

Assignment-6

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Abstract—This document explains the concept of finding the equation of tangent to a circle using linear algebra.

Download all python codes from

<https://github.com/vipulmalik8569/MT-EE5609>

and latex-tikz codes from

<https://github.com/vipulmalik8569/MT-EE5609>

1 PROBLEM

Write down the equation of the tangent to a circle passing through the point \mathbf{p} .

Equation of the circle and positional vector \mathbf{p} is given as :

$$x^2 + y^2 - 3x + 10y = 15 \quad (1.0.1)$$

$$\mathbf{p} = \begin{pmatrix} 4 \\ -11 \end{pmatrix} \quad (1.0.2)$$

2 SOLUTION

General equation of the circle in vector form is :

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

In the vector form (1.0.1) can be written as :

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} -\frac{3}{2} \\ 5 \end{pmatrix}^T \mathbf{x} - 15 = 0 \quad (2.0.2)$$

By comparing (2.0.1) and (2.0.2) we get :

$$\mathbf{u} = \begin{pmatrix} -\frac{3}{2} \\ 5 \end{pmatrix}, f = -15 \quad (2.0.3)$$

We know that the equation of tangent in the form of normal vector $(\mathbf{p} + \mathbf{u})$ and point \mathbf{p} can be written as:

$$(\mathbf{p} + \mathbf{u})^T (\mathbf{x} - \mathbf{p}) = 0 \quad (2.0.4)$$

$$(\mathbf{p} + \mathbf{u})^T \mathbf{x} - \mathbf{p}^T \mathbf{p} - \mathbf{u}^T \mathbf{p} = 0 \quad (2.0.5)$$

Using (2.0.1), (2.0.5) will become :

$$(\mathbf{p} + \mathbf{u})^T \mathbf{x} + \mathbf{u}^T \mathbf{p} + f = 0 \quad (2.0.6)$$

By putting the values of \mathbf{p} , \mathbf{u} and f from (2.0.3) in (2.0.6) we get :

$$\left(\begin{pmatrix} 4 \\ -11 \end{pmatrix} + \begin{pmatrix} -\frac{3}{2} \\ 5 \end{pmatrix} \right)^T \mathbf{x} + \begin{pmatrix} 4 \\ -11 \end{pmatrix}^T \begin{pmatrix} -\frac{3}{2} \\ 5 \end{pmatrix} - 15 = 0 \quad (2.0.7)$$

$$\begin{pmatrix} \frac{5}{2} \\ -6 \end{pmatrix}^T \mathbf{x} - 76 = 0 \quad (2.0.8)$$

Hence the equation of the tangent to the circle passing through the point \mathbf{p} is:

$$\begin{pmatrix} \frac{5}{2} \\ -6 \end{pmatrix}^T \mathbf{x} = 76 \quad (2.0.9)$$

Plot of the tangent to a circle given by equation (2.0.9) is as follows :

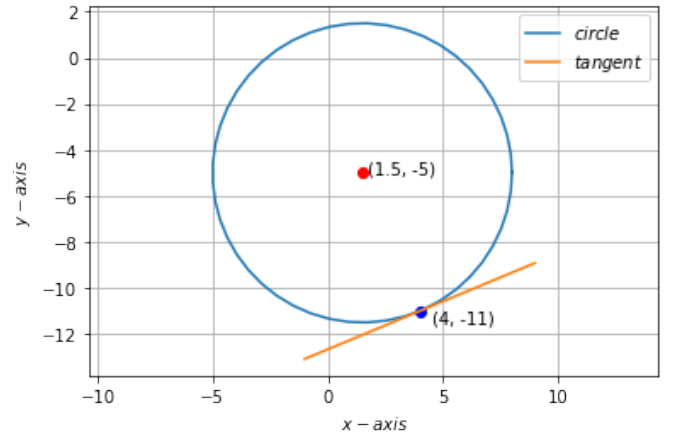


Fig. 0: Tangent to a circle centered at $(1.5, -5)$ with radius 6.5 passing through the point $(4, -11)$.