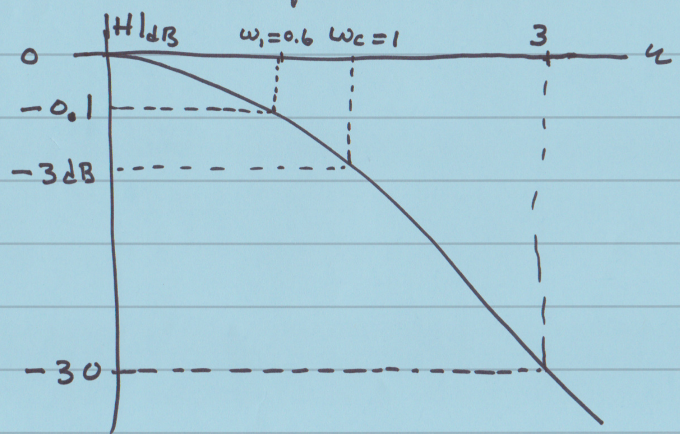
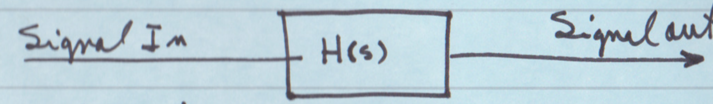
Example: Obtain the order and realize the low pass filter whose specs are given as:



Chapter 10

Analog RLC Filters.



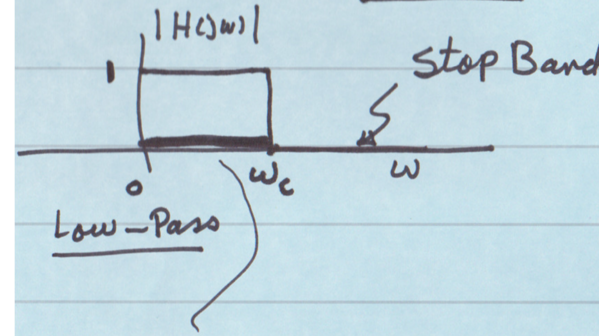


TABLE A8-1 Normalized Low-Pass Butterworth Filters

(a) Pole Locations

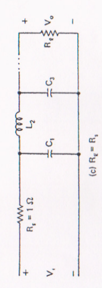
Order n	$p_{1,n}$	$p_{2,n-1}$	$p_{3,n-2}$	$p_{4,n-3}$	p_5
1	-1.00000000				
2	$-0.70710678 \pm j0.70710678$				
3	$-0.50000000 \pm j0.86602540$	-1.00000000			
4	$-0.38268343 \pm j0.92387953$	$-0.92387953 \pm j0.38268343$			
5	$-0.30901699 \pm j0.95105652$	$-0.80901699 \pm j0.58778525$	-1.00000000		
6	$-0.25881905 \pm j0.96592583$	$-0.70710678 \pm j0.70710678$	$-0.96592583 \pm j0.25881905$		
7	$-0.22252093 \pm j0.97492791$	$-0.62348980 \pm j0.78183148$	$-0.90096887 \pm j0.43388374$	-1.00000000	
8	$-0.19509032 \pm j0.98078528$	$-0.55557023 \pm j0.83146961$	$-0.83146961 \pm j0.55557023$	$-0.98078528 \pm j0.19509032$	
9	$-0.17364818 \pm j0.98480775$	$-0.50000000 \pm j0.86602540$	$-0.76604444 \pm j0.64278761$	$-0.93969262 \pm j0.34202014$	-1.00000000

(b) Denominator Polynomials $B(s) = s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \ldots + b_0$

Order n	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
1	1.00000000								
2	1.00000000	1.41421356							
3	1.00000000	2.00000000	2.00000000						
4	1.00000000	2.61312593	3.41421356	2.61312593					
5	1.00000000	3.23606798	5.23606798	5.23606798	3.23606798				
6	1.00000000	3.86370331	7.46410162	9.14162017	7.46410162	3.86370331			
7	1.00000000	4.49395921	10.09783468	14.59179389	14.59179389	10.09783468	4.49395921		
8	1.00000000	5.12583090	13.13707118	21.84615097	25.68835593	21.84615097	13.13707118	5.12583090	
9	1.00000000	5.75877048	16.58171874	31.16343748	41.98638573	41.98638573	31.16343748	16.58171874	5.75877048

(c) Denominator Polynomial Factors $B(s) = B_1(s)\,B_2(s)\,B_3(s)\,B_4(s)\,B_5(s)$

Order n	$B(s)$
1	$(s+1)$
2	$(s^2 + 1.41421356s + 1)$
3	$(s^2 + s + 1)\ (s+1)$
4	$(s^2 + 0.76536686s + 1)\ (s^2 + 1.84775907s + 1)$
5	$(s^2 + 0.61803399s + 1)\ (s^2 + 1.61803399s + 1)\ (s+1)$
6	$(s^2 + 0.51763809s + 1)\ (s^2 + 1.41421356s + 1)\ (s^2 + 1.93185165s + 1)$
7	$(s^2 + 0.44504187s + 1)\ (s^2 + 1.24697960s + 1)\ (s^2 + 1.80193774s + 1)\ (s+1)$
8	$(s^2 + 0.39018064s + 1)\ (s^2 + 1.11114047s + 1)\ (s^2 + 1.66293922s + 1)\ (s^2 + 1.96157056s + 1)$
9	$(s^2 + 0.34729636s + 1)\ (s^2 + s + 1)\ (s^2 + 1.53208889s + 1)\ (s^2 + 1.87938524s + 1)\ (s+1)$



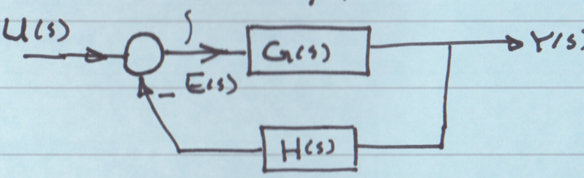
(c) $R_\ell = R_s$

Fig. 8-9 Circuit structures of low-pass Butterworth filters.

TABLE 8-1 Element Values for the Circuit in Fig. 8-9(c)

n	C_1	L_2	C_3	L_4	C_5	L_6	C_7	L_8	C_9
1	2.0000								
2	1.4142	1.4142							
3	1.0000	2.0000	1.0000						
4	0.7654	1.8478	1.8478	0.7654					
5	0.6180	1.6180	2.0000	1.6180	0.6180				
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176			
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450		
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902	
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473

Block-Diagram Manipulation

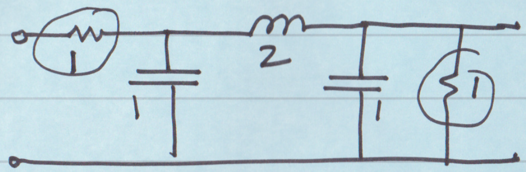


$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{Y(s)}{U(s)}$$

$$Y(s) = G(s)\,.\,E(s)\ , \quad E(s) = U(s) - H(s)\,Y(s)$$

$$Y(s) = G(s)\,[U(s) - H(s)\,Y(s)]$$

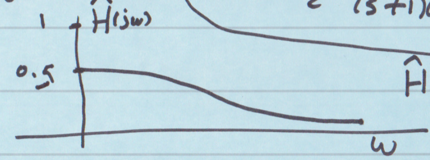
Example Give the realization of a 3^{rd} order Normalized Butterworth Filter



Lossy Filter

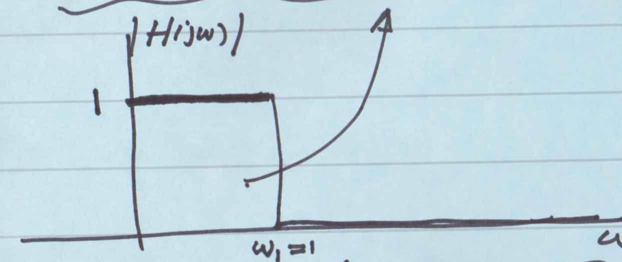
Ideal Transfer Function $H(s) = \frac{1}{(s+1)(s^2+s+1)}$; $H(j0)=1$

Actual Transfer Function $\hat{H}(s) = \frac{1}{2}\cdot\frac{1}{(s+1)(s^2+s+1)}$

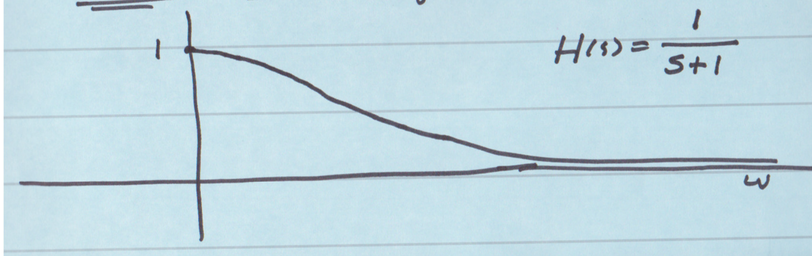


$\hat{H}(j0) = \frac{1}{2}$

Design of RLC Filters



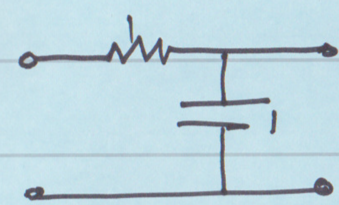
Ideal Normalized Low-Pass



$$|H(j\omega)|^2 = \frac{1}{1+\omega^2},$$

at $\omega = 1$

$$|H(j\omega)|^2 = \frac{1}{2}$$



$$|H(j\omega)| = \frac{\sqrt{2}}{2} = 0.707$$

$$|H(j\omega)|_{dB} = -3\,dB$$

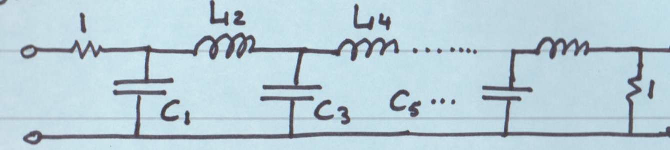
Determining the order of Butterworth Filter

n	
1	$s+1$
2	$s^2+\sqrt{2}\,s+1$
3	$(s+1)(s^2+s+1)$
4	(X
...	

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

RLC Realization of $\frac{1}{D(s)}$

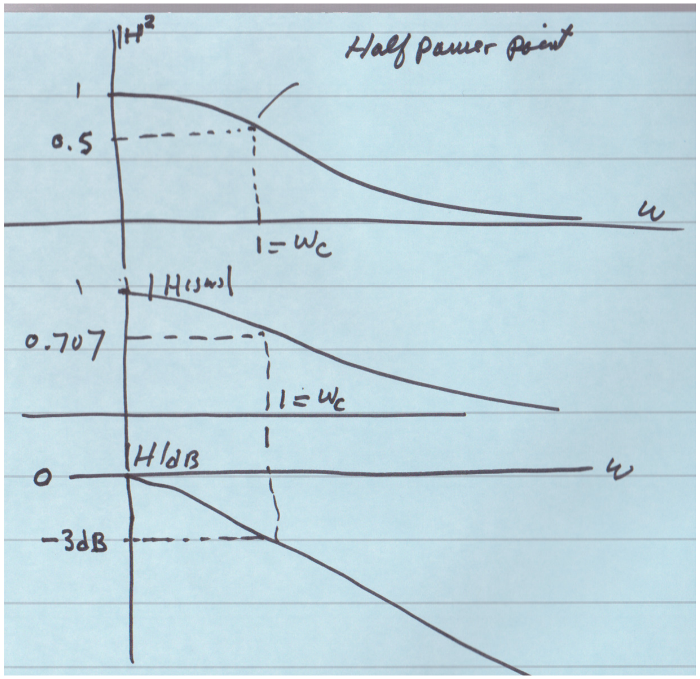
Darlington

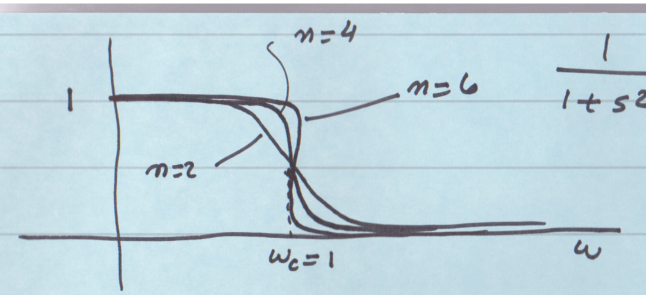


n	C_1	L_2	C_3	L_4	.....
1	2	0 . .	0 . .		
2	$\sqrt{2}$	$\sqrt{2}$			
3	1	2	1		

$$C_m^n = 2\sin\frac{(2m-1)}{2n}\pi$$

$$L_m^n = 2\sin\frac{(2m-1)}{2n}\pi$$





$\frac{1}{1+s^2}$

Butterworth Filter $\quad |H(\omega)|^2 = \frac{1}{1+\omega^{2n}}$

For which: $|H(j0)|^2 = 1 \qquad \omega = \frac{s}{j}$

$|H(j1)|^2 = \frac{1}{2} \Rightarrow \omega_c = 1$ rad/sec.

$$H(s) = \frac{1}{D(s)}$$

For Butterworth

$$|H(j\omega)|^2 = \frac{1}{1+\omega^{2n}}$$

$$|H(j\omega)|_{dB} = 20\log_{10}|H(j\omega)| = 10\log_{10}\frac{1}{1+\omega^{2n}}$$

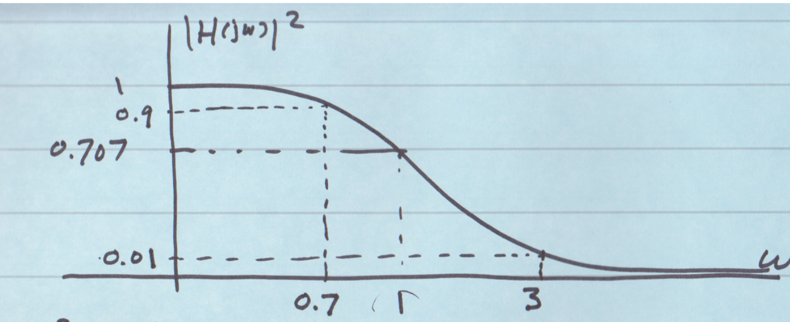
$$|H(j\omega)|_{dB} = -10\log(1+\omega^{2n}) = -\alpha_{dB}$$

$$10\log(1+\omega^{2n}) = \alpha_{dB}$$

$$\therefore \quad 1+\omega^{2n} = 10^{0.1\alpha_{dB}}$$

$$\omega^{2n} = 10^{0.1\alpha_{dB}} - 1$$

$$2n\log\omega_1 = \log\left(10^{0.1\alpha_{dB}} - 1\right)$$



$\alpha^2 = 0.9$

$\beta^2 = 0.01$

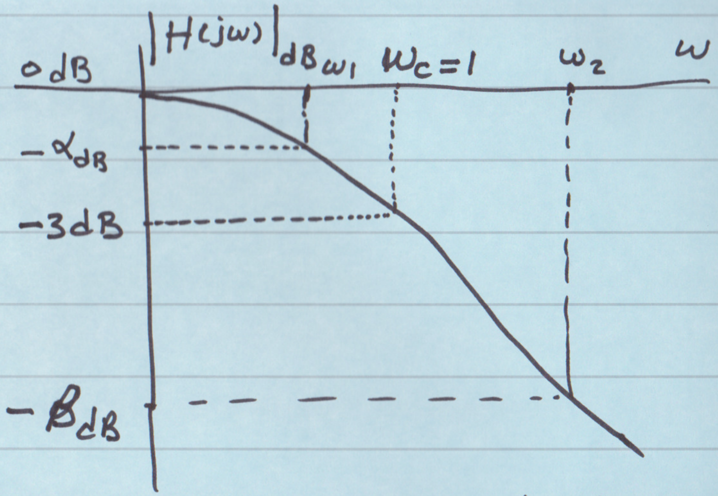
$$n_1 = \frac{\log\left(\frac{1}{0.9}-1\right)}{2\log 0.7} = \frac{-0.954}{-0.309}$$

$\boxed{n_1 = 4}$

$\boxed{n = 4}$

$$n_2 = \frac{\log\left(\frac{1}{0.01}-1\right)}{2\log 3} = \frac{1.99}{0.954}$$

$\boxed{n_2 = 4}$



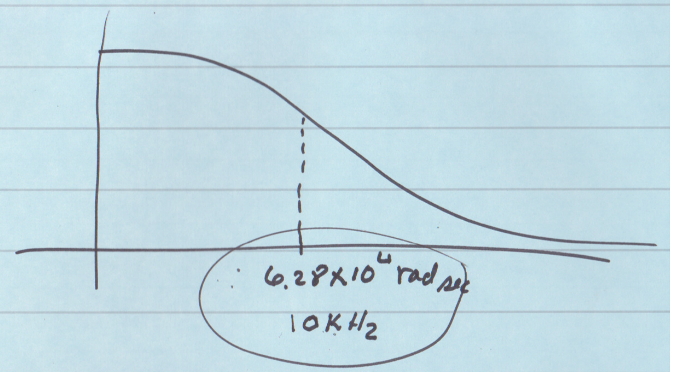
$|H(j\omega_1)|_{dB} \geq -\alpha_{dB}$ $\quad \omega \leq \omega_1$ $\quad$ Low Pass Spec.

$|H(j\omega)|_{dB} \leq -\beta_{dB}$ $\quad \omega \geq \omega_2$ $\quad$ Band Pass Spec.

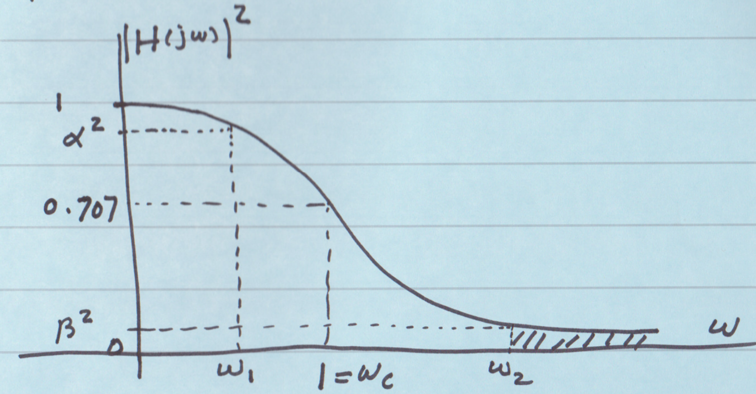
$$\boxed{n_1 = \frac{\log\left(10^{0.1\alpha_{dB}} - 1\right)}{2\log\omega_1}} \quad \text{Passband}$$

$$\boxed{n_2 = \frac{\log\left(10^{0.1\beta} - 1\right)}{2\log\omega_2}} \quad \text{stop-band}$$

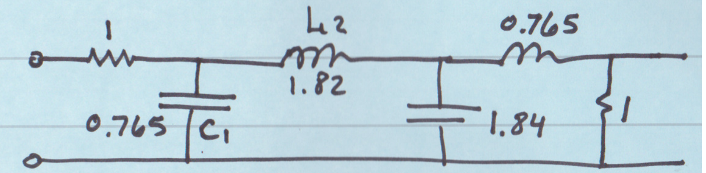
Hence $\quad n = \{\max(n_1, n_2)\}$



If absolute mag. is given



$$H(s) = \frac{1}{(s^2+0.765s+1)(s^2+1.847s+1)}$$



$$C_1 = 2\sin\frac{(2-1)\pi}{8} = 0.765\,F$$

$$L_2 = 2\sin\frac{(4-1)\pi}{8} = 1.82\,H$$

$$\frac{R_2}{R_1}$$

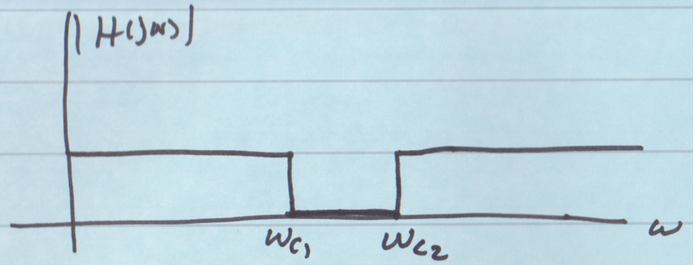
$$|H(jw)|^2 = \frac{1}{1+w^{2n}}$$

For Passband : $\dfrac{1}{1+w_1^{2n}} \geq \alpha^2$

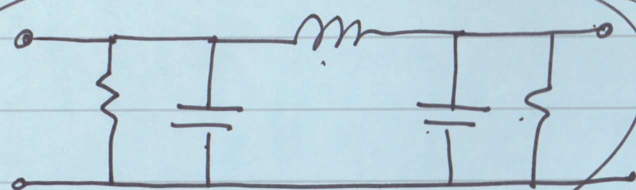
$$\frac{1}{1+w_1^{2n}} = \alpha^2 \Rightarrow 1+w_1^{2n} = \frac{1}{\alpha^2}$$

$$w_1^{2n} = \frac{1}{\alpha^2} - 1$$

$$\boxed{n_1 = \frac{\log(\frac{1}{\alpha^2}-1)}{2\log w_1}}$$

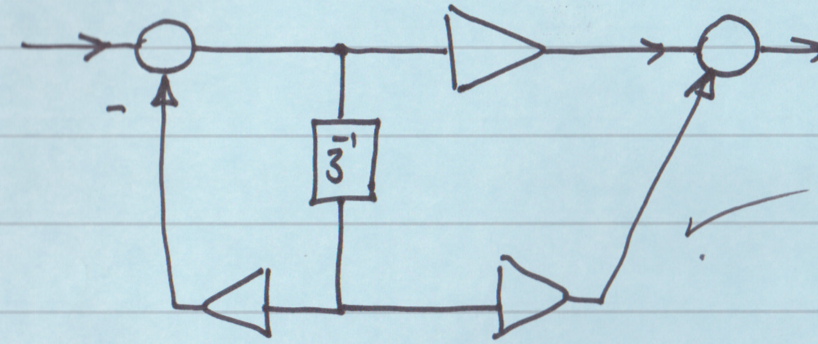


Band stop - Notch Filter



Passive RLC Filter

Digital Filters



To satisfy passband requirement.

$$n_1 = \frac{\log(10^{0.01}-1)}{2\log 0.6} = \frac{-1.632}{-0.443} = 3.678$$

Hence $\quad n_1 = 4$

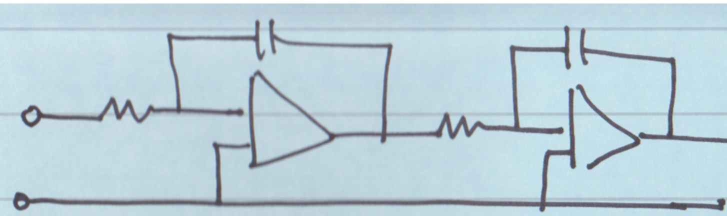
To satisfy stop band :

$$n_2 = \frac{\log(10^{3}-1)}{2\log 3} = \frac{2.99}{0.954} = 3.143$$

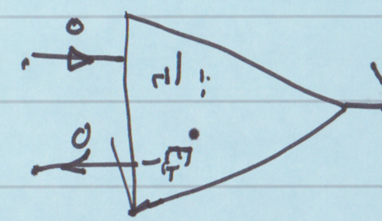
$2.75 \Rightarrow 3$

$n_2 = 4$

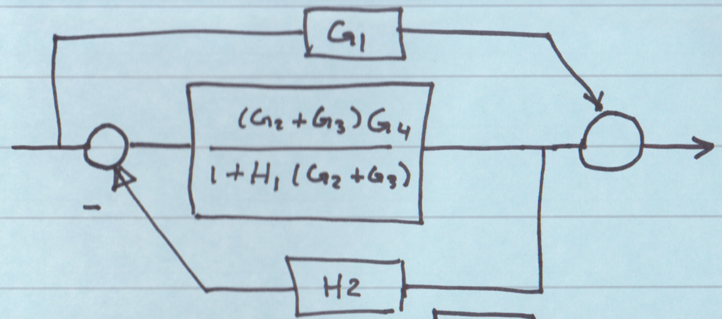
Hence $\boxed{n = 4}$

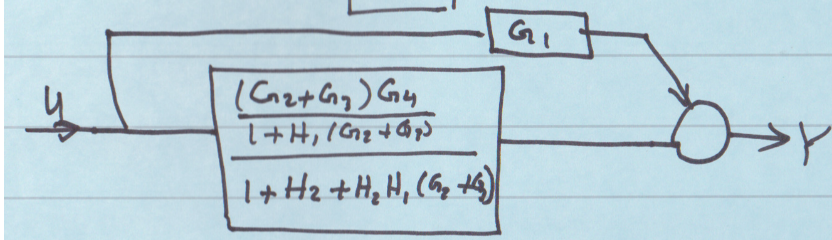


Active RC Filter

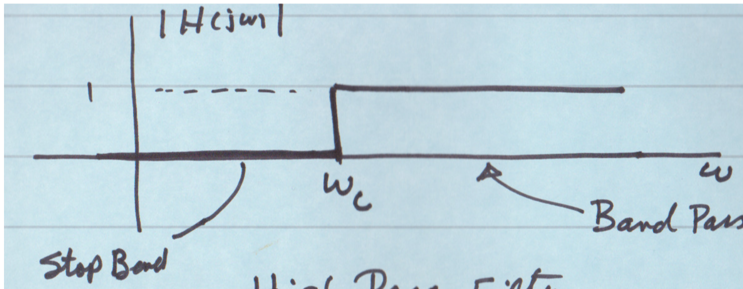


Mid 1950

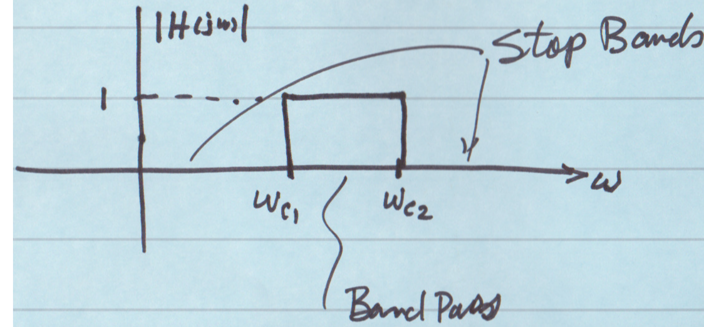




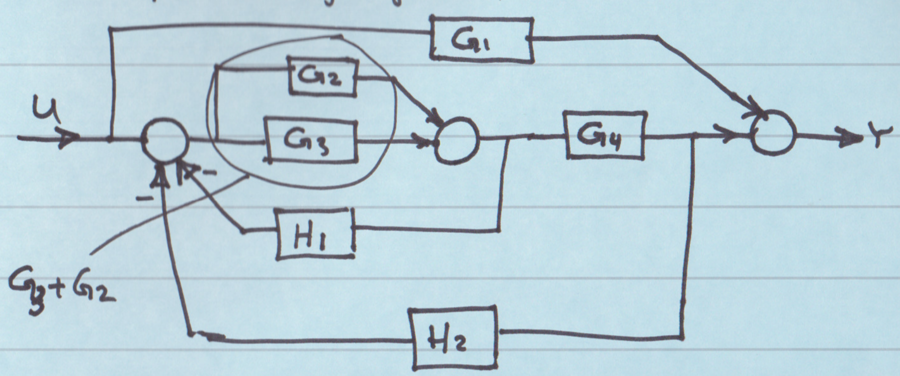
$$Y = \Big[\qquad\Big]U + G_1 U$$



High Pass Filter



Example Find The Transfer Function of the Following systems:



Find $\dfrac{Y}{U}$

$$\frac{Y}{U} = G_1 + \frac{\dfrac{(G_2+G_3)G_4}{1+H_1(G_2+G_3)}}{1+H_2+H_2H_1(G_2+G_3)}$$

Stable iff all Poles of H(s) are located in the L.H. of the s-plane

Example $\quad H(s) = \dfrac{s^4+6s^2+3}{(s+6)(s+2)(s-1)(s+18)}$