Assignment-5

Vipul Kumar Malik

Abstract—**This document explains the concept of finding the equation of circle using linear algebra.**

Download all python codes from

https://github.com/vipulmalik8569/MT−EE5609

and latex-tikz codes from

https://github.com/vipulmalik8569/MT−EE5609

1 Problem

Find the equation of circle passing through the points

$$\mathbf{x_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{x_3} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Vector form of the equation of circle with radius r and centered at $\mathbf{c}$ is :

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}^T\mathbf{x} + \mathbf{c}^T\mathbf{c} - r^2 = 0 \tag{2.0.1}$$

Where

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{c} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \tag{2.0.2}$$

For $\mathbf{x_1}$, $\mathbf{x_2}$ and $\mathbf{x_3}$ equation (2.0.1) can be written as:

$$\mathbf{x_1}^T\mathbf{x_1} - 2\mathbf{c}^T\mathbf{x_1} + \mathbf{c}^T\mathbf{c} - r^2 = 0 \tag{2.0.3}$$
$$\mathbf{x_2}^T\mathbf{x_2} - 2\mathbf{c}^T\mathbf{x_2} + \mathbf{c}^T\mathbf{c} - r^2 = 0 \tag{2.0.4}$$
$$\mathbf{x_3}^T\mathbf{x_3} - 2\mathbf{c}^T\mathbf{x_3} + \mathbf{c}^T\mathbf{c} - r^2 = 0 \tag{2.0.5}$$

In the matrix form this can be written as :

$$\begin{pmatrix} x_1 & y_1 & 1 & \mathbf{x_1}^T\mathbf{x_1} \\ x_2 & y_2 & 1 & \mathbf{x_2}^T\mathbf{x_2} \\ x_3 & y_3 & 1 & \mathbf{x_3}^T\mathbf{x_3} \end{pmatrix} \begin{pmatrix} -2x_0 \\ -2y_0 \\ x_0^2 + y_0^2 - r^2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$

By putting the values of $\mathbf{x_1}, \mathbf{x_2}$ and $\mathbf{x_3}$ in (2.0.6) we get :

$$\begin{pmatrix} 1 & 1 & 1 & 2 \\ 2 & -1 & 1 & 5 \\ 8 & 2 & 1 & 68 \end{pmatrix} \begin{pmatrix} -2x_0 \\ -2y_0 \\ x_0^2 + y_0^2 - r^2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

Using Gaussian Elimination method :

$$\xleftrightarrow[R_3 \leftarrow 8R_1 - R_3]{R_2 \leftarrow 2R_1 - R_2} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & -1 \\ 0 & 6 & 7 & -52 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{3}R_2} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 6 & 7 & -52 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow{R_3 \leftarrow 6R_2 - R_3} \begin{pmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -5 & 50 \end{pmatrix} \tag{2.0.10}$$

Solving for $\mathbf{c}$ and r using (2.0.10) we get :

$$\begin{pmatrix} -2x_0 \\ -2y_0 \\ x_0^2 + y_0^2 - r^2 \\ 1 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \\ 10 \\ 1 \end{pmatrix} \tag{2.0.11}$$

$$\mathbf{c} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} \frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \tag{2.0.12}$$

$$r^2 = 12.5 \tag{2.0.13}$$

Putting these values in (2.0.1), the equation of circle is as follows :

$$\left\| \mathbf{x} - \begin{pmatrix} \frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \right\|^2 - 12.5 = 0 \tag{2.0.14}$$

$$x^2 + y^2 - 9x - 6y + 10 = 0 \tag{2.0.15}$$

Plot of the circle given by equation (2.0.15) is as follows :

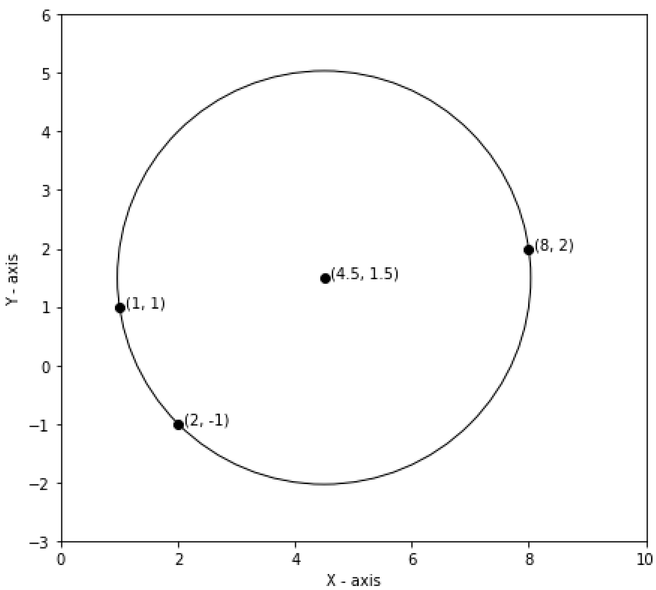

Fig. 0: A circle centered at $(4.5, 1.5)$ with radius 3.53