Assignment-6

Vipul Kumar Malik

***Abstract*—This document explains the concept of finding the equation of tangent to a circle using linear algebra.**

Download all python codes from

https://github.com/vipulmalik8569/MT−EE5609

and latex-tikz codes from

https://github.com/vipulmalik8569/MT−EE5609

1 Problem

Write down the equation of the tangent to a circle passing through the point $\mathbf{p}$.

Equation of the circle and positional vector $\mathbf{p}$ is given as :

$$x^2 + y^2 - 3x + 10y = 15 \tag{1.0.1}$$

$$\mathbf{p} = \begin{pmatrix} 4 \\ -11 \end{pmatrix} \tag{1.0.2}$$

2 Solution

General equation of the circle in vector form is :

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$

In the vector form (1.0.1) can be written as :

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} \frac{-3}{2} \\ 5 \end{pmatrix}^T \mathbf{x} - 15 = 0 \tag{2.0.2}$$

By comparing (2.0.1) and (2.0.2) we get :

$$\mathbf{u} = \begin{pmatrix} -\frac{3}{2} \\ 5 \end{pmatrix}, f = -15 \tag{2.0.3}$$

We know that the equation of tangent in the form of normal vector $(\mathbf{p} + \mathbf{u})$ and point $\mathbf{p}$ can be written as:

$$(\mathbf{p} + \mathbf{u})^T(\mathbf{x} - \mathbf{p}) = 0 \tag{2.0.4}$$

$$(\mathbf{p} + \mathbf{u})^T\mathbf{x} - \mathbf{p}^T\mathbf{p} - \mathbf{u}^T\mathbf{q} = 0 \tag{2.0.5}$$

Using (2.0.1), (2.0.5) will become :

$$(\mathbf{p} + \mathbf{u})^T\mathbf{x} + \mathbf{u}^T\mathbf{p} + f = 0 \tag{2.0.6}$$

By putting the values of $\mathbf{p}$, $\mathbf{u}$ and f from (2.0.3) in (2.0.6) we get :

$$\begin{pmatrix} \frac{5}{2} & -6 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 4 & -11 \end{pmatrix}\begin{pmatrix} \frac{-3}{2} \\ 5 \end{pmatrix} - 15 = 0 \tag{2.0.7}$$

$$\begin{pmatrix} \frac{5}{2} & -6 \end{pmatrix}\mathbf{x} - 76 = 0 \tag{2.0.8}$$

Hence the equation of the tangent to the circle passing through the point $\mathbf{p}$ is:

$$\begin{pmatrix} \frac{5}{2} & -6 \end{pmatrix}\mathbf{x} = 76 \tag{2.0.9}$$

Plot of the tangent to a circle given by equation (2.0.9) is as follows :

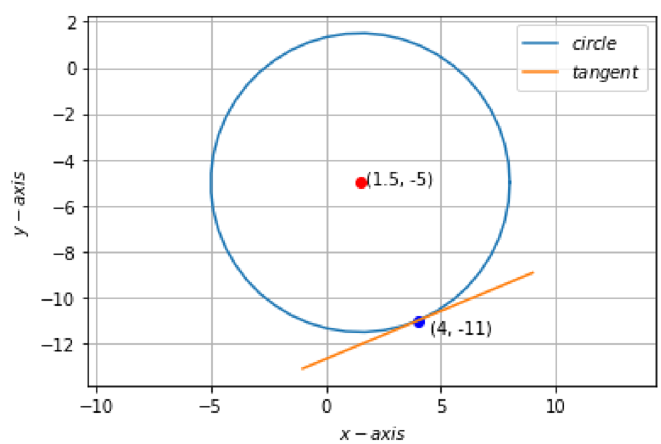

Fig. 0: Tangent to a circle centered at $(1.5, -5)$ with radius 6.5 passing through the point $(4, -11)$.