CHAPTER 9

Rotation

1* · Two points are on a disk turning at constant angular velocity, one point on the rim and the other halfway between the rim and the axis. Which point moves the greater distance in a given time? Which turns through the greater angle? Which has the greater speed? The greater angular velocity? The greater tangential acceleration? The greater angular acceleration? The greater centripetal acceleration?

1. The point on the rim moves the greater distance. 2. Both turn through the same angle. 3. The point on the rim has the greater speed 4. Both have the same angular velocity. 5. Both have zero tangential acceleration. 6. Both have zero angular acceleration. 7. The point on the rim has the greater centripetal acceleration.

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2 · True or false: (*a*) Angular velocity and linear velocity have the same dimensions. (*b*) All parts of a rotating wheel must have the same angular velocity. (*c*) All parts of a rotating wheel must have the same angular acceleration.

(*a*) False (*b*) True (*c*) True

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3 ·· Starting from rest, a disk takes 10 revolutions to reach an angular velocity w. At constant angular acceleration, how many additional revolutions are required to reach an angular velocity of $2w$? (*a*) 10 rev (*b*) 20 rev (*c*) 30 rev (*d*) 40 rev (*e*) 50 rev.

From Equ. 9-9; $w^2 \propto q$ $\quad q_2 = 4q_1$; $\Delta q = 3q_1 = 30$ rev; (*c*)

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4 · A particle moves in a circle of radius 90 m with a constant speed of 25 m/s. (*a*) What is its angular velocity in radians per second about the center of the circle? (*b*) How many revolutions does it make in 30 s?

(*a*) $w = v/r$ $\quad w = (25/90)$ rad/s $= 0.278$ rad/s

(*b*) $q = wt$ $\quad q = 8.33$ rad $= 1.33$ rev.

5* · A wheel starts from rest with constant angular acceleration of 2.6 rad/s^2. After 6 s, (*a*) What is its angular velocity? (*b*) Through what angle has the wheel turned? (*c*) How many revolutions has it made? (*d*) What is the speed and acceleration of a point 0.3 m from the axis of rotation?

(*a*) $w = at$ $\quad w = (2.6 \times 6)$ rad/s $= 15.6$ rad/s

(*b*), (*c*) $q = 1/2\,at^2$ $\quad q = 46.8$ rad $= 7.45$ rev

(*d*) $v = wr$, $a_c = rw^2$, $a_t = ra$; $a = (a_t^2 + a_c^2)^{1/2}$ $\quad v = (15.6 \times 0.3)$ m/s $= 4.68$ m/s;

$a = [(0.3 \times 15.6^2)^2 + (0.3 \times 2.6)^2]^{1/2}$ m/s$^2 = 73$ m/s^2

6 · When a turntable rotating at 33 1/3 rev/min is shut off, it comes to rest in 26 s. Assuming constant angular acceleration, find (*a*) the angular acceleration, (*b*) the average angular velocity of the turntable, and (*c*) the number of revolutions it makes before stopping.

(*a*) $\alpha = \omega/t$ $\qquad$ $\alpha = (33.3 \times 2\pi/60 \times 26)\ \text{rad/s}^2 = 0.134\ \text{rad/s}^2$

(*b*) $\omega_{av} = 1/2\omega_0$ $\qquad$ $\omega_{av} = 1/2(33.3 \times 2\pi/60)\ \text{rad/s} = 1.75\ \text{rad/s}$

(*c*) $\theta = \omega_{av}t$ $\qquad$ $\theta = (1.75 \times 26)\ \text{rad} = 45.4\ \text{rad} = 7.22\ \text{rev}$

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7 · A disk of radius 12 cm, initially at rest, begins rotating about its axis with a constant angular acceleration of 8 rad/s^2. At t = 5 s, what are (*a*) the angular velocity of the disk, and (*b*) the tangential acceleration a_t and the centripetal acceleration a_c of a point on the edge of the disk?

(*a*) $\omega = \alpha t$ $\qquad$ $\omega = (8 \times 5)\ \text{rad/s} = 40\ \text{rad/s}$

(*b*) $a_t = r\alpha$; $a_c = r\omega^2$ $\qquad$ $a_t = (0.12 \times 8)\ \text{m/s}^2 = 0.96\ \text{m/s}^2$;

$a_c = (0.12 \times 40^2)\ \text{m/s}^2 = 192\ \text{m/s}^2$

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8 · Radio announcers who still play vinyl records have to be careful when cuing up live recordings. While studio albums have blank spaces between the songs, live albums have audiences cheering. If the volume levels are left up when the turntable is turned on, it sounds as though the audience has suddenly burst through the wall. If a turntable begins at rest and rotates through 10^o in 0.5 s, how long must the announcer wait before the record reaches the required angular speed of 33.3 rev/min? Assume constant angular acceleration.

1. Determine α; $\theta = 1/2\alpha t^2$ $\qquad$ $(10 \times 360/2\pi) = 1/2\alpha \times 0.5^2$ rad; $\alpha = 1.4\ \text{rad/s}^2$
2. Find $T = \omega/\alpha$ $\qquad$ $T = (33.3 \times 2\pi/60 \times 1.4)\ \text{s} = 2.5\ \text{s}$

9* · A Ferris wheel of radius 12 m rotates once in 27 s. (*a*) What is its angular velocity in radians per second? (*b*) What is the linear speed of a passenger? What is the centripetal acceleration of a passenger?

(*a*) $\omega = 2\pi/27$ rad/s = 0.233 rad/s.

(*b*) $v = r\omega = 12 \times 0.233$ /s = 2.8 m/s. $\quad a_c = r\omega^2 = 12 \times 0.233^2\ \text{m/s}^2 = 0.65\ \text{m/s}^2$.

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10 · A cyclist accelerates from rest. After 8 s, the wheels have made 3 rev. (*a*) What is the angular acceleration of the wheels? (*b*) What is the angular velocity of the wheels after 8 s?

(*a*) $\theta = 1/2\alpha t^2$; $\alpha = 2\theta/t^2$ $\qquad$ $\alpha = (2 \times 3 \times 2\pi/8^2)\ \text{rad/s}^2 = 0.59\ \text{rad/s}^2$

(*b*) $\omega = \alpha t$ $\qquad$ $\omega = (0.59 \times 8)\ \text{rad/s} = 4.72\ \text{rad/s}$

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11 · What is the angular velocity of the earth in rad/s as it rotates about its axis?

$\omega = 2\pi$ rad/day = $(2\pi/24 \times 60 \times 60)$ rad/s = 7.27×10^{-5} rad/s.

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12 · A wheel rotates through 5.0 radians in 2.8 seconds as it is brought to rest with constant angular acceleration. The initial angular velocity of the wheel before braking began was (*a*) 0.6 rad/s. (*b*) 0.9 rad/s. (*c*) 1.8 rad/s. (*d*) 3.6 rad/s. (*e*) 7.2 rad/s.

$\omega_{av} = 1/2\omega_0 = \theta/t$; $\omega_0 = 2\theta/t = 3.57$ rad/s; (*d*)

13* · A circular space station of radius 5.10 km is a long way from any star. Its rotational speed is controllable to some degree, and so the apparent gravity changes according to the tastes of those who make the decisions. Dave the Earthling puts in a request for artificial gravity of 9.8 m/s^2 at the circumference. His secret agenda is to give the Earthlings a home-gravity advantage in the upcoming interstellar basketball tournament. Dave's request would require an angular speed of (*a*) 4.4×10^{-2} rad/s. (*b*) 7.0×10^{-3} rad/s. (*c*) 0.28 rad/s. (*d*) -0.22 rad/s. (*e*) 1300 rad/s.

(*a*) Use $a_c = r\omega^2$ and solve for ω.

14 · A bicycle has wheels of 1.2 m diameter. The bicyclist accelerates from rest with constant acceleration to 24 km/h in 14.0 s. What is the angular acceleration of the wheels?

$a = a_t = r\alpha$; $\alpha = a/r$ $\alpha = (24/3.6 \times 14)/0.6$ rad/s^2 = 0.794 rad/s^2

15 ·· The tape in a standard VHS videotape cassette has a length L = 246 m; the tape plays for 2.0 h (Figure 9-36). As the tape starts, the full reel has an outer radius of about R = 45 mm, and an inner radius of about r = 12 mm. At some point during the play, both reels have the same angular speed. Calculate this angular speed in rad/s and rev/min.

1. At the instant both reels have the same area, $2(R_f^2 - r^2) = R^2 - r^2$
   Solve for R_f R_f = 32.9 mm = 3.29 cm
1. Determine the linear speed v v = 246/2 m/h = 123 m/h = 3.42 cm/s
2. Find $\omega = v/r$ ω = 3.42/3.29 rad/s = 1.04 rad/s = 9.93 rev/min

16 · The dimension of torque is the same as that of (*a*) impulse. (*b*) energy. (*c*) momentum. (*d*) none of the above.

(*b*)

17* · The moment of inertia of an object of mass M (*a*) is an intrinsic property of the object. (*b*) depends on the choice of axis of rotation. (*c*) Is proportional to M regardless of the choice of axis. (*d*) both (*b*) and (*c*) are correct.

(*d*)

18 · Can an object continue to rotate in the absence of torque?

Yes

19 · Does an applied net torque always increase the angular speed of an object?

No; it may cause a rotating object to come to rest.

20 · True or false: (*a*) If the angular velocity of an object is zero at some instant, the net torque on the object must be zero at that instant. (*b*) The moment of inertia of an object depends on the location of the axis of rotation. (*c*) The moment of inertia of an object depends on the angular velocity of the object.

(*a*) False (*b*) True (*c*) False

21* · A disk is free to rotate about an axis. A force applied a distance d from the axis causes an angular acceleration α. What angular acceleration is produced if the same force is applied a distance 2d from the axis? (*a*) α (*b*) 2α (*c*) $\alpha/2$ (*d*) 4α (*e*) $\alpha/4$

(*b*) $\alpha \propto \tau = Fl$.

22 · A disk-shaped grindstone of mass 1.7 kg and radius 8 cm is spinning at 730 rev/min. After the power is shut off, a woman continues to sharpen her ax by holding it against the grindstone for 9 s until the grindstone stops rotating. (*a*) What is the angular acceleration of the grindstone? (*b*) What is the torque exerted by the ax on the grindstone? (Assume constant angular acceleration and a lack of other frictional torques.)

(*a*) $\alpha = \omega/t$ $\alpha = (730 \times 2\pi/60 \times 9)$ rad/s^2 = 8.49 rad/s^2

(*b*) $\tau = I\alpha$; $I = \frac{1}{2}MR^2$ $\tau = \frac{1}{2}(1.7 \times 0.08^2) \times 8.49$ N·m = 0.046 N·m

23 · A 2.5-kg cylinder of radius 11 cm is initially at rest. A rope of negligible mass is wrapped around it and pulled with a force of 17 N. Find (*a*) the torque exerted by the rope, (*b*) the angular acceleration of the cylinder, and (*c*) the angular velocity of the cylinder at $t = 5$ s.

(*a*) $t = Fl$ $t = 17 \times 0.11$ N·m = 1.87 N·m

(*b*) $a = t/I$; $I = {}_{1/2}MR^2$ $a = 1.87/(1/2 \times 2.5 \times 0.11^2)$ rad/s^2 = 124 rad/s^2

(*c*) $w = at$ $w = 124 \times 5$ rad/s = 620 rad/s

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24 ·· A wheel mounted on an axis that is not frictionless is initially at rest. A constant external torque of 50 N·m is applied to the wheel for 20 s, giving the wheel an angular velocity of 600 rev/min. The external torque is then removed, and the wheel comes to rest 120 s later. Find (*a*) the moment of inertia of the wheel, and (*b*) the frictional torque, which is assumed to be constant.

(*a*) $a = w/t = t/I$; $I = tt/w$ $I = (50 \times 20)/(600 \times 2p/60)$ kg·m^2 = 15.9 kg·m^2

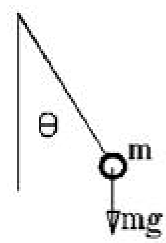
(*b*) $t_{fr} = t/6$ t_{fr} = 2.65 N·m

25* ·· A pendulum consisting of a string of length L attached to a bob of mass m swings in a vertical plane. When the string is at an angle q to the vertical, (*a*) what is the tangential component of acceleration of the bob? (*b*) What is the torque exerted about the pivot point? (*c*) Show that $t = Ia$ with $a_t = La$ gives the same tangential acceleration as found in part (*a*).

(a) The pendulum and the forces acting on it are shown.The tangential force is $mg \sin q$. Therefore, the tangential acceleration is $a_t = g \sin q$.

(b) The tension causes no torque. The torque due to the weight about the pivot is $mgL \sin q$.

(b) Here $I = mL^2$; so $a = mgL \sin q/mL^2 = g \sin q/L$, and $a_t = g \sin q$.



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26 ··· A uniform rod of mass M and length L is pivoted at one end and hangs as in Figure 9-37 so that it is free to rotate without friction about its pivot. It is struck by a horizontal force F_0 for a short time Δt at a distance x below the pivot as shown. (*a*) Show that the speed of the center of mass of the rod just after being struck is given by $v_0 = 3F_0x\Delta t/2ML$. (*b*) Find the force delivered by the pivot, and show that this force is zero if $x = 2L/3$. (*Note:* The point $x = 2L/3$ is called the *center of percussion* of the rod.)

(*a*) The torque due to F_0 is $F_0x = Ia = (ML^2/3)a$; thus, $a = 3F_0x/ML^2$, and $w = a\Delta t = 3F_0x\Delta t/ML^2$. The center of mass is a distance $L/2$ from the pivot, so $v_{cm} = wL/2 = 3F_0x\Delta t/2ML$.

(*b*) Let P_p be the impulse exerted by the pivot on the rod. Then $P_p + F_0\Delta t = Mv_{cm}$ and $P_p = Mv_{cm} - F_0\Delta t$. Using the result from part (*a*) one finds that $P_p = F_0\Delta t(3x/2L - 1)$ and $F_p = F_0(3x/2L - 1)$. If $x = 2L/3$, $F_p = 0$.

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27 ··· A uniform horizontal disk of mass M and radius R is rotating about its vertical axis with an angular velocity w. When it is placed on a horizontal surface, the coefficient of kinetic friction between the disk and surface is m_k. (*a*) Find the torque dt exerted by the force of friction on a circular element of radius r and width dr. (*b*) Find the total torque exerted by friction on the disk. (*c*) Find the time required to bring the disk to a halt.

(a) The force of friction, df_k, is the product of m_k and $g\,dm$, where $dm = 2prs\,dr$, and s is the mass per unit area. Here $s = M/pR^2$. The torque exerted by the friction force df_k is rdf_k. Combining these quantities we find that: $dt = 2pm_k sgr^2\,dr = 2(M/R^2)m_k gr^2\,dr$.

(b) To obtain the total torque we have to integrate dt: $t = 2(M/R^2)\, m_k g \int_0^R r^2 dr = (2/3)MR\, m_k g$.

(c) $t = w/a$, and $a = t/I$, where $I = {}_{1/2}MR^2$. So $t = 3Rw/4m_k g$.

28 · The moment of inertia of an object about an axis that does not pass through its center of mass is ____ the moment of inertia about a parallel axis through its center of mass. (a) always less than (b) sometimes less than (c) sometimes equal to (d) always greater than

(d)

29* · A tennis ball has a mass of 57 g and a diameter of 7 cm. Find the moment of inertia about its diameter. Assume that the ball is a thin spherical shell.

$I = (2/3)MR^2$ (see Table 9-1) $\quad I = (2/3) \times 0.057 \times 0.035^2 \text{ kg·m}^2 = 4.66 \times 10^{-5} \text{ kg·m}^2$

30 · Four particles at the corners of a square with side length L = 2 m are connected by massless rods (Figure 9-38). The masses of the particles are $m_1 = m_3 = 3$ kg and $m_2 = m_4 = 4$ kg. Find the moment of inertia of the system about the z axis.

Use Equ. 9-17 $\quad I = [2 \times 3 \times 2^2 + 4 \times (2\sqrt{2}\,)^2] \text{ kg·m}^2 = 56 \text{ kg·m}^2$

31 · Use the parallel-axis theorem and your results for Problem 30 to find the moment of inertia of the four-particle system in Figure 9-38 about an axis that is perpendicular to the plane of the masses and passes through the center of mass of the system. Check your result by direct computation.

1. Distance to center of mass = $\sqrt{2}$ m; M = 14 kg; by parallel axis theorem $I_{cm} = (56 - 2 \times 14) \text{ kg·m}^2 = 28 \text{ kg·m}^2$.

2. By direct computation: $I_{cm} = (4+4+3+3) \times (\sqrt{2}\,)^2 \text{ kg·m}^2 = 2 \times 14 \text{ kg·m}^2 = 28 \text{ kg·m}^2$.

32 · For the four-particle system of Figure 9-38, (a) find the moment of inertia I_x about the x axis, which passes through m_3 and m_4, and (b) Find I_y about the y axis, which passes through m_1 and m_4.

(a) $I_x = (3 \times 2^2 + 4 \times 2^2) \text{ kg·m}^2 = 28 \text{ kg·m}^2$. ($b$) By symmetry, $I_y = I_x = 28 \text{ kg·m}^2$.

33* · Use the parallel-axis theorem to find the moment of inertia of a solid sphere of mass M and radius R about an axis that is tangent to the sphere (Figure 9-39).

$I_{cm} = (2/5)MR^2$ (see Table 9-1); use Equ. 9-21 $\quad I = (2/5)MR^2 + MR^2 = (7/5)MR^2$

34 ·· A 1.0-m-diameter wagon wheel consists of a thin rim having a mass of 8 kg and six spokes each having a mass of 1.2 kg. Determine the moment of inertia of the wagon wheel for rotation about its axis.

Use Table 9-1 for I_{rim} and I_{spoke} and add $\quad I = [(8 \times 0.5^2) + (6 \times 1.2 \times 0.5^2/3)] \text{ kg·m}^2 = 2.6 \text{ kg·m}^2$

35 ·· Two point masses m_1 and m_2 are separated by a massless rod of length L. (a) Write an expression for the moment of inertia about an axis perpendicular to the rod and passing through it at a distance x from mass m_1. (b) Calculate dI/dx and show that I is at a minimum when the axis passes through the center of mass of the system.

(a) $I = m_1x^2 + m_2(L - x)^2$.

(b) $dI/dx = 2m_1x + 2m_2(L - x)(-1) = 2(m_1x + m_2x - m_2L)$; $dI/dx = 0$ when $x = m_2L/(m_1+m_2)$. This is, by definition, the distance of the center of mass from m_1.

36 ·· A uniform rectangular plate has mass m and sides of lengths a and b. (a) Show by integration that the moment of inertia of the plate about an axis that is perpendicular to the plate and passes through one corner is $m(a^2 + b^2)/3$. (b) What is the moment of inertia about an axis that is perpendicular to the plate and passes through its center of mass?

(*a*) The element of mass is $\sigma\, dxdy$, where $\sigma = m/ab$. The distance of the element dm from the corner, which we designate as our origin, is given by $r^2 = x^2 + y^2$.

$$I = \sigma \int_0^a \int_0^b (x^2 + y^2)dx\, dy = \frac{1}{3}\sigma\,(a^3 b + ab^3) = \frac{1}{3}m\,(a^2 + b^2).$$

(*b*) The distance from the origin to the center of mass is $d = [(1/2a)^2 + (1/2b)^2]^{1/2}$. Using Equ. 9-21 one obtains: $I_{cm} = (1/3)m(a^2 + b^2) - (1/4)m(a^2 + b^2) = (1/12)m(a^2 + b^2)$.

37* ·· Tracey and Corey are doing intensive research on theoretical baton-twirling. Each is using "The Beast" as a model baton: two uniform spheres, each of mass 500 g and radius 5 cm, mounted at the ends of a 30-cm uniform rod of mass 60 g (Figure 9-40). Tracey and Corey want to calculate the moment of inertia of The Beast about an axis perpendicular to the rod and passing through its center. Corey uses the approximation that the two spheres can be treated as point particles that are 20 cm from the axis of rotation, and that the mass of the rod is negligible. Tracey, however, makes her calculations without approximations. (*a*) Compare the two results. (*b*) If the spheres retained the same mass but were hollow, would the rotational inertia increase or decrease? Justify your choice with a sentence or two. It is not necessary to calculate the new value of *I*.

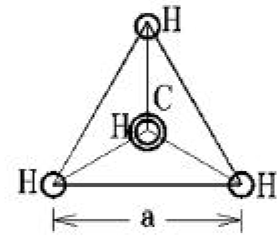
(*a*) 1. Use point mass approximation for I_{app} $\quad I_{app} = (2 \times 0.5 \times 0.2^2)\ \text{kg·m}^2 = 0.04\ \text{kg·m}^2$

2. Use Table 9-1 and Equ. 9-21 to find *I* $\quad I = [2(2/5)(0.5\times0.05^2) + I_{app} + (1/12)(0.06\times0.3^2)]\ \text{kg·m}^2$
$= 0.04145\ \text{kg·m}^2$; $I_{app}/I = 0.965$

(*b*) The rotational inertia would increase because I_{cm} of a hollow sphere > I_{cm} of a solid sphere.

38 ·· The methane molecule (CH_4) has four hydrogen atoms located at the vertices of a regular tetrahedron of side length 1.4 nm, with the carbon atom at the center of the tetrahedron (Figure 9-41). Find the moment of inertia of this molecule for rotation about an axis that passes through the carbon atom and one of the hydrogen atoms.

1. The axis of rotation passes through the center of the base of the tetrahedron. The carbon atom and the hydrogen atom at the apex of the tetrahedron do not contribute to *I* because the distance of their nuclei from the rotation axis is zero.



2. From the geometry, the distance of the three H nuclei from the rotation axis is $a/\sqrt{3}$, where *a* is the side length of the tetrahedron.

3. Apply Equ. 9-17 with $m = 1.67 \times 10^{-27}$ kg $\quad I = 3m(a/\sqrt{3})^2 = ma^2 = 3.27 \times 10^{-45}\ \text{kg·m}^2$

39 ··· A hollow cylinder has mass *m*, an outside radius R_2, and an inside radius R_1. Show that its moment of inertia about its symmetry axis is given by $I = 1/2m(R_2^2 + R_1^2)$.

Let the element of mass be $dm = \rho\, dV = 2\pi\rho hr\, dr$, where *h* is the height of the cylinder. The mass *m* of the hollow cylinder is $m = \pi\rho h(R_2^2 - R_1^2)$, so $\rho = m/[\pi h(R_2^2 - R_1^2)]$. The element $dI = r^2\, dm = 2\pi\rho hr^3\, dr$. Integrate *dI* from R_1 to R_2 and obtain $I = 1/2\pi\rho h(R_2^4 - R_1^4) = 1/2\pi\rho h(R_2^2+R_1^2)(R_2^2-R_1^2) = 1/2m(R_2^2+R_1^2)$.

40 ··· Show that the moment of inertia of a spherical shell of radius *R* and mass *m* is $2mR^2/3$. This can be done by direct integration or, more easily, by finding the increase in the moment of inertia of a solid sphere when its radius changes. To do this, first show that the moment of inertia of a solid sphere of density ρ is $I = (8/15)\pi\rho R^5$. Then compute the change *dI* in *I* for a change *dR*, and use the fact that the mass of this shell is $dm = 4\pi R^2\rho\, dR$.

From Table 9-1, $I = (2/5)mR^2$, and $m = (4/3)\pi\rho R^3$. So $I = (8/15)\pi\rho R^5$. Then, $dI = (8/3)\pi\rho R^4\, dR$. We can express this in terms of the mass increase $dm = 4\pi\rho R^2\, dR$: $dI = (2/3)R^2\, dm$. Therefore, the moment of inertia of the spherical shell of mass m is $(2/3)mR^2$.

41* ··· The density of the earth is not quite uniform. It varies with the distance r from the center of the earth as $r = C(1.22 - r/R)$, where R is the radius of the earth and C is a constant. (a) Find C in terms of the total mass M and the radius R. (b) Find the moment of inertia of the earth. (See Problem 40.)

$$(a)\ \ M = \int dm = \int_0^R 4p\, rr^2 dr = 4pC\int_0^R 1.22r^2 dr - \frac{4pC}{R}\int_0^R r^3 dr = \frac{4p}{3}1.22CR^3 - pCR^3.$$ C = 0.508 M/R^3.

$$(b)\ I = \int dI = \frac{8p}{3}\int_0^R rr^4 dr = \frac{8p \times 0.508M}{3R^3}\left[\int_0^R 1.22r^4 dr - \frac{1}{R}\int_0^R r^5 dr\right] = \frac{4.26M}{R^3}\left[\frac{1.22}{5}R^5 - \frac{1}{6}R^5\right] = 0.329\ MR^2.$$

42 ·· Use integration to determine the moment of inertia of a right circular homogeneous cone of height H, base radius R, and mass density r about its symmetry axis.

We take our origin at the apex of the cone, with the z axis along the cone's symmetry axis. Then the radius, a distance z from the apex is $r = zR/H$. Consider a disk at z of thickness dz. Its mass is $p\, r r^2\, dz$ and its mass is

$$M = p\, r\int_0^H r^2 dz = p\, r\int_0^H \frac{R^2}{H^2}z^2 dz = \frac{p\, rR^2 H}{3}.$$

Likewise,

$$I = 1/2 p\, r\int_0^H r^4 dz = \frac{p\, rR^4}{2H^4}\int_0^H z^4 dz = \frac{p\, rR^4 H}{10} = \frac{3}{10}MR^2.$$

43 ·· Use integration to determine the moment of inertia of a hollow, thin-walled, right circular cone of mass M, height H, and base radius R about its symmetry axis.

Use the same coordinates as in Problem 9-42. The element of length along the cone is $[(H^2+R^2)^{1/2}/H]\ dz$, so

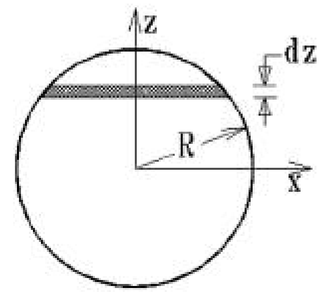
$$M = \frac{2p\, sR\sqrt{H^2+R^2}}{H^2}\int_0^H z dz = p\, sR\sqrt{H^2+R^2};$$

likewise,

$$I = \frac{2p\, sR^3\sqrt{H^2+R^2}}{H^4}\int_0^H z^3 dz = 1/2p\, sR^2\sqrt{H^2+R^2}.$$ Thus $I = 1/2MR^2$.

44 ·· Use integration to determine the moment of inertia of a thin uniform disk of mass M and radius R for rotation about a diameter. Check your answer by referring to Table 9-1.

The element of mass, dm is $2s\sqrt{R^2 - z^2}\,dz$ (See the Figure)



The moment of inertia about the diameter is then

$$I = 2s\int_{-R}^{R} z^2\sqrt{R^2 - z^2}\,dz = \frac{spR^4}{4} = \frac{1}{4}MR^2.$$

in agreement with the expression given in Table 9-1 for a cylinder of length $L = 0$.

45* ··· Use integration to determine the moment of inertia of a thin circular hoop of radius R and mass M for rotation about a diameter. Check your answer by referring to Table 9-1.

Here, $dm = lR\,dq$, and $dI = z^2\,dm$, where $z = R\sin q$. Thus, $I = lR^3\int_{-p}^{p}\sin^2 q\,dq = lpR^3 = \frac{1}{2}MR^2$, in agreement with Table 9-1 for a hollow cylinder of length $L = 0$.

46 ·· A roadside ice-cream stand uses rotating cones to catch the eyes of travelers. Each cone rotates about an axis perpendicular to its axis of symmetry and passing through its apex. The sizes of the cones vary, and the owner wonders if it would be more energy-efficient to use several smaller cones or a few big ones. To answer this, he must calculate the moment of inertia of a homogeneous right circular cone of height H, base radius R, and mass density r. What is the result?

The element of mass is, as in Problem 9-42, $dm = prr^2\,dz$. Each elemental disk rotates about an axis that is parallel to its diameter but removed from it by a distance z. We can now use the result of Problem 9-44 and the parallel axis theorem to obtain the expression for the element dI; as before, $r = Rz/H$.

$$dI = pr\left[\frac{1}{4}\left(\frac{R^2z^2}{H^2}\right)^2 + \frac{R^2z^4}{H^2}\right]dz\ .$$

Integrate from $z = 0$ to $z = H$ and use the result $M = prR^2H/3$: $I = 3M(H^2/5 + R^2/20)$.

47 · A constant torque acts on a merry-go-round. The power input of the torque is (*a*) constant. (*b*) proportional to the angular speed of the merry-go-round. (*c*) zero. (*d*) none of the above.

(*b*)

48 · The particles in Figure 9-42 are connected by a very light rod whose moment of inertia can be neglected. They rotate about the y axis with angular velocity $w = 2$ rad/s. (*a*) Find the speed of each particle, and use it to calculate the kinetic energy of this system directly from $\Sigma\, 1/2\, m_i v_i^2$. (*b*) Find the moment of inertia about the y axis, and calculate the kinetic energy from $K = 1/2\, Iw^2$.

(*a*) 1. Use $v = rw$	$v_3 = (0.2 \times 2)$ m/s $= 0.4$ m/s; $v_1 = (0.4 \times 2)$ m/s $= 0.8$ m/s
2. Find K	$K = (2 \times 1/2 \times 3 \times 0.4^2 + 2 \times 1/2 \times 1 \times 0.8^2)$ J $= 1.12$ J
(*b*) 1. Find I using Equ. 9-2	$I = (2 \times 3 \times 0.2^2 + 2 \times 1 \times 0.4^2)$ kg·m^2 $= 0.56$ kg·m^2
2. Find $K = 1/2\, Iw^2$	$K = 1/2 \times 0.56 \times 2^2$ J $= 1.12$ J

49* · Four 2-kg particles are located at the corners of a rectangle of sides 3 m and 2 m as shown in Figure 9-43. (*a*) Find the moment of inertia of this system about the *z* axis. (*b*) The system is set rotating about this axis with a kinetic energy of 124 J. Find the number of revolutions the system makes per minute.

(*a*) Use Equ. 9-2 — $I = 2[2^2 + 3^2 + (2^2 + 3^2)]$ kg·m^2 = 52 kg·m^2

(*b*) Find $\omega = (2K/I)^{1/2}$ — $\omega = (2 \times 124/52)^{1/2}$ rad/s = 2.18 rad/s = 20.9 rev/min

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50 · A solid ball of mass 1.4 kg and diameter 15 cm is rotating about its diameter at 70 rev/min. (*a*) What is its kinetic energy? (*b*) If an additional 2 J of energy are supplied to the rotational energy, what is the new angular speed of the ball?

(*a*) $I = (2/5)MR^2$; $K = {}^1\!/_2 I\omega^2 = MR^2\omega^2/5$ — $K = (1.4 \times 0.075^2 \times 7.33^2/5)$ J = 0.0846 J

(*b*) $K = 2.0846$; $\omega \propto K^{1/2}$ — $\omega = [70 \times (2.0846/0.0846)^{1/2}]$ rev/min = 347 rev/min

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51 · An engine develops 400 N·m of torque at 3700 rev/min. Find the power developed by the engine.

Use Equ. 9-27 — $P = (400 \times 3700 \times 2\pi/60)$ W = 155 kW

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52 ·· Two point masses m_1 and m_2 are connected by a massless rod of length L to form a dumbbell that rotates about its center of mass with angular velocity ω. Show that the ratio of kinetic energies of the masses is $K_1/K_2 = m_2/m_1$.

Let r_1 and r_2 be the distances of m_1 and m_2 from the center of mass. Then, by definition, $r_1m_1 = r_2m_2$. Since $K \propto mr^2\omega^2$, $K_1/K_2 = m_1r_1^2/m_2r_2^2 = m_2/m_1$.

53* ·· Calculate the kinetic energy of rotation of the earth, and compare it with the kinetic energy of motion of the earth's center of mass about the sun. Assume the earth to be a homogeneous sphere of mass 6.0×10^{24} kg and radius 6.4×10^6 m. The radius of the earth's orbit is 1.5×10^{11} m.

1. Find K_{rot}; use result of Problem 9-11 and Table 9-1 — $K_{rot} = (1/2 \times 0.4 \times 6 \times 10^{24} \times 6.4^2 \times 10^{12} \times 7.27^2 \times 10^{-10})$ J = 2.6×10^{29} J

2. Find K_{orb}; $I = M_ER_{orb}{}^2$; $\omega_{orb} = 2\pi/3.156 \times 10^7$ rad/s — $K_{orb} = (1/2 \times 6 \times 10^{24} \times 1.5^2 \times 10^{22} \times 2^2 \times 10^{-14})$ J = 2.7×10^{33} J

$K_{orb} \approx 10^4 K_{rot}$

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54 ·· A 2000-kg block is lifted at a constant speed of 8 cm/s by a steel cable that passes over a massless pulley to a motor-driven winch (Figure 9-44). The radius of the winch drum is 30 cm. (*a*) What force must be exerted by the cable? (*b*) What torque does the cable exert on the winch drum? (*c*) What is the angular velocity of the winch drum? (*d*) What power must be developed by the motor to drive the winch drum?

(*a*) $T = mg$ — $T = (2000 \times 9.81)$ N = 19.62 kN

(*b*) $\tau = Tr$ — $\tau = (19.62 \times 0.3)$ kN·m = 5.89 kN·m

(*c*) $\omega = v/r$ — $\omega = (0.08/0.3)$ rad/s = 0.267 rad/s

(*d*) $P = Fv = Tv$ — $P = (19620 \times 0.08)$ W = 1.57 kW

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55 ·· A uniform disk of mass M and radius R is pivoted such that it can rotate freely about a horizontal axis through its center and perpendicular to the plane of the disk. A small particle of mass m is attached to the rim of the disk at the top, directly above the pivot. The system is given a gentle start, and the disk begins to rotate. (*a*) What is the angular velocity of the disk when the particle is at its lowest point? (*b*) At this point, what force must be exerted on the particle by the disk to keep it on the disk?

(*a*) Use energy conservation for ω; $I = 1/2MR^2 + mR^2$ $\qquad 2mgR = 1/2[1/2MR^2+mR^2]\omega^2$; $\omega = \sqrt{\dfrac{8mg/R}{2m+M}}$

(*b*) $F = mg + mR\omega^2$ $\qquad F = mg\left(1+\dfrac{8m}{2m+M}\right)$

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56 ·· A ring 1.5 m in diameter is pivoted at one point on its circumference so that it is free to rotate about a horizontal axis. Initially, the line joining the support and center is horizontal. (*a*) If released from rest, what is its maximum angular velocity? (*b*) What must its initial angular velocity be if it is to just make a complete revolution?

(*a*) Apply energy conservation $\qquad mgR = 1/2I\omega^2$; $I = 2mR^2$;

Solve for ω; $R = 0.75$ m $\qquad \omega = \sqrt{g/R} = 3.62$ rad/s

(*b*) Now CM must rise a height R $\qquad 1/2I\omega_i^2 = mgR$; $\omega_i = 3.62$ rad/s

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57* ·· You set out to design a car that uses the energy stored in a flywheel consisting of a uniform 100-kg cylinder of radius R. The flywheel must deliver an average of 2 MJ of mechanical energy per kilometer, with a maximum angular velocity of 400 rev/s. Find the least value of R such that the car can travel 300 km without the flywheel having to be recharged.

1. Find total energy $\qquad K = (2 \times 10^6 \times 300)\text{ J} = 6 \times 10^8\text{ J} = 1/2 \times 50 \times R^2 \times \omega^2$

2. Solve for R with $\omega = 800\pi$ rad/s $\qquad R = \sqrt{24\times10^6/(800\pi)^2}$ m = 1.95 m

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58 ·· A ladder that is 8.6 m long and has mass 60 kg is placed in a nearly vertical position against the wall of a building. You stand on a rung with your center of mass at the top of the ladder. Assume that your mass is 80 kg. As you lean back slightly, the ladder begins to rotate about its base away from the wall. Is it better to quickly step off the ladder and drop to the ground or to hold onto the ladder and step off just before the top end hits the ground?

We shall solve this problem for the general case of a ladder of length L, mass M, and person of mass m. If the person falls off the ladder at the top, the speed with which he strikes the ground is given by $v_f^2 = 2gL$. Now consider what happens if the person holds on and rotates with the ladder. We shall use conservation of energy. This gives $(m + M/2)gL = 1/2(m + M/3)L^2\omega^2 = 1/2(m + M/3)v_r^2$. We find that the ratio $v_r^2/v_f^2 = (m+M/2)/(m+M/3)$. Evidently, unless M, the mass of the ladder, is zero, $v_r > v_f$. It is therefore better to let go and fall to the ground.

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59 ··· Consider the situation in Problem 58 with a ladder of length L and mass M. Find the ratio of your speed as you hit the ground if you hang on to the ladder to your speed if you immediately step off as a function of the mass ratio M/m, where m is your mass.

See Problem 9-58. We obtain $\dfrac{v_r}{v_f} = \sqrt{\dfrac{1+M/2m}{1+M/3m}}$, where v_r is the speed for hanging on, v_f for stepping off the ladder.

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60 ·· A 4-kg block resting on a frictionless horizontal ledge is attached to a string that passes over a pulley and is attached to a hanging 2-kg block (Figure 9-45). The pulley is a uniform disk of radius 8 cm and mass 0.6 kg. (*a*) Find the speed of the 2-kg block after it falls from rest a distance of 2.5 m. (*b*) What is the angular velocity of the pulley at this time?

(*a*) Use energy conservation — $mgh = \frac{1}{2}(M+m)v^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(M+m+I/R^2)v^2$

Solve for and evaluate v; $m = 2$ kg, $M = 4$ kg, $h = 2.5$ m, and $I/R^2 = \frac{1}{2}M_p = 0.3$ kg — $v = \sqrt{\dfrac{2mgh}{M+m+\frac{1}{2}M_p}} = 3.95$ m/s

(*b*) $\omega = v/R$, where $R = 0.08$ m — $\omega = (3.95/0.08)$ rad/s $= 49.3$ rad/s

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61* ·· For the system in Problem 60, find the linear acceleration of each block and the tension in the string.

1. Write the equations of motion for the three objects — $4a = T_1$; $2a = 2g - T_2$; $0.08(T_2 - T_1) = \frac{1}{2} \times 0.6 \times 0.08^2\alpha$
2. Use $\alpha = a/r$ and solve for a — $T_2 - T_1 = 2g - 6a = 0.3a$; $a = 2g/6.3 = 3.11$ m/s^2
3. Find T_1 (acting on 4 kg) and T_2 (acting on 2 kg). — $T_1 = 12.44$ N; $T_2 = T_1 + 0.3a = 13.37$ N

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62 ·· Work Problem 60 for the case in which the coefficient of friction between the ledge and the 4-kg block is 0.25.

(*a*) Use energy conservation; see Problem 9-60 — $mgh = \frac{1}{2}(M+m+I/R^2)v^2 + \mu_k Mgh$

Solve for and evaluate v for $m = 2$ kg, $M = 4$ kg, $h = 2.5$ m, $M_p = 0.6$ kg, $\mu_k = 0.25$ — $v = \sqrt{\dfrac{2h(mg-\mu_k Mg)}{M+m+\frac{1}{2}M_p}} = 2.79$ m/s

(*b*) $\omega = v/R$, where $R = 0.08$ m — $\omega = 34.6$ rad/s

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63 ·· Work Problem 61 for the case in which the coefficient of friction between the ledge and the 4-kg block is 0.25.

1. Now $4a = T_1 - 0.25 \times 4g$; also see Prob. 9-61 — $4a = T_1 - g$; $2a = 2g - T_2$; $T_2 - T_1 = 0.3a$
2. Solve for a — $a = g/6.3 = 1.56$ m/s^2
3. Find T_1 and T_2 — $T_1 = 16.\ 0$ N; $T_2 = 16.5$ N

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64 ·· In 1993, a giant yo-yo of mass 400 kg and measuring about 1.5 m in radius was dropped from a crane 57 m high. Assuming the axle of the yo-yo had a radius of $r = 0.1$ m, find the velocity of the descent v at the end of the fall.

1. Write the equations of motion — $ma = mg - T$; $a = r\alpha = r\tau/I = r^2T/\frac{1}{2}mR^2$; $T = mR^2a/2r^2$
2. Solve for and evaluate a; $m = 400$ kg, $R = 1.5$ m — $a = g/(1 + R^2/2r^2) = 0.0872$ m/s^2
3. Use $v = (2as)^{1/2}$ — $v = (2 \times 0.0872 \times 57)^{1/2}$ m/s $= 3.15$ m/s

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65* ·· A 1200-kg car is being unloaded by a winch. At the moment shown in Figure 9-46, the gearbox shaft of the winch breaks, and the car falls from rest. During the car's fall, there is no slipping between the (massless) rope, the pulley, and the winch drum. The moment of inertia of the winch drum is 320 kg·m^2 and that of the pulley is 4 kg·m^2. The radius of the winch drum is 0.80 m and that of the pulley is 0.30 m. Find the speed of the car as it hits the water.

1. Use energy conservation and $\omega = v/r$ — $mgh = \frac{1}{2}mv^2+\frac{1}{2}I_w\omega_w^2+\frac{1}{2}I_p\omega_p^2 = \frac{1}{2}v^2(m+I_w/r_w^2+I_p/r_p^2)$
2. Solve for and evaluate v — $v = [2mgh/(m+I_w/r_w^2+I_p/r_p^2)]^{1/2} = 8.2$ m/s

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66 ·· The system in Figure 9-47 is released from rest. The 30-kg block is 2 m above the ledge. The pulley is a uniform disk with a radius of 10 cm and mass of 5 kg. Find (*a*) the speed of the 30-kg block just before it hits the ledge, (*b*) the angular speed of the pulley at that time, (*c*) the tensions in the strings, and (*d*) the time it takes for the 30-kg block to reach the ledge. Assume that the string does not slip on the pulley.

(*a*) 1. m_1=20 kg, m_2=30 kg; use energy conservation — $m_2gh = m_1gh + \frac{1}{2}(m_1v^2 + m_2v^2 + I\omega^2)$

2. $I = \frac{1}{2}mr^2$; $\omega^2=v^2/r^2$; so $I\omega^2 = \frac{1}{2}mv^2$; $m = 5$ kg — $v = [2gh(m_2-m_1)/(m_1+m_2+\frac{1}{2}m)]^{1/2} = 2.73$ m/s

(*b*) Use $\omega = v/r$ — $\omega = (2.73/0.1)$ rad/s = 27.3 rad/s

(*c*) 1. Find acceleration; $a = v^2/2h$ — $a = 1.87$ m/s^2

2. $T_1 = m_1(g + a)$; $T_2 = m_2(g - a)$ — $T_1 = 234$ N; $T_2 = 238$ N

(*d*) Use $t = h/v_{av} = 2h/v$ — $t = (4/2.73)$ s = 1.47 s

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67 ·· A uniform sphere of mass M and radius R is free to rotate about a horizontal axis through its center. A string is wrapped around the sphere and is attached to an object of mass m as shown in Figure 9-48. Find (*a*) the acceleration of the object, and (*b*) the tension in the string.

(*a*) The equations of motion for the two objects are $mg - T = ma$ and $I\alpha = \tau$. Now $\tau = RT$, $I = (2/5)MR^2$, and $\alpha = a/R$. Thus, $T = (2/5)Ma$ and $a = g/[1 + (2M/5m)]$.

(*b*) As obtained in (*a*), $T = (2/5)Ma = 2mMg/(5m+2M)$.

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68 ·· An Atwood's machine has two objects of mass m_1 = 500 g and m_2 = 510 g, connected by a string of negligible mass that passes over a frictionless pulley (Figure 9-49). The pulley is a uniform disk with a mass of 50 g and a radius of 4 cm. The string does not slip on the pulley. (*a*) Find the acceleration of the objects. (*b*) What is the tension in the string supporting m_1? In the string supporting m_2? By how much do they differ? (*c*) What would your answers have been if you had neglected the mass of the pulley?

Note that this problem is identical to Problem 9-66. We use the result for v^2 and $a = v^2/2h$.

(*a*) $a = (m_2 - m_1)g/(m_1 + m_2 + \frac{1}{2}m)$; use the given values — $a = (10/1035)g = 9.478$ cm/s^2

(*b*) $T_1 = m_1(a + g)$; $T_2 = m_2(g - a)$ — $T_1 = 4.9524$ N; $T_2 = 4.9548$ N; $\Delta T = 0.0024$ N

(*c*) If $m = 0$; $a = (m_2-m_1)g/(m_1+m_2)$; $T_1 = T_2$ — $a = 9.713$ cm/s^2; $T = 4.9536$ N; $\Delta T = 0$

69* ·· Two objects are attached to ropes that are attached to wheels on a common axle as shown in Figure 9-50. The total moment of inertia of the two wheels is 40 kg·m^2. The radii of the wheels are R_1 = 1.2 m and R_2 = 0.4 m. (*a*) If m_1 = 24 kg, find m_2 such that there is no angular acceleration of the wheels. (*b*) If 12 kg is gently added to the top of m_1, find the angular acceleration of the wheels and the tensions in the ropes.

(a) Find τ_{net} and set equal to 0 — $\tau = m_1gR_1 - m_2gR_2 = 0$; $m_2 = m_1R_1/R_2 = 72$ kg

(b) 1. Write the equations of motion — $T_1 = m_1(g - R_1\alpha)$; $T_2 = m_2(g + R_2\alpha)$; $\alpha = (T_1R_1 - T_2R_2)/I$

2. Solve for and find α with $m_1 = 36$ kg, $m_2 = 72$ kg — $\alpha = (m_1R_1 - m_2R_2)g/(m_1R_1^2 + m_2R_2^2 + I) = 1.37$ rad/s^2

3. Substitute $\alpha = 1.37$ rad/s^2 to find T_1 and T_2 — $T_1 = 294$ N; $T_2 = 745$ N

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70 ·· A uniform cylinder of mass M and radius R has a string wrapped around it. The string is held fixed, and the cylinder falls vertically as shown in Figure 9-51. (a) Show that the acceleration of the cylinder is downward with a magnitude $a = 2g/3$. (b) Find the tension in the string.

(a) The equation of motion is $\tau = I\alpha = RT = \frac{1}{2}MR^2a/R$; $T = \frac{1}{2}Ma$. But $Mg - T = Ma$. Thus, $a = (2/3)g$.

(b) $T = \frac{1}{2}Ma = Mg/3$.

Note that we could have obtained the result also from Problem 9-64, setting $r = R$.

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71 ·· The cylinder in Figure 9-51 is held by a hand that is accelerated upward so that the center of mass of the cylinder does not move. Find (a) the tension in the string, (b) the angular acceleration of the cylinder, and (c) the acceleration of the hand.

(a) Since $a = 0$, $T = Mg$. (b) Use $\alpha = RT/I = RMg/\frac{1}{2}MR^2 = 2g/R$. ($c$) $a = R\alpha = 2g$.

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72 ·· A 0.1-kg yo-yo consists of two solid disks of radius 10 cm joined together by a massless rod of radius 1 cm and a string wrapped around the rod. One end of the string is held fixed and is under constant tension T as the yo-yo is released. Find the acceleration of the yo-yo and the tension T.

See Problem 9-64 — $a = g(1 + R^2/2r^2) = 0.192$ m/s^2; $T = m(g-a) = 0.902$ N

73* ·· A uniform cylinder of mass m_1 and radius R is pivoted on frictionless bearings. A massless string wrapped around the cylinder connects to a mass m_2, which is on a frictionless incline of angle θ as shown in Figure 9-52. The system is released from rest with m_2 a height h above the bottom of the incline. (a) What is the acceleration of m_2? (b) What is the tension in the string? (c) What is the total energy of the system when m_2 is at height h?
(d) What is the total energy when m_2 is at the bottom of the incline and has a speed v? (e) What is the speed v?
(f) Evaluate your answers for the extreme cases of $\theta = 0^o$, $\theta = 90^o$, and $m_1 = 0$.

(a) 1. Write the equations of motion — $m_2a = m_2g\sin\theta - T$; $\tau = RT = \frac{1}{2}m_1R^2\alpha$; $T = \frac{1}{2}m_1a$

2. Solve for a — $a = (g\sin\theta)/(1 + m_1/2m_2)$

(b) Solve for T — $T = (\frac{1}{2}m_1g\sin\theta)/(1 + m_1/2m_2)$

(c) Take $U = 0$ at $h = 0$ — $E = K + U = m_2gh$

(d) This is a conservative system — $E = m_2gh$

(e) $U = 0$; $E = K = \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$; $\omega = v/R$ — $m_2gh = \frac{1}{2}(m_2 + \frac{1}{2}m_1)v^2$; $v = \sqrt{(2gh)/(1 + m_1/2m_2)}$

(f) 1. For $\theta = 0$ — $a = T = 0$

2. For $\theta = 90^o$ — $a = g/(1 + m_1/2m_2)$; $T = \frac{1}{2}m_1a$; $v = \sqrt{(2gh)/(1 + m_1/2m_2)}$

3. For $m_1 = 0$ — $a = g\sin\theta$, $T = 0$, $v = \sqrt{2gh}$

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74 ·· A device for measuring the moment of inertia of an object is shown in Figure 9-53. A circular platform has a concentric drum of radius 10 cm about which a string is wound. The string passes over a frictionless pulley to a weight of mass M. The weight is released from rest, and the time for it to drop a distance D is measured. The system is then rewound, the object placed on the platform, and the system again released from rest. The time required for the weight to drop the same distance D then provides the data needed to calculate I. With M = 2.5 kg, and D = 1.8 m, the time is 4.2 s. (*a*) Find the combined moment of inertia of the platform, drum, shaft, and pulley. (*b*) With the object placed on the platform, the time is 6.8 s for D = 1.8 m. Find I of that object about the axis of the platform.

Let r be the radius of the concentric drum (10 cm) and let I_0 be the moment of inertia of the drum plus platform.

(*a*) 1. Write the equations of motion, empty platform	$Ma = Mg - T$; $rT = I_0\alpha = I_0a/r$; $T = I_0a/r^2$
2. Solve for I_0	$I_0 = Mr^2(g - a)/a$
3. Use $a = 2D/t^2$ and evaluate I_0	$I_0 = 1.177\ \mathrm{kg{\cdot}m^2}$
(*b*) Now $I_{tot} = I_0 + I$; $I_{tot} = Mr^2(g-a)/a$; $a = 2D/t^2$	$I_{tot} = 3.125\ \mathrm{kg{\cdot}m^2}$; $I = 1.948\ \mathrm{kg{\cdot}m^2}$

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75 · True or false: When an object rolls without slipping, friction does no work on the object.

True

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76 · A wheel of radius R is rolling without slipping. The velocity of the point on the rim that is in contact with the surface, relative to the surface, is (*a*) equal to $R\omega$ in the direction of motion of the center of mass. (*b*) equal to $R\omega$ opposite the direction of motion of the center of mass. (*c*) zero. (*d*) equal to the velocity of the center of mass and in the same direction. (*d*) equal to the velocity of the center of mass but in the opposite direction.

(*c*)

77* ·· A solid cylinder and a solid sphere have equal masses. Both roll without slipping on a horizontal surface. If their kinetic energies are the same, then (*a*) the translational speed of the cylinder is greater than that of the sphere. (*b*) the translational speed of the cylinder is less than that of the sphere. (*c*) the translational speeds of the two objects are the same. (*d*) (*a*), (*b*), or (*c*) could be correct depending on the radii of the objects.

$K_c = (3/4)mv_c^2$; $K_s = (7/10)mv_s^2$. If $K_c = K_s$, then $v_c < v_s$. (*b*)

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78 ·· Starting from rest at the same time, a coin and a ring roll down an incline without slipping. Which of the following is true? (*a*) The ring reaches the bottom first. (*b*) The coin reaches the bottom first. (*c*) The coin and ring arrive at the bottom simultaneously. (*d*) The race to the bottom depends on their relative masses. (*e*) The race to the bottom depends on their relative diameters.

$K_r = K_c$; $m_rv_r^2 = m_rgh$; $v_r^2 = gh$. For the coin, $v_c^2 = (4/3)gh$. $v_c > v_r$. (*b*)

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79 ·· For a hoop of mass M and radius R that is rolling without slipping, which is larger, its translational kinetic energy or its rotational kinetic energy? (*a*) Translational kinetic energy is larger. (*b*) Rotational kinetic energy is larger. (*c*) Both are the same size. (*d*) The answer depends on the radius. (*e*) The answer depends on the mass.

(*c*)

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80 ·· For a disk of mass M and radius R that is rolling without slipping, which is larger, its translational kinetic energy or its rotational kinetic energy? (*a*) Translational kinetic energy is larger. (*b*) Rotational kinetic energy is larger. (*c*) Both are the same size. (*d*) The answer depends on the radius. (*e*) The answer depends on the mass.

(*a*)

81* ·· A ball rolls without slipping along a horizontal plane. Show that the frictional force acting on the ball must be zero. *Hint:* Consider a possible direction for the action of the frictional force and what effects such a force would have on the velocity of the center of mass and on the angular velocity.

Let us assume that $f \neq 0$ and acts along the direction of motion. Now consider the acceleration of the center of

$t = 0$ since $l = 0$, so $a = 0$. But $a = 0$ is not consistent with $a_{cm} \neq 0$. Consequently, $f = 0$.

82 · A homogeneous solid cylinder rolls without slipping on a horizontal surface. The total kinetic energy is K. The kinetic energy due to rotation about its center of mass is (*a*) $1/2K$. (*b*) $1/3K$. (*c*) $4/7K$. (*d*) none of the above.

(*b*)

83 · A homogeneous cylinder of radius 18 cm and mass 60 kg is rolling without slipping along a horizontal floor at 5 m/s. How much work is needed to stop the cylinder?

$W = K = (3/4)mv^2$ $\quad$ $W = (0.75 \times 60 \times 5^2)$ J = 1125 J

84 · Find the percentages of the total kinetic energy associated with rotation and translation, respectively, for an object that is rolling without slipping if the object is (*a*) a uniform sphere, (*b*) a uniform cylinder, or (*c*) a hoop.

(*a*) For a sphere, $K_{tot} = 0.7mv^2$; $K_{trans} = 0.5mv^2$ $\quad$ $K_{trans} = 71.4\%\ K_{tot}$; $K_{rot} = 28.6\%\ K_{tot}$

(*b*) For a cylinder, $K_{tot} = 0.75mv^2$; $K_{trans} = 0.5mv^2$ $\quad$ $K_{trans} = 66.7\%\ K_{tot}$; $K_{rot} = 33.3\%\ K_{tot}$

(*c*) For a hoop, $K_{tot} = mv^2$ $\quad$ $K_{trans} = 50\%\ K_{tot}$; $K_{rot} = 50\%\ K_{tot}$

85* · A hoop of radius 0.40 m and mass 0.6 kg is rolling without slipping at a speed of 15 m/s toward an incline of slope 30^o. How far up the incline will the hoop roll, assuming that it rolls without slipping?

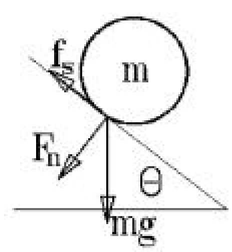
1. Find the energy at the bottom of the slope $\quad$ $K = mv^2$

2. Use energy conservation; $mgL \sin 30^o = K$ $\quad$ $L = 2v^2/g = 45.9$ m

86 · A ball rolls without slipping down an incline of angle q. The coefficient of static friction is m_s. Find (*a*) the acceleration of the ball, (*b*) the force of friction, and (*c*) the maximum angle of the incline for which the ball will roll without slipping.

We assume that the ball is a solid sphere.

The free-body diagram is shown. Note that both the force $m\boldsymbol{g}$ and the normal reaction force $\boldsymbol{F}_n$ act through the center of mass, so their torque about the center of mass is zero.



(*a*) 1. Write the equations of motion; use $a = a/r$ $\quad$ $ma = mg \sin q - f_s$; $t = f_s r = Ia = (2/5)mr^2$; $f_s = (2/5)ma$

2. Solve for a $\quad$ $a = (5/7)g \sin q$

(*b*) Find f_s using the results in part (*a*) $\quad$ $f_s = (2/7)mg \sin q$

(*c*) Use $f_{s,max} = m_s F_n = m_s mg \cos q$ $\quad$ $m_s \cos q = (2/7) \sin q$; $q_{max} = \tan^{-1}(7m_s/2)$

87 ·· An empty can of total mass $3M$ is rolling without slipping. If its mass is distributed as in Figure 9-54, what is the value of the ratio of kinetic energy of translation to the kinetic energy of rotation about its center of mass?

1. Find the total moment of inertia $\quad I = 2(\frac{1}{2}MR^2) + MR^2 = 2MR^2$

2. $K_{trans} = \frac{1}{2}(3Mv^2)$; $K_{rot} = \frac{1}{2}(2MR^2)v^2/R^2 = Mv^2$ $\quad K_{trans}/K_{rot} = 3/2$

---

88 ·· A bicycle of mass 14 kg has 1.2-m diameter wheels, each of mass 3 kg. The mass of the rider is 38 kg. Estimate the fraction of the total kinetic energy of bicycle and rider associated with rotation of the wheels.

Assume the wheels are hoops, i.e., neglect the mass of the spokes. Then the total kinetic energy is $K = \frac{1}{2}Mv^2 + 2(\frac{1}{2}I_w\omega^2) = \frac{1}{2}Mv^2 + m_wv^2 = [\frac{1}{2}(52) + 3]v^2 = 29v^2$. $K_{rot} = 3v^2$. $K_{rot}/K = 3/29 \approx 10\%$.

89* ·· A hollow sphere and uniform sphere of the same mass m and radius R roll down an inclined plane from the same height H without slipping (Figure 9-55). Each is moving horizontally as it leaves the ramp. When the spheres hit the ground, the range of the hollow sphere is L. Find the range L' of the uniform sphere.

1. Find v of each object as it leaves ramp. Use energy conservation. $\quad mgH = \frac{1}{2}mv_h^2 + \frac{1}{2}(2/3)mv_h^2$; $v_h^2 = 6gH/5$; $\quad mgH = \frac{1}{2}mv_u^2 + \frac{1}{2}(2/5)mv_u^2$; $v_u^2 = 10gH/7$

2. Since distance $\propto v$, $L'/L = v_u/v_h$ $\quad L' = L(25/21)^{1/2} = 1.09L$

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90 ·· A hollow cylinder and a uniform cylinder are rolling horizontally without slipping. The speed of the hollow cylinder is v. The cylinders encounter an inclined plane that they climb without slipping. If the maximum height they reach is the same, find the initial speed v' of the uniform cylinder.

Since they climb the same height, $K_h = \frac{1}{2}m_hv^2 + \frac{1}{2}I_h\omega_h^2 = m_hv_h^2 = m_hgh = K_u = \frac{1}{2}m_u(v')^2 + \frac{1}{2}I_u\omega_u^2 = (3/4)m_u(v')^2 = m_ugh$. Consequently, $v' = \sqrt{4/3}v$.

---

91 ·· A hollow, thin-walled cylinder and a solid sphere start from rest and roll without slipping down an inclined plane of length 3 m. The cylinder arrives at the bottom of the plane 2.4 s after the sphere. Determine the angle between the inclined plane and the horizontal.

1. Find a_c and a_s; see Problem 9-86. $\quad a_s = (5/7)g\sin\theta$; similarly, one obtains $a_c = \frac{1}{2}g\sin\theta$

2. Use $s = \frac{1}{2}at^2$ $\quad a_s t_s^2 = a_c t_c^2$; $t_c^2 = (t_s + 2.4)^2 = t_s^2 + 4.8t_s + 5.76$

3. Write the quadratic equation for t_s $\quad t_s^2 + 4.8t_s + 5.76 = (10/7)t_s^2$

4. Solve for t_s $\quad t_s = 12.3$ s

5. Use steps 1 and 2 and solve for θ $\quad \sin\theta = 42/(5 \times 9.81 \times 12.3^2) = 0.00567$; $\theta = 0.325^\circ$

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92 ·· A uniform solid sphere of radius r starts from rest at a height h and rolls without slipping along the loop-the-loop track of radius R as shown in Figure 9-56. (a) What is the smallest value of h for which the sphere will not leave the track at the top of the loop? (b) What would h have to be if, instead of rolling, the ball slides without friction?

We shall assume that h is the initial height of the center of the sphere of radius r. To just remain in contact with the track, the centripetal acceleration of the sphere's center of mass must equal mg.

(a) 1. Note radius of loop for center of mass = $R - r$ $\quad mv^2/(R - r) = mg \quad (1)$

2. Use energy conservation $\quad mg(h - 2R + r) = \frac{1}{2}mv^2 + \frac{1}{2}(2mv^2/5) \quad (2)$

3. Use Equ. (1) for mv^2 and solve for h $\quad h = 2.7R - 1.7r$

(b) Now $\frac{1}{2}(2mv^2/5)$ term in (2) is absent. $\quad h = 2.5R - 1.5r$

93* ··· A wheel has a thin 3.0-kg rim and four spokes each of mass 1.2 kg. Find the kinetic energy of the wheel when it rolls at 6 m/s on a horizontal surface.

1. Find I of the wheel	$I = M_{rim}R^2 + 4[(1/3)M_{spoke}R^2]$
2. Write $K = K_{trans} + K_{rot}$; use $v = Rw$	$K = 1/2(7.8 + 3 + 1.6) \times 6^2$ J = 223 J

94 ·· Two uniform 20-kg disks of radius 30 cm are connected by a short rod of radius 2 cm and mass 1 kg. When the rod is placed on a plane inclined at 30°, such that the disks hang over the sides, the assembly rolls without slipping. Find (*a*) the linear acceleration of the system, and (*b*) the angular acceleration of the system. (*c*) Find the kinetic energy of translation of the system after it has rolled 2 m down the incline starting from rest. (*d*) Find the kinetic energy of rotation of the system at the same point.

(*a*) 1. As in Problem 9-86, $t = fr$. Write the equations of motion.	$Mg \sin q - f = Ma$, where M = 41 kg; $fr = Ia$, where $a = a/r$
2. Write a, eliminating f	$a = (Mg \sin q)/(M + I/r^2)$
3. Determine I	$I = (2 \times 1/2 \times 20 \times 0.3^2 + 1/2 \times 1 \times 0.02^2)$ kg·m^2 = 1.80 kg·m^2
4. Evaluate a	$a = (41 \times 9.81 \times 0.5)/(41 + 1.80/0.02^2)$ m/s^2 = 0.0443 m/s^2
(*b*) $a = a/r$	a = (0.0443/0.02) rad/s^2 = 2.21 rad/s^2
(*c*) Use $v^2 = 2as$ and $K_{trans} = 1/2Mv^2$	$K_{trans} = (1/2 \times 41 \times 2 \times 0.0443 \times 2)$ J = 3.63 J
(*d*) $K_{rot} = Mgh - K_{trans}$; $h = 2 \sin 30^\circ$ m = 1 m	$K_{rot} = [(41 \times 9.81 \times 1) - 3.63]$ J = 399 J

95 ·· A wheel of radius R rolls without slipping at a speed V. The coordinates of the center of the wheel are X, Y. (*a*) Show that the x and y coordinates of point P in Figure 9-57 are $X + r_0 \cos q$ and $R + r_0 \sin q$, respectively,. (*b*) Show that the total velocity $\mathbf{v}$ of point P has the components $v_x = V + (r_0V \sin q)/R$ and $v_y = -(r_0V \cos q)/R$. (*c*) Show that at the instant that $X = 0$, $\mathbf{v}$ and $\mathbf{r}$ are perpendicular to each other by calculating $\mathbf{v}\cdot\mathbf{r}$. (*d*) Show that $v = rw$, where $w = V/R$ is the angular velocity of the wheel. These results demonstrate that, in the case of rolling without slipping, the motion is the same as if the rolling object were instantaneously rotating about the point of contact with an angular speed $w = V/R$.

(*a*) From the figure it is evident that $x = r_0 \cos q$ and $y = r_0 \sin q$ relative to the center of the wheel. Therefore, if the coordinates of the center are X and R, those of point P are as stated.

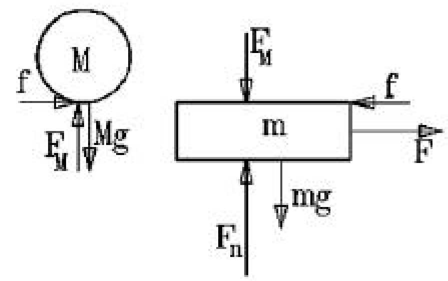
(*b*) $v_{Px} = d(X + r_0 \cos q)/dt = dX/dt - r_0 \sin q\, dq/dt$. Note that $dX/dt = V$ and $dq/dt = -w = -V/R$; therefore, $v_{Px} = V + (r_0V \sin q)/R$, $v_{Py} = d(R + r_0 \sin q)/dt = r_0 \cos q\, dq/dt$ $(dR/dt = 0)$. Again, $dq/dt = -w$, so $v_{Py} = -(r_0V \cos q)/R$.

(*c*) $\mathbf{v}\cdot\mathbf{r} = v_{Px}r_x + v_{Py}r_y = (V + r_0V \sin q/R)(r_0 \cos q) - (r_0V \cos q/R)(R + r_0 \sin q) = 0$.

(*d*) $v^2 = v_x^2 + v_y^2 = V^2[1 + (2r_0/R) \sin q + r_0^2/R^2]$; $r^2 = r_x^2 + r_y^2 = R^2[1 + (2r_0/R) \sin q + r_0^2/R^2]$; so $v/r = V/R = w$.

96 ·· A uniform cylinder of mass M and radius R is at rest on a block of mass m, which in turn rests on a horizontal, frictionless table (Figure 9-58). If a horizontal force $\mathbf{F}$ is applied to the block, it accelerates and the cylinder rolls without slipping. Find the acceleration of the block.

We begin by drawing the two free-body diagrams.



For the block,

$F - f = ma_B$ (1)

For the cylinder,

$f = Ma_C$ (2)

Also, $fR = \frac{1}{2}MR^2\alpha$ and $f = \frac{1}{2}MR\alpha$. But $a_C = a_B - R\alpha$ or $R\alpha = a_B - a_C$.

Using Equs. (1) and (2) we now obtain $2f/M = a_B - f/M$ and

$3f/M = 3a_C = a_B$ (3)

Equs. (1) and (3) yield $F - Ma_B/3 = ma_B$ and solving for a_B we obtain

$a_B = 3F/(M + 3m)$ and $a_C = F/(M + 3m)$.

97* ··· (*a*) Find the angular acceleration of the cylinder in Problem 96. Is the cylinder rotating clockwise or counterclockwise? (*b*) What is the cylinder's linear acceleration relative to the table? Let the direction of $\boldsymbol{F}$ be the positive direction. (*c*) What is the linear acceleration of the cylinder relative to the block?

(*a*) From Problem 9-96, $\alpha = (a_B - a_C)/R = 2F/[R(M + 3m)]$. From the free body diagram of the preceding problem it is evident that the torque and, therefore, α is in the counterclockwise direction.

(*b*) The linear acceleration of the cylinder relative to the table is $a_C = F/(M + 3m)$. (see Problem 96)

(*c*) The acceleration of the cylinder relative to the block is $a_C - a_B = -2F/(M + 3m)$.

98 ··· If the force in Problem 96 acts over a distance *d*, find (*a*) the kinetic energy of the block, and (*b*) the kinetic energy of the cylinder. (*c*) Show that the total kinetic energy is equal to the work done on the system.

(*a*) $K_m = \frac{1}{2}mv_m^2 = ma_m d = Fdm/(m + \frac{1}{2}M)$.

(*b*) $K_{cyl} = K_{trans} + K_{rot} = \frac{1}{2}Mv_M^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}FdM/(m + \frac{1}{2}M) + (1/4)FdM/(m + \frac{1}{2}M)$.

(*c*) The total $K = Fd$ which is the work done by the force *F*.

99 ··· A marble of radius 1 cm rolls from rest without slipping from the top of a large sphere of radius 80 cm, which is held fixed (Figure 9-59). Find the angle from the top of the sphere to the point where the marble breaks contact with the sphere.

Use energy conservation to find $v^2(\theta)$. $\Delta U = -mg(R + r)(1 - \cos\theta) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2(1 + 2/5) = 7mv^2/10$. $v^2 = 10g(R + r)(1 - \cos\theta)/7$. The marble will separate from the sphere when $mg\cos\theta = mv^2/(R+r)$. The condition is $\cos\theta = 10/17$; $\theta = 54^o$. (Note that θ does not depend on the radii of the sphere and marble.)

100 · True or false: When a sphere rolls and slips on a rough surface, mechanical energy is dissipated.

True

101*· A cue ball is hit very near the top so that it starts to move with topspin. As it slides, the force of friction (*a*) increases v_{cm}. (*b*) decreases v_{cm}. (*c*) has no effect on v_{cm}.

(*a*)

102 · A bowling ball of mass *M* and radius *R* is thrown such that at the instant it touches the floor it is moving horizontally with a speed v_0 and is not rotating. It slides for a time t_1 a distance s_1 before it begins to roll without slipping. (*a*) If μ_k is the coefficient of sliding friction between the ball and the floor, find s_1, t_1, and the final speed v_1 of the ball. (*b*) Find the ratio of the final mechanical energy to the initial mechanical energy of the ball. (*c*) Evaluate these quantities for $v_0 = 8$ m/s and $\mu_k = 0.06$.

Part (a) of this problem is identical to Example 9-16. From Example 9-16, we have:

(a) $s_1 = (12/49)v_0^2/\mu_k g$; $t_1 = 2v_0/7\mu_k g$; $v_1 = (5/2)\mu_k g t_1 = 5v_0/7$.

(b) $K_f = {}_{1/2}Mv_1^2 + {}_{1/2}[(2/5)Mv_1^2] = (7/10)Mv_1^2 = (5/14)Mv_0^2$; $K_i = {}_{1/2}Mv_0^2$. $K_f/K_i = 5/7$.

(c) Inserting the appropriate numerical values: $s_1 = 26.6$ m; $t_1 = 3.88$ s; $v_1 = 5.71$ m/s.

---

103 ·· A cue ball of radius r is initially at rest on a horizontal pool table (Figure 9-60). It is struck by a horizontal cue stick that delivers a force of magnitude P_0 for a very short time Δt. The stick strikes the ball at a point h above the ball's point of contact with the table. Show that the ball's initial angular velocity ω_0 is related to the initial linear velocity of its center of mass v_0 by $\omega_0 = 5v_0(h - r)/2r^2$.

The translational impulse $P_t = P_0\Delta t = mv_0$. The rotational impulse about the center of mass is

$P_t = P_t(h - r) = I\omega_0$.With $I = (2/5)mr^2$ one then obtains $\omega_0 = 5v_0(h-r)/2r^2$.

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104 ·· A uniform spherical ball is set rotating about a horizontal axis with an angular speed ω_0 and is placed on the floor. If the coefficient of sliding friction between the ball and the floor is μ_k, find the speed of the center of mass of the ball when it begins to roll without slipping.

1. f_k gives the ball a forward acceleration a	$a = \mu_k g$; $v = at = \mu_k gt$
2. The torque $\tau = f_k r$ results in a reduction of ω	$\omega = \omega_0 - \alpha t$; $\alpha = \tau/I = \mu_k mrg/[(2/5)mr^2] = (5/2)\mu_k g/r$
3. The ball rolls without slipping when $\omega r = v$	$\omega_0 r - (5/2)\mu_k gt = \mu_k gt$; $t = 2r\omega_0/7\mu_k g$
4. Find v at $t = 2r\omega_0/7\mu_k g$	$v = 2r\omega_0/7$

105* ·· A uniform solid ball resting on a horizontal surface has a mass of 20 g and a radius of 5 cm. A sharp force is applied to the ball in a horizontal direction 9 cm above the horizontal surface. The force increases linearly from 0 to a peak value of 40,000 N in 10^{-4} s and then decreases linearly to 0 in 10^{-4} s. (a) What is the velocity of the ball after impact? (b) What is the angular velocity of the ball after impact? (c) What is the velocity of the ball when it begins to roll without sliding? (d) For how long does the ball slide on the surface? Assume that $\mu_k = 0.5$.

(a) Find the translational impulse; then use $P_t = mv$	$F_{av} = 20{,}000$ N, $\Delta t = 2 \times 10^{-4}$ s; $v_0 = (4/0.02)$ m/s $= 200$ m/s
(b) Proceed as in Problem 9-103	$\omega_0 = 5 \times 200 \times (.09 - .05)/(2 \times .05^2)$ rad/s $= 8000$ rad/s
(c), (d) Note that $\omega_0 r = 400$ m/s $> v_0$; proceed as in Problem 9-104	$\omega = \omega_0 - (5/2)\mu_k gt/r$; $v = v_0 + \mu_k gt$; set $\omega r = v$; find t $t = 2(\omega_0 r - v_0)/7\mu_k g = 11.6$ s; $v = 257$ m/s

---

106 ·· A 0.3-kg billiard ball of radius 3 cm is given a sharp blow by a cue stick. The applied force is horizontal and passes through the center of the ball. The initial velocity of the ball is 4 m/s. The coefficient of kinetic friction is 0.6. (a) For how many seconds does the ball slide before it begins to roll without slipping? (b) How far does it slide? (c) What is its velocity once it begins rolling without slipping?

Since the impulse passes through the CM, $\omega_0 = 0$. We use the results of Problem 9-102.

(a) $t = 2v_0/7\mu_k g = 0.194$ s. (b) $s = 12v_0^2/49\mu_k g = 0.666$ m. (c) $v = 5v_0/7 = 2.86$ m/s.

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107 ·· A billiard ball initially at rest is given a sharp blow by a cue stick. The force is horizontal and is applied at a distance $2R/3$ below the centerline, as shown in Figure 9-61. The initial speed of the ball is v_0, and the coefficient of kinetic friction is μ_k. (a) What is the initial angular speed ω_0? (b) What is the speed of the ball once it begins to roll without slipping? (c) What is the initial kinetic energy of the ball? (d) What is the frictional work done as it slides on the table?

(a) Use rotation impulse, $P_t = mv_0r$; $r = 2R/3$	$P_t = I\omega_0$; $\omega_0 = (2mv_0R/3)/[(2/5)mR^2] = 5v_0/3R$
(b) Since F is below the center line, the spin is backward, i.e., the ball will slow down. Proceed as in Problem 9-105, with $\omega_0 = -5v_0/R$.	$\omega = \omega_0 + (5/2)\mu_k gt/R$; $v = v_0 - \mu_k gt$; set $\omega R = v$ $v_0 - \mu_k gt = -(5/3)v_0 + (5/2)\mu_k gt$; $t = (16/21)v_0/\mu_k g$ $v = (5/21)v_0 = 0.238v_0$
(c) $K_i = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2$	$K_i = \frac{1}{2}mv_0^2 + \frac{1}{2}(50/45)mv_0^2 = (19/18)mv_0^2 = 1.056mv_0^2$
(d) Find K_f; then $W_{fr} = K_i - K_f$	$K_f = (7/10)mv^2 = (0.7 \times 0.238^2)mv_0^2 = 0.0397mv_0^2$ $W_{fr} = 1.016mv_0^2$

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108 ·· A bowling ball of radius R is given an initial velocity v_0 down the lane and a forward spin $\omega_0 = 3v_0/R$. The coefficient of kinetic friction is μ_k. (a) What is the speed of the ball when it begins to roll without slipping? (b) For how long does the ball slide before it begins to roll without slipping? (c) What distance does the ball slide down the lane before it begins rolling without slipping?

(a) Apply conditions for rolling; see Problem 9-108	$v = v_0 + \mu_k gt$; $\omega R = 3v_0 - (5/2)\mu_k gt = v$
(a) and (b) Find t and v	$t = 2v_0/3.5\mu_k g$; $v = 1.57v_0$
(c) $s = v_{av}t = \frac{1}{2}(v + v_0)t$	$s = 0.735v_0^2/\mu_k g$

109* ·· A solid cylinder of mass M resting on its side on a horizontal surface is given a sharp blow by a cue stick. The applied force is horizontal and passes through the center of the cylinder so that the cylinder begins translating with initial velocity v_0. The coefficient of sliding friction between the cylinder and surface is μ_k. (a) What is the translational velocity of the cylinder when it is rolling without slipping? (b) How far does the cylinder travel before it rolls without slipping? (c) What fraction of its initial mechanical energy is dissipated in friction?

This Problem is identical to Example 9-16 except that now $I = \frac{1}{2}MR^2$. Follow the same procedure.

(a) Set $\omega R = v$; $v = v_0 - \mu_k gt$; $\omega R = 2\mu_k gt$	$t = v_0/3\mu_k g$; $v = (2/3)v_0$
(b) $s = v_{av}t$	$s = 5v_0^2/18\mu_k g$
(c) $W_{fr}/K_i = (K_i - K_f)/K_i$	$K_i = \frac{1}{2}mv_0^2$; $K_f = (3/4)mv^2 = (1/3)mv_0^2$; $W_{fr}/K_i = 1/3$

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110 · The torque exerted on an orbiting communications satellite by the gravitational pull of the earth is (a) directed toward the earth. (b) directed parallel to the earth's axis and toward the north pole. (c) directed parallel to the earth's axis and toward the south pole. (d) directed toward the satellite. (e) zero.

(e)

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111 · The moon rotates as it revolves around the earth so that we always see the same side. Use this fact to find the angular velocity (in rad/s) of the moon about its axis. (The period of revolution of the moon about the earth is 27.3 days.)

$\omega = 1/27.3$ rev/day $= 2\pi/(27.3 \times 24 \times 60 \times 60)$ rad/s $= 2.7 \times 10^{-6}$ rad/s.

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112 · Find the moment of inertia of a hoop about an axis perpendicular to the plane of the hoop and through its edge.

Use the parallel axis theorem $\quad I = MR^2 + MR^2 = 2MR^2$

113* ·· The radius of a park merry-go-round is 2.2 m. To start it rotating, you wrap a rope around it and pull with a force of 260 N for 12 s. During this time, the merry-go-round makes one complete rotation. (a) Find the angular acceleration of the merry-go-round. (*b*) What torque is exerted by the rope on the merry-go-round? (*c*) What is the moment of inertia of the merry-go-round?

(*a*) $\alpha = 2\theta/t^2$	$\alpha = 4\pi/12^2$ rad/s^2 = 0.0873 rad/s^2
(*b*) $\tau = Fr$	$\tau = (260 \times 2.2)$ N·m = 572 N·m
(*c*) $I = \tau/\alpha$	$I = (572/0.0873)$ kg·m^2 = 6552 kg·m^2

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114 ·· A uniform disk of radius 0.12 m and mass 5 kg is pivoted such that it rotates freely about its central axis (Figure 9-62). A string wrapped around the disk is pulled with a force of 20 N. (*a*) What is the torque exerted on the disk? (*b*) What is the angular acceleration of the disk? (*c*) If the disk starts from rest, what is its angular velocity after 5 s? (*d*) What is its kinetic energy after 5 s? (*e*) What is the total angle θ that the disk turns through in 5 s? (*f*) Show that the work done by the torque $\tau\Delta\theta$ equals the kinetic energy.

(*a*) $\tau = FR$	$\tau = (20 \times 0.12)$ N·m = 2.4 N·m
(*b*) $\alpha = \tau/I$; $I = \frac{1}{2}MR^2$	$\alpha = 2\tau/MR^2 = 66.7$ rad/s^2
(*c*) $\omega = \alpha t$	$\omega = 333$ rad/s
(*d*) $K = \frac{1}{2}I\omega^2$	$K = (\frac{1}{2} \times 0.036 \times 333^2)$ J = 2000 J
(*e*) $\theta = \frac{1}{2}\alpha t^2$	$\theta = (\frac{1}{2} \times 66.7 \times 5^2)$ rad = 833 rad
(*f*) Express K in terms of τ and θ	$K = \frac{1}{2}(\tau/\alpha)(\alpha t)^2 = \frac{1}{2}\alpha\tau t^2 = \tau\theta$; Q.E.D.

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115 ·· A 0.25-kg rod of length 80 cm is suspended by a frictionless pivot at one end. It is held horizontal and released. Immediately after it is released, what is (*a*) the acceleration of the center of the rod, and (*b*) the initial acceleration of a point on the end of the rod? (*c*) Find the linear velocity of the center of mass of the rod when it is vertical.

(*a*) 1. Find τ and I about the pivot	$\tau = (0.25 \times 9.81 \times 0.4)$ N·m = 0.981 N·m
	$I = (0.25 \times 0.8^2/3)$ kg·m^2 = 0.0533 kg·m^2
2. Find α and $a = \alpha l = \alpha L/2$	$\alpha = \tau/I = 18.4$ rad/s^2; $a_{cm} = (18.4 \times 0.4)$ m/s^2 = 7.36 m/s^2
(*b*) $a_{end} = L\alpha$	$a_{end} = (18.4 \times 0.8)$ m/s^2 = 14.7 m/s^2
(*c*) 1. Use energy conservation; $\frac{1}{2}I\omega^2 = mg\Delta h$	$\omega = (2 \times 0.25 \times 9.81 \times 0.4/0.0533)^{1/2}$ rad/s = 6.07 rad/s
2. $v = R\omega = \frac{1}{2}L\omega$	$v = (0.4 \times 6.07)$ m/s = 2.43 m/s

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116 ·· A uniform rod of length 3*L* is pivoted as shown in Figure 9-63 and held in a horizontal position. What is the initial angular acceleration α of the rod upon release?

1. The CM is 0.5*L* from the support; find τ and I	$\tau = 0.5mgL$; $I = m(3L)^2/12 + m(0.5L)^2 = mL^2$
2. $\alpha = \tau/I$	$\alpha = 0.5mgL/mL^2 = 0.5g/L$

117* ·· A uniform rod of length *L* and mass *m* is pivoted at the middle as shown in Figure 9-64. It has a load of mass 2*m* attached to one of the ends. If the system is released from a horizontal position, what is the maximum velocity of the load?

1. Find I	$I = mL^2/12 + 2mL^2/4 = 7mL^2/12$
2. $\frac{1}{2}I\omega^2 = 2mgL/2$; $v = \omega L/2$; solve for v	$v = (2mgL/I)^{1/2}(L/2) = (6gL/7)^{1/2}$

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118 ·· A marble of mass M and radius R rolls without slipping down the track on the left from a height h_1 as shown in Figure 9-65. The marble then goes up the *frictionless* track on the right to a height h_2. Find h_2.

1. Find K at the bottom; find v^2 at the bottom	$K = Mgh_1 = \frac{1}{2}Mv^2 + (1/5)Mv^2$; $v^2 = 10gh_1/7$
2. There is no friction; so $v^2 = 2gh_2$	$h_2 = v^2/2g = 5h_1/7$

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119 ·· A uniform disk with a mass of 120 kg and a radius of 1.4 m rotates initially with an angular speed of 1100 rev/min. (*a*) A constant tangential force is applied at a radial distance of 0.6 m. What work must this force do to stop the wheel? (*b*) If the wheel is brought to rest in 2.5 min, what torque does the force produce? What is the mag-nitude of the force? (*c*) How many revolutions does the wheel make in these 2.5 min?

(*a*) Find K_i; $W = K_i$	$W = \frac{1}{2}I\omega^2 = [\frac{1}{2}(\frac{1}{2} \times 120 \times 1.4^2)(1100 \times 2\pi/60)^2]$ J $= 780$ kJ
(*b*) Use $P_{av} = \tau\omega_{av} = W/t$;	$\tau = [780 \times 10^3/2.5 \times 60 \times \frac{1}{2} \times (1100 \times 2\pi/60)]$ N·m $= 90.4$ N·m
$F = \tau/R$	$F = (90.4/0.6)$ N $= 150.7$ N
(*c*) $\theta = \omega_{av}t$	$\theta = [2.5 \times 60 \times \frac{1}{2}(1100/60)]$ rev $= 1375$ rev

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120 ·· A park merry-go-round consists of a 240-kg circular wooden platform 4.00 m in diameter. Four children running alongside push tangentially along the platform's circumference until, starting from rest, the merry-go-round reaches a steady speed of one complete revolution every 2.8 s. (*a*) If each child exerts a force of 26 N, how far does each child run? (*b*) What is the angular acceleration of the merry-go-round? (*c*) How much work does each child do? (*d*) What is the kinetic energy of the merry-go-round?

(*a*) Use energy conservation; $K_f = 4Fs = \frac{1}{2}I\omega^2$	$s = I\omega^2/8F = [\frac{1}{2} \times 240 \times 4 \times (2\pi/2.8)^2/8 \times 26]$ m $= 11.6$ m
(*b*) $\alpha = \tau/I$; $\tau = 4FR$	$\alpha = (4 \times 26 \times 2/480)$ rad/s$^2 = 0.433$ rad/s^2
(*c*) W per child $= Fs$	$W = (26 \times 11.6)$ J $= 302$ J
(*d*) $K = 4Fs$	$K = 1208$ J

121* ·· A hoop of mass 1.5 kg and radius 65 cm has a string wrapped around its circumference and lies flat on a horizontal frictionless table. The string is pulled with a force of 5 N. (*a*) How far does the center of the hoop travel in 3 s? (*b*) What is the angular velocity of the hoop about its center of mass after 3 s?

(*a*) $F_{net} = F = ma_{cm}$; $s = \frac{1}{2}a_{cm}t^2 = Ft^2/2m$	$s = (5 \times 3^2/2 \times 1.5)$ m $= 15$ m
(*b*) $\alpha = \tau/I$; $\omega = \alpha t = FRt/mR^2 = Ft/mR$	$\omega = (5 \times 3/1.5 \times 0.65)$ rad/s $= 15.4$ rad/s

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122 ·· A vertical grinding wheel is a uniform disk of mass 60 kg and radius 45 cm. It has a handle of radius 65 cm of negligible mass. A 25-kg load is attached to the handle when it is in the horizontal position. Neglecting friction, find (*a*) the initial angular acceleration of the wheel, and (*b*) the maximum angular velocity of the wheel.

(*a*) Find I and τ; $I = \frac{1}{2}MR^2 + mr^2$; $\tau = mgr$; $\alpha = \tau/I$.	$I = (\frac{1}{2} \times 60 \times 0.45^2 + 25 \times 0.65^2)$ kg·m$^2 = 16.64$ kg·m^2
Here $M = 60$ kg, $m = 25$ kg, $R = 0.45$ m,	$\tau = (25 \times 9.81 \times 0.65)$ N·m $= 159.4$ N·m; $\alpha = 9.58$ rad/s^2

$r = 0.65$ m.

(b) Use energy conservation; $mgr = \frac{1}{2}I\omega^2$ $\omega = [2 \times 25 \times 9.81 \times 0.65/16.64]^{1/2}$ rad/s = 4.38 rad/s

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123 ·· In this problem, you are to derive the perpendicular-axis theorem for planar objects, which relates the moments of inertia about two perpendicular axes in the plane of Figure 9-66 to the moment of inertia about a third axis that is perpendicular to the plane of figure. Consider the mass element dm for the figure shown in the xy plane. (a) Write an expression for the moment of inertia of the figure about the z axis in terms of dm and r. (b) Relate the distance r of dm to the distances x and y, and show that $I_z = I_y + I_x$. (c) Apply your result to find the moment of inertia of a uniform disk of radius R about a diameter of the disk.

(a), (b) $I_z = \int r^2 dm = \int (x^2 + y^2) dm = \int x^2 dm + \int y^2 dm = I_y + I_x$.

(c) Let the z axis be the axis of rotation of the disk. By symmetry, $I_x = I_y$. So $I_x = \frac{1}{2}I_z = (1/4)MR^2$.

(see Table 9-1)

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124 ·· A uniform disk of radius R and mass M is pivoted about a horizontal axis parallel to its symmetry axis and passing through its edge such that it can swing freely in a vertical plane (Figure 9-67). It is released from rest with its center of mass at the same height as the pivot. (a) What is the angular velocity of the disk when its center of mass is directly below the pivot? (b) What force is exerted by the pivot at this time?

(a) Use energy conservation; $\frac{1}{2}I\omega^2 = Mgh = Mgr$ $I = \frac{1}{2}Mr^2 + Mr^2 = 3Mr^2/2$; $\omega = \sqrt{4g/3r}$ rad/s

(b) $F = Mg + Mr\omega^2$ $F = Mg + 4Mg/3 = 7Mg/3$

125* ·· A spool of mass M rests on an inclined plane at a distance D from the bottom. The ends of the spool have radius R, the center has radius r, and the moment of inertia of the spool about its axis is I. A long string of negligible mass is wound many times around the center of the spool. The other end of the string is fastened to a hook at the top of the inclined plane such that the string always pulls parallel to the slope as shown in Figure9-68. (a) Suppose that initially the slope is so icy that there is *no* friction. How does the spool move as it slips down the slope? Use energy considerations to determine the speed of the center of mass of the spool when it reaches the bottom of the slope. Give your answer in terms of $M, I, r, R, g, D,$ and θ. (b) Now suppose that the ice is gone and that when the spool is set up in the same way, there is enough friction to keep it from slipping on the slope. What is the direction and magnitude of the friction force in this case?

(a) The spool will move down the plane at constant acceleration, spinning in a counterclockwise direction as string unwinds. From energy conservation, $MgD \sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$; $v = r\omega$.

$$v = \sqrt{\frac{2MgD\sin\theta}{M + I/r^2}} .$$

(b) 1. The direction of the friction force is up along the plane

2. Since $a_{cm} = 0$ and $\alpha = 0$, $F_{net} = 0$ and $\tau = 0$ $Mg \sin\theta = T + f_s$; $Tr = f_s R$

3. Solve for f_s $f_s = (Mg \sin\theta)/(1 + R/r)$

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126 ·· Ian has suggested another improvement for the game of hockey. Instead of the usual two-minute penalty, he would like to see an offender placed in a barrel at mid-ice and then spun in a circle by the other team. When the offender is silly with dizziness, he is put back into the game. Assume that a penalized player in a barrel approximates a uniform, 100-kg cylinder of radius 0.60 m, and that the ice is smooth (Figure 9-69). Ropes are wound around the barrel, so that pulling them causes rotation. If two players simultaneously pull the ropes with forces of 40 N and 60 N for 6 s, describe the motion of the barrel. Give its acceleration, velocity, and the position of its center of mass as functions of time.

The barrel will translate to the right and rotate as indicated in the figure. We first consider $t \leq 6$ s.

1. $a_{cm} = F_{net}/m$; $v_{cm} = a_{cm}t$; $x_{cm} = 1/2 a_{cm}t^2$ — $a_{cm} = 0.2$ m/s^2; $v_{cm} = 0.2t$ m/s; $x_{cm} = 0.1t^2$ m
2. $a = t/I$; $w = at$; $q = 1/2 at^2$ — $t = 100 \times 0.6$ N·m = 60 N·m; $I = 50 \times 0.6^2$ kg·m^2 = 18 kg·m^2; $a = 3.33$ rad/s^2; $w = 3.33t$ rad/s; $q = 1.67t^2$ rad

For $t > 6$ s: $a_{cm} = a = 0$; $v_{cm} = 1.2$ m/s; $x_{cm} = [3.6 + 1.2(t - 6)]$ m; $w = 20$ rad/s; $q = [60 + 20(t - 6)]$ rad.

127 ·· A solid metal rod 1.5 m long is free to rotate without friction about a fixed, horizontal axis perpendicular to the rod and passing through one end. The other end is held in a horizontal position. Small coins of mass m are placed on the rod 25 cm, 50 cm, 75 cm, 1 m, 1.25 m, and 1.5 m from the bearing. If the free end is now released, calculate the initial force exerted on each coin by the rod. Assume that the mass of the coins may be neglected in comparison to the mass of the rod.

1. Determine a; $a = t/I$; $I = ML^2/3$ — $a = (MgL/2)/(ML^2/3) = 3g/2L = |g|$ rad/s^2
2. Determine $a(x)$, where x = distance from pivot — $a(x) = gx$
3. $ma = mg - F$; $F = m(g - a)$ — $F(0.25) = 0.75mg$; $F(0.50) = 0.5mg$; $F(0.75) = 0.25mg$; $F(1.0) = F(1.25) = F(1.5) = 0$

128 ·· A thin rod of length L and mass M is supported in a horizontal position by two strings, one attached to each end as shown in Figure 9-70. If one string is cut, the rod begins to rotate about the point where it connects to the other string (point A in the figure). (*a*) Find the initial acceleration of the center of mass of the rod. (*b*) Show that the initial tension in the string is $mg/4$ and that the initial angular acceleration of the rod about an axis through the point A is $3g/2L$. (*c*) At what distance from point A is the initial linear acceleration equal to g?

It is tempting to assume that the tension in the string above A is the same as before the other string is cut, namely $Mg/2$. However, the tension can change instantaneously. What cannot change instantaneously due to its inertia is the position of the rod. Thus point A is momentarily fixed.

(*a*), (*b*), and (*c*) From Problem 9-127 we have $a = 3g/2L$. The center of mass of the rod is at a distance $L/2$ from A; consequently, $a_{cm} = aL/2 = 3g/4$. Now $Ma_{cm} = Mg - T$, and solving for T one obtains $T = Mg/4$. To find the distance from A where $a = g$, set $ax = g$ and solve for x: $x = g/a = 2L/3$.

129* ·· Figure 9-71 shows a hollow cylinder of length 1.8 m, mass 0.8 kg, and radius 0.2 m. The cylinder is free to rotate about a vertical axis that passes through its center and is perpendicular to the cylinder's axis. Inside the cylinder are two masses of 0.2 kg each, attached to springs of spring constant k and unstretched lengths 0.4 m. The inside walls of the cylinder are frictionless. (*a*) Determine the value of the spring constant if the masses are located 0.8 m from the center of the cylinder when the cylinder rotates at 24 rad/s. (*b*) How much work was needed to bring the system from $w = 0$ to $w = 24$ rad/s?

Let $m = 0.2$ kg mass, $M = 0.8$ kg mass of cylinder, $L = 1.8$ m, and x = distance of m from center = $x_0 + \Delta x$.

(a) We have $k\Delta x = m(x_0 + \Delta x)\omega^2$; solve for k — $k = (0.2 \times 0.8 \times 24^2/0.4)$ N/m = 230.4 N/m

(b) $K = K_{rot} + 1/2k\Delta x^2$; determine I of system when $x = 0.8$ m — $I_M = 1/2Mr^2 + ML^2/12 = 0.232$ kg·m^2

$I_{2m} = 2(mr^2/4 + mx^2) = 0.13$ kg·m^2; $I = 0.362$ kg·m^2

Evaluate $K = 1/2I\omega^2 + 1/2k\Delta x^2 = W$ — $W = (1/2 \times 0.362 \times 24^2 + 1/2 \times 230.4 \times 0.4^2)$ J = 122.7 J

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130 ·· Suppose that for the system described in Problem 129, the spring constants are each $k = 60$ N/m. The system starts from rest and slowly accelerates until the masses are 0.8 m from the center of the cylinder. How much work was done in the process?

1. Proceed as in Problem 9-129a and find ω — $\omega = [(60 \times 0.4)/(0.2 \times 0.8)]^{1/2}$ rad/s = 12.25 rad/s

2. Determine W as in Problem 9-129b — $W = (1/2 \times 0.362 \times 12.25^2 + 1/2 \times 60 \times 0.4^2)$ J = 32 J

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131 ·· A string is wrapped around a uniform cylinder of radius R and mass M that rests on a horizontal frictionless surface. The string is pulled horizontally from the top with force F. (a) Show that the angular acceleration of the cylinder is twice that needed for rolling without slipping, so that the bottom point on the cylinder slides backward against the table. (b) Find the magnitude and direction of the frictional force between the table and cylinder needed for the cylinder to roll without slipping. What is the acceleration of the cylinder in this case?

(a) The only force is F; therefore, $a_{cm} = F/M$. The torque about the center of mass is $\tau = FR$ and $I = 1/2MR^2$. Thus $\alpha = \tau/I = 2F/MR$. If the cylinder rolls without slipping, $a_{cm} = \alpha R$. Here, $\alpha = 2a_{cm}/R$.

(c) Take the point of contact with the floor as the "pivot" point. The torque about that point is $\tau = 2FR$ and the moment of inertia about that point is $I = 1/2MR^2 + MR^2 = 3MR^2/2$. Thus, $\alpha = \tau/I = 4F/3MR$, and the linear acceleration of the center of the cylinder is $\alpha R = a_{cm} = 4F/3M$. But $Ma_{cm} = F + f$, where f is the frictional force. We find that the frictional force is $f = F/3$, and is in the same direction as F.

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132 ·· Figure 9-72 shows a solid cylinder of mass M and radius R to which a hollow cylinder of radius r is attached. A string is wound about the hollow cylinder. The solid cylinder rests on a horizontal surface. The coefficient of static friction between the cylinder and surface is μ_s. If a light tension is applied to the string in the vertical direction, the cylinder will roll to the left; if the tension is applied with the string horizontally, the cylinder rolls to the right. Find the angle of the string with the horizontal that will allow the cylinder to remain stationary when a small tension is applied to the string.

1. First, we note that if the tension is small, then there can be no slipping, and the system must roll.

2. Now consider the point of contact of the cylinder with the surface as the "pivot" point. If τ about that point is zero, the system will not roll. This will occur if the line of action of the tension passes through the pivot point. We see from the figure that the angle θ is given by $\theta = \cos^{-1}(r/R)$.

133* ··· A heavy, uniform cylinder has a mass m and a radius R (Figure 9-73). It is accelerated by a force $\boldsymbol{T}$, which is applied through a rope wound around a light drum of radius r that is attached to the cylinder. The coefficient of static friction is sufficient for the cylinder to roll without slipping. (a) Find the frictional force. (b) Find the acceleration a of the center of the cylinder. (c) Is it possible to choose r so that a is greater than T/m? How?

(d) What is the direction of the frictional force in the circumstances of part (c)?

(a) 1. Write the equations for translation and rotation — $T + f = ma$ (1)

$Tr - fR = Ia = \frac{1}{2}mRa$ (2)

2. Solve (2) for f — $f = Tr/\mathrm{R} - \frac{1}{2}ma$
3. Use (3) in (1) to find a — $a = (2T/3m)(1 + r/R)$ (4)
4. Use (4) in (3) to find f in terms of T, r, and R — $f = (T/3)(2r/R - 1)$

(*b*) See Equ. (4) above — Note: for $r = R$, results agree with Problem 9- 131*b*

(*c*) Find r so that $a > T/m$ — From Equ. (4) above, $a > T/m$ if $r > \frac{1}{2}R$

(*d*) If $r > \frac{1}{2}R$ then $f > 0$, i.e., in the direction of T

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134 ··· A uniform stick of length L and mass M is hinged at one end. It is released from rest at an angle q_0 with the vertical. Show that when the angle with the vertical is q, the hinge exerts a force F_r along the stick and a force F_t perpendicular to the stick given by $F_r = \frac{1}{2}Mg(5 \cos q - 3 \cos q_0)$ and $F_t = (Mg/4) \sin q$.

The system is shown in the drawing in two positions, with angles q_0 and q with the vertical. We also show all the forces that act on the stick. These forces result in a rotation of the stick—and its center of mass—about the pivot, and a tangential acceleration of the center of mass given by $a_t = \frac{1}{2}La$. As the stick's angle changes from q_0 to q, its potential energy decreases by Mgh, where h is the distance the center of mass falls. Using energy conservation and $I = ML^2/3$, we obtain $\frac{1}{2}(ML^2/3)w^2 = (MgL/2)(\cos q - \cos q_0)$. Thus we have $w^2 = (3g/L)(\cos q - \cos q_0)$. The centripetal force that must act radially on the center of mass is $\frac{1}{2}MLw^2$. This is part of the radial component of the force at the pivot. In addition to the centripetal force, gravity also acts on the center of mass. The radial component of $M\mathbf{g}$ is $Mg \cos q$. Hence the total radial force at the hinge is $F_r = \frac{1}{2}ML(3g/L)(\cos q - \cos q_0) + Mg \cos q = \frac{1}{2}Mg(5 \cos q - 3 \cos q_0)$. The mass M times the tangential acceleration of the center of mass must equal the sum of the tangential component of $M\mathbf{g}$ and the tangential component of the force at the pivot. The tangential acceleration of the center of mass is $a_t = \frac{1}{2}La$, where $a = t/I = (\frac{1}{2}MgL \sin q)/(ML^2/3) = (3g \sin q)/2L$. Thus, $a_t = (3/4)g \sin q = g \sin q + F_t/M$, which gives $F_t = -(1/4)Mg \sin q$. Here the minus sign indicates that the force F_t is directed opposite to the tangential component of $M\mathbf{g}$.