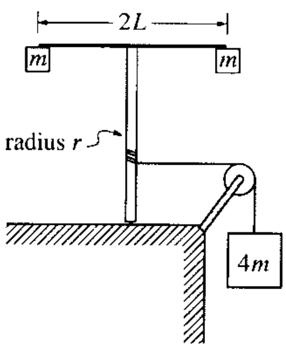
AP PHYSICS C – HOMEWORK 7, ROTATIONAL MECHANICS (due 12/8-9/2014)

(Show work and answers on other sheets of paper to turn in)

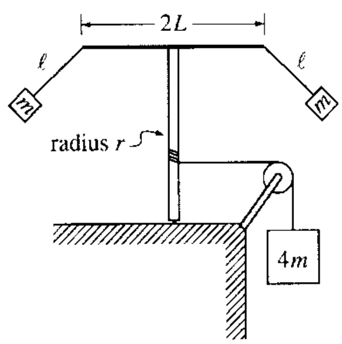
1. Starting from rest, a disk takes 20 revolutions to reach an angular velocity ω (at constant angular acceleration). How many additional revolutions are needed to reach an angular velocity of 2ω?
2. A record is rotating at a rate of 30 revolutions per minute when it begins to slow to a stop at a constant angular acceleration. If it takes 20 seconds to come to rest,
    a. What is the angular acceleration, α?
    b. What is the average angular velocity?
    c. How many revolutions does the record make while stopping?
3. Four particles are at the corners of a square-shaped, massless wire with sides of length 2 meters. Two of the particles are 3kg and on opposite corners, while the remaining particles are 5kg and on the remaining two opposite corners. Find the moment of inertia around an axis perpendicular to the plane of the square that,
    a. passes through one of the corners with a 5 kg mass.
    b. passes through the center of mass.
    c. passes through one of the corners with a 3 kg mass.
4. Show that $I = ½ M(R_1^2 + R_2^2)$ about an axis through the center of a ring formed from a cylinder of height h and radius R_2 that has had another cylinder of smaller radius, R_1 removed from it (thus, the ring is a hollow cylinder). You can assume the ring has a mass of M and a uniform mass density. (note: you can find the result through integration or using the fact that $I = ½ mr^2$ for a solid cylinder of mass m and radius r).
5. A solid, circular disk of mass M and radius R has a non-uniform mass density that is a function of the distance from the center given by, $\sigma(r) = \sigma_0(1+r/R)$. In terms of M and R, find the moment of inertia about an axis through the center of the disk.
6. A wagon wheel 2 meters in diameter consists of a thin rim of mass 10kg and six 1.5 kg spokes. Determine the moment of inertia of the wagon wheel about its axis.
7. A wheel is on an axis that is <u>not</u> frictionless. Starting from rest, a constant torque of 100 Nm is applied to the wheel for 10 seconds, giving it an angular velocity of 60 rad/s. The torque is then removed, and the wheel comes to rest 80 seconds later.
    a. Find the moment of inertia of the wheel about the axis.
    b. Find the torque due to friction (which is constant).
8. A uniform rod of mass M and length L hangs vertically by a frictionless pivot at one end. It is struck by a horizontal force, F, for a short time, t, at a distance of x below the pivot.
    a. Show that the speed of the center of mass of the rod just after being struck is $v = 3Fxt/2ML$.
    b. Find the horizontal component of the force delivered by the pivot and show that it is zero if $x = 2L/3$.
9. A uniform cylinder of mass M and radius R has a string wrapped around it as a make-shift yo-yo. The end of the string is held in place and the yo-yo is allowed to fall straight down, with the string unwinding.
    a. Show that the acceleration of the cylinder is downward with $a = 2g/3$.
    b. Find the tension in the string.
10. A loop of radius .5 m and mass of .8 kg rolls without slipping at a speed of 10 m/s toward an incline of slope 30°. What distance up the incline will the hoop roll (assuming it rolls without slipping)?
11. A thin ring of mass M and radius R and a solid disk of the same mass and radius are placed at the top of an inclined ramp. If they are released at the same time and roll without slipping,
    a. which gets to the bottom first?
    b. how do their total kinetic energies compare when they reach the bottom of the incline?
    c. what fraction of the kinetic energy of the ring is due to rotation?
    d. what fraction of the kinetic energy of the disk is due to rotation?
12. An Atwood's machine has two objects of mass 4 kg and 6 kg connected by a massless string that passes over a pulley consisting of a uniform disk of mass 2 kg and radius .1 m. The string does not slip on the pulley.
    a. Find the magnitude of the acceleration of the objects.
    b. What is the tension in the string supporting the 4 kg mass?
    c. What is the tension in the string supporting the 6 kg mass?

13. A solid disk mass 2 kg and radius 2 m is given a horizontal push of 20N at a point .3 m above its center.
    a. What is the minimum μ_s between the disk and the floor to allow rolling without slipping?
    b. If the disk rolls without slipping, what is,
        i. the acceleration of the center of mass
        ii. the linear speed of the center of mass after 5 seconds
        iii. the total kinetic energy after 5 seconds
    c. If μ_s = .2 and μ_k = .1, what is,
        i. the acceleration of the center of mass?
        ii. the linear speed of the center of mass after 5 seconds
        iii. the total kinetic energy after 5 seconds

14. A bowling ball of mass M and radius R is thrown such that at the instant it touches the floor it is moving horizontally with a speed v_0 and is not rotating. It slides for a time t_1 a distance s_1 before it rolls without slipping and moves with a speed of v_1.
    a. If μ_k is the coefficient of sliding friction between the ball and floor, find t_1, s_1, and v_1.
    b. Let μ_k = .06 and v_0 = 8 m/s. Now find find t_1, s_1, and v_1.
    c. Find the ratio of the final kinetic energy to the initial kinetic energy for the ball.



Experiment A



Experiment B

15. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length 2L, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.
    a. Determine the rotational inertia of the rod-and-block apparatus attached to the pole.
    b. Determine the downward acceleration of the large block.
    c. When the large block has descended a distance *D*, how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD*? (greater than, less than, or equal to)

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

d. When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c.? (greater than, less than, or equal to that in C before)?

Answers

1. $160\pi/\omega$
2. (a) $-\pi/20$ rad/s^2 (b) $\pi/2$ rad/s (c) 5 revolutions
3. (a) 64 kgm^2 (b) 32 kgm^2 (c) 64 kgm^2
4. N/A
5. (27/50) MR^2
6. 13 kgm^2
7. (a) 14.8 kgm^2 (b) 11.1 Nm
8. (a) N/A (b) $F(3x/2L - 1)$
9. (a) $2g/3$ (b) $Mg/3$
10. 20 m along incline
11. (a) disk (b) same (c) ½ (d) 1/3
12. (a) 2/11 g or 1.8 m/s^2 (b) 47.2 N (c) 49.2 N (using g = 10 m/s^2)
13. (a) .24 (b) i. 7.7 m/s^2 ii. 38 m/s iii. 2200 J (c) i. 9 m/s^2 ii. 45 m/s iii. 2340 J
14. (a) $(2/7)\, v_0/g\mu_k$, $(12/49)v_0^2/g\mu_k$, $(5/7)v_0$ (b) 3.9 s, 26.7 m, 5.7 m/s^2 (c) .51
15. (a) $2mL^2$ (b) $2gr^2/(L^2 + 2r^2)$ (c) equal to 4mgD (d) less than