$$\frac{3+5s}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$= \frac{(A+B)s + 2A}{s(s+2)} \equiv \frac{5s+3}{}$$

$$2A = 3 \quad , \quad A+B = 5$$

$$A = 1.5 \quad B = 3.5$$

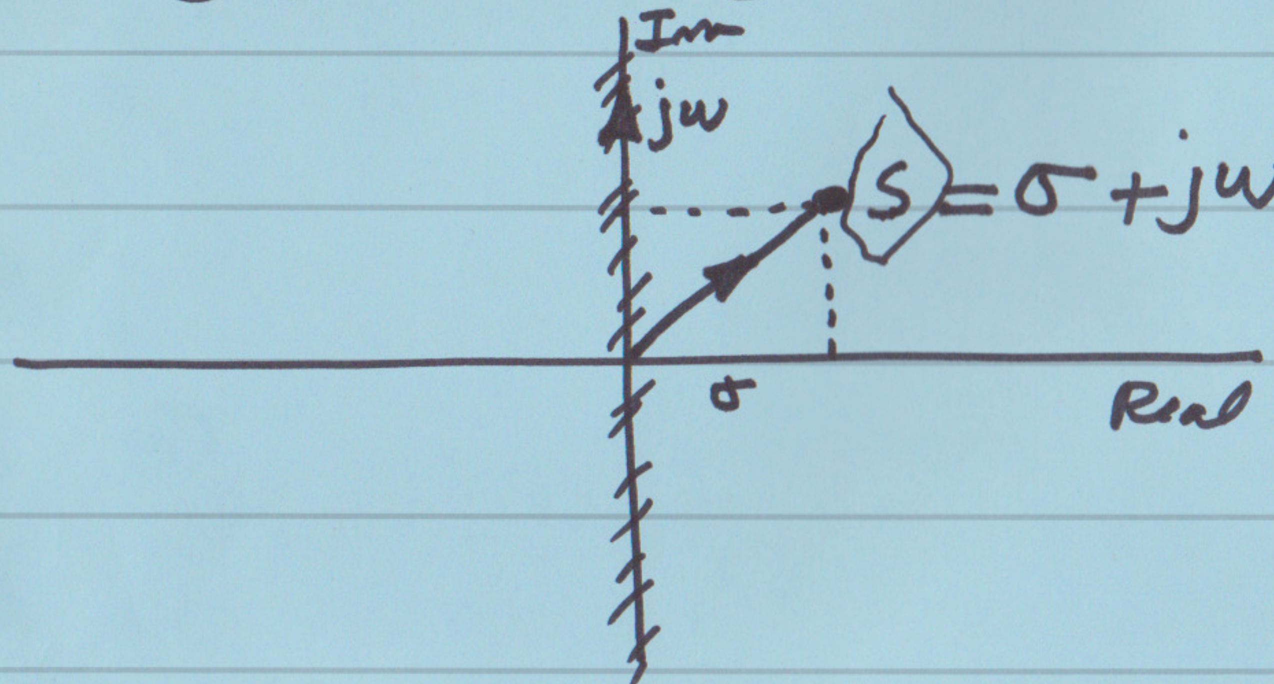
$$X(s) = \frac{1.5}{s} + \frac{3.5}{s+2}$$

$$x(t) = 1.5u_0(t) + 3.5e^{-2t}u_0(t)$$

$$\boxed{x(t) = (1.5 + 3.5e^{-2t})u_0(t)}$$

Chapter 5

Laplace Transform and its application in System Analysis



Application of $\mathcal{L}$-Transform

Soln of Diff. Eqns.

e.g.

$$\begin{cases} \dfrac{dx}{dt} + 2x = 3u_0(t) \\ x(0^-) = 5 \end{cases}$$

$\mathcal{L}$

$$\left(s\,X(s) - 5\right) + 2\,X(s) = 3 \times \frac{1}{s}$$

$$(s+2)\,X(s) = \frac{3}{s} + 5$$

$$X(s) = \frac{\frac{3}{s} + 5}{s+2} = \frac{3+5s}{s(s+2)}$$

$$x(t) \xleftarrow{\mathcal{L}^{-1}} \frac{3+5s}{s(s+2)}$$

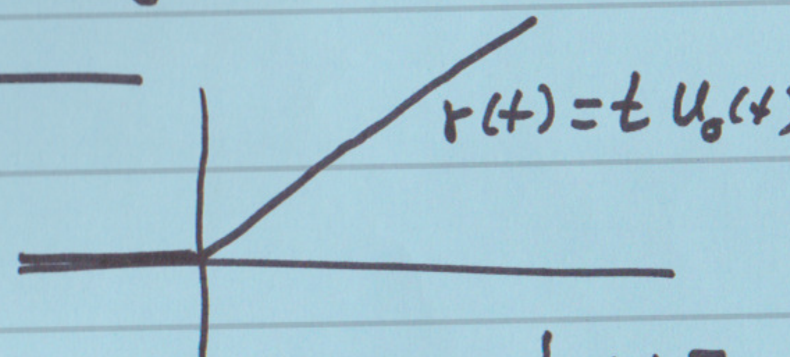
Given $f(t)\ ;\ 0 \longrightarrow \infty$

Then

$$\boxed{F(s) = \int_{0^-}^{\infty} f(t)\cdot e^{-st}\,dt}$$

e.g. $f(t) = \delta(t)$ $\qquad F(s) = \int_{0^-}^{0^+} \delta(t)\, e^{-st}\, dt = 1$

$f(t)$	$F(s)$
$\delta(t)$	1
$u_0(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$



$$R(s) = -\frac{d}{ds}\left[\frac{1}{s}\right] = \frac{1}{s^2}$$

4.5

$e^{-j\omega t}$	$\frac{1}{s+j\omega}$
$e^{j\omega t}$	$\frac{1}{s-j\omega}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$t\,u_0(t)$	$\frac{1}{s^2}$

$$\sin \omega t = \frac{1}{2j}\left[e^{j\omega t} - e^{-j\omega t}\right]$$

$$\frac{1}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

$$\frac{1}{2j}\left[\frac{\cancel{s}+j\omega - \cancel{s}+j\omega}{s^2+\omega^2}\right]$$

$$\frac{\cancel{2}\cancel{j}\omega}{\cancel{2}\cancel{j}(s^2+\omega^2)}$$

Properties of $\mathcal{L}$- Transform

1) $\mathcal{L}\{k_1 f_1(t) + k_2 f_2(t)\} = k_1 F_1(s) + k_2 F_2(s)$

2) $\mathcal{L}\left\{\frac{d}{dt} f(t)\right\} = sF(s) - f(0^-)$

Proof, integration by parts of def. of $\mathcal{L}$. T.

3) $\mathcal{L}\left\{\frac{d^2}{dt^2} f(t)\right\} = s^2 F(s) - sf(0^-) - f'(0^-)$

$\mathcal{L}\left\{\frac{d^n}{dt^n} f(t)\right\} = s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) \ldots\ldots f^{n-1}(0^-)$

3) $\mathcal{L}\left\{ \int_{0^-}^{t} f(\sigma)\,d\sigma \right\} = \frac{1}{s}F(s) + \frac{1}{s}\int_{0}^{0^-} f(t)\,dt$

4) $\mathcal{L}\left\{ e^{-at} f(t) \right\} = F(s+a)$

5) $\mathcal{L}\left\{ f(t-T)\,U_0(t-T) \right\} = e^{-sT} F(s)$

6) $\mathcal{L}\left\{ t \cdot f(t) \right\} = -\frac{d}{ds} F(s)$

$\mathcal{L}\left\{ t^n \cdot f(t) \right\} = (-1)^n \frac{d^n}{ds^n} F(s)$

$\mathcal{L}\left\{ t^n \cdot \right\} = (-1)^n \frac{d^n}{ds^n} \left[\frac{1}{s} \right] = \frac{n!}{s^{n+1}}$

1) $\mathcal{L}\left\{ f(at) \right\} = \frac{1}{a} F\left(\frac{s}{a}\right)$

Example Find $\mathcal{L}\left\{ t \cos \omega t \right\} = -\frac{d}{ds}\left[\frac{s}{s^2+\omega^2}\right]$

$f(t) = \cos \omega t$ Rule #6 $\Rightarrow$

$F(s) = \frac{s}{s^2+\omega^2}$,

$$-\frac{(s^2+\omega^2) - 2s^2}{(s^2+\omega^2)^2} = \frac{s^2-\omega^2}{(s^2+\omega^2)^2} \checkmark$$

$$\frac{\cancel{-s^2} - w^2 \cancel{+2s^2}}{(\quad)^2} = \frac{s^2 - w^2}{(s^2+w^2)^2} \qquad 8$$

$$\{(t-2)\cos(t-2)\,u_0(t-2)\} \quad ; \; T$$

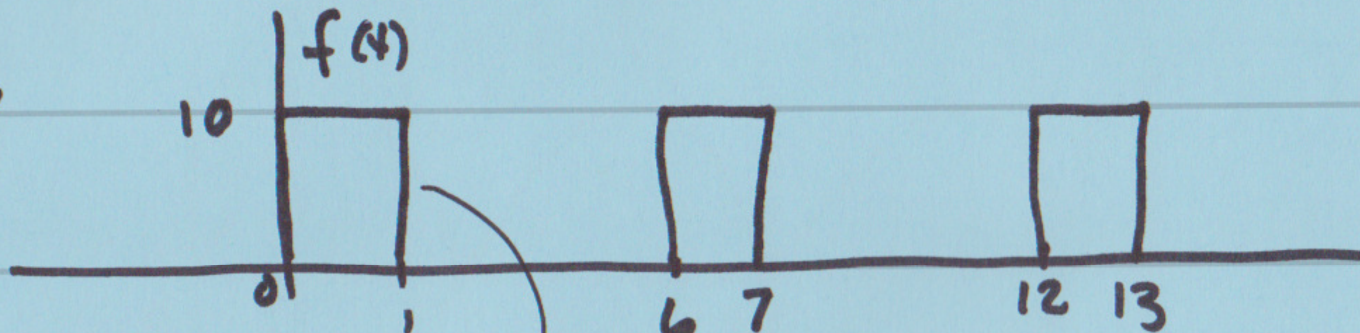
$$\{t \cos wt\, u_0(t)\} = \frac{s^2 - w^2}{(s^2+w^2)^2}$$

$$\mathcal{L}\{f(t-T)\,u_0(t-T)\} = e^{-sT} F(s)$$

$$= e^{-2s} \cdot \frac{s^2 - w^2}{(s^2+w^2)^2}$$

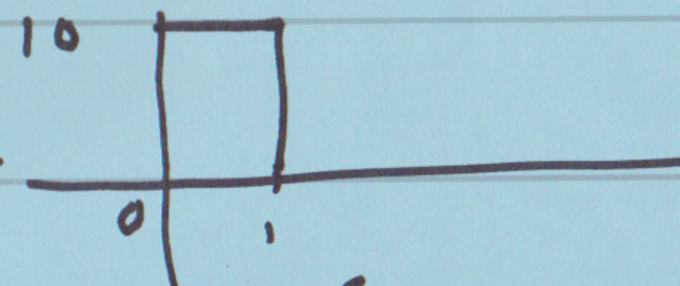
8) $\boxed{F(s) = \dfrac{F_1(s)}{1 - e^{-sT}}}$

Example



Find $F(s)$

$f_1(t) =$

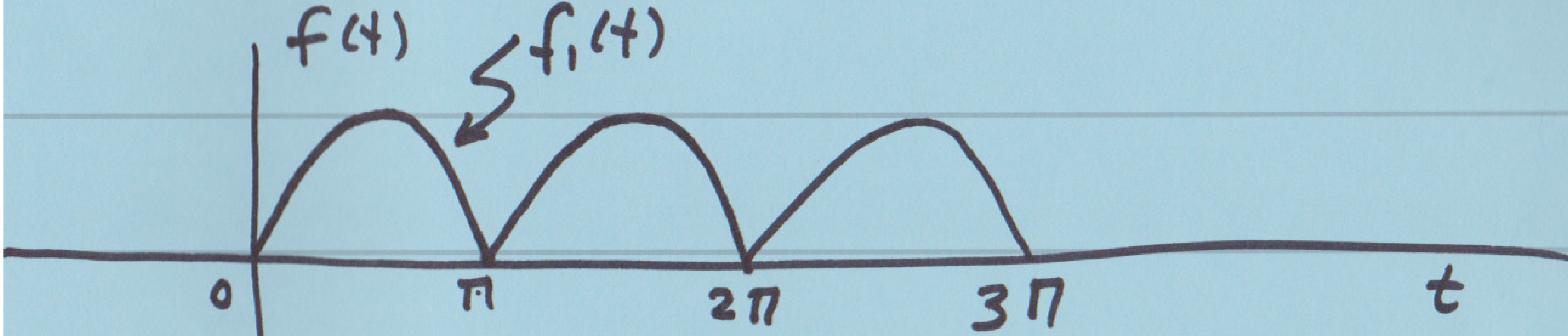


$$F_1(s) = \frac{10}{s} - \frac{10e^{-s}}{s} = 10\,\frac{1-e^{-s}}{s}$$

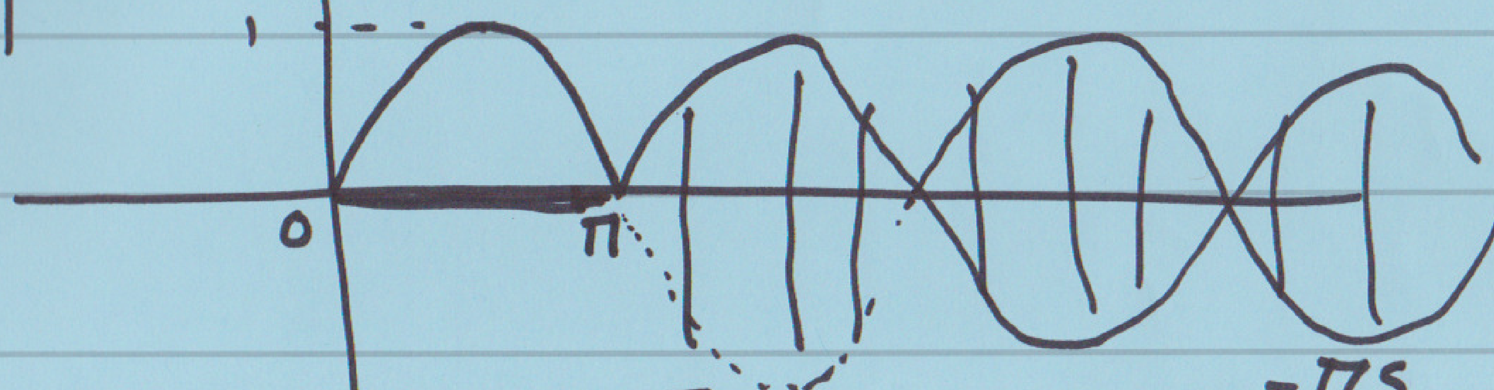
Then $$F(s) = \frac{10\,\dfrac{1-e^{-s}}{s}}{1 - e^{-6s}}$$

$$F(s) = \frac{10(1-e^{-s})}{s(1-e^{-6s})}$$

10



$f_1(t) = \sin t + \sin(t-\pi)$

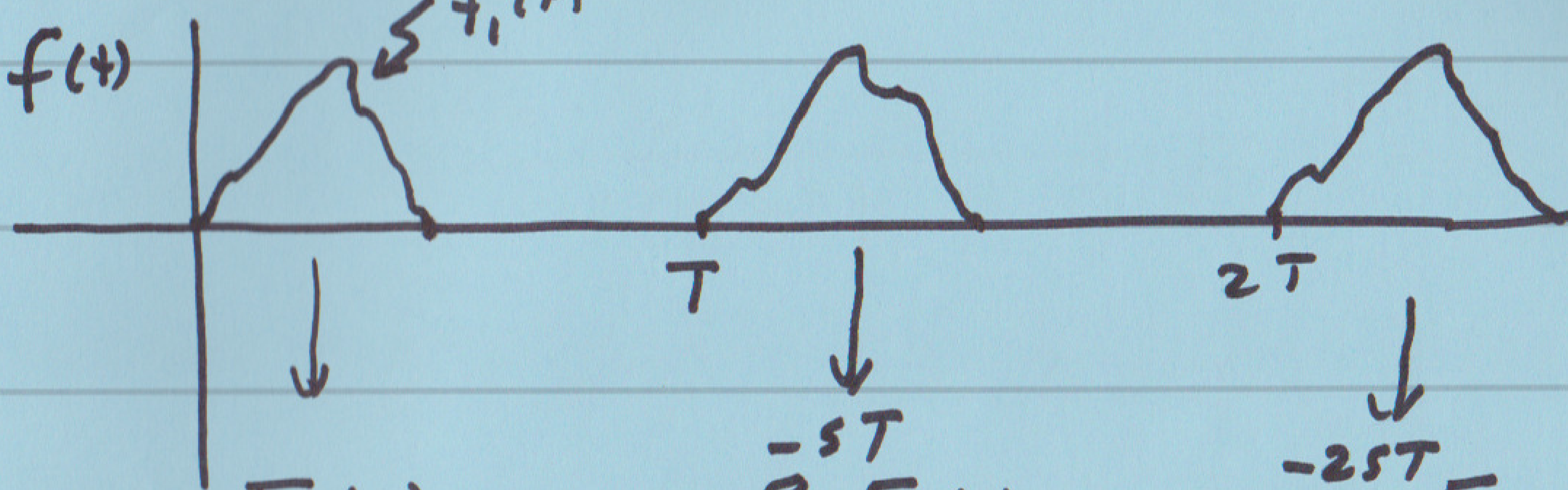


$$F_1(s) = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1} = \frac{1+e^{-\pi s}}{s^2+1}$$

j. $F(s) = \dfrac{F_1(s)}{1-e^{-\pi s}}$

$$\boxed{F(s) = \dfrac{1+e^{-\pi s}}{(s^2+1)(1-e^{-\pi s})}}$$

$\mathcal{L}\{f(t)\}$; $f(t)=f(t+kT)$; $k=0,1,2,\dots$



$F_1(s)$ $\qquad$ $e^{-sT}F_1(s)$ $\qquad$ $e^{-2sT}F(s)$ .....

$$F(s)=F_1(s)\left[1+e^{-sT}+e^{-2sT}+e^{-3sT}+\dots\dots\right]$$

$$\left[\underbrace{1+x+x^2+x^3+x^4\cdots}_{\Downarrow \atop \frac{1}{1-x}}\right.$$