Assignment-4

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***Abstract*—This document explains the concept of congruence of triangles in a quadrilateral.**

Download all latex-tikz codes from

https://github.com/vipulmalik8569/MT−EE5609

1 Problem

In quadrilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$. Show that $BC = BD$.

2 Figure

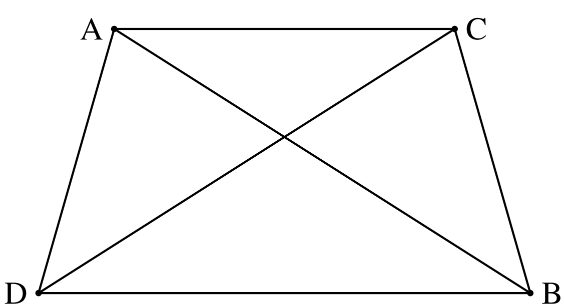

Fig. 0: Quadrilateral ACBD

3 Solution

In Quadrilateral ACBD it is given that :

$$AC = AD \tag{3.0.1}$$
$$\angle CAB = \angle DAB \tag{3.0.2}$$

By taking cosine on both sides of (3.0.1) we get

$$\cos \angle CAB = \cos \angle DAB \tag{3.0.3}$$
$$\frac{(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{A}-\mathbf{B}\|} = \frac{(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{A}-\mathbf{C}\|\,\|\mathbf{A}-\mathbf{B}\|} \tag{3.0.4}$$

From (3.0.1)

$$\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{A}-\mathbf{C}\| \tag{3.0.5}$$

Therefore (3.0.4) will become

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{B}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) \tag{3.0.6}$$
$$\|\mathbf{A}-\mathbf{D}\|^2 - (\mathbf{A}-\mathbf{D})^T(\mathbf{B}-\mathbf{D}) = \|\mathbf{A}-\mathbf{C}\|^2 - (\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.7}$$

Using (3.0.5) we get

$$(\mathbf{B}-\mathbf{D})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{B}-\mathbf{C})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.8}$$
$$\|\mathbf{B}-\mathbf{D}\|^2 + (\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{D}) = \|\mathbf{B}-\mathbf{C}\|^2 + (\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.9}$$
$$\|\mathbf{B}-\mathbf{D}\|^2 + (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{B}) = \|\mathbf{B}-\mathbf{C}\|^2 + (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) \tag{3.0.10}$$

Using (3.0.6) we get :

$$\|\mathbf{B}-\mathbf{D}\|^2 = \|\mathbf{B}-\mathbf{C}\|^2 \tag{3.0.11}$$
$$\|\mathbf{B}-\mathbf{D}\| = \|\mathbf{B}-\mathbf{C}\| \tag{3.0.12}$$
$$BD = BC \tag{3.0.13}$$