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Task 3 (CE6002) : Linear classification

Create your own implementation of linear classification to perform a classification of the dataset provided in the *Etivity3_LinearClassification.ipynb* notebook without adding extra features to those provided. Use normal linear regression with $\text{sign}(wTx)$ to obtain a classification.

Observe your results and explain why these results seem disappointing (record your thoughts in a Markdown cell in your notebook).

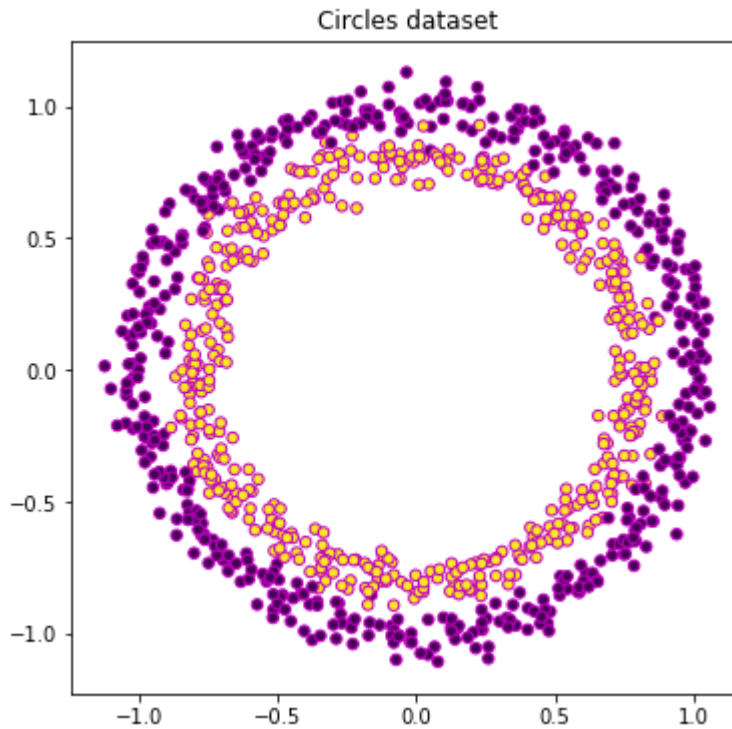
Now choose suitable new features and use these in your linear classification algorithm to improve the classification performance. Observe and explain (use plots where appropriate) why the classification performance has improved. Try a few different ones and note the differences!

Also, use scikit-learn's Logistic Regression algorithm and compare the performance with your algorithm. It is useful to spend some time thinking about the difference in approach taken in Logistic Regression.

```
In [1]: 1 import matplotlib.pyplot as plt
        2 import numpy as np
        3 from sklearn.linear_model import LogisticRegression
        4 from sklearn.model_selection import train_test_split
        5 %matplotlib inline
```

```
In [2]: 1 from sklearn.datasets.samples_generator import make_circles
        2 X, y = make_circles(n_samples=1000, noise = 0.05)
        3 y = [yy if yy == 1 else -1 for yy in y]
```

```
In [3]: 1 plt.figure(figsize=(6,6))
2 plt.scatter(X[:,0], X[:,1], c=y, marker='o', s=30, edgecolors='m')
3 plt.title("Circles dataset")
4 plt.show()
```



```
In [4]: 1 X.shape
```

```
Out[4]: (1000, 2)
```

Linear Classification Implementation

- Matrix X and vector y with bias term of $x_0=1$
- Calculate the pseudo inverse for matrix X using "np.linalg.pinv"
- Get the weights $w = [\text{inv}(\text{trans}(X) X) \text{trans}(X)] * y$

```

In [5]: 1 def get_weights(X, y):
2         # Function to find the weight matrix
3         weight_matrix = np.zeros(1 + X.shape[1])
4         pseudo_inv_matrix = np.linalg.pinv(X)
5         weight_matrix = pseudo_inv_matrix.dot(y)
6         return weight_matrix
7
8
9 def y_hat(X, weight_matrix):
10        # Function to calculate the y_hat
11        return (np.dot(X, np.transpose(weight_matrix)))
12
13
14 def calc_error(actual, predicted):
15        # Function to calculate the classification error
16        errors = 0
17        for x,y in zip(predicted, actual):
18            if (x !=y):
19                errors+=1
20        return errors / len(predicted)
21
22
23 def predict_class(X, weight_matrix):
24        # Function to predict the classification label for the input data X
25        return np.sign(y_hat(X, weight_matrix))
26
27
28 # Thanks Michel for the help here
29 def point_on_decision_boundary(x, weights):
30        # Fucntion to find the y-pos on the boundary based on x-pos
31        return -(weights[0] + weights[1]*x) / weights[2]
32
33
34 def plot_classifier(X, Y, weights):
35        # Function to plot the decision boundary
36
37        # plot the data samples
38        plt.scatter(X[:,1],X[:,2], c=Y, marker='o', s=30, edgecolors='m', label='
39
40        # Getting the X and Y position for the classification boundary
41        x_min, x_max = X.min(), X.max()
42        X_pos = [x_min, x_max]
43        Y_pos = [point_on_decision_boundary(x_min, weights), point_on_decision_bo
44
45        # Plot the decision boundary
46        plt.plot(X_pos, Y_pos, 'bx-.', label='descision boundary')
47        plt.legend(loc='best')
48

```

```

In [6]: 1 def linear_classification(X, y, title=""):
2         # Function to calculate the weights and plot the classification boundary
3
4         # Insert bias to the input data X; x_0=1
5         X = np.insert(X, 0, 1, axis=1)
6
7         # Split data in train and test set with 20% samples as test data
8         X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
9
10        # Get the weights for linear classifier from the train data
11        weights = get_weights(X_train, y_train)
12
13        # Calculate the classification error
14        y_hat = predict_class(X_test, weights)
15        error = calc_error(y_test, y_hat)
16        print('Classification error for test data: {:.4f}'.format(error))
17        print('Classification Score for test data: {:.4f}'.format(1-error))
18
19
20        # Create figure for plotting
21        plt.figure(figsize=(16, 7))
22
23        # Decision boundary for train data
24        plt.subplot(1,2,1)
25        plt.title('Linear Classification: Train data '+title)
26        plot_classifier(X_train, y_train, weights)
27
28        # Decision boundary for test data
29        plt.subplot(1,2,2)
30        plt.title('Linear Classification: Test data '+title)
31        plot_classifier(X_test, y_test, weights)
32

```

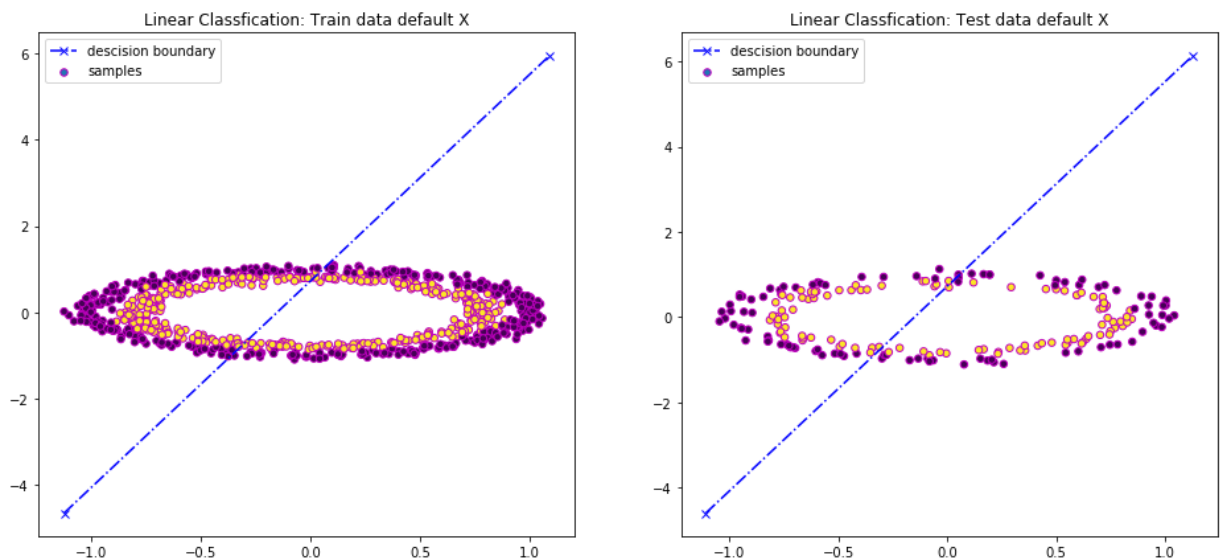
```

In [7]: 1 # Run the classification on Original data
2        linear_classification(X, y, title="default X")

```

Classification error for test data:0.5150

Classification Score for test data:0.4850



B. Observation on results:

The Eout is very high (0.59) for the in house linear classification with the default X data.

The results are disappointing as the **dataset is not linearly separable** due to which the **linear classification algorithm is not able to split to data properly**.

We can see from the plot that in this case almost 50% of data has a chance of being misclassified and leads to a high classification error.

To improve the Linear Classification, I will transform the data so that it can be linearly separable and then apply the linear classification in Z space.

C. Choose suitable new features and use these in your linear classification algorithm to improve the classification performance. Observe and explain (use plots where appropriate) why the classification performance has improved

- Try a few different ones and note the differences! ($Z=[1, X_1, X_2, X_1_{sq}, X_2_{sq}]$)

References on non-linear transformation:

1. <https://people.revoledu.com/kardi/tutorial/Regression/nonlinear/NonLinearTransformation.htm> (<https://people.revoledu.com/kardi/tutorial/Regression/nonlinear/NonLinearTransformation.htm>)
2. <https://towardsdatascience.com/machine-learning-with-python-easy-and-robust-method-to-fit-nonlinear-data-19e8a1ddbd49> (<https://towardsdatascience.com/machine-learning-with-python-easy-and-robust-method-to-fit-nonlinear-data-19e8a1ddbd49>)

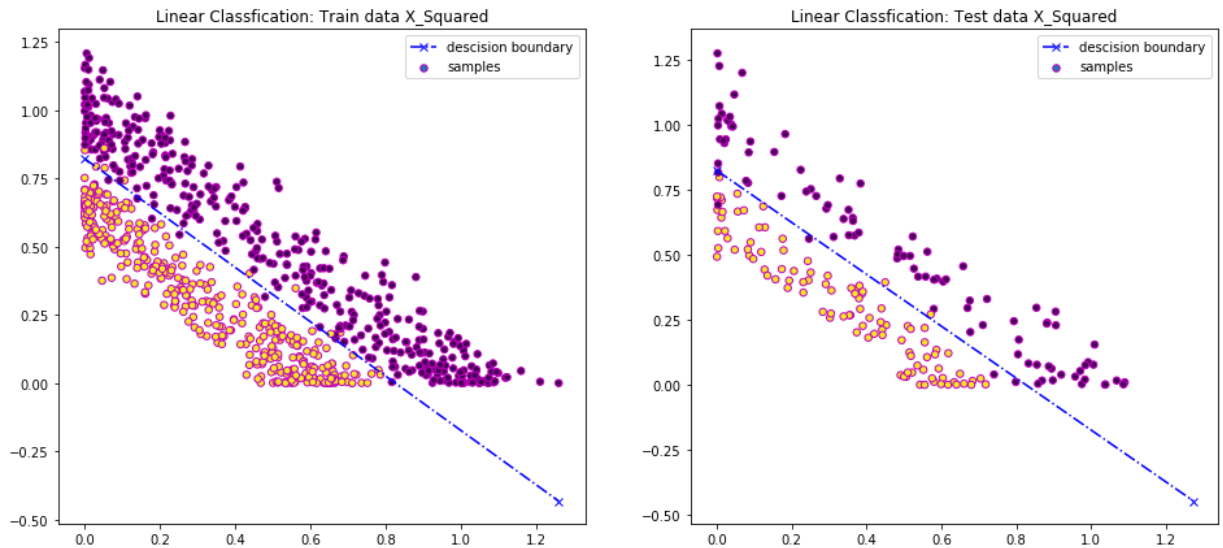
1. Square Transformation on the data

Lets do some non-linear transformation on the data feature and move from X space to Z space and then apply the Linear classification algorithm.

```
In [8]: 1 # Square the X data
2 X_square = np.square(X)
3
4 linear_classification(X_square, y, title="X_Squared")
```

Classification error for test data:0.0350

Classification Score for test data:0.9650



Observation of "Square" feature transformation on the classification:

After we apply the square transformation on the original data (non-linearly separable), the **new feature space is linearly separable**.

The **linear classification algorithm** gives a low Eout of 0.035 on the linearly separable data in **Z space**

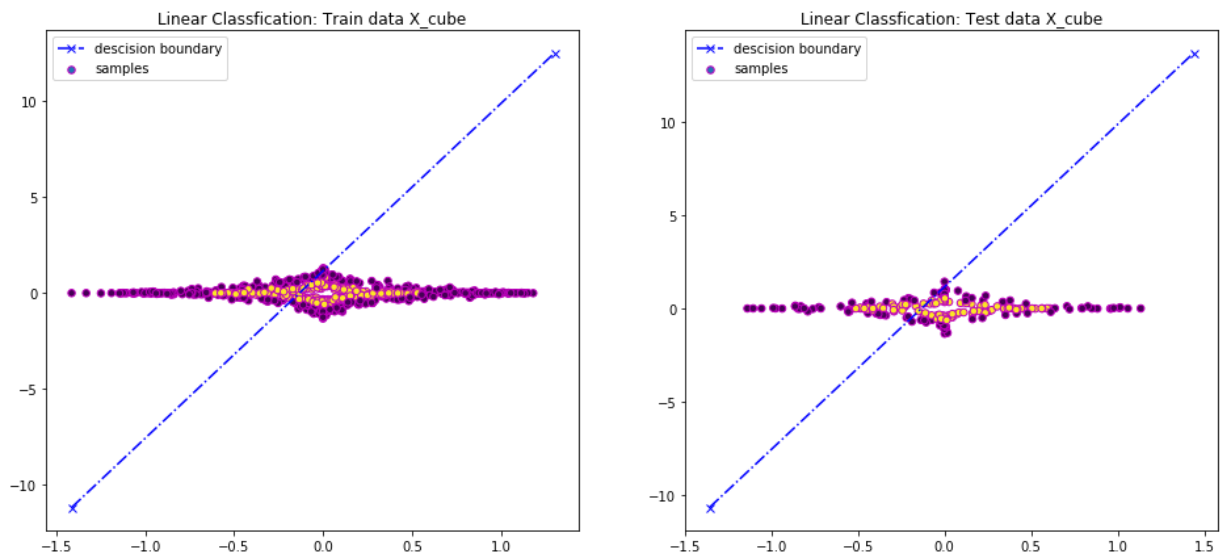
This is a much better score, **however we cannot 'guarantee' that this will generalise well as we have broken the VC bound by data snooping**. I will complete some more transformations now to compare.

2. Cube Transformation of data

```
In [9]: 1 # Cube the X data
        2 X_cube = np.power(X, 3)
        3 linear_classification(X_cube, y, title="X_cube")
```

Classification error for test data:0.5500

Classification Score for test data:0.4500



The error is very high ($E_{out}=0.685$) if we use the cubic transformation on the data and leads to bad classification performance

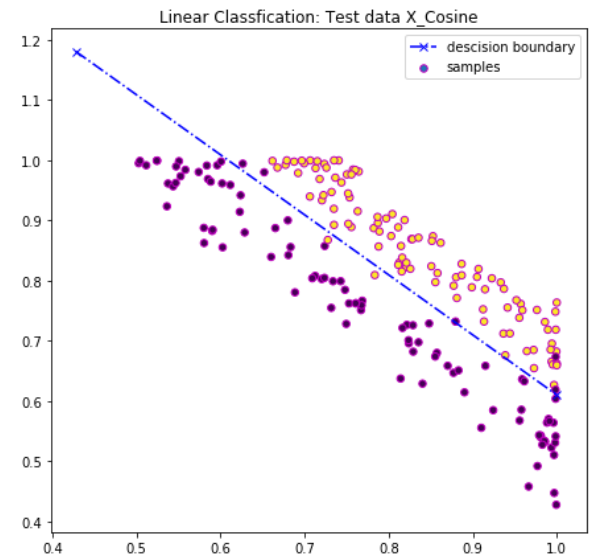
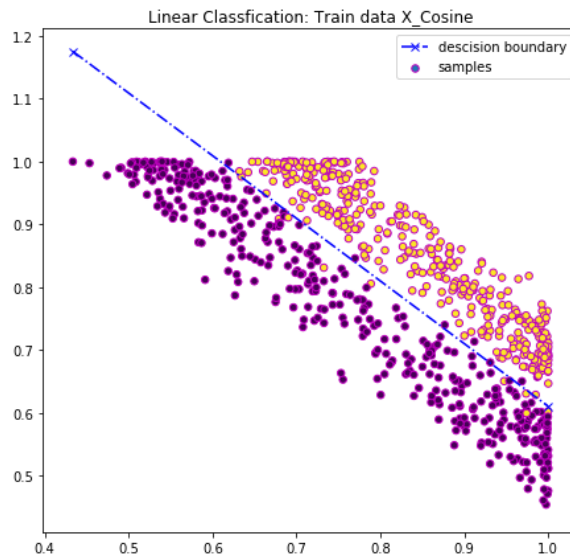
Trigonometric Transformations on the data

3. Cosine Transformation of data

```
In [10]: 1 # Cosine the X data
2 X_cos = np.cos(X)
3 linear_classification(X_cos, y, title="X_Cosine")
```

Classification error for test data:0.0350

Classification Score for test data:0.9650



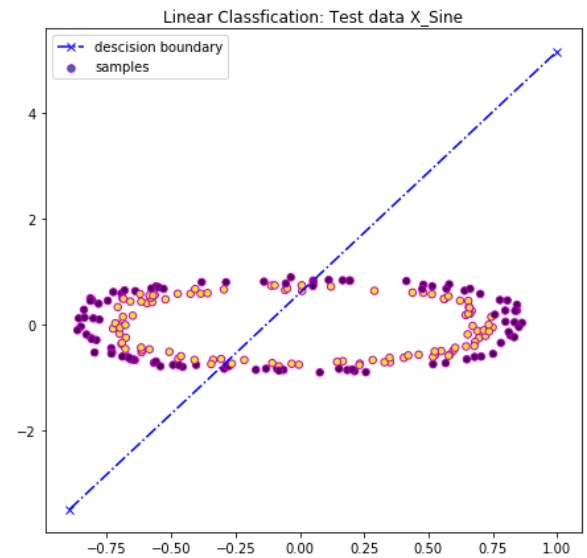
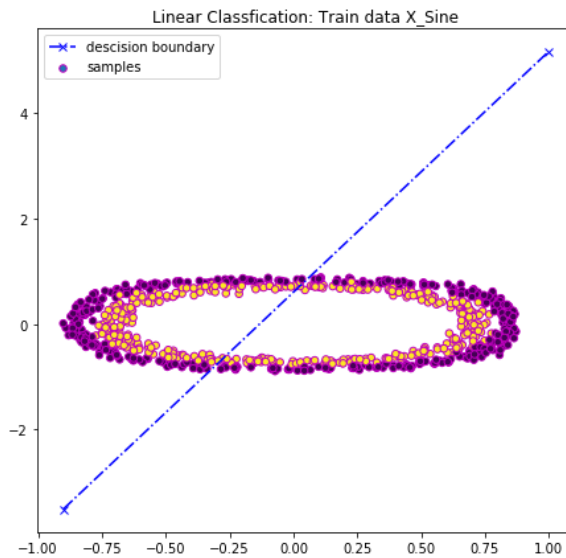
The error is low ($E_{out}=0.035$) if we use the cosine transformation and give equivalent results of square transformed data. This is probably because we are using data with circle pattern. I know that I am guilty of data snooping here!

4. Sine Transformation of data


```
In [11]: 1 # Sine the X data
          2 X_sin = np.sin(X)
          3 weights = linear_classification(X_sin, y, title="X_Sine")
```

Classification error for test data:0.5100

Classification Score for test data:0.4900



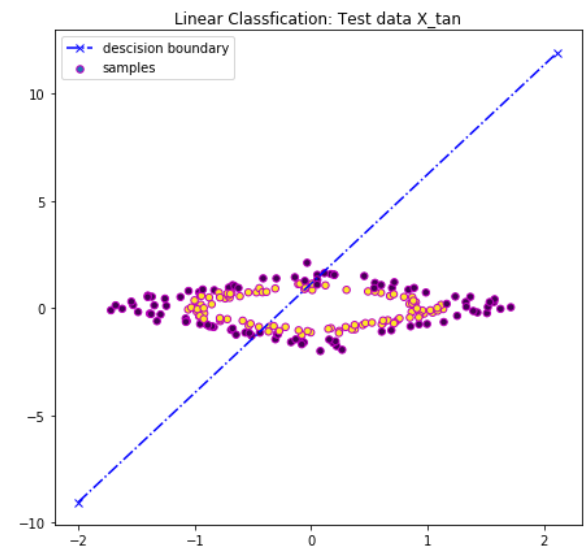
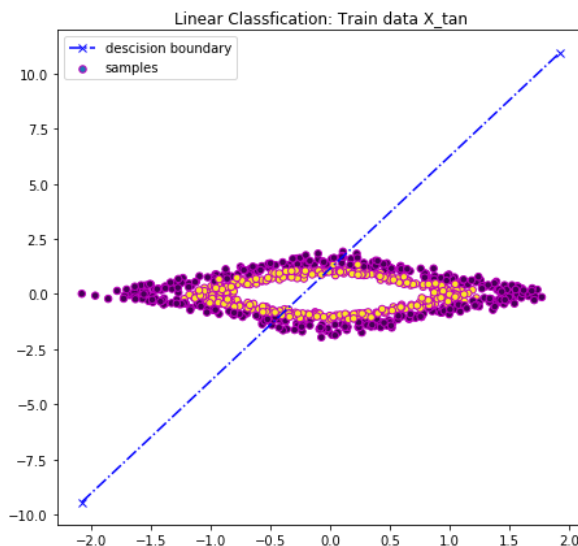
The error is very high (Eout=0.56) if we use the sine transformation on the data and leads to bad classification performance

5. Tan Transformation of data

```
In [12]: 1 # Tanget on the X data
          2 X_tan = np.tan(X)
          3 weights = linear_classification(X_tan, y, title="X_tan")
```

Classification error for test data:0.5300

Classification Score for test data:0.4700



The error is high ($E_{out}=0.6$) if we use the tan transformation on the data and leads to bad classification performance

Observation and Summary:

After the bad classification results from original X space (with X data), I went ahead with non-linear transformation on the data to perform classification in Z space.

The Square and Cosine transformation on the data gives a very good classification performance. The cubic transformation and sine, tan trigonometric transform doesn't give better results.

Note: In this case we have looked at the data which was a circle and hence squaring method was used to improve the classification error, this may lead to data snooping and makes the VC generalization bound invalid

D. Use scikit-learn's Logistic Regression algorithm and compare the performance with your algorithm

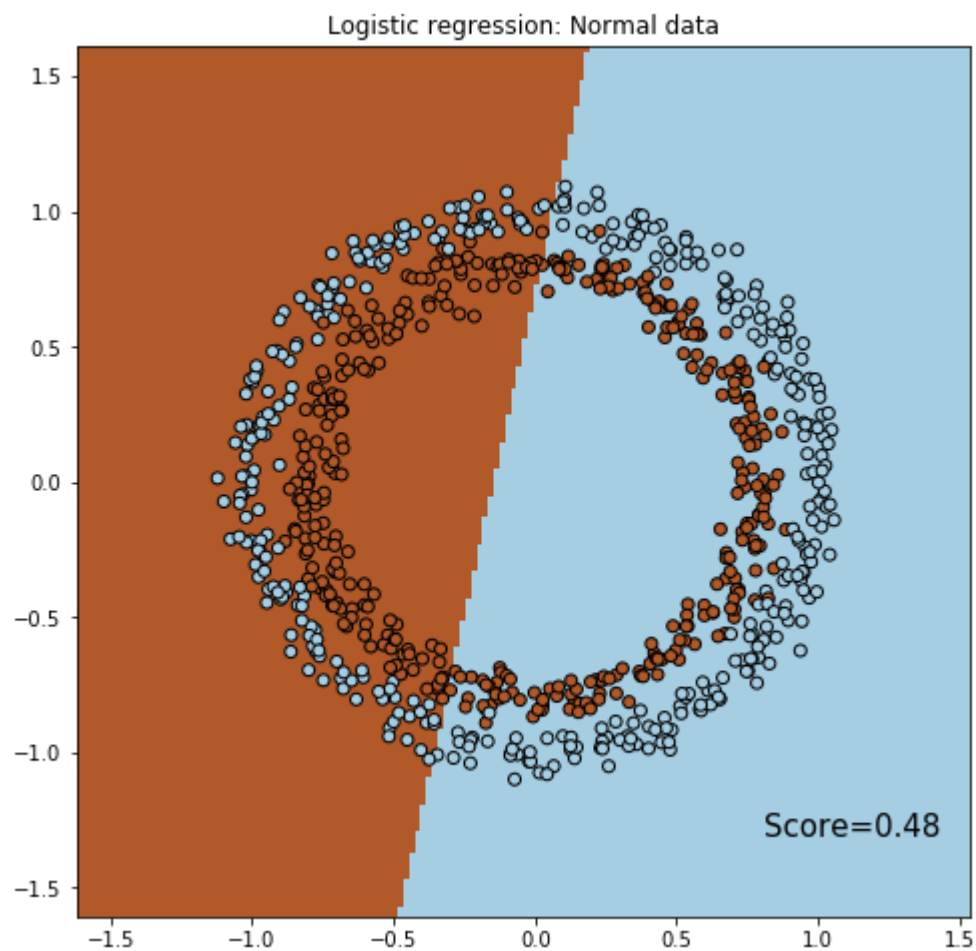
- Think about the difference in approach taken in Logistic Regression

Scikit-Learn Logistic Regression

Reference: https://scikit-learn.org/stable/auto_examples/linear_model/plot_iris_logistic.html
(https://scikit-learn.org/stable/auto_examples/linear_model/plot_iris_logistic.html)

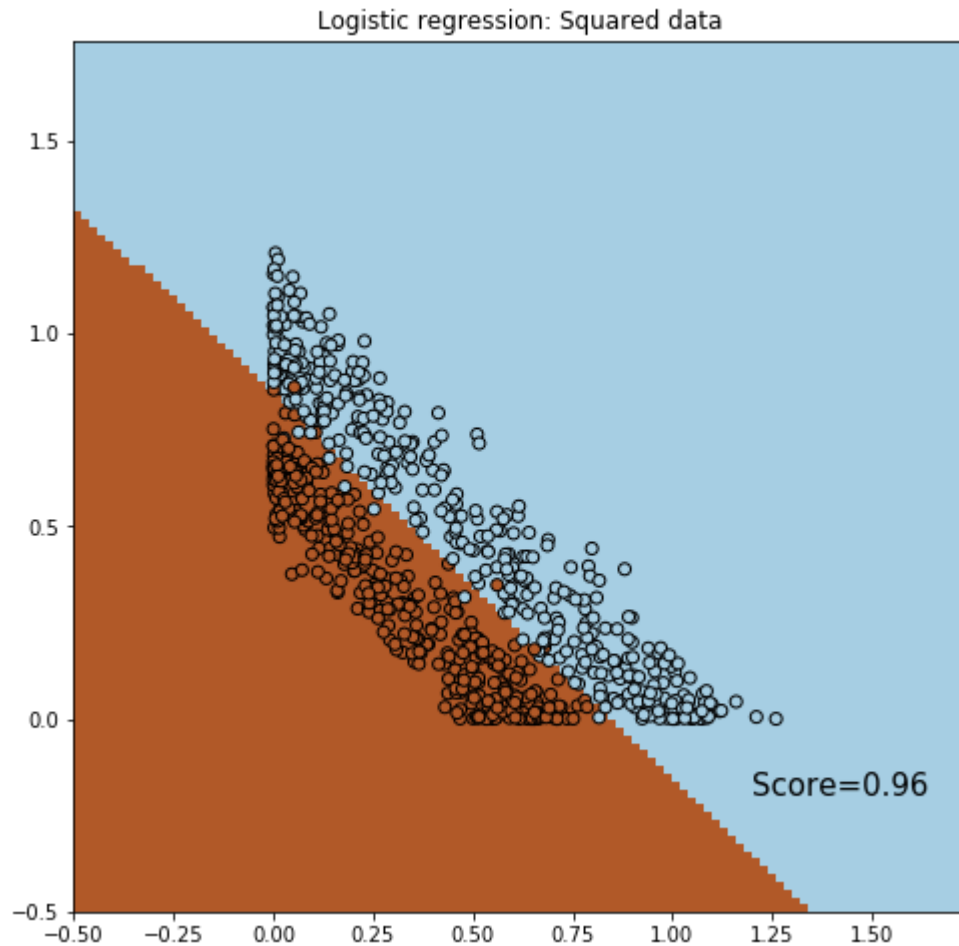
```
In [13]: 1 def plot_logistic_regression(X, y, title=''):
2
3     # Split data in train and test set with 20% samples as test data
4     X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
5
6     log_reg = LogisticRegression(solver='lbfgs')
7     log_reg.fit(X_train, y_train)
8
9     y_pred=log_reg.predict(X_test)
10    test_score = log_reg.score(X_test, y_test)
11
12    # Plot the decision boundary and assign a color to each point in the mesh
13    x_min, x_max = X[:, 0].min() - .5, X[:, 0].max() + .5
14    y_min, y_max = X[:, 1].min() - .5, X[:, 1].max() + .5
15    h = .02 # step size in the mesh
16    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max,
17    Z = log_reg.predict(np.c_[xx.ravel(), yy.ravel()]))
18
19    # Put the result into a color plot
20    Z = Z.reshape(xx.shape)
21    plt.figure(figsize=(8, 8))
22    plt.pcolormesh(xx, yy, Z, cmap=plt.cm.Paired)
23
24    # Plot also the training points
25    plt.scatter(X_train[:, 0], X_train[:, 1], c=y_train, edgecolors='k', cmap
26    # Print the classification score on test data
27    plt.text(xx.max()-.1, yy.min()+.3, ('Score=%.2f' %test_score), size=15, h
28
29    plt.title(title)
30    plt.xlim(xx.min(), xx.max())
31    plt.ylim(yy.min(), yy.max())
32    plt.show()
33
34    return test_score, log_reg
35
```

```
In [14]: 1 # Logistic regression on Original data
2 raw_test_score, clf = plot_logistic_regression(X, y, title="Logistic regression: Normal data")
3 print("Logistic regression: Mean accuracy score on Raw Test data =", raw_test_score)
4
```



Logistic regression: Mean accuracy score on Raw Test data = 0.485

```
In [15]: 1 # Logistic regression on Squared data
2 squared_test_score, clf = plot_logistic_regression(X_square, y, title="Logis
3 print("Logistic regression: Mean accuracy score on Squared transformed Test d
4
```



Logistic regression: Mean accuracy score on Squared transformed Test data = 0.965

Observations:

Logistic regression uses a probabilistic function for classification. We can see significant improvement in classification performance while using the square transformed data in logistic regression classification.

The classification score on the linear regression and logistic regression match very well in the X space and Z space(with square transformed data)

```
In [ ]: 1
```