Assignment-6

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Abstract—**This document explains the concept of finding the equation of tangent to a circle using linear algebra.**

Download all python codes from

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https://github.com/vipulmalik8569/MT−EE5609
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and latex-tikz codes from

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https://github.com/vipulmalik8569/MT−EE5609
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1 Problem

Write down the equation of the tangent to a circle passing through the point $\mathbf{p}$.

Equation of the circle and positional vector $\mathbf{p}$ is given as :

$$x^2 + y^2 - 3x + 10y = 15 \tag{1.0.1}$$

$$\mathbf{p} = \begin{pmatrix} 4 \\ -11 \end{pmatrix} \tag{1.0.2}$$

2 Solution

We know that the equation of the tangent in parametric form with slope vector $\mathbf{s}$ is:

$$\mathbf{x} = t\mathbf{s} + \mathbf{p} \tag{2.0.1}$$

General equation of the circle in vector form is :

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

In the vector form (1.0.1) can be written as :

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} \frac{-3}{2} \\ 5 \end{pmatrix}^T \mathbf{x} - 15 = 0 \tag{2.0.3}$$

By comparing (2.0.2) and (2.0.3) we get :

$$\mathbf{u} = \begin{pmatrix} -\frac{3}{2} \\ 5 \end{pmatrix}, f = -15 \tag{2.0.4}$$

Using (2.0.4) the center $\mathbf{c}$ and radius r of the circle can be calculated as :

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} \frac{3}{2} \\ -5 \end{pmatrix} \tag{2.0.5}$$

$$r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \sqrt{\frac{109}{4} + 15} = \frac{13}{2} \tag{2.0.6}$$

Displacement vector from the center $\mathbf{c}$ to the point $\mathbf{p}$ is :

$$\mathbf{d} = \mathbf{c} - \mathbf{p} = \begin{pmatrix} \frac{-5}{2} \\ 6 \end{pmatrix} \tag{2.0.7}$$

We know that the displacement vector $\mathbf{d}$ is perpendicular to the slope vector $\mathbf{s}$ of the tangent. Therefore,

$$\mathbf{s} = \begin{pmatrix} 6 \\ \frac{5}{2} \end{pmatrix} \tag{2.0.8}$$

By putting the values of $\mathbf{s}$ and $\mathbf{p}$ in equation (2.0.1) we get :

$$\mathbf{x} = t\begin{pmatrix} 6 \\ \frac{5}{2} \end{pmatrix} + \begin{pmatrix} 4 \\ -11 \end{pmatrix} \tag{2.0.9}$$

$$x = 6t + 4 \tag{2.0.10}$$

$$y = \frac{5}{2}t - 11 \tag{2.0.11}$$

Using (2.0.10) and (2.0.11) the equation of the tangent to a circle passing through the point $\mathbf{p}$ is

$$y = \frac{5}{2}x - \frac{71}{6} \tag{2.0.12}$$

Plot of the tangent to a circle given by equation (2.0.12) is as follows :

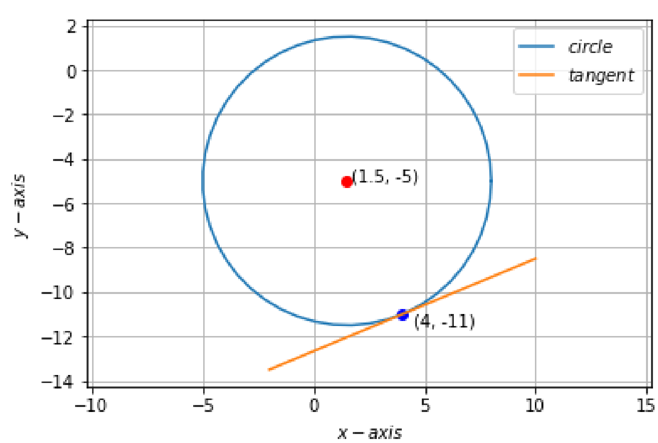

Fig. 0: Tangent to a circle centered at $(1.5, -5)$ with radius 6.5 passing through the point $(4, -11)$.