Assignment-5

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Abstract—This document explains the concept of finding the equation of circle using linear algebra.

1 PROBLEM

Find the equation of circle passing through the points $(1, 1), (2, −1)$ and $(8, 2)$.

2 SOLUTION

We know that the general equation of circle is :

$$x^2 + y^2 + Ax + By + C = 0 \qquad (2.0.1)$$

Therefore by using the three given points we need to find A, B and C.

In the matrix form this can be written as :

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = -\begin{pmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ x_3^2 + y_3^2 \end{pmatrix} \qquad (2.0.2)$$

By putting the values of $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) in (2.0.2) we get :

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 8 & 2 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -68 \end{pmatrix} \qquad (2.0.3)$$

Using Gaussian Elimination method :

$$\xrightarrow[\substack{R_3 \leftarrow 8R_1 - R_3}]{R_2 \leftarrow 2R_1 - R_2} \begin{pmatrix} 1 & 1 & 1 : & -2 \\ 0 & 3 & 1 : & 1 \\ 0 & 6 & 7 : & 52 \end{pmatrix} \qquad (2.0.4)$$

$$\xrightarrow{R_2 \leftarrow \frac{1}{3}R_2} \begin{pmatrix} 1 & 1 & 1 : & -2 \\ 0 & 1 & \frac{1}{3} : & \frac{1}{3} \\ 0 & 6 & 7 : & 52 \end{pmatrix} \qquad (2.0.5)$$

$$\xrightarrow{R_3 \leftarrow 6R_2 - R_3} \begin{pmatrix} 1 & 1 & 1 : & -2 \\ 0 & 1 & \frac{1}{3} : & \frac{1}{3} \\ 0 & 0 & -5 : & -50 \end{pmatrix} \qquad (2.0.6)$$

Solving for A, B and C using (2.0.6) we get :

$$\mathbf{X} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \\ 10 \end{pmatrix} \qquad (2.0.7)$$

Putting these values in (2.0.1), the equation of circle is as follows :
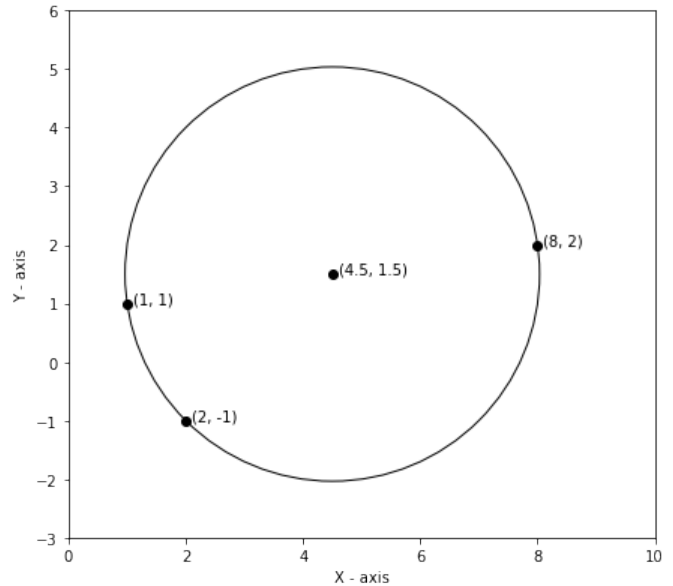
$$x^2 + y^2 - 9x - 3y + 10 = 0 \qquad (2.0.8)$$

Fig. 0: Plot of the circle given by equation (2.0.8)