Calculus Problems, Solutions, and Tricks

Part 1: Basic Concepts

1. Definition of Differentiation:

Differentiation finds the rate of change of a function with respect to a variable.

Example: If y = x^2, then dy/dx = 2x.

2. Definition of Integration:

Integration calculates the area under the curve of a function.

Example: If integrate(x^2 dx), the result is (x^3)/3 + C, where C is the constant of integration.

Part 2: Tricks and Shortcuts

1. Trick for Differentiation of Products:

Use the product rule:

d/dx[u * v] = u' * v + u * v'.

Example: d/dx[x^2 * e^x] = 2x * e^x + x^2 * e^x.

2. Quick Integration of Polynomials:

Add 1 to the power and divide by the new power.

Example: integrate(x^3 dx) = (x^(3+1))/(3+1) = x^4/4 + C.

3. Shortcut for Integration by Parts:

Remember the formula:

integrate(u * v dx) = u * integrate(v dx) - integrate(u' * integrate(v dx) dx).

Choose u as the function that simplifies on differentiation.

Part 3: Problems and Solutions

1. Problem 1 (Differentiation):

Differentiate $f(x) = 3x^3 - 5x^2 + 4x - 7$.

Solution:

Using the power rule: $f'(x) = 9x^2 - 10x + 4$.

2. Problem 2 (Integration):

Solve integrate($2x^3 - 4x^2 + 5$ dx).

Solution:

Integrate term by term: integrate($2x^3$ dx) = $(2x^4)/4$, integrate($-4x^2$ dx) = $(-4x^3)/3$, integrate(5 dx) = 5x.

Final answer: $x^4/2 - 4x^3/3 + 5x + C$.

3. Problem 3 (Trigonometric Integration):

Solve integrate(sin(x) dx).

Solution:

The integral of sin(x) is -cos(x) + C.

Part 4: Advanced Tricks

1. Substitution Method:

If the integral is integrate(sin(2x) dx), use substitution:

Let u = 2x, so du = 2dx.

Rewrite:

integrate(sin(2x) dx) = (1/2)integrate(sin(u) du) = -(1/2)cos(u) + C = -(1/2)cos(2x) + C.

2. Partial Fractions for Rational Functions:

Example: integrate($1/(x^2 - 1)$ dx).

Decompose:

$1/(x^2 - 1) = 1/((x-1)(x+1)) = A/(x-1) + B/(x+1)$.

Solve for A and B, then integrate term by term.