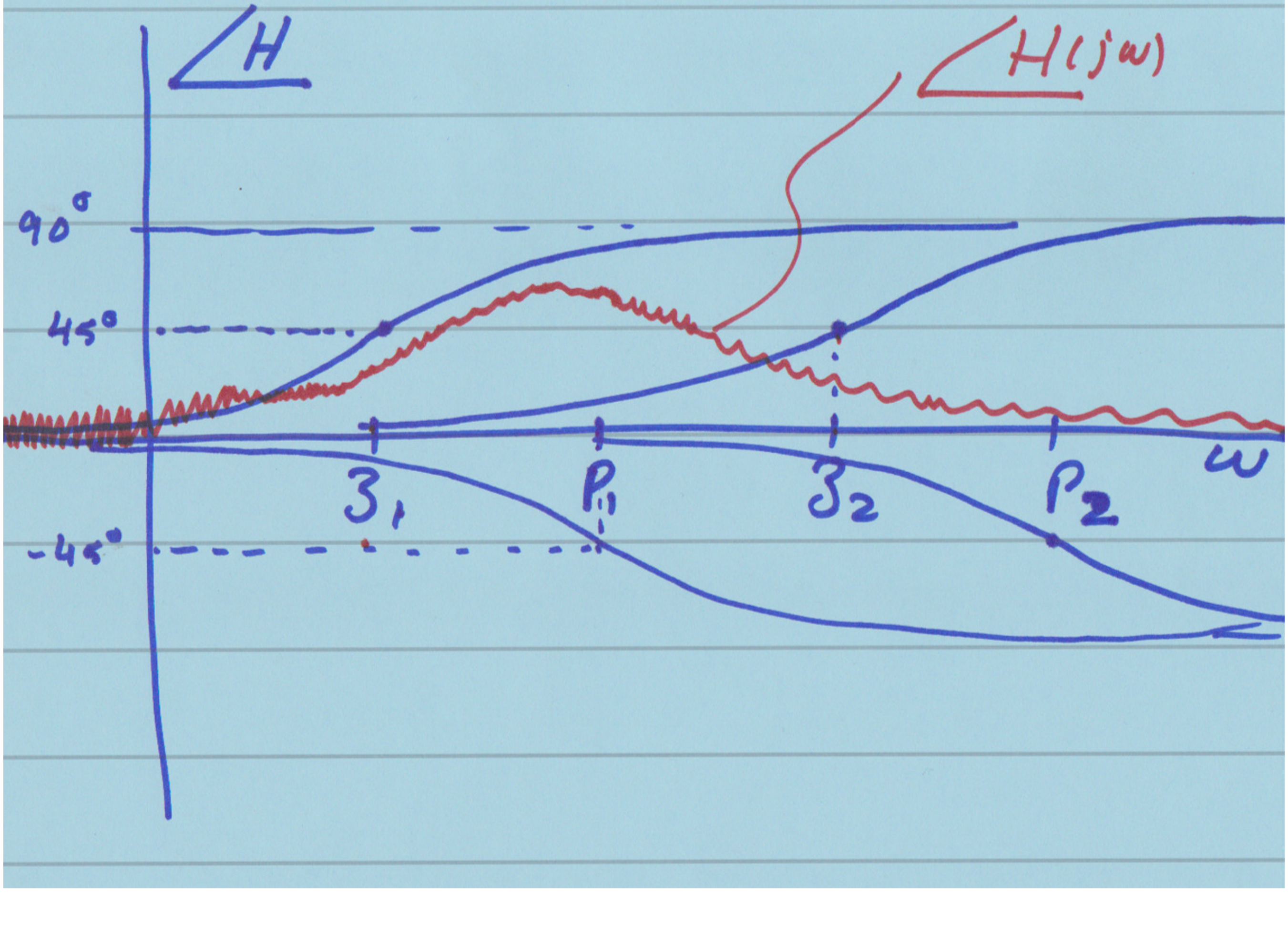
$$H(s) = K \frac{(s+z_1)(s+z_2)\cdots}{(s+p_1)(s+p_2)\cdots}$$

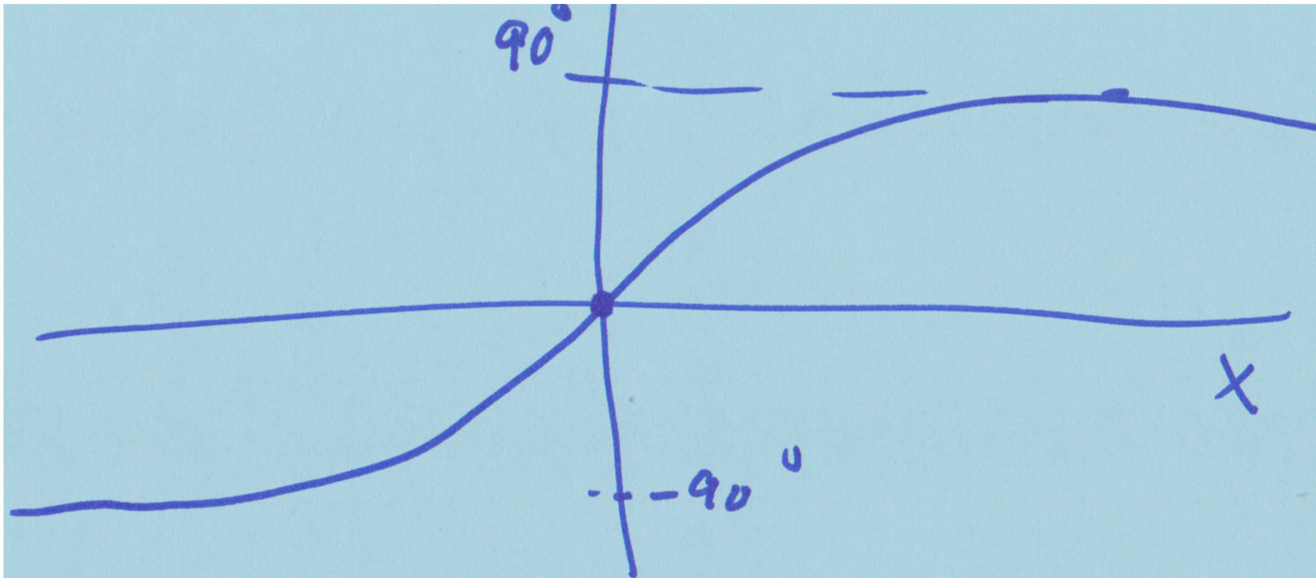
$$H(j\omega) = K \frac{\left(1+j\frac{\omega}{z_1}\right)\left(1+j\frac{\omega}{z_2}\right)\cdots}{\left(1+j\frac{\omega}{p_1}\right)\left(1+j\frac{\omega}{p_2}\right)\cdots}$$

$$\angle H(j\omega) = \underset{0 \text{ or } 180^\circ}{\angle K} + \angle\left(1+j\frac{\omega}{z_1}\right) + \angle\left(1+\frac{\omega}{z_2}\right)$$

$$- \angle\left(1+j\frac{\omega}{p_1}\right) - \angle\left(1+j\frac{\omega}{p_2}\right) \cdots$$

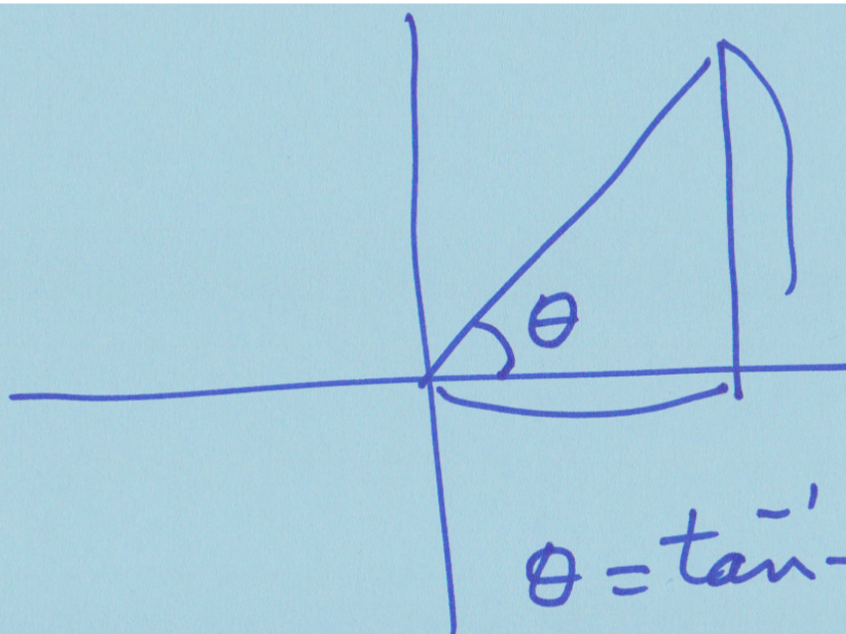
$$\angle\left(1+j\frac{\omega}{z_1}\right) = \tan^{-1}\frac{\omega}{z_1}$$



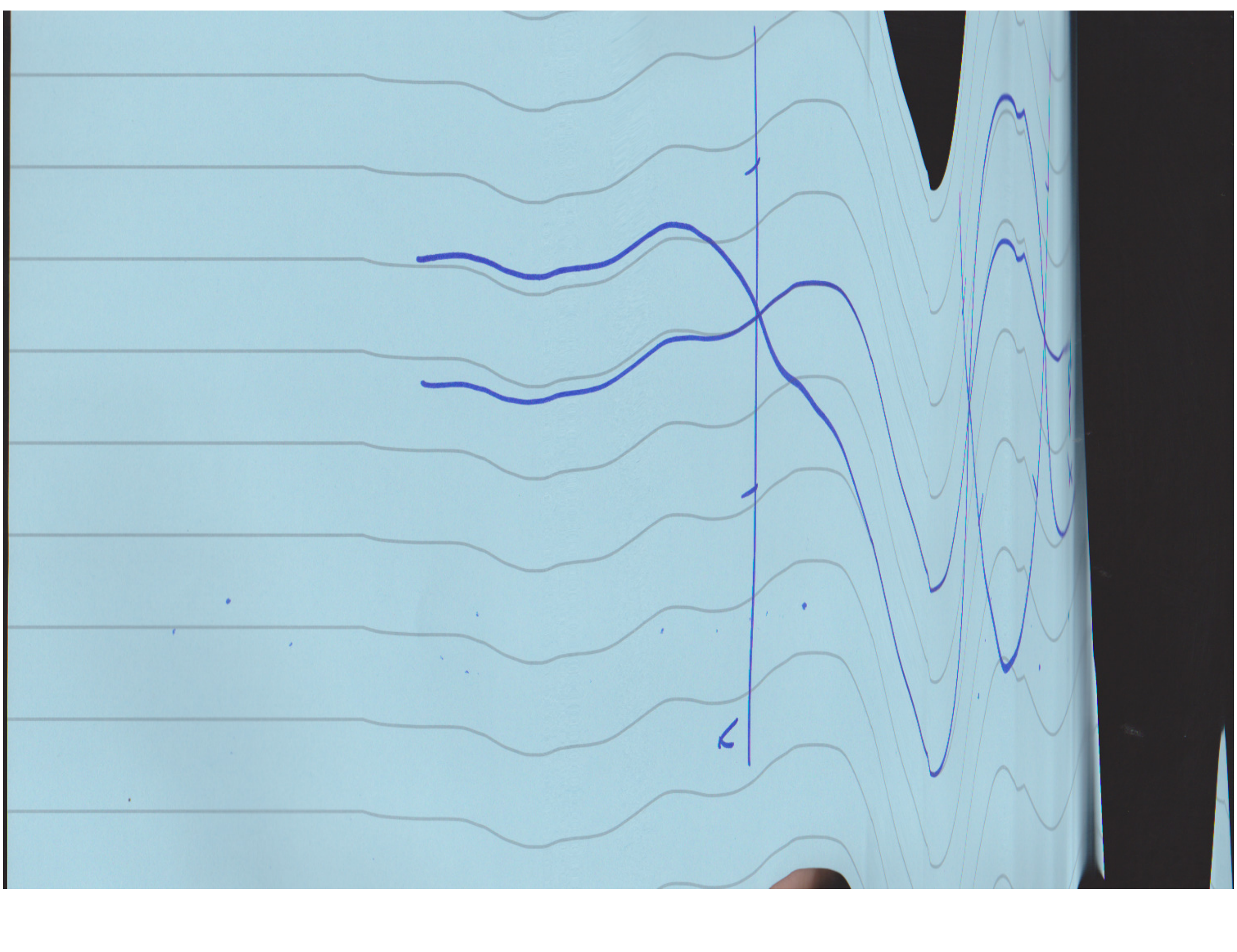


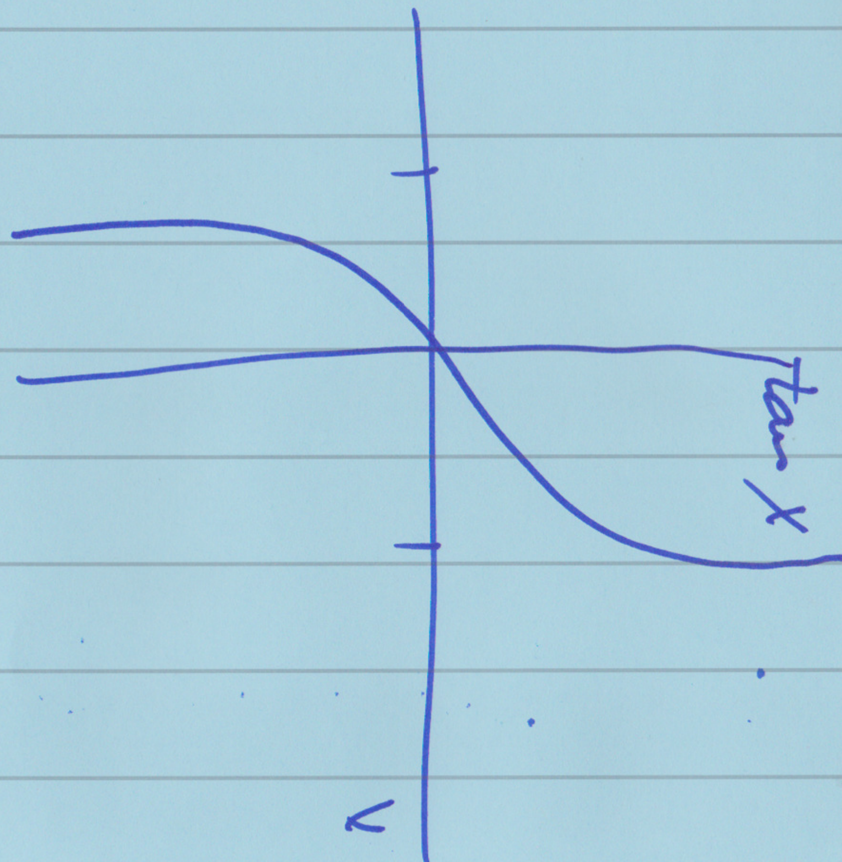
$$H(s) = \frac{s^2+s+1}{s^2+2s+1} \quad \text{Notch Filter}$$

bode([1 1 1], [1 2 1])



$$\theta = \tan^{-1} \frac{Im}{Real}$$



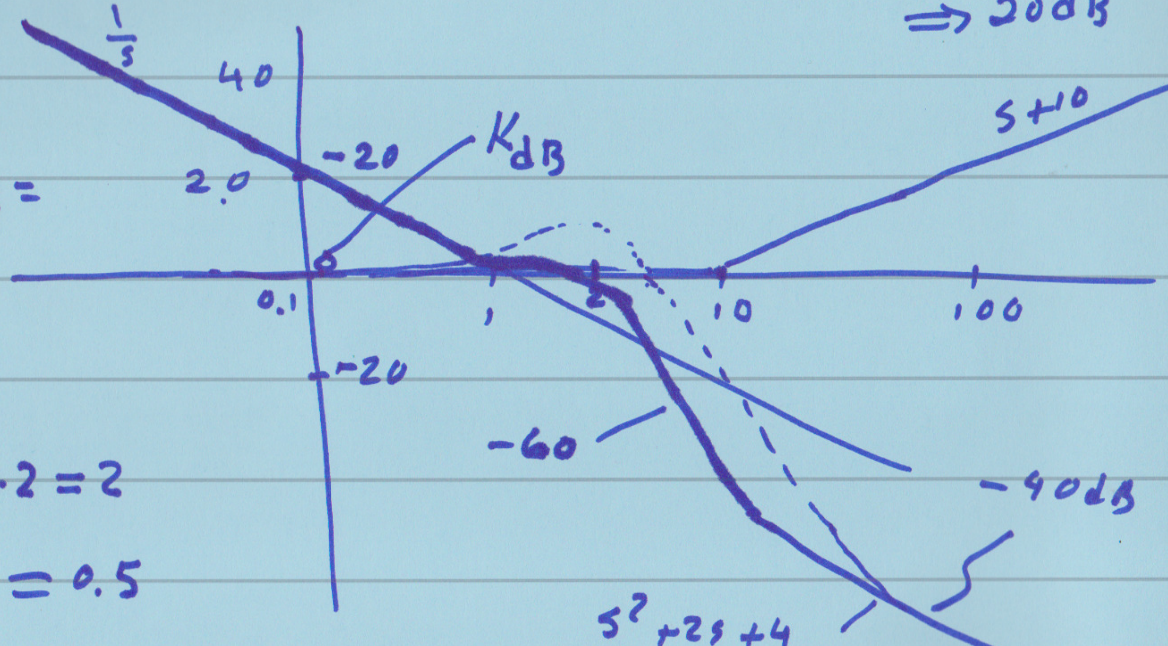


Example $H(s) = \dfrac{s+10}{s(s^2+2s+4)}$, $\omega_n^2 = 4$, $\omega_n = 2$

$\dfrac{10}{4} \cdot \dfrac{1+\frac{s}{10}}{s\left(\frac{s^2}{4}+\frac{s}{2}+1\right)}\Bigg|_{s=0.1} = \dfrac{1}{0.1} = 10$

$\Rightarrow 20\,dB$

$K_{dB} =$

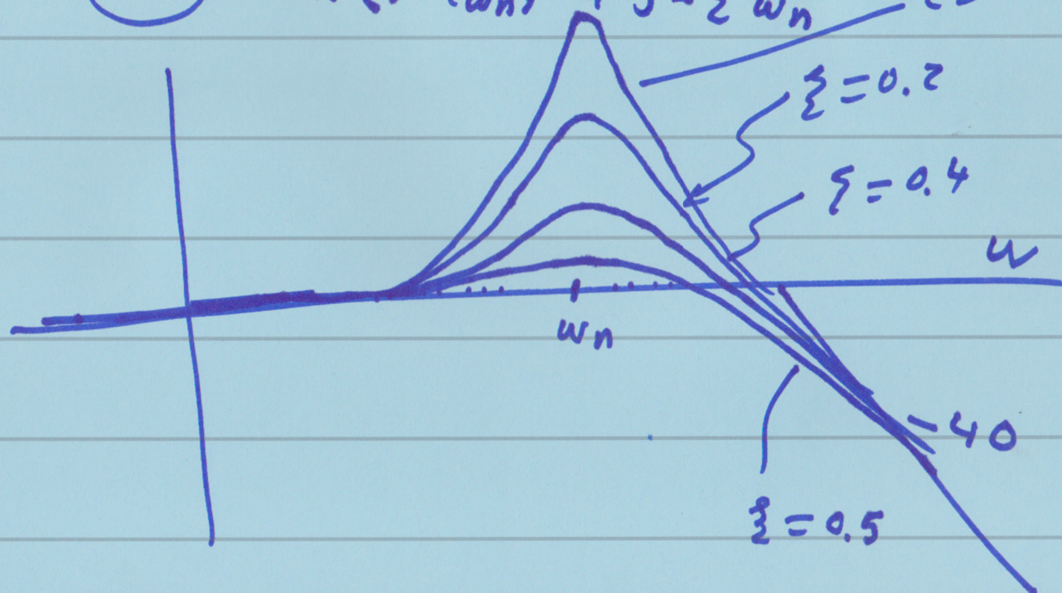


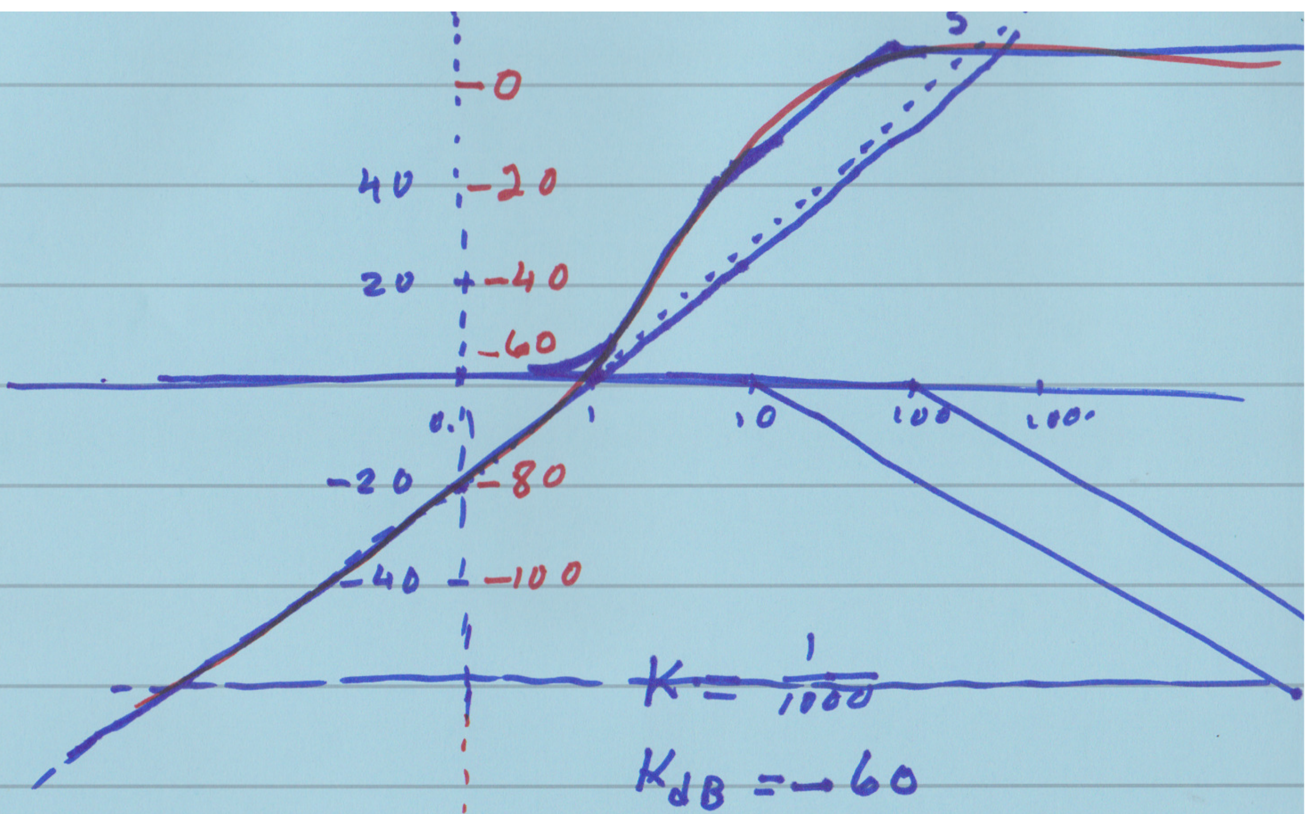
$2\zeta \cdot 2 = 2$

$\zeta = 0.5$

$$H(s) = \frac{1}{s^2 + 2\underbrace{\xi}_{\text{damping ratio}} \omega_n s + \omega_n^2}$$

$$= \frac{1}{\omega_n^2} \cdot \frac{1}{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + j 2 \xi \frac{\omega}{\omega_n}}$$





$K = \frac{1}{1000}$

$K_{dB} = -60$

Example

$z_1 = 0 \quad p_1 = 10$

$z_2 = 1 \quad p_2 = 100$

$$H(s) = \frac{s(s+1)}{(s+10)(s+100)}$$

$$H(s) = \frac{1}{1000} \cdot \underbrace{\frac{s\left(1+\frac{s}{1}\right)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)}}_{\hat{H}}$$

$\hat{H}(s=0.1)$

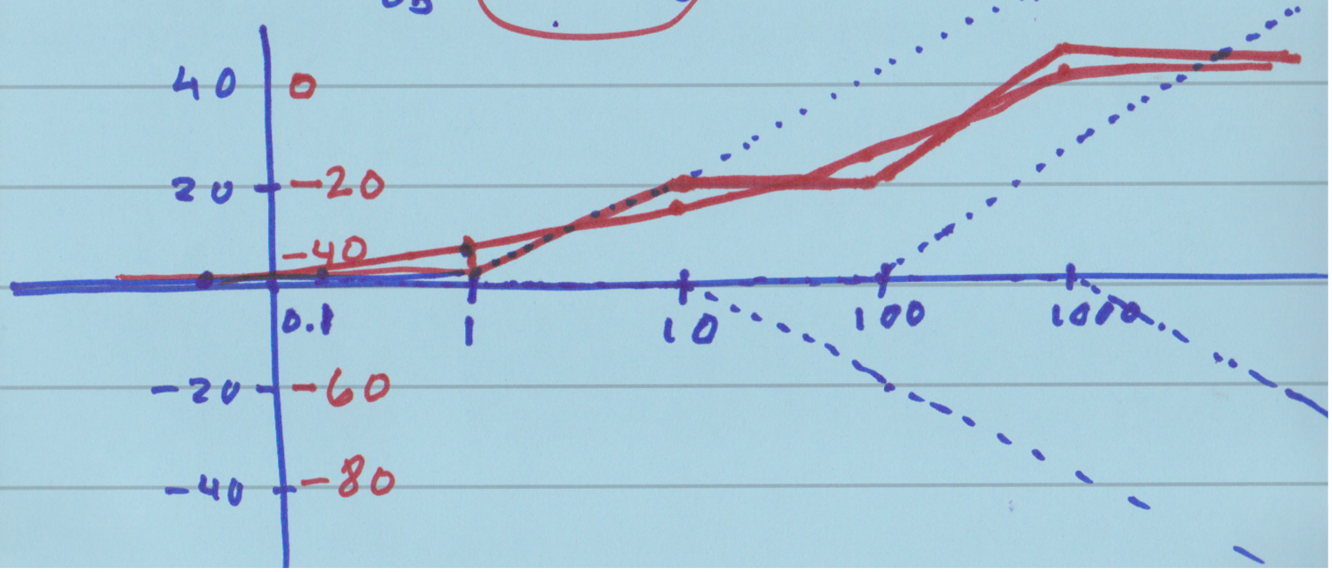
$$= \frac{0.1\left(1+\frac{0.1}{1}\right)}{\left(1+\frac{0.1}{10}\right)\left(1+\frac{0.1}{100}\right)} \cong 0.1 \Rightarrow -20\,dB$$

Example

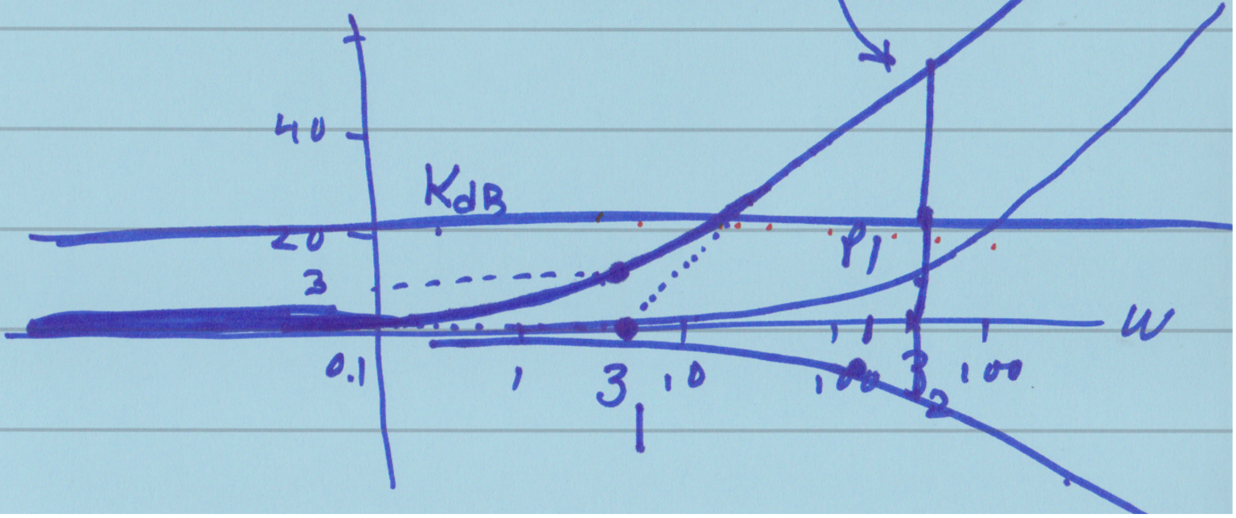
$$H(s) = \frac{(s+1)(s+100)}{(s+10)(s+1000)}$$

$$H(s) = \frac{100}{10^4} \cdot \frac{\left(1+\frac{s}{1}\right)\left(1+\frac{s}{100}\right)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{1000}\right)}$$

$K = 10^{-2} \Rightarrow K_{dB} = -40\,dB$



$$10\log\left(1+\frac{w^2}{z_1^2}\right) = \begin{cases} w = z_1 \Rightarrow 3\,dB \\ w \gg z_1 \quad = 20\log\frac{w}{z_1} \end{cases}$$



$$|H(jw)| = |K| \cdot \frac{\sqrt{1+\frac{w^2}{z_1^2}} \cdot \sqrt{1+\frac{w^2}{z_2^2}} \cdots\cdots}{\sqrt{1+\frac{w^2}{p_1^2}} \cdot \sqrt{1+\frac{w^2}{p_2^2}} \cdots\cdots}$$

<u>**Step 3**</u>

$$|H(jw)|_{dB} = K_{dB} + 10\log\left(1+\frac{w^2}{z_1^2}\right) + 10\log\left(1+\frac{w^2}{z_2^2}\right)$$
$$- 10\log\left(1+\frac{w^2}{p_1^2}\right) - 10\log\left(1+\frac{w^2}{p_2^2}\right) \cdots$$

Now Consider $\quad 10\log\left(1+\frac{w^2}{z_1^2}\right)$

$p_k, z_k \triangleq$ Corner frequencies

Consider

$$H(s) = \frac{(s+z_1)(s+z_2)(s+z_3)\cdots}{(s+p_1)(s+p_2)(s+p_4)\cdots\cdots}$$

<u>step 1</u> Factor all z_k and p_k out

$$H(s) = \underbrace{\frac{z_1 \cdot z_2 \cdot z_3 \cdots}{p_1 \cdot p_2 \cdot p_3 \cdots}}_{K} \cdot \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\left(1+\frac{s}{z_3}\right)\cdots}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\left(1+\frac{s}{p_3}\right)\cdots}$$

<u>Step 2</u> Replace s by jw and take mag.

$$|H(jw)| = |K| \cdot \frac{\left|1+j\frac{w}{z_1}\right| \cdot \left|1+j\frac{w}{z_2}\right| \cdots\cdots}{\left|1+j\frac{w}{p_1}\right| \cdot \left|1+j\frac{w}{p_2}\right| \cdots\cdots}$$