

Assignment-5

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Abstract—This document explains the concept of finding the equation of circle using linear algebra.

Download all python codes from

<https://github.com/vipulmalik8569/MT-EE5609>

and latex-tikz codes from

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Using Gaussian Elimination method :

$$\begin{array}{c} \xleftrightarrow{R_1 \leftarrow -\frac{1}{2}R_1} \\ \xleftrightarrow{R_2 \leftarrow R_2 - 4R_1} \end{array} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 \\ 0 & -6 & -1 & -1 \\ 16 & 4 & 1 & -68 \end{pmatrix} \quad (2.0.7)$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 16R_1} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 \\ 0 & -6 & -1 & -1 \\ 0 & -12 & -7 & -52 \end{pmatrix} \quad (2.0.8)$$

$$\begin{array}{c} \xleftrightarrow{R_2 \leftarrow -\frac{1}{6}R_2} \\ \xleftrightarrow{R_3 \leftarrow R_3 + 12R_2} \end{array} \begin{pmatrix} 1 & 1 & \frac{1}{2} & -1 \\ 0 & 1 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & -5 & -50 \end{pmatrix} \quad (2.0.9)$$

Using (2.0.6) and (2.0.9) we get :

$$\begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\frac{9}{2} \\ -\frac{3}{2} \\ 10 \end{pmatrix} \quad (2.0.10)$$

$$\mathbf{u} = \begin{pmatrix} -\frac{9}{2} \\ -\frac{3}{2} \end{pmatrix} \quad (2.0.11)$$

$$f = 10 \quad (2.0.12)$$

By putting the values of \mathbf{u} and f in (2.0.1) we get :

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} -\frac{9}{2} \\ -\frac{3}{2} \end{pmatrix}^T \mathbf{x} + 10 = 0 \quad (2.0.13)$$

$$x^2 + y^2 - 9x - 6y + 10 = 0 \quad (2.0.14)$$

Plot of the circle given by equation (2.0.14) is as follows :

1 PROBLEM

Find the equation of circle passing through the points

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

Vector form of the equation of circle is :

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{x}^T \mathbf{u} + f = 0 \quad (2.0.1)$$

For \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_3 equation (2.0.1) can be written as:

$$\mathbf{x}_1^T \mathbf{x}_1 + 2\mathbf{x}_1^T \mathbf{u} + f = 0 \quad (2.0.2)$$

$$\mathbf{x}_2^T \mathbf{x}_2 + 2\mathbf{x}_2^T \mathbf{u} + f = 0 \quad (2.0.3)$$

$$\mathbf{x}_3^T \mathbf{x}_3 + 2\mathbf{x}_3^T \mathbf{u} + f = 0 \quad (2.0.4)$$

In matrix form this can be written as :

$$\begin{pmatrix} 2\mathbf{x}_1^T & 1 \\ 2\mathbf{x}_2^T & 1 \\ 2\mathbf{x}_3^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -\mathbf{x}_1^T \mathbf{x}_1 \\ -\mathbf{x}_2^T \mathbf{x}_2 \\ -\mathbf{x}_3^T \mathbf{x}_3 \end{pmatrix} \quad (2.0.5)$$

By putting the values of \mathbf{x}_1 , \mathbf{x}_2 and \mathbf{x}_3 from (1.0.1) in (2.0.5) we get :

$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & -2 & 1 \\ 16 & 4 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -68 \end{pmatrix} \quad (2.0.6)$$

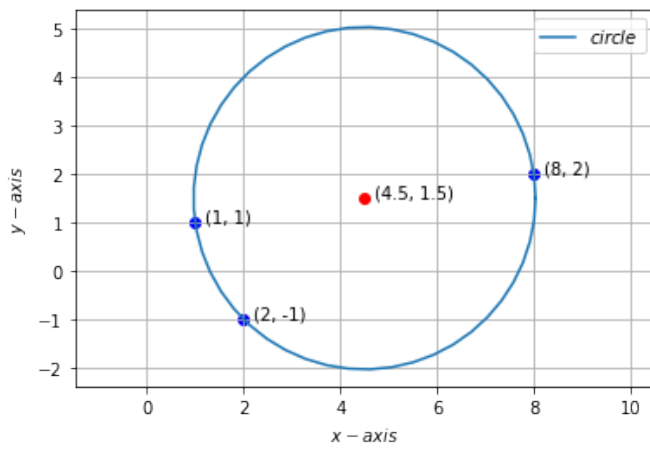


Fig. 0: A circle centered at (4.5, 1.5) with radius 3.53.