Assignment-5

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***Abstract*—This document explains the concept of finding the equation of circle using linear algebra.**

Download all python codes from

https://github.com/vipulmalik8569/MT−EE5609

and latex-tikz codes from

https://github.com/vipulmalik8569/MT−EE5609

1 Problem

Find the equation of circle passing through the points

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{x_3} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Vector form of the equation of circle is :

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{x}^T\mathbf{u} + f = 0 \tag{2.0.1}$$

For $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ equation (2.0.1) can be written as:

$$\mathbf{x_1}^T\mathbf{x_1} + 2\mathbf{x_1}^T\mathbf{u} + f = 0 \tag{2.0.2}$$

$$\mathbf{x_2}^T\mathbf{x_2} + 2\mathbf{x_2}^T\mathbf{u} + f = 0 \tag{2.0.3}$$

$$\mathbf{x_3}^T\mathbf{x_3} + 2\mathbf{x_3}^T\mathbf{u} + f = 0 \tag{2.0.4}$$

In matrix form this can be written as :

$$\begin{pmatrix} 2\mathbf{x_1}^T & 1 \\ 2\mathbf{x_2}^T & 1 \\ 2\mathbf{x_3}^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\mathbf{x_1}^T\mathbf{x_1} \\ -\mathbf{x_1}^T\mathbf{x_1} \\ -\mathbf{x_3}^T\mathbf{x_3} \end{pmatrix} \tag{2.0.5}$$

By putting the values of $\mathbf{x_1}, \mathbf{x_2}$ and $\mathbf{x_3}$ from (1.0.1) in (2.0.5) we get :

$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & -2 & 1 \\ 16 & 4 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -68 \end{pmatrix} \tag{2.0.6}$$

Using Gaussian Elimination method :

$$\xleftrightarrow[R_2 \leftarrow R_2 - 4R_1]{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 \\ 0 & -6 & -1 & -1 \\ 16 & 4 & 1 & -68 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 16R_1} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 \\ 0 & -6 & -1 & -1 \\ 0 & -12 & -7 & -52 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 + 124R_2]{R_2 \leftarrow -\frac{1}{6}R_2} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & -5 & -50 \end{pmatrix} \tag{2.0.9}$$

Using (2.0.6) and (2.0.9) we get :

$$\begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\frac{9}{2} \\ -\frac{3}{2} \\ 10 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{u} = \begin{pmatrix} -\frac{9}{2} \\ -\frac{3}{2} \end{pmatrix} \tag{2.0.11}$$

$$f = 10 \tag{2.0.12}$$

By putting the values of $\mathbf{u}$ and f in (2.0.1) we get :

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -\frac{9}{2} \\ -\frac{3}{2} \end{pmatrix}^T \mathbf{x} + 10 = 0 \tag{2.0.13}$$

$$x^2 + y^2 - 9x - 6y + 10 = 0 \tag{2.0.14}$$

Plot of the circle given by equation (2.0.14) is as follows :

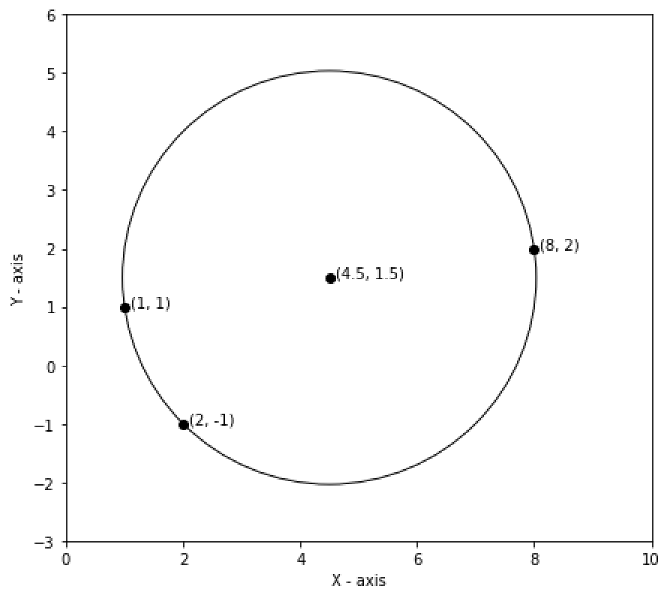

Fig. 0: A circle centered at $(4.5, 1.5)$ with radius 3.53.