Assignment-5

Vipul Kumar Malik

***Abstract*—This document explains the concept of finding the equation of circle using linear algebra.**

Download all python codes from

https://github.com/vipulmalik8569/MT−EE5609

and latex-tikz codes from

https://github.com/vipulmalik8569/MT−EE5609

1 Problem

Find the equation of circle passing through the points

$$\mathbf{X_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{X_2} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{X_3} = \begin{pmatrix} 8 \\ 2 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Vector form of the equation of circle with radius r and centered at $\mathbf{C}$ is :

$$(\mathbf{X} - \mathbf{C})^T(\mathbf{X} - \mathbf{C}) = r^2 \tag{2.0.1}$$

Where

$$\mathbf{X} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \tag{2.0.2}$$

For $\mathbf{X_1}$, $\mathbf{X_2}$ and $\mathbf{X_3}$ (2.0.1) can be written as :

$$(\mathbf{X_1} - \mathbf{C})^T(\mathbf{X_1} - \mathbf{C}) = r^2 \tag{2.0.3}$$
$$(\mathbf{X_2} - \mathbf{C})^T(\mathbf{X_2} - \mathbf{C}) = r^2 \tag{2.0.4}$$
$$(\mathbf{X_3} - \mathbf{C})^T(\mathbf{X_3} - \mathbf{C}) = r^2 \tag{2.0.5}$$

In the matrix form this can be written as :

$$\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \begin{pmatrix} -2x_0 \\ -2y_0 \\ x_0^2 + y_0^2 - r^2 \end{pmatrix} = - \begin{pmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ x_3^2 + y_3^2 \end{pmatrix} \tag{2.0.6}$$

By putting the values of $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) in (2.0.6) we get :

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 8 & 2 & 1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \\ -68 \end{pmatrix} \tag{2.0.7}$$

$$\mathbf{A} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} -2x_0 \\ -2y_0 \\ x_0^2 + y_0^2 - r^2 \end{pmatrix} \tag{2.0.8}$$

Using Gaussian Elimination method :

$$\xleftrightarrow[R_3 \leftarrow 8R_1 - R_3]{R_2 \leftarrow 2R_1 - R_2} \begin{pmatrix} 1 & 1 & 1 : & -2 \\ 0 & 3 & 1 : & 1 \\ 0 & 6 & 7 : & 52 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{3}R_2} \begin{pmatrix} 1 & 1 & 1 : & -2 \\ 0 & 1 & \frac{1}{3} : & \frac{1}{3} \\ 0 & 6 & 7 : & 52 \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow{R_3 \leftarrow 6R_2 - R_3} \begin{pmatrix} 1 & 1 & 1 : & -2 \\ 0 & 1 & \frac{1}{3} : & \frac{1}{3} \\ 0 & 0 & -5 : & -50 \end{pmatrix} \tag{2.0.11}$$

Solving for A, B and C using (2.0.11) we get :

$$\mathbf{A} = \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \\ 10 \end{pmatrix} \tag{2.0.12}$$

Using (2.0.8) we get :

$$\mathbf{C} = \begin{pmatrix} \frac{9}{2} \\ \frac{3}{2} \end{pmatrix}, r^2 = 12.5 \tag{2.0.13}$$

Putting these values in (2.0.1), the equation of circle is as follows :

$$\left\| \mathbf{X} - \begin{pmatrix} \frac{9}{2} \\ \frac{3}{2} \end{pmatrix} \right\|^2 = 12.5 \tag{2.0.14}$$

Plot of the circle given by equation (2.0.14) is as follows :

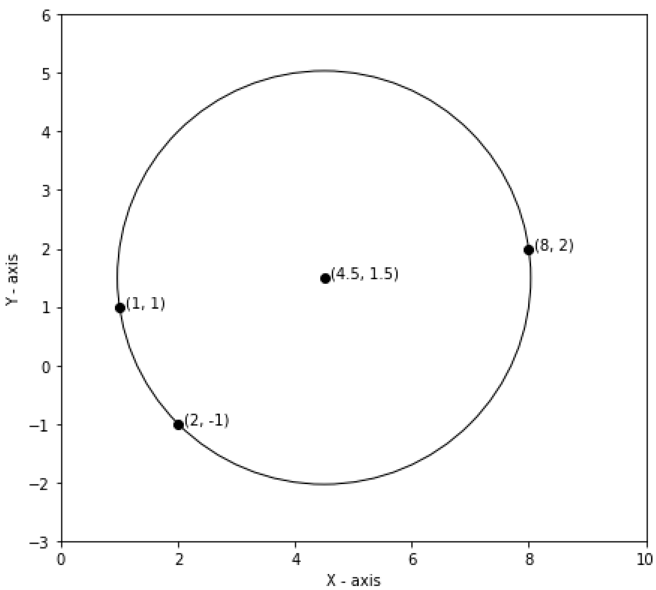

Fig. 0: A circle centered at $(4.5, 1.5)$ with radius 3.53