Chapter 1

Introduction

1.1 Introduction

The dynamic vibration absorber (DVA) or tuned-mass damper (TMD) is a widely used passive vibration control device. Dynamic Vibration Absorbers (DVA) are based on the concept of attaching a secondary mass to a primary vibrating system such that the secondary mass dissipates the energy and thus reduce the amplitude of vibration of the primary system. A simple DVA consists of a mass and a spring. When a mass spring system or a primary system is excited by a harmonic force, its vibration can be suppressed by attaching a DVA as shown in Fig.1.1.
However, adding a DVA to a one-degree-of-freedom (dof) system results in a new 2-dof system. If the exciting frequency coincides with one of the two natural frequencies of the new system, the system will be at resonance. To overcome this problem, a damper is added to DVA. Figure 1.1 shows a primary system attached by a damped DVA.

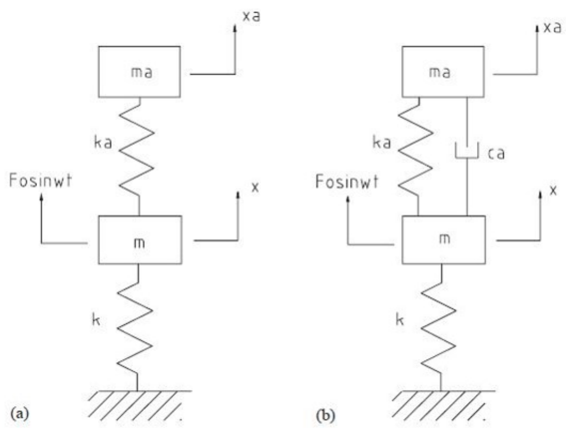


Figure 1.1: A simple dynamic vibration absorber

Equations of motion of the system are given as:

$$m_1\ddot{x_1} + k_1x_1 + k_2(x_1 - x_2) + c_2(\dot{x_1} - \dot{x_2}) = F_0sin(\omega * t)$$
$$m_2\ddot{x_2} + k_2(x_2 - x_1) + c_2(\dot{x_2} - \dot{x_1}) = 0$$

However, adding a Dynamic Vibration Absorber (DVA) to a one-degree-of-freedom (dof) system results in a new two-degree-of-freedom system. If the exciting frequency coincides one of the two natural frequencies of the new system, the system will be at resonance.

The vibration response of a single DOF system is compared with a system having a DVA without a damper, as shown in Fig 1.2 and a graph showing the effect of mass ratio(absorber mass by main mass) on the creation of two new natural frequencies is shown in Fig 1.3. It can be seen that as mass ratio increases the two new frequencies are pushed further apart.

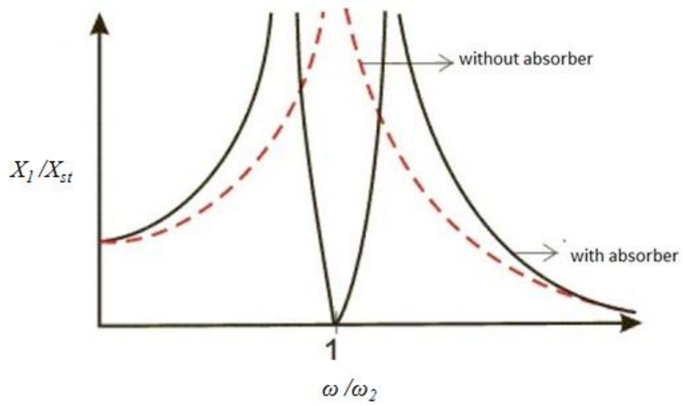


Figure 1.2: Change in response on addition of DVA without a damper

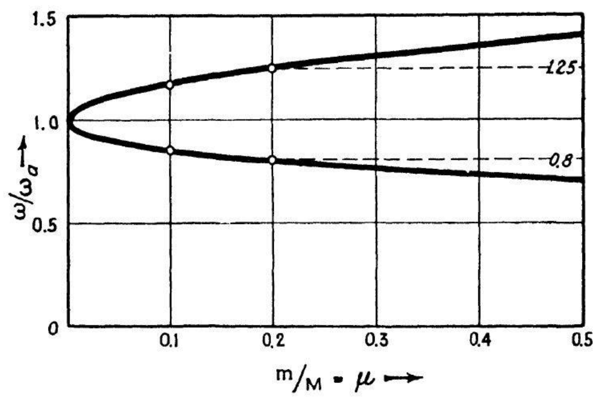


Figure 1.3: Effect of mass ratio on natural frequencies

1.2 Applications:

Tuned mass dampers are largely used in vibration control of crankshafts, hand-held devices and transmission cables.

1.2.1 Damping of hand-held devices

Hair clipper, dry shavers, and hand-held devices alike use electromagnetic motors to power themselves. Usually, the motor operates at a fixed frequency such as 60Hz. The figure 1.4

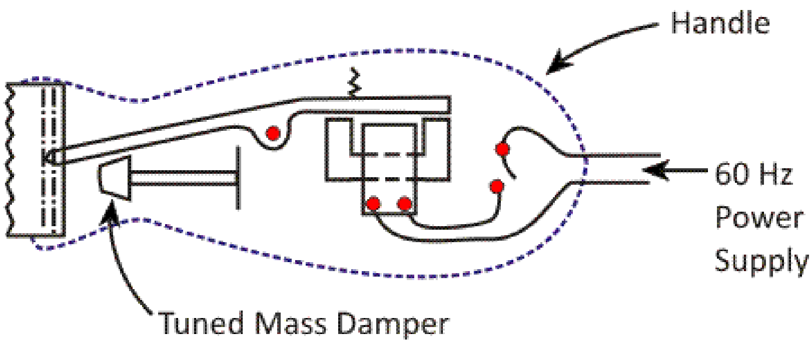


Figure 1.4: A typical electrical hair clipper

shows a typical electric hair clipper. In such hair-clippers, an electro-magnet is used to develop vibrating force for cutting. However, this also generates an unpleasant vibration of the housing. This vibration is neutralized by the application of a pair of mass dampers fixed to the housing at two different points.

1.2.2 Floor vibration control

Wide column spans along with the use of high strength material (less of which would provide the needed structural integrity) tend to make modern composite floors flexible and oscillatory. Human activities (walking, running, dancing, etc.) and operating machines can induce high levels of vibration in such floors.
Floor vibration in a building are considered harmful due to several reasons:

1. Impair the operation of sensitive instruments
2. Affect human psychology for acceleration level even greater than 0.005g
3. Motion having peak amplitude greater than 1 mm is not desirable in active environments

Traditional methods for improving the environment include:

1. Adding extra columns
2. Adding extra thickness to the floors and
3. Increasing structural stiffening

Figure 1.5: Two tuned mass dampers installed underneath a floor

Reactive damping, provided by attaching tuned mass dampers (TMDs) to the floor, is commonly used for treating vibrating floors. Negligible weight penalty, low cost, and ease of installation make TMDs the most practical, cost-effective, and least disruptive floor vibration control solution for both new and existing floor systems.

1.2.3 Houdaille damper

A tuned viscous torsional damper referred to as the Houdaille damper or viscous Lanchester damper can be used to reduce the torsional oscillations of the crankshaft. A Houdaille damper

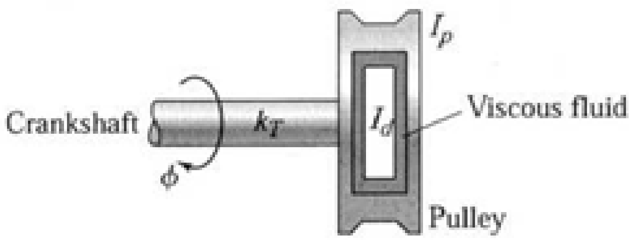


Figure 1.6: Figure showing Houdaille damper

can be used to reduce the vibration in rotating systems, such as in engine installations where the operation speed may vary over a wide range. As shown in figure 1.6, the damper consists of a disk. The disc is free to rotate inside a housing which is attached to the rotating shaft, and the housing and the rotating shaft are assumed to have equivalent mass moment of inertia. The space between the housing and the disk is filled with viscous fluid. In most cases, the fluid is a silicon oil whose viscosity is of similar magnitude to oil but which does not change significantly when the temperature changes. The damping effect is produced by the viscosity of the oil and is proportional to the relative angular velocity between the housing and the disk.

1.2.4 Vibration control of structures

Mass damper is a device mounted on structures such as buildings or bridges to reduce the amplitude of the structures due to the vibrational motions induced by periodic or non-periodic dynamic loading. It operates effectively to prevent internal discomfort and damage as well as the outright structural failures. The installed mass damper moves in opposition to the resonance frequency oscillations of the structure by means of pendulum, spring or fluid. Mass damper that counter-reacts the movement of building guarantees the safety of buildings when subjected to strong wind blow or light seismic wave. These passive vibration control systems

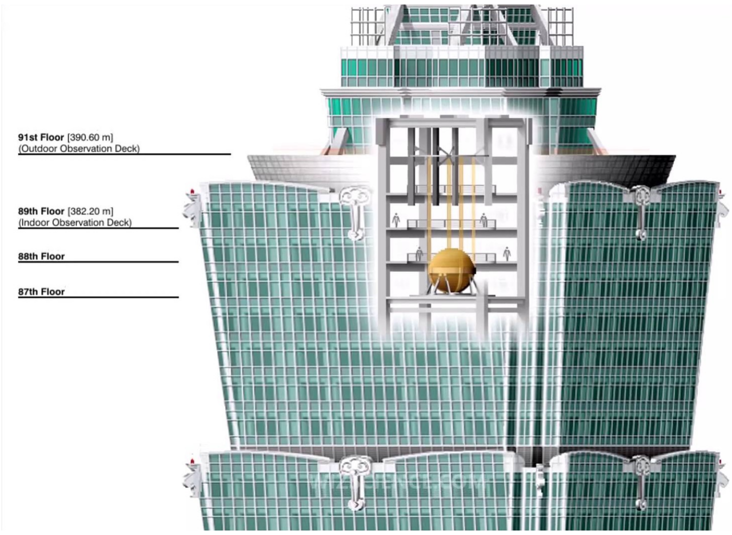


Figure 1.7: Tuned mass damper atop Taipei 101

have been widely utilized in many high-rise building structures particularly those located at seismically active zone. These include Taipei 101 at Taiwan and One Rincon Hill at U.S.A.

1.2.5 Wind turbines

A standard tuned mass damper for wind turbines consists of an auxiliary mass which is attached to the main structure by means of springs and dashpot elements. The natural frequency of the tuned mass damper is basically defined by its spring constant and the damping ratio determined by the dashpot. The tuned parameter of the tuned mass damper enables the auxiliary mass to oscillate with a phase shift with respect to the motion of the structure. In a typical configuration an auxiliary mass hung below the nacelle of a wind turbine supported by dampers or friction plates.

1.2.6 Optical Disk Drives

Demands for higher read/write speeds of optical disk drives (ODDs) have resulted in an increase in the rotational speed of the drive, which has led to greater vibration from the spindle motor. This, in turn, requires a higher servo gain of the pickup actuator, which leads to write failures in CDs and DVDs. The increased vibration also produces an unpleasant sensation for users of mobile equipment with integrated ODDs. Thus, additional mechanisms, such as dynamic vibration absorber (DVA), mass balancer, or auto-ball balancer, are required to decrease the vibration. The DVA is the most popular of these because of its low price and its effectiveness in reducing vibration.

1.3 Suitable name for what is done in thesis

There are four design variables involved while designing a DVA, objective being to minimize the vibration amplitude of the main mass over the operating frequency.The four design variables involved are the mass ratio(μ), stiffness of the spring(k_2), the damping between primary mass and the absorber mass(ζ_2) and the damping in the primary mass(ζ_1) which is usually independent and is specified over which designer doesn't have much control over.

The basic outline of this thesis is:

1. In the absence of damper in the primary system,the optimum design parameters for the DVA. Den Hartog proposed a closed form solution for this case,Den Hartogs derivation is based on a very peculiar observation that for different values of secondary mass damping the curve always passes through 2 fixed points.

2. In second case when DVA is present in both primary and absorber mass,a search for optimal parameters is done.But in this case a closed form solution cannot be obtained,hence

numerical methods are adapted for selection of optimized parameters.

3. In the third instance the effect of configuration of attachment of secondary damper as proposed by Kifu Liu and Jie Liu has been considered.

4. In previous instances,damping is usually viscous but we have considered the addition of coulomb damper between the primary mass and he absorber mass.A numerical search was done to find the optimum frictional force and the FRF generated in this case was compared with the FRF of optimum viscous damper.

Chapter 2

Literature Review

2.1 Linear Dynamic Vibration Absorber

2.1.1 Randall and Taylor (1981) - Journal of Mechanical Design

- Randall and Taylor in this paper proposed a numerical search of optimal parameters when primary mass damping is present.
- The maximum amplitude response is a function of frequency and four design parameters f(ζ_1, ζ_2, μ, f) ,$\zeta_1$is an independent parameter specified by the primary system.
- Of the three remaining parameters (ζ_2, μ, f) the mass ratio is the most probable to be specified in a design and will therefore is considered as a second independent parameter and search for optimal ζ_2 and f is carried out.

As shown below in fig 2.1,Randall and Taylor did a 3-dimensional search for optimal values and the results are shown below-

2.1.2 Rana and Soong(1998) - Engineering Structures, Elsevier

This paper takes into account both primary and secondary mass damping.They studied the effects of detuning of some of the optimal DVA parameters on the performance of the vibration absorber.

Some of the conclusions are:

- The detuning effect of natural frequency of absorber is more pronounced than that of absorber damping.

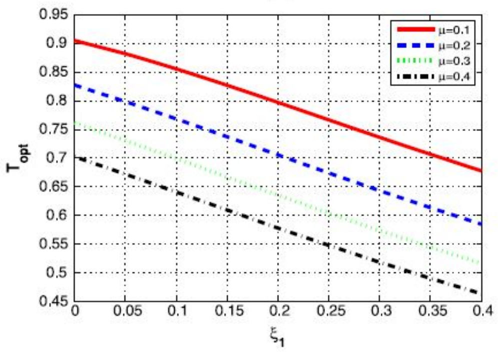


Figure 2.1: optimal f and ζ_1 as a function of μ

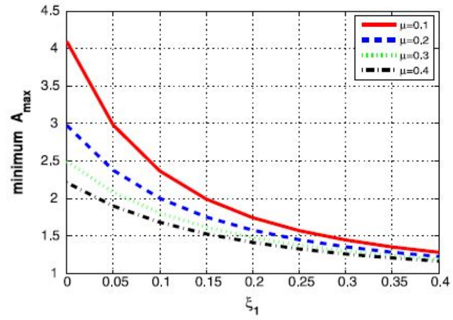


Figure 2.2: Maximum vibrational amplitude for optimum ζ_1

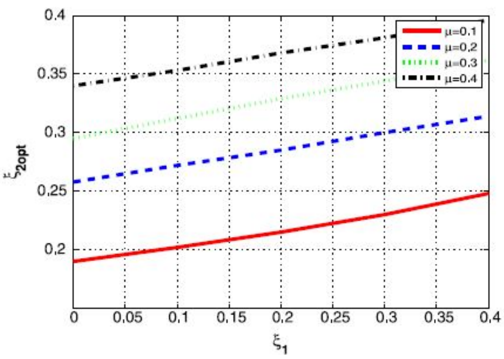


Figure 2.3: optimal ζ_2 and ζ_1 as a function of μ

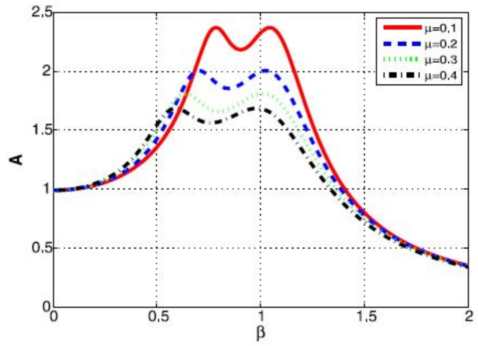


Figure 2.4: FRF

- With increasing damping of the main mass, the effect of detuning becomes less severe.
- With increasing mass ratio also, the effect of detuning becomes less severe.

2.1.3 R.L. Mayes And N. A. Mowbray (1974) - Earthquake Engineering and Structural Dynamics

Paper takes a combination of Coulomb friction and viscous damping and equates the work done per cycle of coloumb friction to equivalent viscous damper,this is the method that we had adapted while working with Coulomb damper.

Some of the conclusions are:

- This paper gives us a method to determine the fraction of viscous and coulomb damping present in the structure and compares the effect of taking equivalent viscous damping.

- The effect on response of structure by combined damping is 10-48 percent higher than the result obtained through equivalent damping.

2.1.4 Kifu Liu and Jie Liu(2004) - Journal of Sound and Vibration

Studies in the field of DVA has tried to reduce the vibration of primary system,that is by looking for possibilities such as changing the type of damping or spring that is used.Benchmark being the optimum solution for FRF as suggested by Den Hartog without primary mass damping,which was later extended to system with primary mass damping,as discussed in earlier sections.

In 2004 Kifu Liu and Jie Liu came up with an optimum FRF for a system that was better than the Optimum FRF as suggested by Den Hartog.They worked on the two configuration of damper,instead of introducing a damper between the primary mass and the secondary mass,they attached the damper between ground and the secondary mass as shown in 2.5.This configuration of attachment is called as skyhook damper and groundhook damper based on the inertial frame where the other end of the damper is attached.

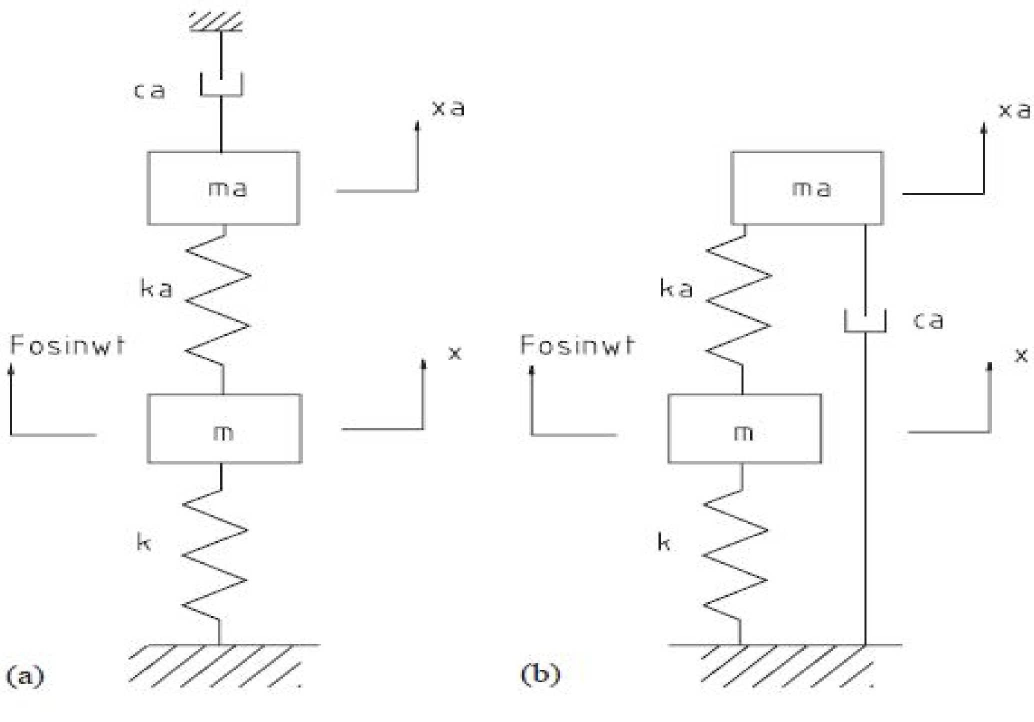


Figure 2.5: Configuration of skyhook and ground hook damper

They studied the groundhook and skyhook damper and compared the optimal value for vibration suppression with the Den Hartog optimal value for a simple DVA.They used the same concept of fixed point theory for deriving analytical solution.It was found out that,overall by changing the configuration,the results were better as shown in 2.6

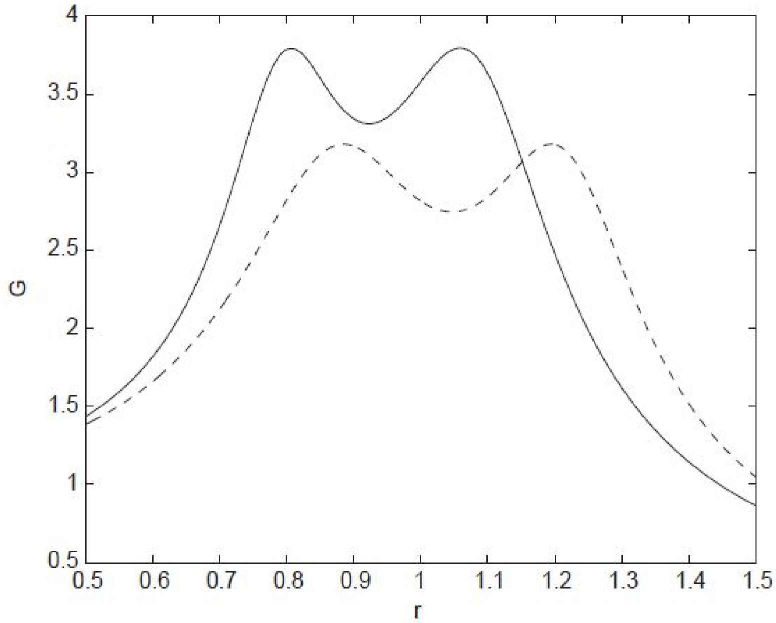


Fig. 5. Comparison of the optimum model A and model B: model A (solid line); model B (dashed line).

Figure 2.6: Comparision of FRF's

In this thesis our work follows along the same line,we have explored the effect that coulomb damper has on the vibration of primary mass and compared the optimum FRF's of coulomb and viscous dampers,then we had introduced cubic non-linear spring in both primary and secondary system,obtained their optimum FRF's.We have explored the various parameters that a designer can exploit to suppress vibration as much as possible.

2.2 Non-linearity in the System

2.2.1 J.C. Ji, N.Zhang(2009) - Journal of Sound and Vibration

The paper presented found that the primary resonance response of a nonlinear oscillator can be suppressed by a linear vibration absorber which consists of a relatively light mass attached to the nonlinear oscillator by a linear damper and a linear spring. The small attachment of light mass can absorb vibrational energy without significantly modifying the nonlinear oscillator and adversely affecting its performance. The stiffness of the linked spring is much lower than the linear stiffness of the nonlinear oscillator itself. The contributions of the absorber stiffness and damping to the linear stiffness and damping of the nonlinear primary system can be considered as a perturbation. Thus the linearized natural frequencies of the nonlinear primary oscillator before and after addition of vibration absorber change only slightly.

The effects of the parameters of the massspringdamper absorber on the vibration suppression of the nonlinear oscillator have been studied. It has been found that a larger coupling damping results in a larger reduction of primary resonance vibrations.

2.2.2 R.A. Borges, V.Steffen (2007) - 19th International Congress of Mechanical Engineering, Brasilia

In this work, a damped nonlinear dynamic vibration absorber is studied. The nonlinear effect is introduced in the system by nonlinear springs. Then, the main purpose is to verify the nonlinear effects, intended to increase the efficiency of the DVA into the frequency band of interest. Equation of motion of the nonlinear DVA are presented. The steady state response of the system is determined using Krylov-Bogoliubov method, which yields four nonlinear equations that are solved to obtain the solution. The effect of adding nonlinear dynamic vibration absorber with hardening and softening spring is illustrated. The effect of changing non-dimensional coefficient of nonlinearity (ϵ_1 and ϵ_2) is shown in figure 2.7. In the end some numerical examples are presented to evaluate the performance of the optimal nonlinear DVA.

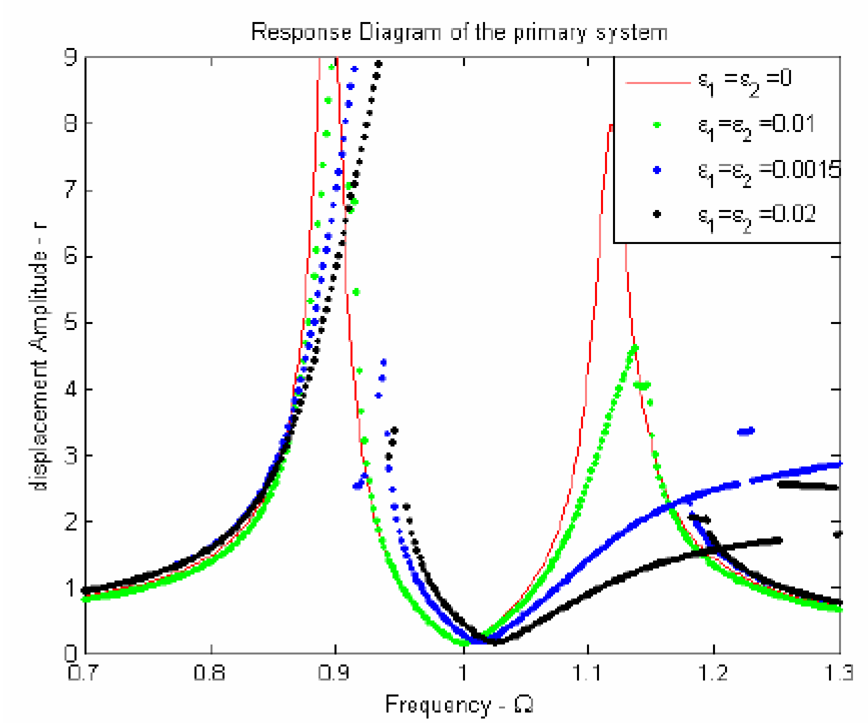


Figure 2.7: Effect of ζ_1 and ζ_2 on the system response

2.2.3 Yung-Sheng Hsu, Neil S Ferguson (2013) - Conference and Exposition on Structural Dynamics

The aforementioned paper investigates the physical behaviour and effectiveness of a nonlinear dynamic vibration absorber(NLDVA). The nonlinear absorber considered involves a nonlinear hardening spring which was designed and attached to a cantilever beam excited by a shaker. The cantilever beam can be considered at low frequencies as a linear single degree-of-freedom system. The nonlinear attachment is designed to behave as a hardening Duffing oscillator. It investigated the potential for vibration reduction of the system. Analytical and numerical results are presented and compared. From the measured results it was observed that the NLDVA had a much wider effective bandwidth compared to a linear absorber. The frequency response curve of the NLDVA has the effect of moving the second resonant peak to a higher frequency away from the tuned frequency so that the device is robust to mistuning. Experimental results have been presented to compare with the model derived.

2.2.4 S. Natsiavas (1992) - Journal of Sound and Vibration

The paper presents an analysis for determining the steady state response of dynamic vibration absorbers, for which both the machine and the absorber are assumed to possess a non-linear, Duffing type, stiffness. The harmonic steady state response is first determined by assuming weakly non-linear conditions and applying the method of averaging. The analysis is applied and numerical results are obtained for several combinations of the system parameters, in an effort to gain a better understanding of the effect of these parameters on the system response. Utilizing information from the amplitude of the solutions obtained, as well as their stability characteristics, it is shown that proper selection of the system parameters results in substantial improvements of the technical performance of non-linear absorbers and avoids dangerous effects that are possible to occur due to the presence of the non-linearities. It is shown that absorbers with hardening/softening springs may result in considerable reduction of the vibration amplitudes in frequency ranges greater/less than the original resonance. The present study is useful in trying to eliminate instability effects systematically, without losing the advantages brought about by the non-linearities, by properly adjusting the parameters of the system.

Chapter 3

Linear Dynamic Vibration Absorber

3.1 Dynamic vibration absorber without primary mass damping

The first mathematical theory on the damped DVA was presented by Ormondroyd and Den Hartog.The mathematical model of a primary mass system attached with a damped DVA is given by-

$$m_1\ddot{x_1} + k_1x_1 + k_2(x_1 - x_2) + c_2(\dot{x_1} - \dot{x_2}) = F_0sin(\omega * t) \tag{3.1}$$

$$m_2\ddot{x_2} + k_2(x_2 - x_1) + c_2(\dot{x_2} - \dot{x_1}) = 0 \tag{3.2}$$

Den Hartog derived closed form expressions for optimum damper parameters. He assumed no damping to be present in the main mass to facilitate the derivations.Den Hartogs derivation is based on a very peculiar observation that for different values of secondary mass damping the curve always passes through 2 fixed points as can be seen from 3.1. Firstly,the closed form solution for the vibration response of primary system was derived and this was later used to obtain closed form optimal solution.

The closed form solution was obtained in the non-dimensional parameters as it helped in understanding how the vibrational response depended on various parameters,for example-the effect on x_1 if F_0 is doubled and also by introducing non-dimensional parameters,the number of variable can be reduced.

$$\frac{x_1}{x_{st}} = \sqrt{\frac{(2\frac{c}{c_c}g)^2 + (g^2 - f^2)^2}{(2\frac{c}{c_c}g)^2(g^2 - 1 + \mu g^2)^2 + [\mu f^2g^2 - (g^2 - 1)(g^2 - f^2)]^2}}$$

where μ = m/M = absorber mass / main mass

w_a = k/m = natural frequency of absorber

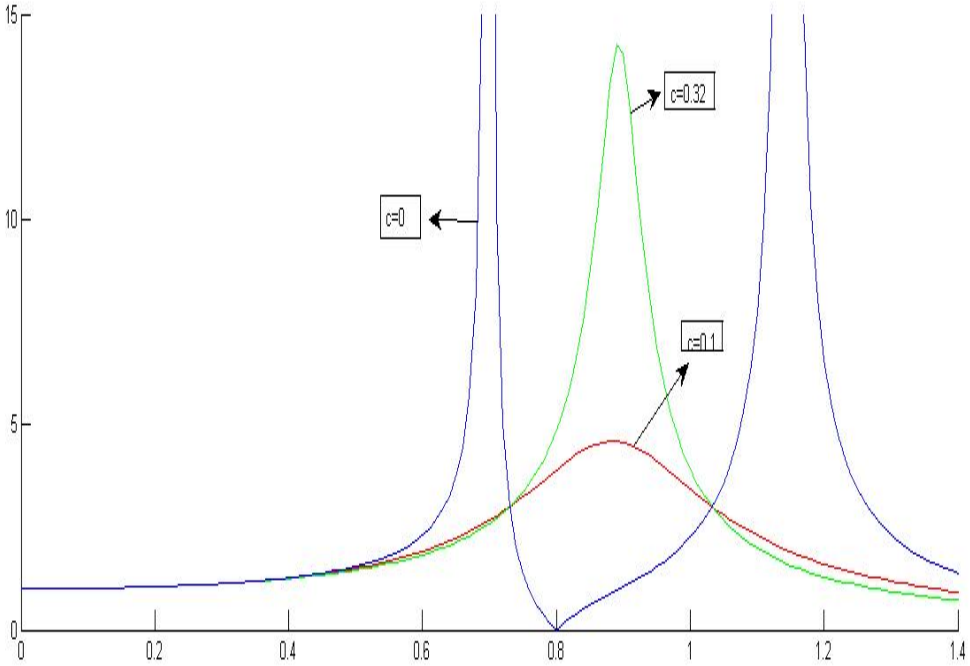


Figure 3.1: For three values of C_2 it can be shown that FRF's pass through 2 fixed points

Ω_n^2 =K/M = natural frequency of main system
$f = w_a/\Omega_n$= frequency ratio(natural frequency)
$g = w/\Omega_n$ = forced frequency ratio
$x_{st} = P_0/K$ = static deflection system
$c_c = 2m\Omega_n$="critical" damping

That is the amplitude ratio $\frac{x_1}{x_{st}}$ of the main mass is a function of four variables μ f , g and $\frac{c}{c_c}$ It is interesting to follow what happens for increasing damping.When the damping becomes infinite, the two masses are virtually clamped together and we have a single-degree-of-freedom system. In adding the absorber to the system, the objective is to bring the resonant peak of the amplitude down to its lowest possible value. With $c_2 = 0$, the peak is infinite at the resonant frequency(g=1); with c = ∞ it is again infinite. Somewhere in between there must be a value of c for which the peak becomes a minimum. As can be seen from 3.1,irrespective of damping the FRF always pass through two points and this is no accident. If we can calculate their location, our problem is practically solved, because the most favourable curve is the one which passes with a horizontal tangent through the highest of the two fixed points P or Q. The best obtainable "resonant amplitude" (at optimum damping) is the ordinate of that point. Even this is not all that can be done. By changing the relative tuning f = $frac\omega_a\omega_n$ of the damper with respect to the main system, the two fixed points P and Q can be shifted up and down the curve for c = 0. By changing f, one point goes up and the other down. Clearly the most favourable case is such that first by a proper choice of f the two fixed points are adjusted to equal heights, and second by a proper choice of $\frac{c}{c_c}$ the curve is adjusted to pass with a horizontal tangent through one of them. By adapting the above said procedure,the frequency ratio,damping co-efficient and amplitude ratio is given by:

$$\mathrm{f} = \frac{1}{1+\mu}$$
$$\frac{x_1}{x_{st}} = \sqrt{\frac{2}{1+\mu}}$$

3.2 Dynamic vibration absorber with primary mass damping

When we add a viscous damper in the primary system, alongside the viscous damper in DVA, the response of the system changes. The resultant system is shown in Fig

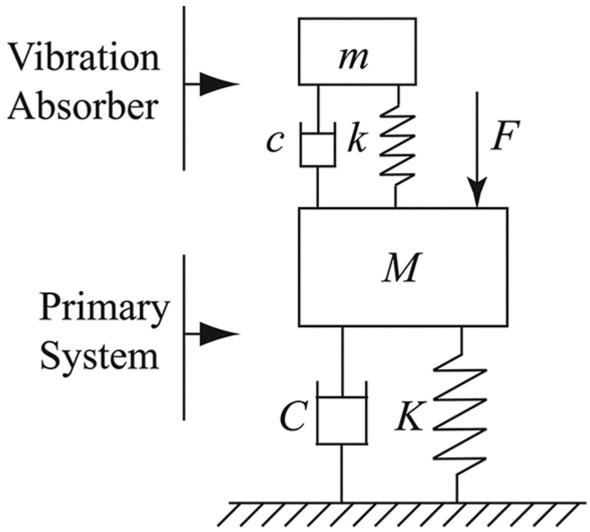


Figure 3.2: The main mass and the DVA, both are damped

The govering equations yield the solution:

$$
\begin{aligned}
\frac{X_1}{X_{st}} &= \frac{\sqrt{(1-r_{f2}^2)^2 + 4(\zeta_2 r_{f2})^2}}{\Delta} \\
\frac{X_2}{X_{st}} &= \frac{\sqrt{1 + 4(\zeta_2 r_{f2})^2}}{\Delta} \\
\text{where} \quad \Delta &= \Big\{ \left[r_{f_1}^2 r_{f_2}^2 - (r_{f_2}^2 + r_{f_1}^2(1+\mu) + 4\zeta_1\zeta_2 r_{f_1} r_{f_2}) + 1 \right]^2 \\
&\quad +4\left[\zeta_1 r_{f_1} + \zeta_1 r_{f_1} - (\zeta_1 r_{f_1} r_{f_2}^2 + \zeta_2 r_{f_2} r_{f_1}^2 (1+\mu))^2 \right] \Big\}^{\frac{1}{2}} \\
r_{f_1} &= \frac{\omega}{\sqrt{K_1/m_1}} \\
r_{f_2} &= \frac{\omega}{\sqrt{K_2/m_2}} \\
\mu &= \frac{m_2}{m_1} \\
\zeta_1 &= \frac{c_1}{2\sqrt{K_1/m_1}} \\
\zeta_2 &= \frac{c_2}{2\sqrt{K_2/m_2}}
\end{aligned}
$$

As we can expect the addition of viscous damping in main mass bring additional terms involving main mass damping (ζ_1) in the solution. If we set these terms to zero ($\zeta_1 = 0$), we obtain the previously mentioned analytical solution when primary mass damping is zero.

Using the analytical solution obtained above, Frequency Response Function (FRF) is plotted which is shown in Fig. 3.3

The most important observation from the plot is that: the graphs are not passing through a fixed point. Hence, we cannot use ‘fixed point theory’ for finding analytically, the optimal parameters to the system, as we had done earlier for the case with damping present only in DVA.

In absence of an analytical solution, a numerical approach is adopted. The basic premise of the numerical technique is to evaluate the response of the system over a range of frequency ratio (f) and secondary mass damping(ζ_2). The primary mass damping (ζ_1) is kept constant because real life systems do not provide control over their damping. Hence, we have to work with whatever damping is present in the primary system. Mass ratio is also kept constant.
The numerical search is visually shown through the surface generated as shown in Fig. 3.4. The lowest point on the surface represents optimal solution for the given system whose parameters are given in Table 3.2, along with optimum values of f and ζ_2 obtained from numerical search.

The optimum frequency response function obtained has the following characteristics:

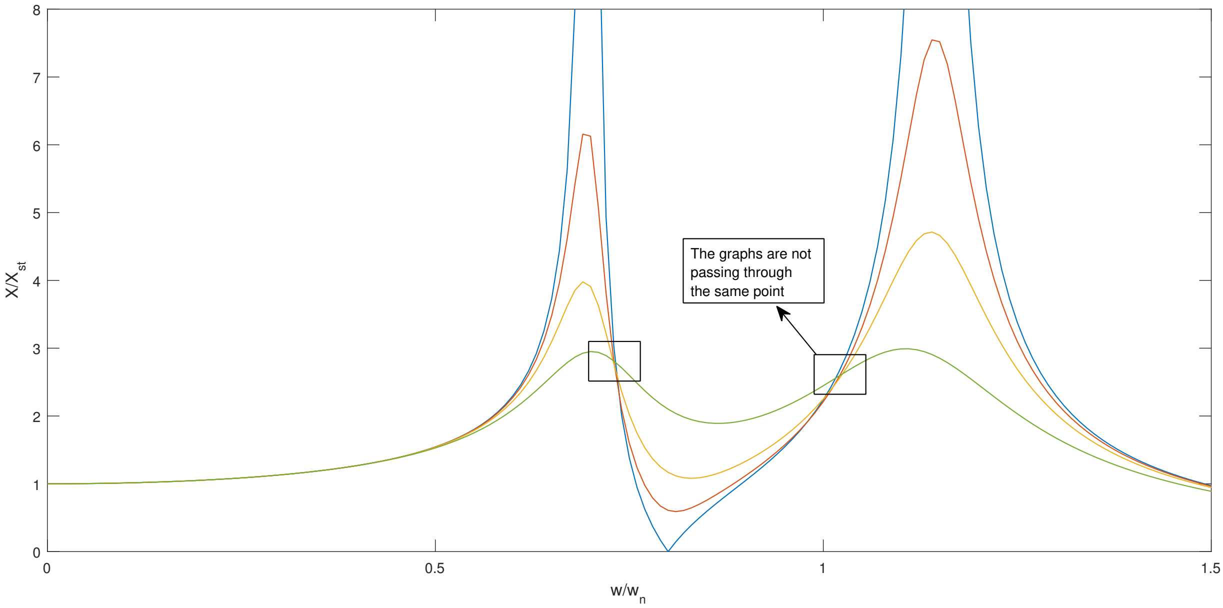


Figure 3.3: The graphs are not passing through a fixed point

PARAMETER	VALUE
Main Mass	1 kg
Mass of Absorber	0.25kg
Stiffness in main mass	$10^3 N/m$
Optimum frequency ratio(f_{opt})	0.8
Optimum secondary mass damping(ζ_2)	0.04

Table 3.1: System parameters with optimum values from numerical search

- The two maximum amplitudes obtained in the FRF are approximately equal. Table **??** shows the forcing frequency ratio at which the maximum amplitudes occur.

Forcing Frequency ratio	Amplitude
0.7	2.951
1.11	2.991

3.3 Coloumb Damping

Friction is difficult to model in multi-degree of freedom mechanical systems.Experimental studies of damping in vibrating mechanical systems typically use modal damping factors to represent the damping.Energy lost by friction is included in these factors. One of the drawbacks is that Friction between contacting surfaces can dissipate energy and cause damage by wear in many

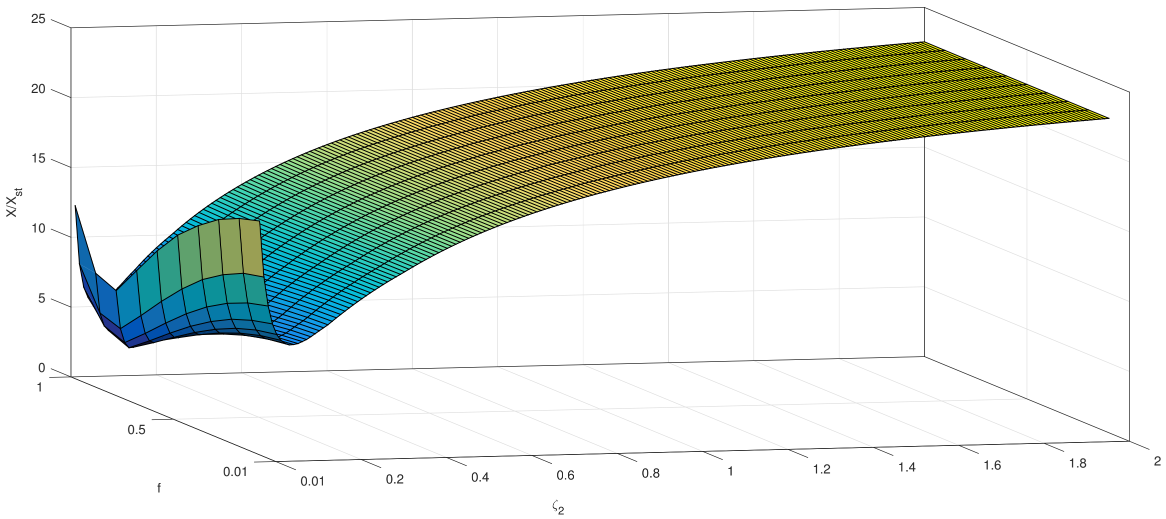


Figure 3.4: Numerical search. Lowest point on the surface represents optimal solution

engineering systems.The purpose of the present study is to explore the possibility of equivalent friction modal damping for a dynamic vibration absorber and to compare its performance with the viscous damping. The equation of motion when coloumb damping is present is given by-

$$m_1\ddot{x_1} + k_1 x_1 + k_2(x_1 - x_2) + c_1(\dot{x_1}) = F_0 sin(\omega * t)$$
$$m_2\ddot{x_2} + k_2(x_2 - x_1) + del * signum(\dot{x_2} - \dot{x_1}) = 0$$

where del is the frictional force acting on the absorber mass.
In figure 3.6 as frictional force is varied between 0 to 200 N,it can be seen that the FRF retains some of the essential characteristics of a viscous damper,for 2-DOF system and for a smaller frictional force there are 2 local peaks and as in case of viscous damping if it is increased after a certain point,one of the peaks dies out.FRF's would be smother if the resolution of ω used is smaller for numerical solution.

We would be varying the frictional force to obtain an optimum solution that minimizes the vibration of the main mass.For this the following steps are done-

- FRF is obtained by first evaluating steady state value from time history at a given friction force and forcing frequency.
- the maximum amplitude from this particular FRF is stored.
- The above 2 steps are repeated by varying Frictional force over the range from 0-500 N and 0-200 N.

The results are as shown in figure 3.7. Now the FRF when the frictional force is 390 N,the FRF obtained is compared with the optimal viscous FRF. Important observations:

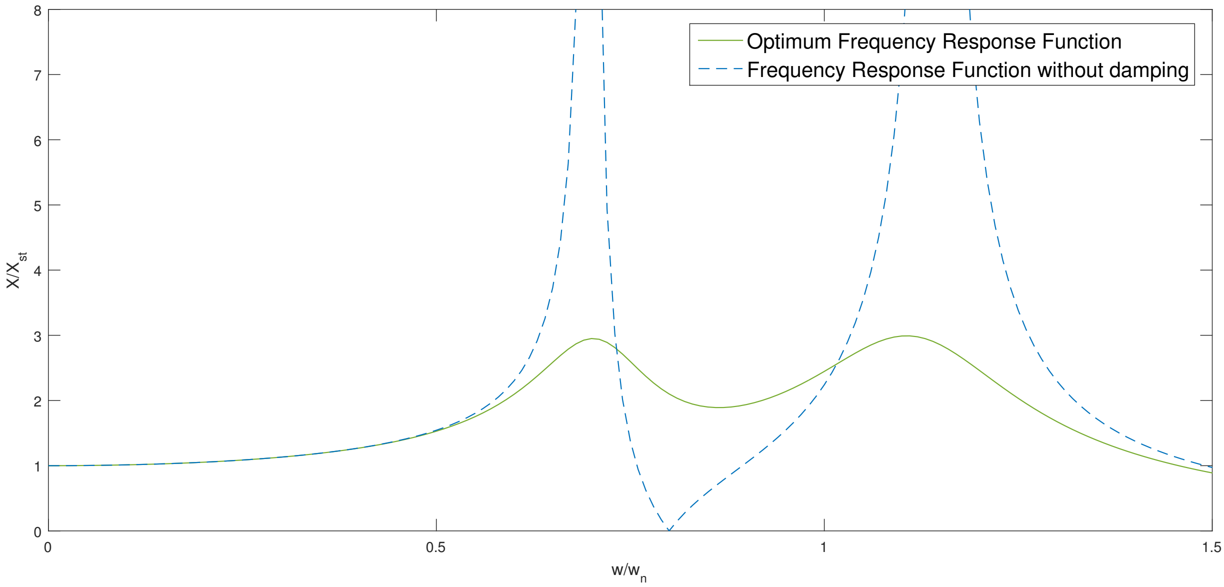


Figure 3.5: Frequency Response Function for optimum values found by numerical search

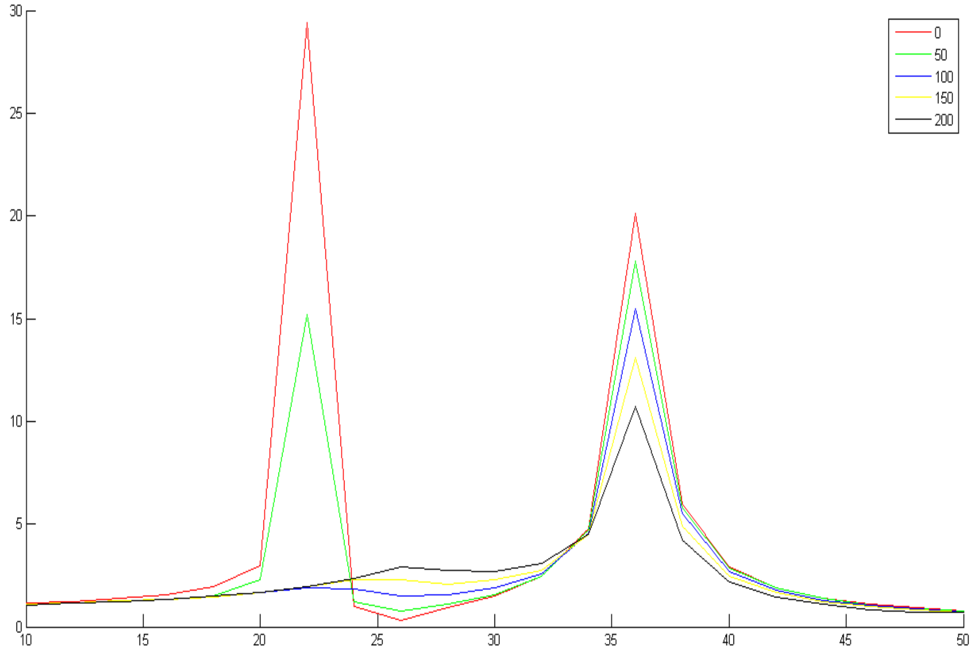


Figure 3.6: FRF in case of coloumb damping.

- It was observed that the optimum value of friction force is close to 400 N(390N to be precise,and this is valid for a particular excitation force(500N))
- Over certain range of frequency that is lower band ($\frac{\omega}{\omega_n} < 0.77$) and higher band ($\frac{\omega}{\omega_n}$

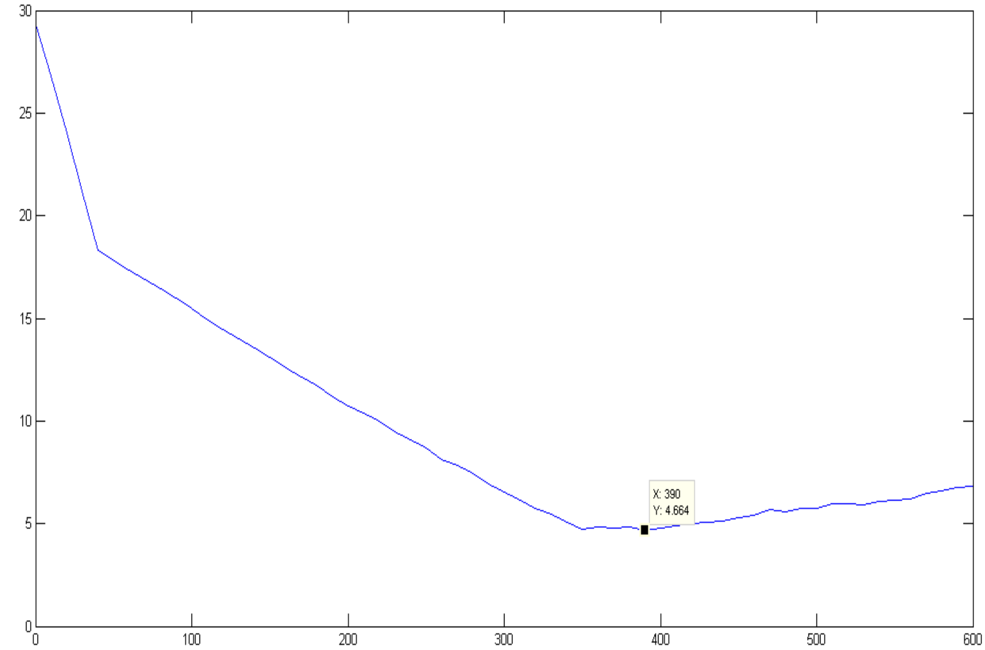


Figure 3.7: Search for best possible coloumb damping.

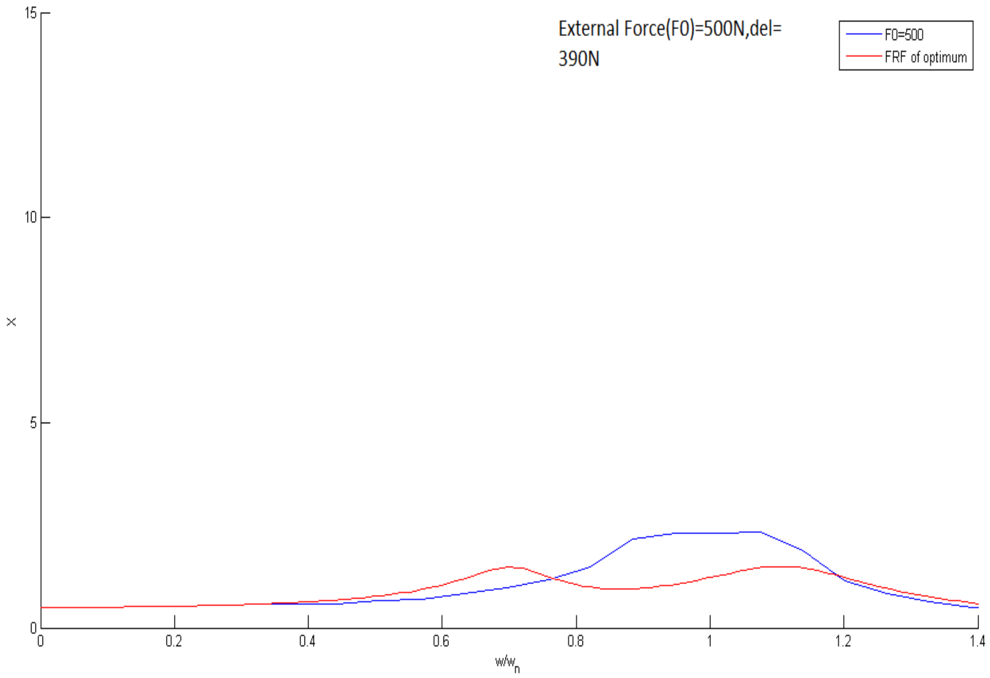


Figure 3.8: Comparison of Optimum viscous with the best FRF in case of coloumb damping .

> 1.19), coloumb damping is slightly better than viscous damping.

- For ($0.77 < \dfrac{\omega}{\omega_n} < 1.19$), viscous damping is found to be more suitable.
- The above observations are by keeping optimal parameters in case of viscous damping. Further improvement may be possible by changing frequency ratio(f).

Important Note: Coulomb damping doesn't offer much benefit over viscous damping,but it is also responsible for wear and tear of mechanical system.Also it can be seen that Search for the best coulomb damping is difficult as no analytical solutions are available,and performing numerical search has its limitations because in case of coulomb damping the response is a function of external force F0 and the frictional force del.Also as shown in figure 3.7,addition of coulomb damping becomes beneficial only for somewhat large frictional force,which in addition to causing wear and tear,from a practical perspective it can become difficult to add such an high frictional force.

Chapter 4

Non-linearity in the System

4.1 Background

The characteristics of the non linear primary system attached by the linear absorber change only slightly in terms of the values of its new linearizied natural frequency, damping coefficient and frequency interval for primary resonance, because the vibration absorber is a small attachment and does not contribute significantly to the change of these parameters (linear stiffness and damping coefficient). Two ratios, namely attenuation ratio and desensitisation ratio, will be defined to indicate the effectiveness of the linear absorber in Suppressing the primary resonance vibrations. The attenuation ratio will be defined by the ratio of the maximum Amplitude of vibrations of the non linear primary system after and before adding the linear vibration absorber under a given value of the amplitude of excitation.The desensitisation ratio will be given by the ratio of the critical values of the amplitude of external excitations presented in the non linear primary system after and before the linear vibration absorber is attached. The critical value of the excitation amplitude refers to here as a certain value of external excitation that results in the occurrence of saddle-node bifurcations and jump phenomena in the frequencyresponse curve. Below this critical value, the frequencyresponse curve of the primary resonance vibrations does not show saddle-node bifurcations (and jump phenomena) and will exhibit saddle-node bifurcations and jump phenomena if the amplitude of excitation exceeds the critical value.

The forced oscillations of a two degree-of-freedom non linear system having cubic non linearities have been studied by many researchers The attention of these studies has focused on the case of internal resonances when $\omega_1 = \omega_2 or \omega_3 = \omega_1$. Specifically, Nayfeh and Mook considered the forced oscillations of cubically non linear systems without linear coupling terms under internal resonances $\omega_2 = 3\omega_1$. Natsiavas studied the steady-state oscillations and stability of the non linear system having cubic non linearities under one-to-one internal resonances $\omega_1 = \omega_2$. It was shown that the presence of one-to one internal resonances in the nonlinear system of dynamic vibration absorber may result in instability of the periodic response and quasi-periodic oscillations with much higher amplitudes. The non-linear system considered is as shown in figure Below we included the figures of FRF when a hardening spring(cubic stiffness is positive)

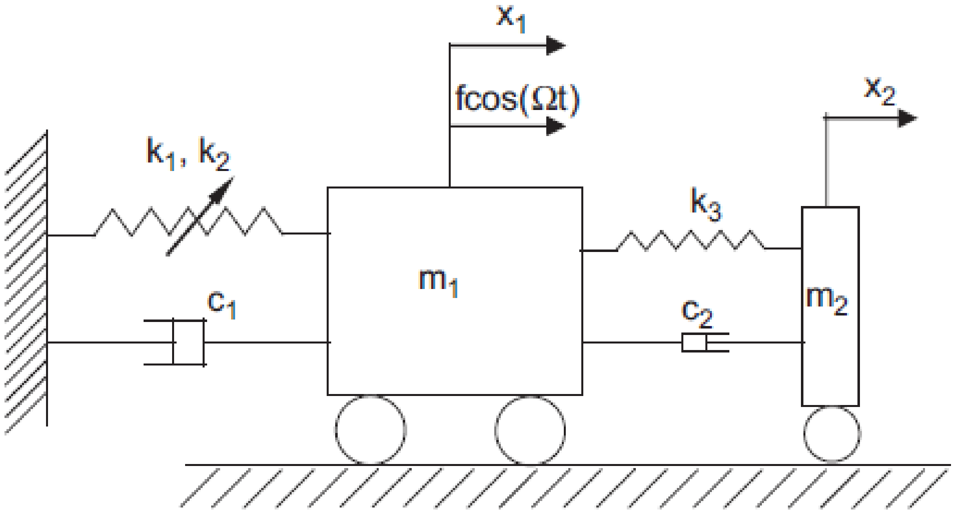


Figure 4.1: Non-linearity added in the primary system.

is included in the system.In figure 4.3 the FRF's are drawn without absorber and then to this system a cubic hardening spring is included without the absorber (green line),it is seen that FRF shifts towards right,it is seen that as non-linearity increases more the shift occurs.
Similar effects are seen when the non-linearity is introduced in the absorber system,in this case also the graphs lean to right at both first and second peak and with increase in non-linearity same leaning effect is seen.

Given the fairly limited literature concerning the nonlinear dynamic vibration absorber (NDVA), the specific objectives of the study were as follows: To develop a complete analytical expression and verify the numerical solutions that describe the performance of the particular NDVA under harmonic excitation. Subsequently, investigate the response and its sensitivity to the various physical parameters of the absorber namely the absorber mass, damping and stiffness. The latter possesses restoring force contributions which are both linear and nonlinear cubic powers of the displacement.

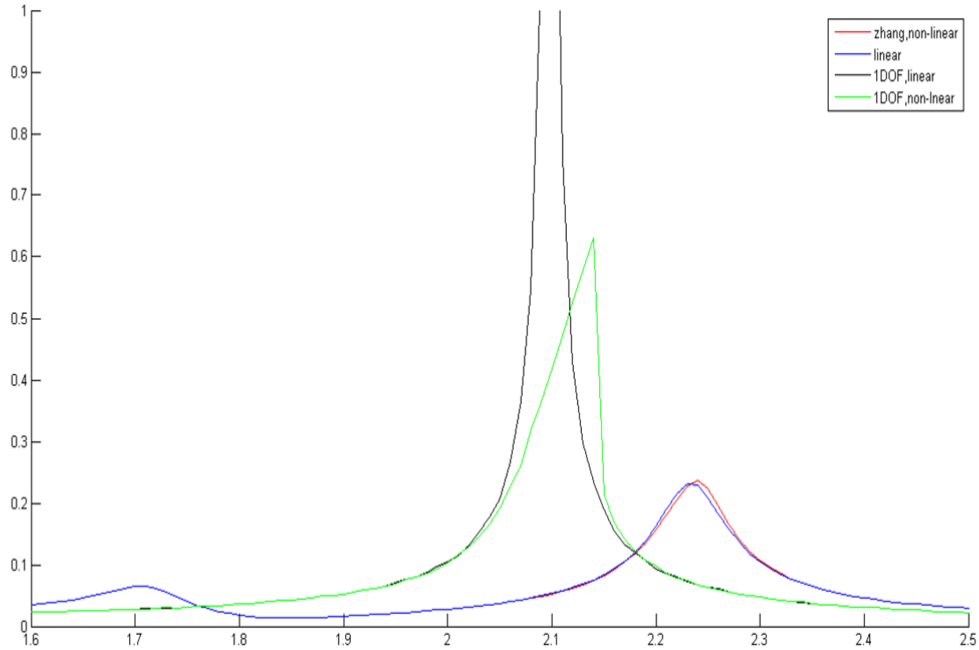


Figure 4.2: non-linearity is added in secondary system.

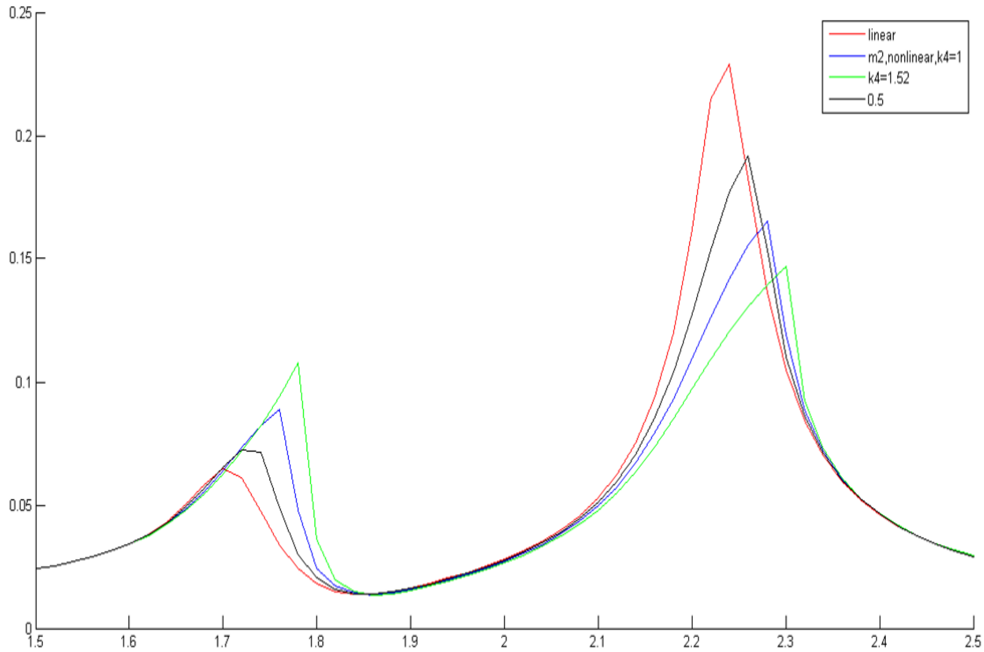


Figure 4.3: 2-DOF non linear system.

4.2 Non-linear spring in primary system: Optimum solution

4.2.1 Effect of varying stiffness of spring of the absorber

Initially an FRF for the linear paraameters

$$m1 = 10kg \quad m2 = 0.6kg \quad k1 = 44N/m \quad c1 = 0.1Ns/m$$

is drawn.In this case it can be seen that the two peaks are equal.Now a non-linearity is added in the primary system by addition of a cubic spring with $k1n = 44N/m^3$.After the addition of a cubic spring,the effect of change in the stiffness of the spring of absorber on the two peaks in the FRF and the bandwidth is to be studied. From the Figure 4.4 it can be seen that on

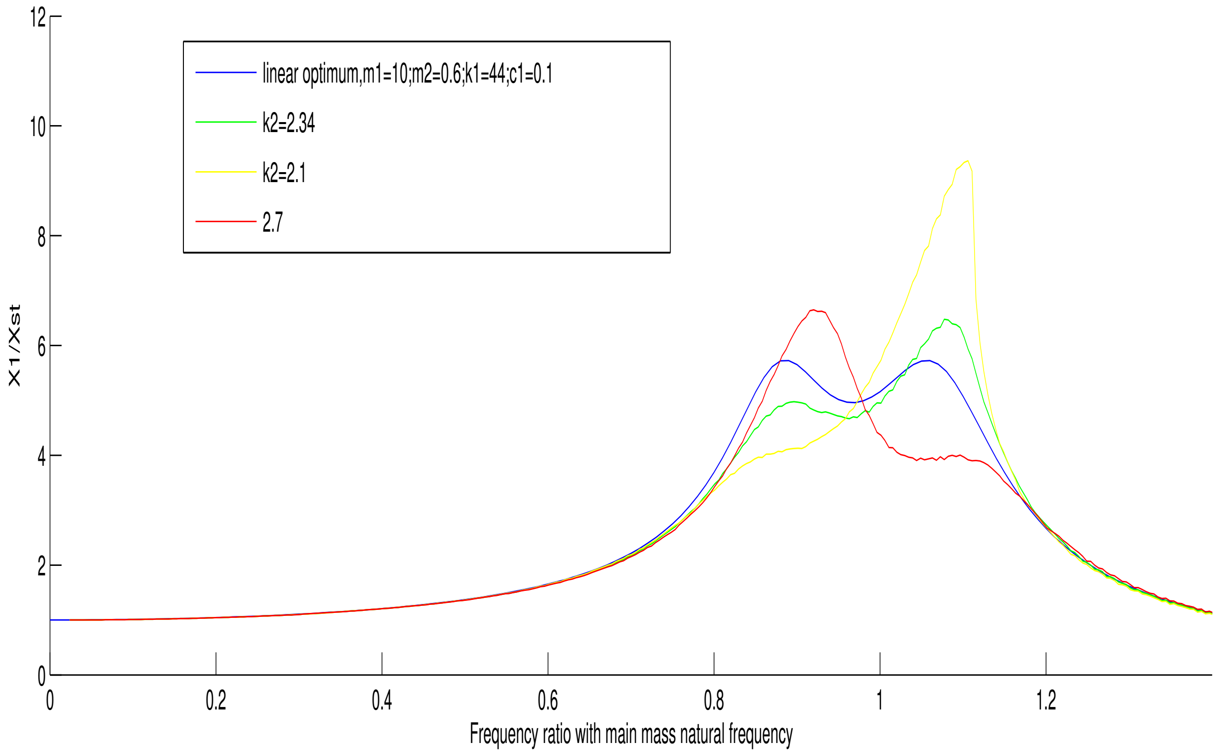


Figure 4.4: Effect of varying stiffnes of absorber.

increasing the stiffness of absorber spring, the 1st peak or mode -I peak increases whereas the 2nd or Mode -II peak goes on decreasing as k2 is increased.It is difficult to conclusively predict the effect on bandwidth that is the distance between two peaks,as both the peaks are shifted towards right. The optimum solution by varying k2,would have both mode I and mode II peaks as equal,also it can be seen that if k2 goes on decreasing the mode II peak almost disappears and only mode-I becomes prominent.

By conducting name of the search the optimal solution was found to be $k2 = 2.43$ It can be seen that when a non-linear spring is added the optimal solution changes,in this case only k2 is varied,but in the sections that follow we have explored the effect that secondary damping have on the peaks and the combined the effects of secondary stiffness and damping to provide a best case scenario.

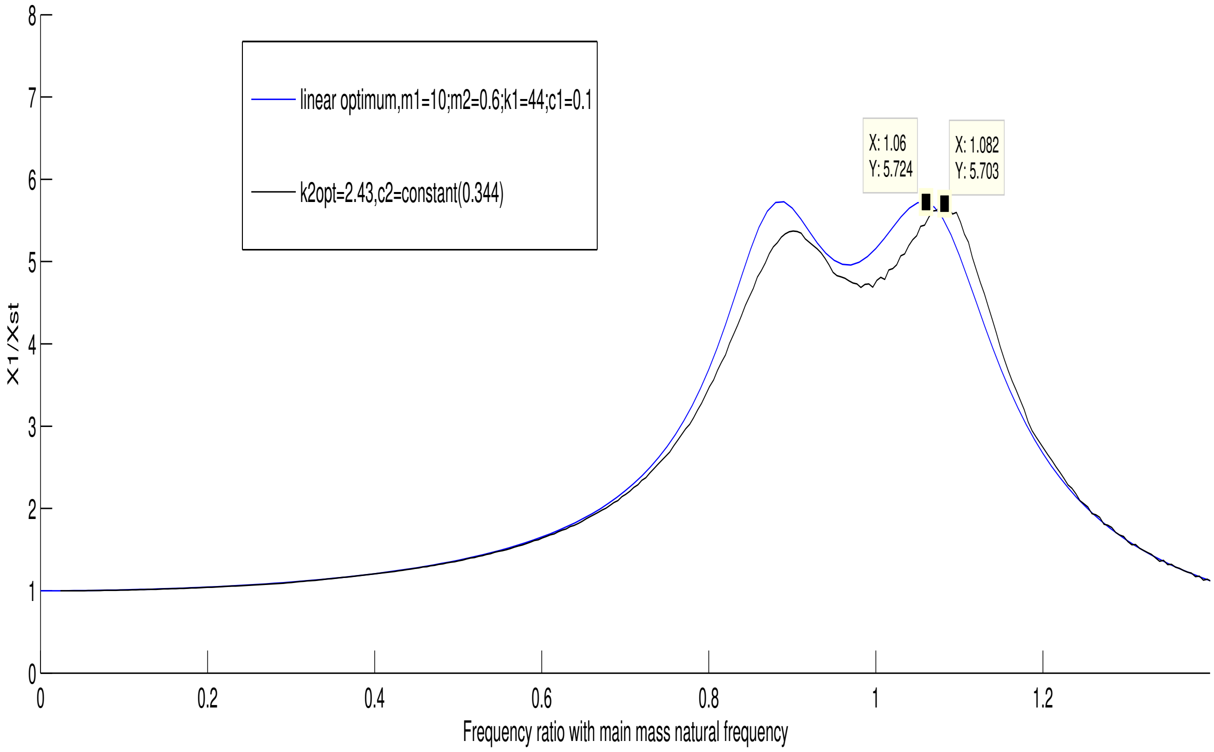


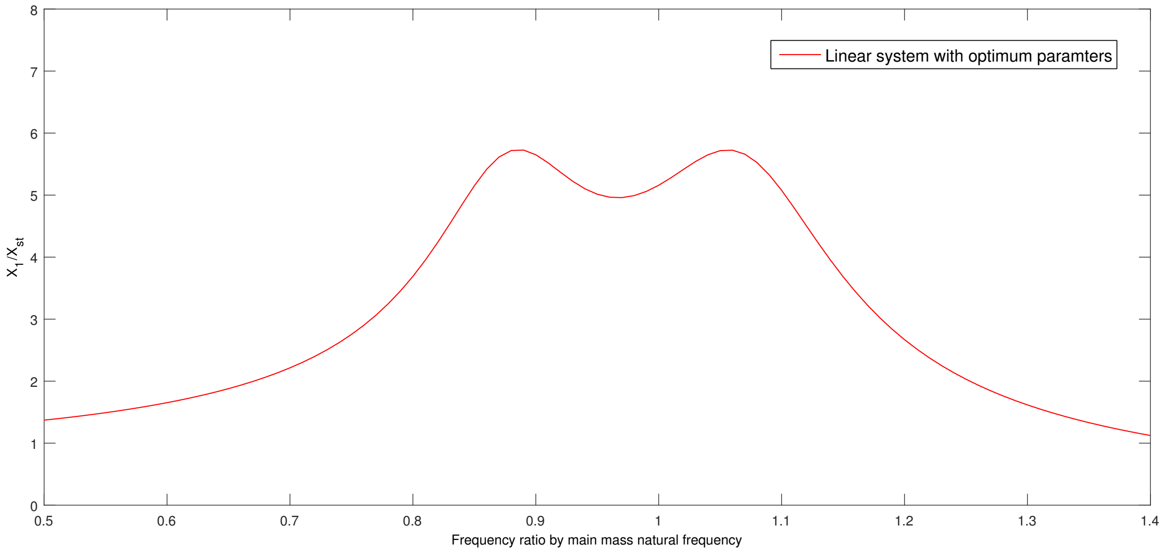
Figure 4.5: Comparision of optimum FRF of linear and non-linear spring .

It is to be noted that the optimum case of non-linearity is obtained in this case by varying only the stiffness of spring attached to the secondary mass.We were able to obtain an FRF that was as good as the FRF for optimized linear case and as stated earlier there is not much change in the bandwidth.

4.2.2 Effect of varying damping of the absorber

The initial system is having all its components to be linear. When the linear system is designed having optimum parameters, its Frequency Response Characteristic is as shown below.

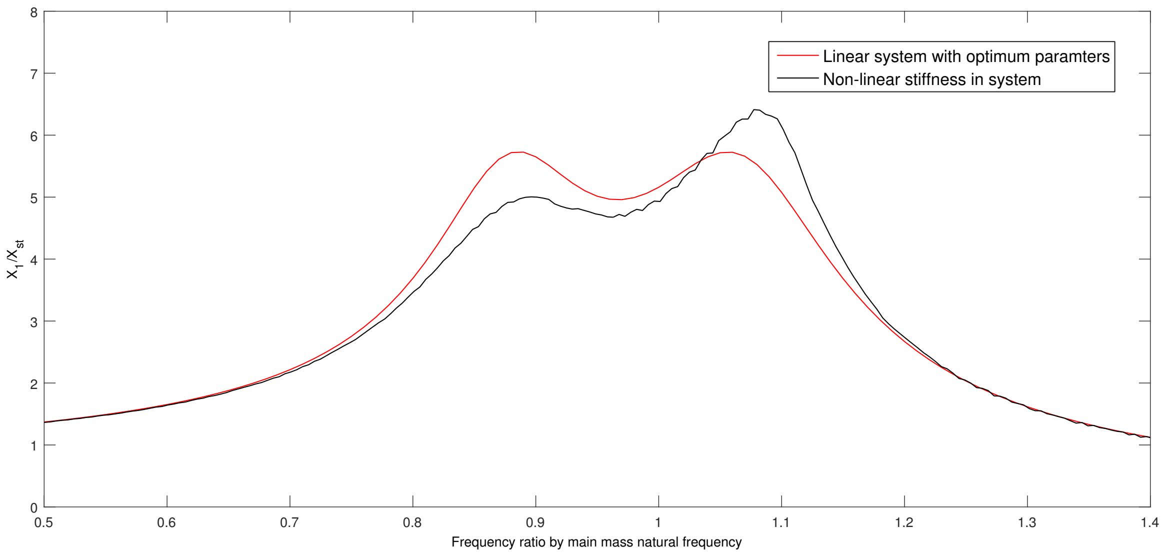
The two peaks have almost same value and they are at the minimum possible value. Hence, the system as a whole is said to have the best or optimum response over a range of frequency.



If now we add a non-linearity in the primary mass, the FRF changes with a reduction in maximum value of left peak and increase in maximum value of right peak.

$$m_1 = 10kg \quad m_2 = 0.6kg$$
$$k_1 = 44N/m \quad k_{1N} = 1000N/m^3 \quad k_2 = 2.347N/m$$
$$c_1 = 0.1N \cdot s/m \quad c_2 = 0.344N \cdot s/m$$

Once it is established that addition of non-linearity to the primary mass disturbs the linear



system under optimal conditions, we studied the effect of changing the secondary mass damping, on the system.

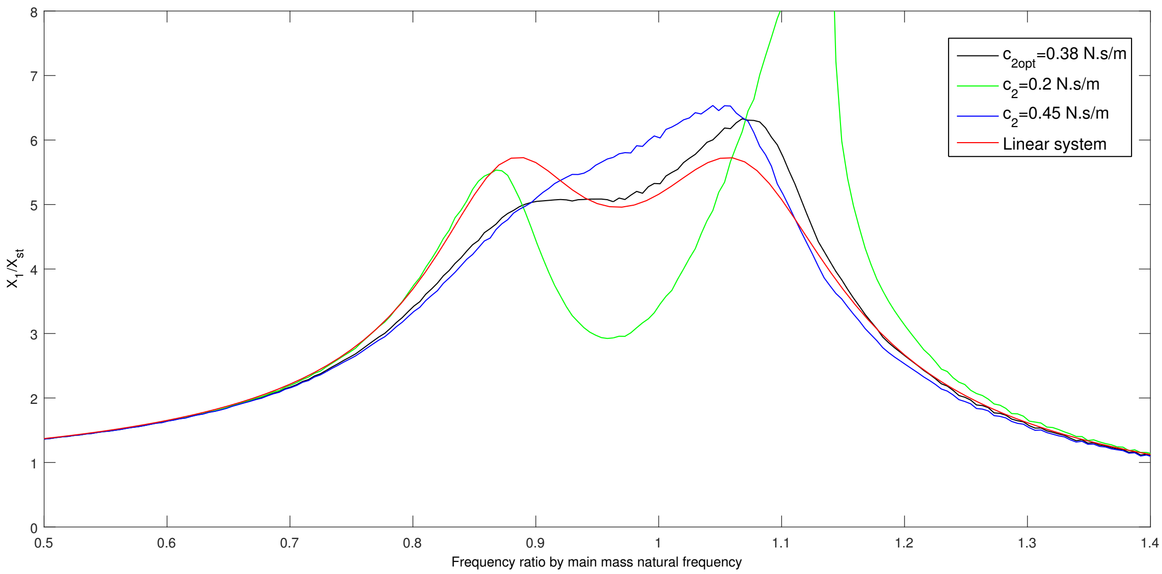
The graph show under is plotted with varying values of secondary mass damping (ζ_2). The

	Linear	Non-Linear	Difference
Peak 1	5.726	5.004	0.722
Peak 2	5.724	6.412	0.688

Table 4.1: Comparing maximum values of ¡write something here¿

linear plot is retained for comparison purpose.Since we are concerned with getting the best response from the system, we need to minimize its maximum amplitude over the whole range. This is achieved when both the peaks have approximately the same maximum value.

From the above figure, the following points are to be noted:



- If non-linearity is decreased, the right peak increases whereas the left peak comes down and vice-versa when non-linearity is increased.
- The value of $c_2 = 0.38N \cdot s/m$ gives the best case scenario when we are changing the secondary mass damping. It is certainly not the best case, if we were also allowed to vary other system parameters. However, the maximum of the two peaks are close to each other.
- The value $c_2 = 0.38N \cdot s/m$ is not too offset from the linear system's optimal secondary mass damping value $c_2 = 0.344N \cdot s/m$. In the non-linear case, however, the response is better near the first peak and worse off near the second peak, as compared to the linear system.

4.2.3 Optimum Frequency Response Function for Nonlinear spring in Primary system

In this section the combined effects of variation secondary mass spring stiffness and damping co-efficient is used to obtain the best possible solution and is used to compare with the optimal linear solution,to see if there is a reductionn in the maximum amplitude of vibration of the primary mass system and how the bandwidth between mode -I and mode-II of the optimal linear and non-linear FRF differs. It is observed that after performing name of search by

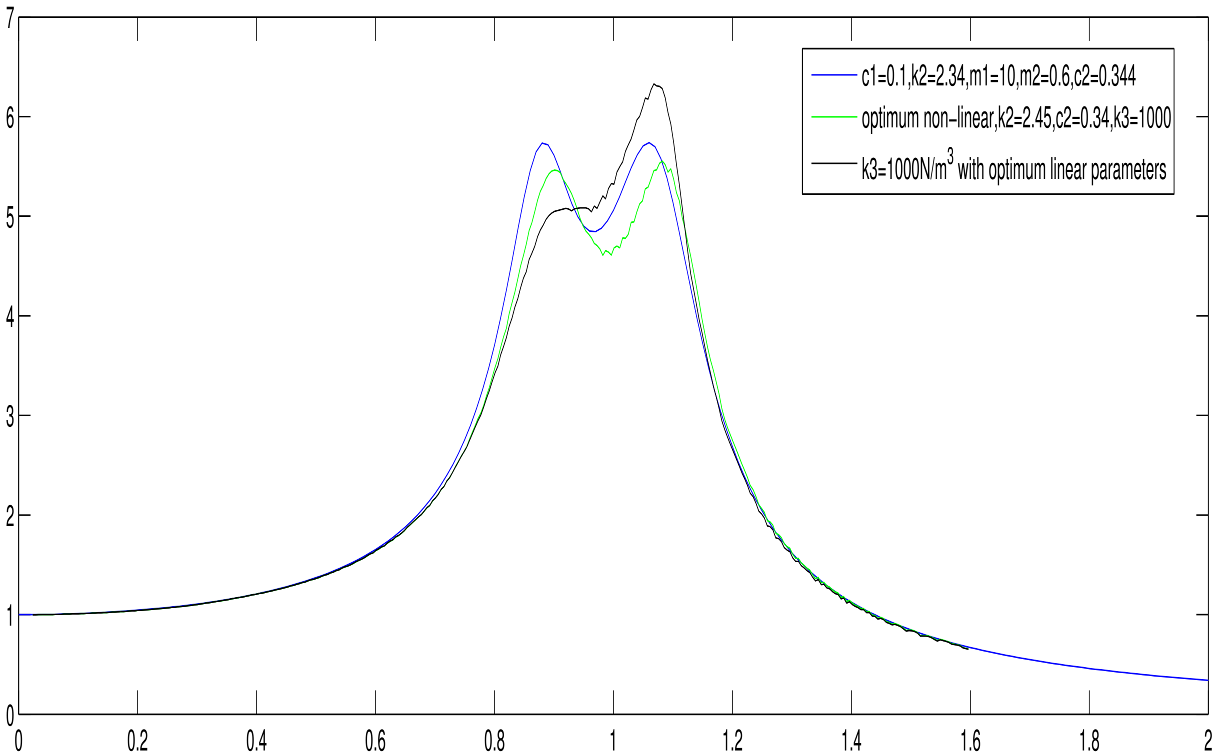


Figure 4.6: Comparision of optimum FRF's for linear and cubic spring.

varying both stiffness and damping co-efficient,the non-linear FRF obtained as can be seen from figure 4.6 is better than the optimum linear FRF,both Mode-I and mode-II peaks can be brought down.So if non-linearity is already present in the primary system,designers have an opportunity to bring down the maximum amplitude of vibration further down.Also as seen from figure 4.6 the vibration response at the resonant frequency($\omega = 1$)is also brought down as hardening spring has an effect of shifting the FRF towards right.

As such it is difficult to conclusively predict the difference in bandwidth between peaks of linear and non-linear optimum solution.

Below is the comparison of the parameters:

Case	$\left(\frac{X_1}{X_{stat}}\right)_{max}$	Bandwidth
Linear optimum FRF	5.733	0.18
Non-Linear optimum FRF	5.452	0.1857

4.2.4 Other Cases

Case 1

Mass of primary system, m_1= 20kg

Mass of secondary system, m_2 = 1.6kg

Mass ratio, μ = 0.08

Primary mass stiffness k_1= 8000 N/m

Primary mass damping ratio ζ_1 = 0.02

Optimum Frequency ratio, f_{opt} = 0.914

Optimum secondary mass damping, ζ_{2opt} = 0.013

Stiffness of secondary mass spring, k_2 = 534.65 N/m

Stiffness of non-linear spring, k_{1n} = 10420 N/m^3

Optimum stiffness of secondary mass spring with added non-linearity = 566 N/m

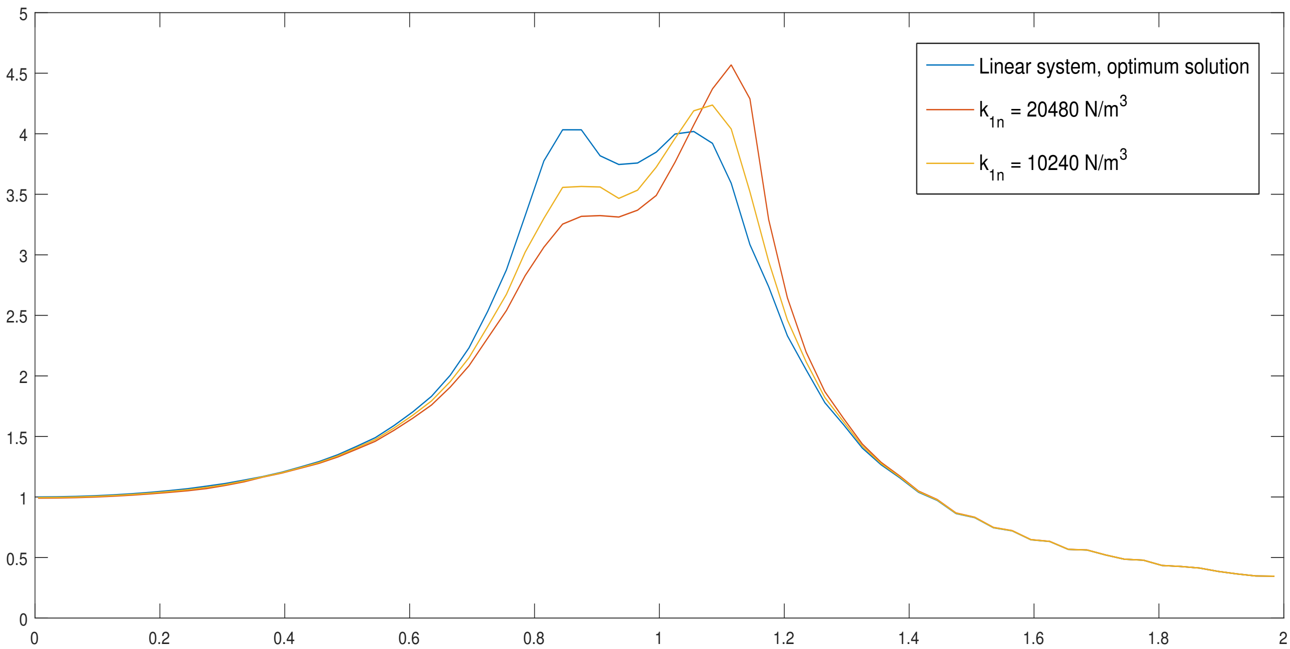


Figure 4.7: Effect of adding non-linearity in the primary system

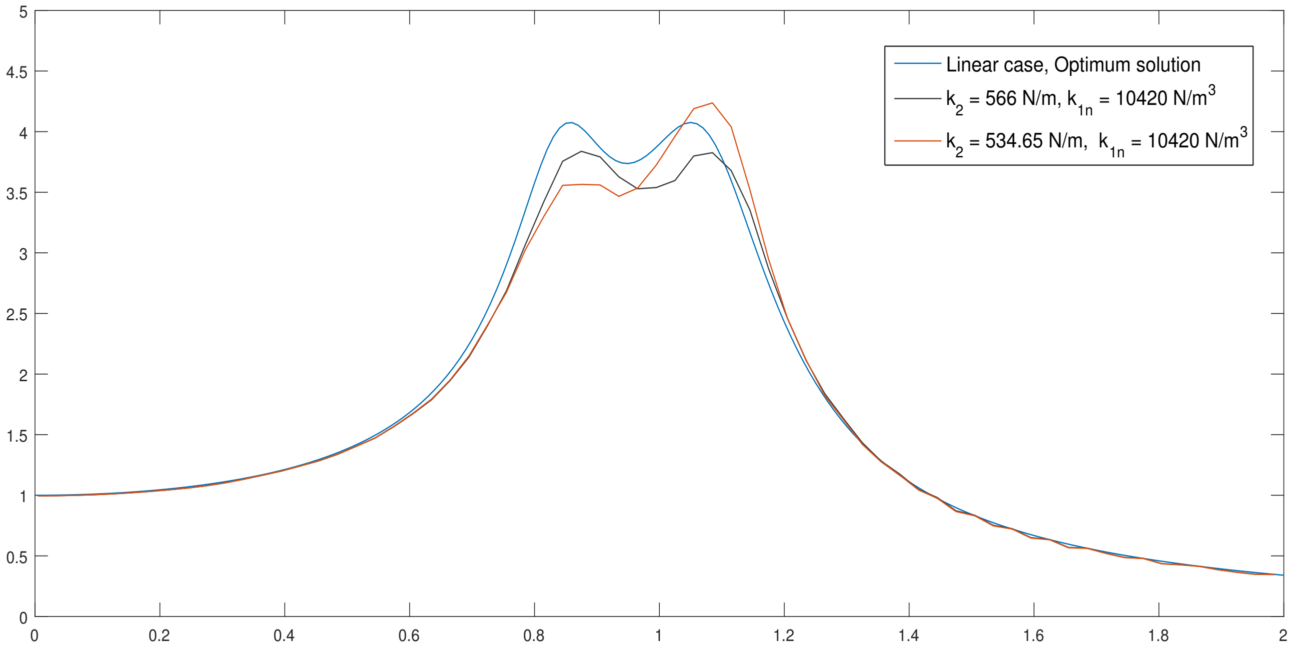


Figure 4.8: Effect of changing stiffness of absorber

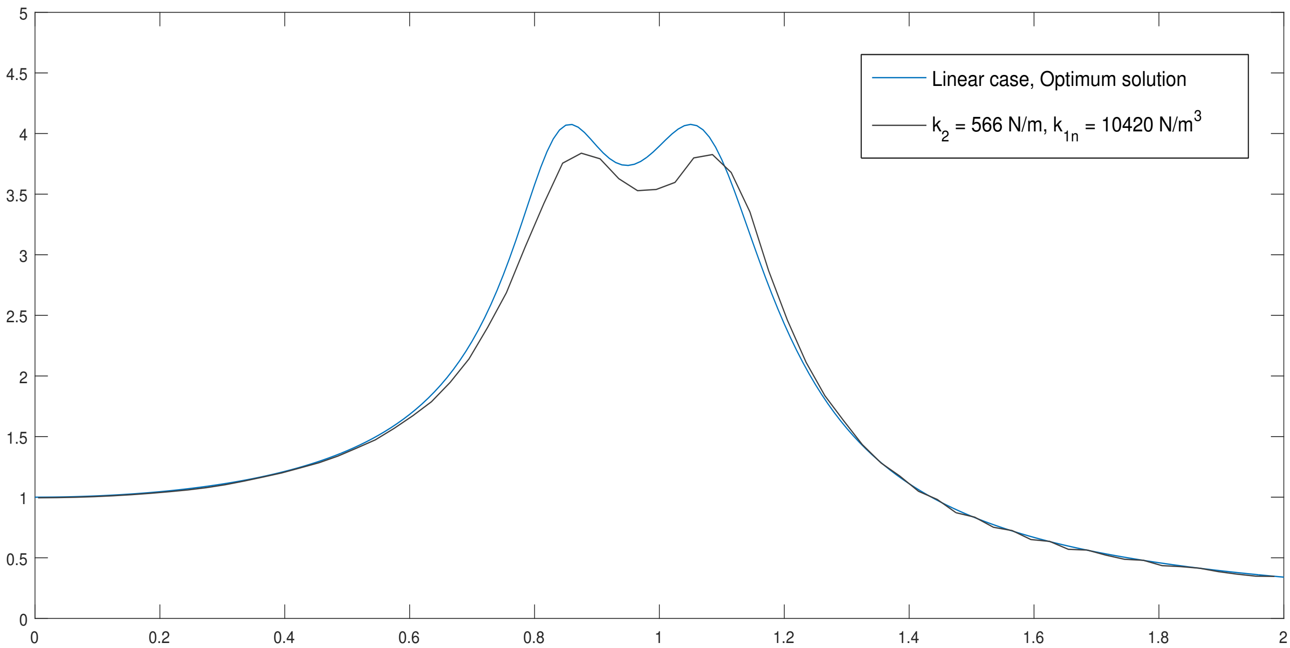


Figure 4.9: Comparison of non-linear and linear optimal solution

Case 2

Mass of primary system, m_1= 50kg

Mass of secondary system, m_2 = 5kg

Mass ratio, μ = 0.1

Primary mass stiffness k_1= 11250 N/m

Case	$\left(\frac{X_1}{X_{stat}}\right)_{max}$	Bandwidth
Linear optimum FRF	4.075	0.21
Non-Linear optimum FRF	3.838	0.22

Primary mass damping ratio $\zeta_1 = 0.02$

Optimum Frequency ratio, $f_{opt} = 0.914$

Optimum secondary mass damping, $\zeta_{2opt} = 0.017$

Stiffness of secondary mass spring, $k_2 = 913.27$ N/m

Stiffness of non-linear spring, $k_{1n} = 14238$ N/m^3

Optimum stiffness of secondary mass spring with added non-linearity = 1010 N/m

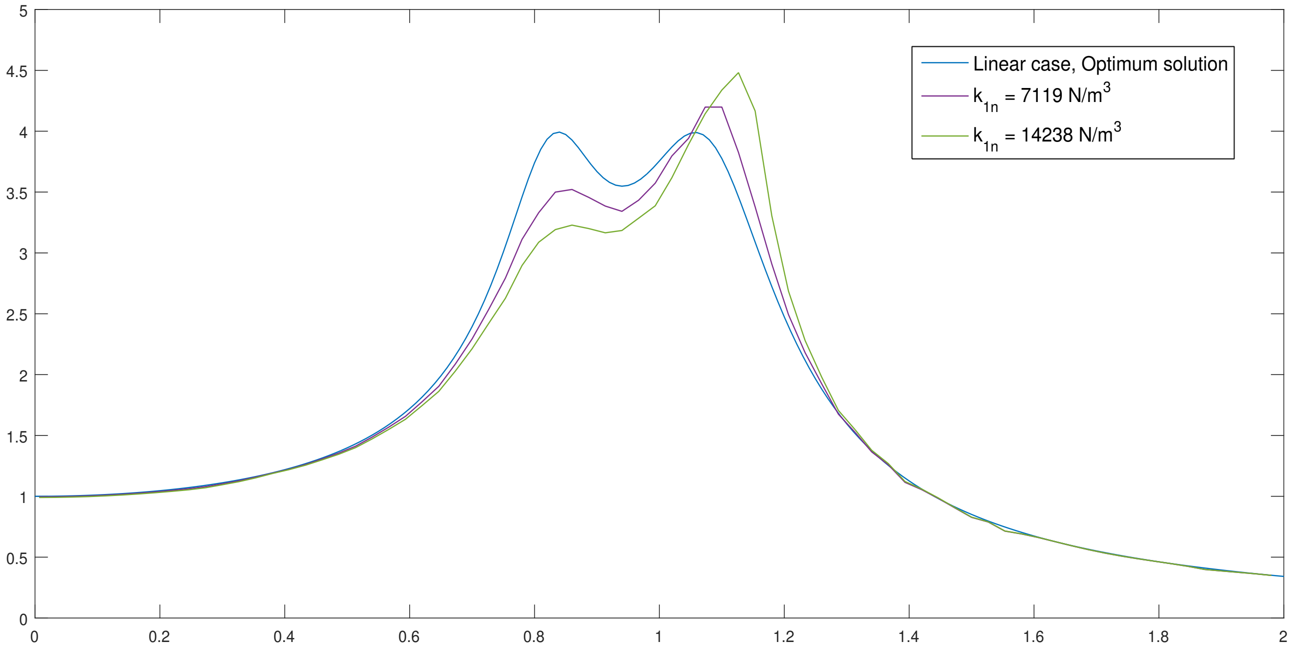


Figure 4.10: Effect of adding non-linearity in the primary system

Case	$\left(\frac{X_1}{X_{stat}}\right)_{max}$	Bandwidth
Linear optimum FRF	3.993	0.22
Non-Linear optimum FRF	3.636	0.267

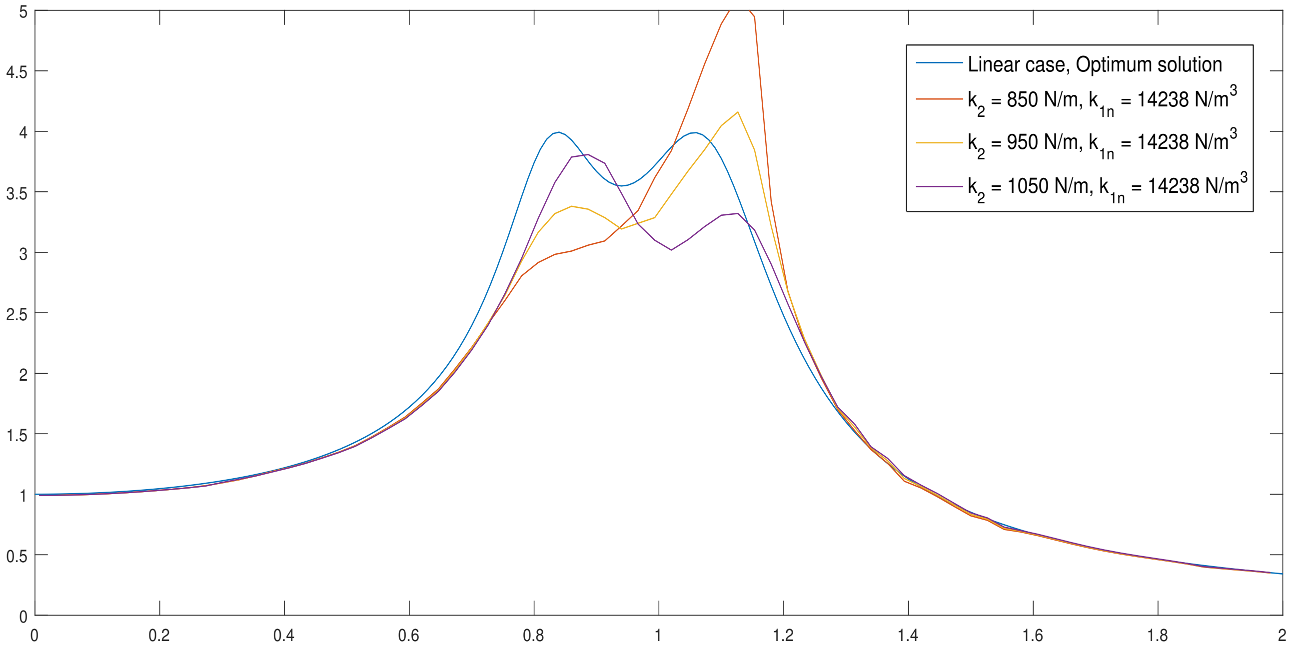


Figure 4.11: Effect of changing stiffness of absorber

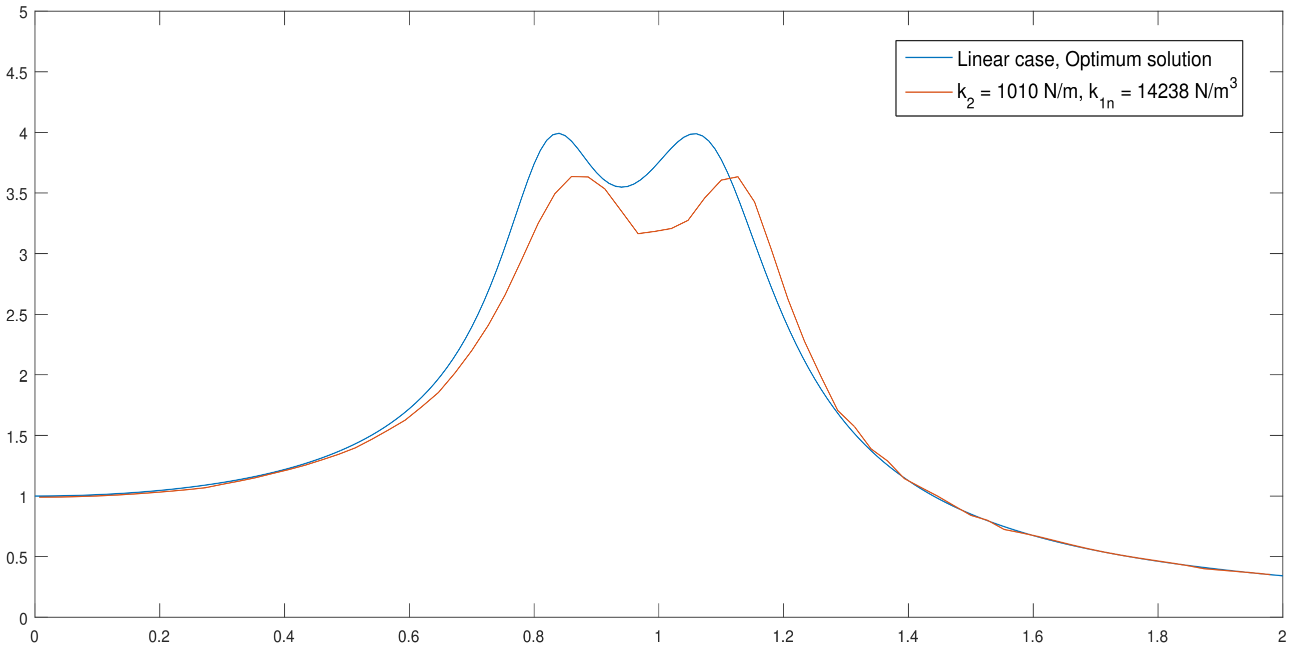


Figure 4.12: Comparison of non-linear and linear optimal solution

4.3 Non-linear spring in the Secondary system: Optimum solution

4.3.1 Effect of varying stiffness of spring of the absorber

In the sections below we would analyse and tabulate the effects of varying stiffness,damping and try to find the optimum solution when a cubic spring is present in between the main mass and the absorber mass.The procedure adapted in this section is similar to previous sections, qualitative results are presented on varying the independent parameters.

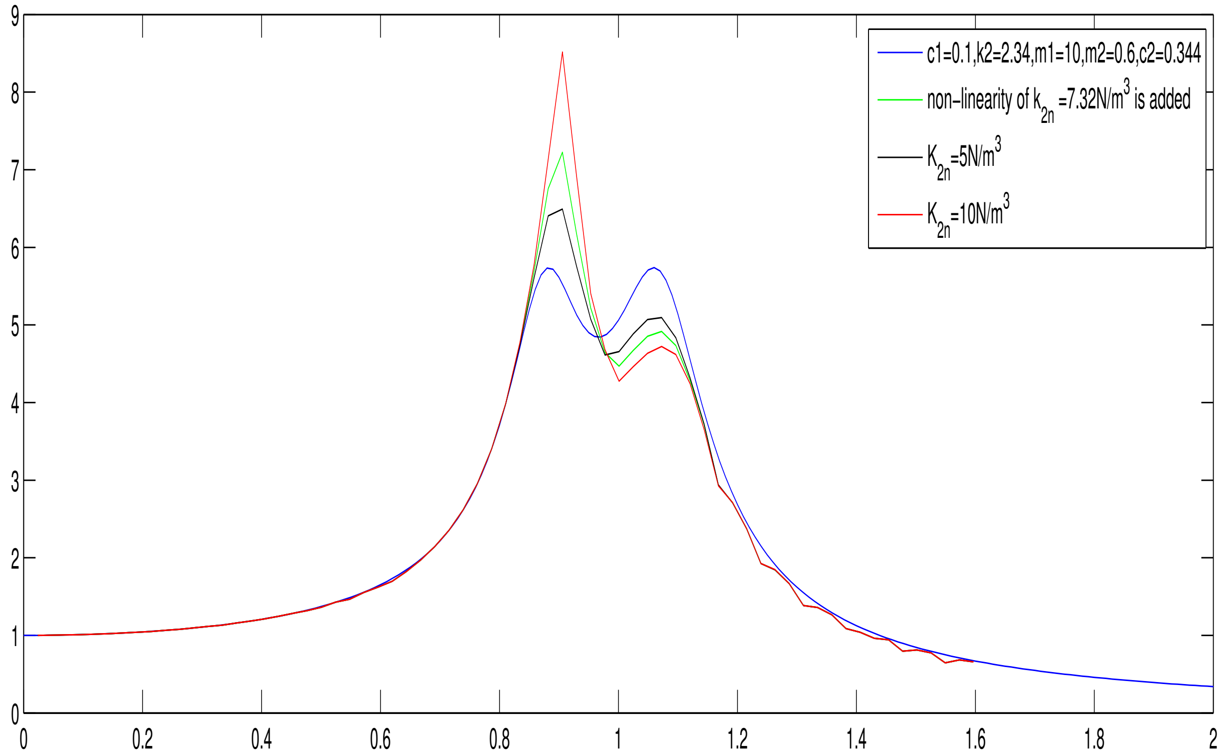


Figure 4.13: Effect of increasing non-linear stiffness of absorber.

It is to be noted that for secondary mass system,we have control over the amount of non-linearity that is to be introduced and the linear stiffness and damping.For a designer one more design variable is introduced.So we will start with the effect of non-linearity,from 4.13 it can be qualitatively seen that as the cubic non-linearity increases the the mode-I or first peak goes on increasing whereas the mode-II peak decreases and both Mode-I and II are shifted towards right.

Now for $k2_n = 7.32N/m^3$ we would change the linear stiffness value of the secondary spring and note the effects.

- On increasing the linear stiffness K_2 mode-I peak goes up whereas mode-II goes down.
- The effect is similar to the cubic stiffness,so increasing both linear and non-linear stiffness would have an adverse effect.
- Bandwidth between 2 peaks is essentially same as we vary linear stiffness.
- To obtain an optimum solution,the two peaks should be equal,so if we increase K_{2n},we need to reduce the value of linear stiffness.

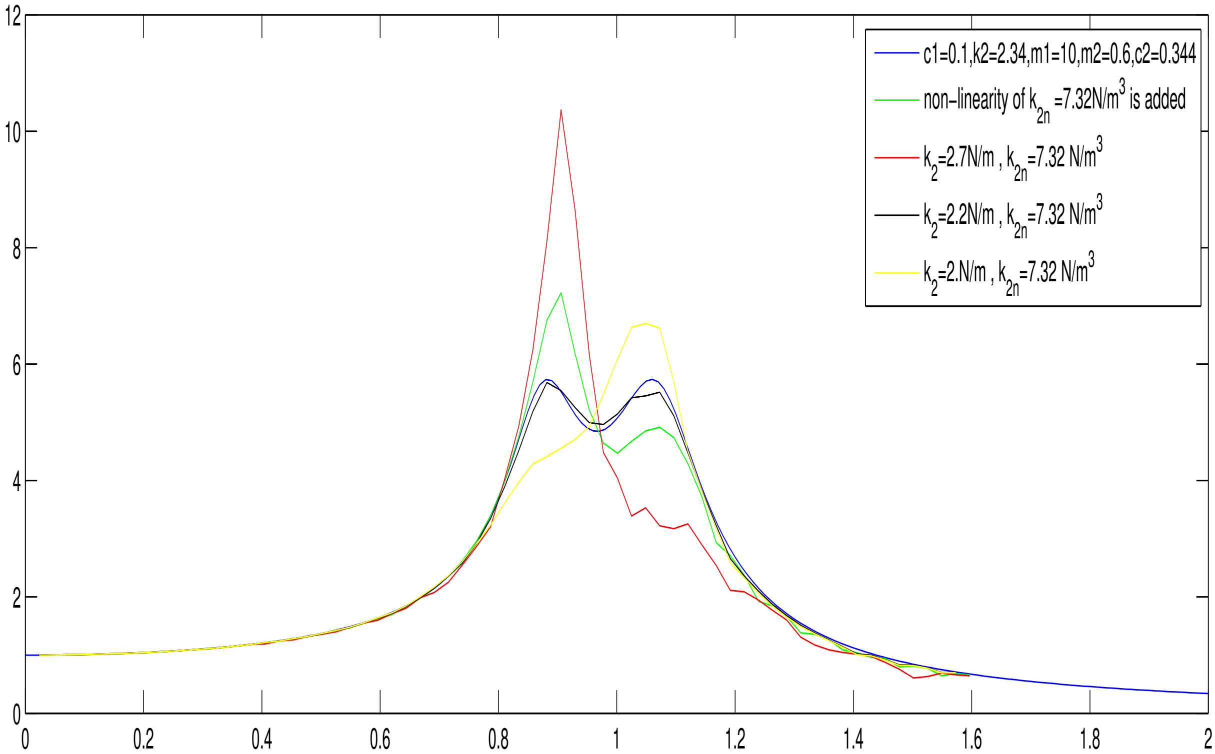


Figure 4.14: Effect of increasing linear stiffness if cubic non-linearity is present in absorber.

4.3.2 Effect of varying the damping of secondary system

We would be keeping $K_{2n} = 7.34N/m^3$ and $k_2 = 2.34N/m^3$ but we note the effects of varying the damping in the secondary system.The idea is to combine the effects that are noted for change in spring stiffness and the damping,to obtain an optimum solution in case when non-linearity is introduced in the secondary mass and compare it with the linear optimum solution,this excercise gives us an idea of whether we can further reduce the vibrational amplitude of primary mass. The observations made on varying the damping are:

- The variation damping has a major effect on the Mode-I peak.
- On increasing the damping Mode-I peak decreases
- Increase of damping decreases the mode-II peak but the rate of decrease is much smaller,if damping is increase beyond certain level mode-II peaks dies out.
- From the above two points,it is wrong to infer that more the damping lesser the vibration.
- As can be seen in 4.15 for $C_2 = 0.6$ only one peak remains but the maximum amplitude in FRF is greater than for $C_2 = 0.4$.So after certain point,increasing damping would be counter productive.

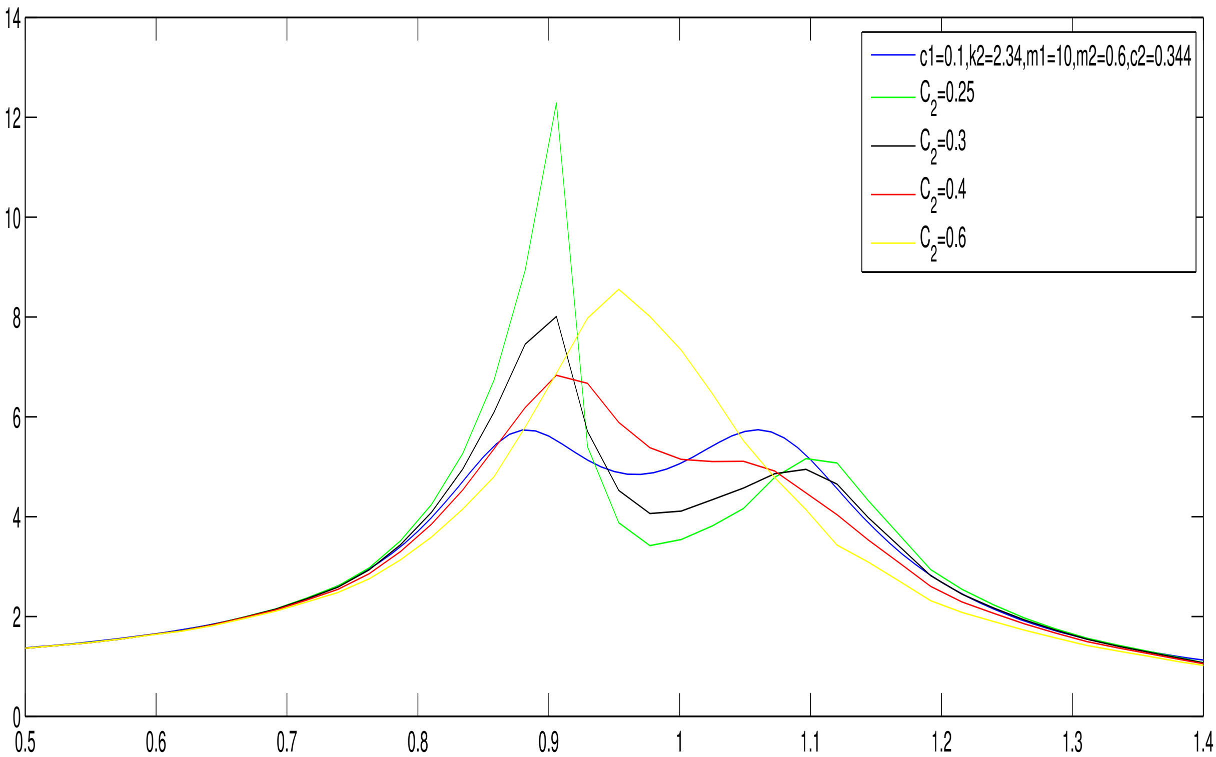
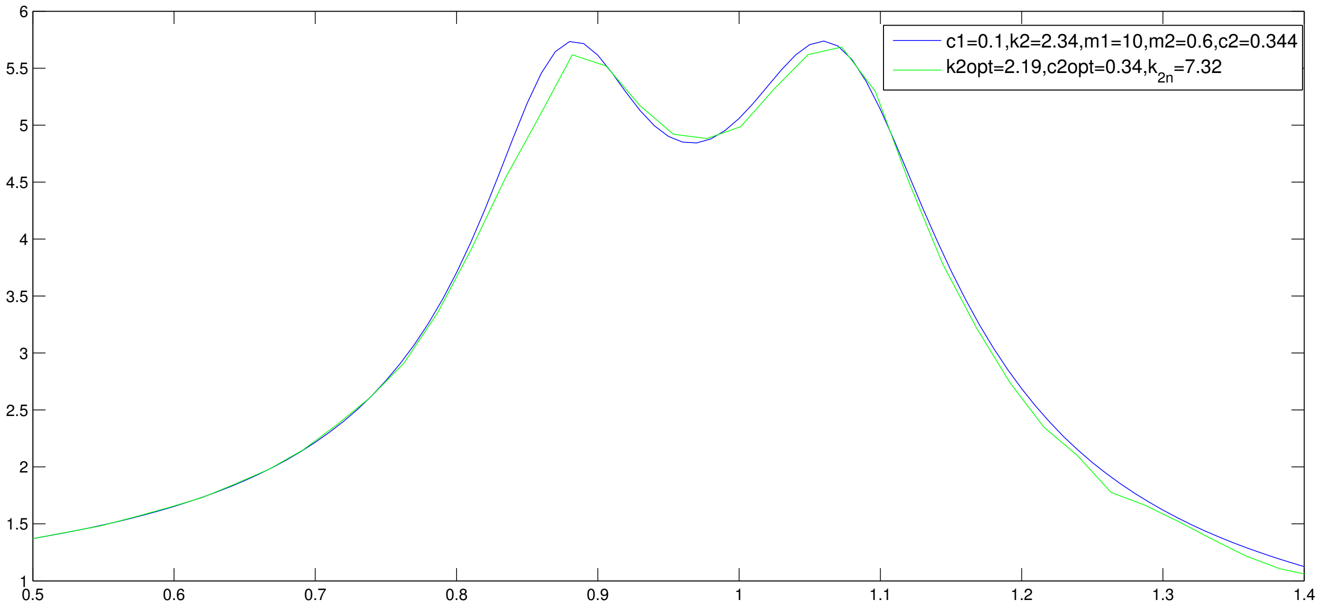


Figure 4.15: Effect of increasing damping if cubic non-linearity is present in absorber.

4.3.3 Comparison of optimum solution when non-linearity is introduced with the linear optimum solution

It is observed that after performing mini-max search by varying both stiffness and damping co-efficient,the non-linear FRF obtained is as shown in figure 4.6 and is almost similar to linear optimum FRF,this can be attributed to smaller value of cubic non-linear spring,but on introduction of non-linear spring the optimum value for linear stiffness changes but optimum damping almost remains same



c1=0.1,k2=2.34,m1=10,m2=0.6,c2=0.344
k2opt=2.19,c2opt=0.34,k2n=7.32
6
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4

Chapter 5

Analytical study of non-linearity in single DOF system

Harmonic balance method is selected for analytical study because it is not restricted to weakly nonlinear problems and, for smooth systems, the assumed harmonic solutions always converge to the exact solution. The mathematical analysis can be conducted relatively easily due to it not producing complicated mathematical expressions for higher order terms. However, the HBM has some limitations in accuracy. In the interpretation considered here, one chooses just to use the response at the excitation frequency, i.e., by assuming that sub and super harmonics are negligible compared to the fundamental harmonic and that the system is only under periodic excitation. In addition, the stability characteristics require a separate analysis. We will be considering the non-linear system governed by duffing equation.Here we will be considering the forced vibration of a single degree of freedom system with viscous damping.the eqation of motion of such a system is given by

$$m\ddot{x} + \alpha x + c(\dot{x}) + \beta x^3 = F_0 sin(\omega t) \tag{5.1}$$

We will be considering only forced vibration and neglecting the transient vibration which decays after a certain interval of time because of presence of damping in the system.The steady state forced vibrations will have harmonics corresponding to forcing frequency ω and higher harmonics corresponding to odd multiples of $\omega, 3\omega, 5\omega etc$ The forced vibration will be dominant in first harmonic and decreases rapidly for higher harmonics. Here,for harmonic balance method,we would be restricting ourselves to the first harmonic.

$$\ddot{x} + p^2x + 2\zeta p(\dot{x}) + p^2\mu x^3 = F_0 sin(\omega t) \tag{5.2}$$

$$p^2 = \alpha/m \tag{5.3}$$

$$\mu = \beta/\alpha \tag{5.4}$$

$$\zeta = c/2mp \tag{5.5}$$

$$\tau = pt \tag{5.6}$$

$$r = \omega/p \tag{5.7}$$

$$x = asin(r\tau) + bcos(r\tau) \tag{5.8}$$

We use harmonic balance method in which each harmonic is balanced by separating out the terms in LHS and RHS.The third harmonic and higher order harmonics are neglected. Equating the co-efficients of $cos(r\tau)andsin(r\tau)$ we get:

$$-ar^2 - 2\zeta br + a + 3/4\mu a^3 + 3/4\mu ab^2 = 0 \tag{5.9}$$

$$-br^2 - 2\zeta ar + b + 3/4\mu b^3 + 3/4\mu a^2b = 0 \tag{5.10}$$

$$X^2 = a^2 + b^2 \tag{5.11}$$

rearranging the above equations-

$$9/16(\mu)^2X^6 + 3/2\mu(1 - r^2)X^4 + ()r^4 - 2r^2 + 1)X^2 - X_{st}^2 = 0 \tag{5.12}$$

$$\tag{5.13}$$

For constant force excitation,FRF is plotted by varying r and for $\mu = 9, X_{st} = 10, \zeta = 0.1$ A comparision of Linear and non-linear system-

- In non-linear system frequency of free vibration depends on amplitude of vibration.
- For the linear system, the amplitude-frequency relationship for free vibration is a vertical line on the forced response diagram at the excitation frequency equal to the natural frequency. The forced vibration response occurs to the left and right of this vertical resonance line. For a nonlinear system this relationship for a hardening spring bends to the right and for a softening spring, it bends to the left. The forced response follows above and below the free vibration characteristic, giving rise to the jump phenomenon.
- The forced vibration response in a linear system with harmonic excitation force is also harmonic with the same frequency as the excitation frequency. In a nonlinear system with harmonic excitation, however, the response is nonharmonic but periodic with the first and higher harmonics of the fundamental period. For symmetric stiffness characteristics, only odd harmonics are present.

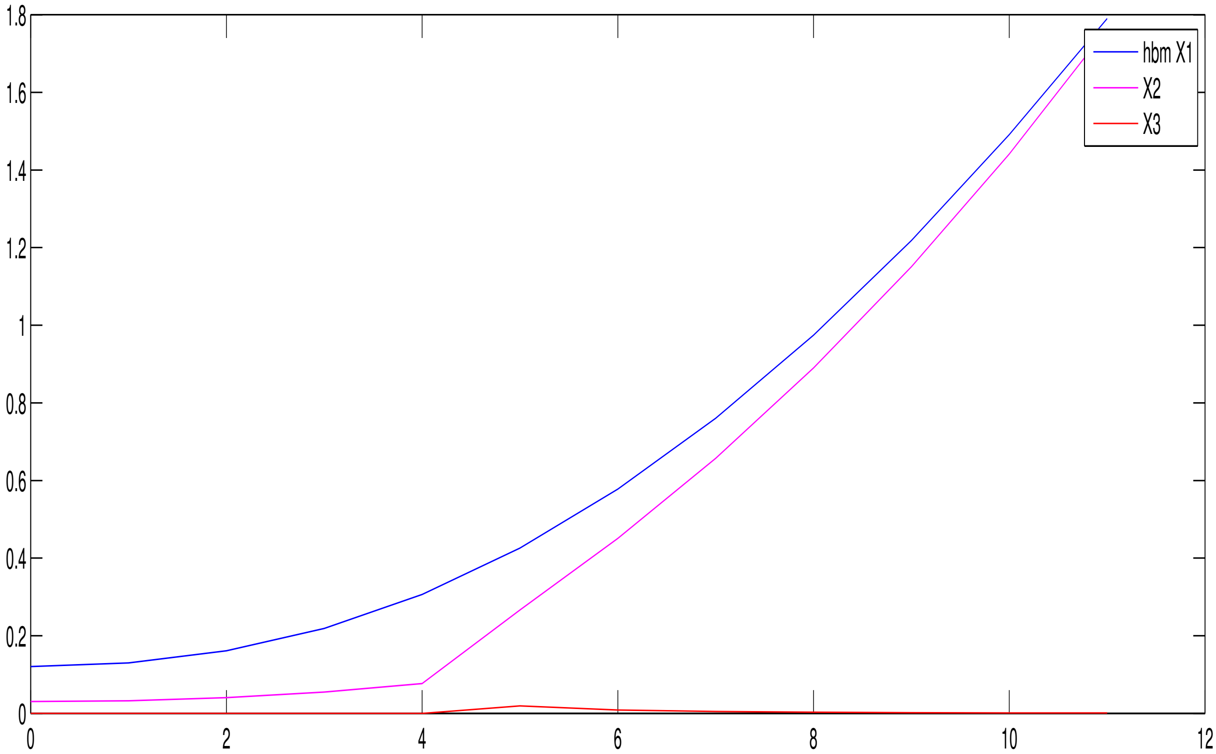


Figure 5.1: Response by harmonic balance method.

- For linear systems, the response is directly proportional to the magnitude of the excitation force at a given frequency of excitation. For nonlinear systems, however a sudden upward jump in the amplitude takes place when the force is increased gradually and reaches a specific point. Similarly, if the magnitude of the force is decreased gradually. a sudden downward jump in amplitude of vibration takes place.

- In a linear system, the forced vibration response gradually increases until resonance and decreases gradually as the frequency is increased keeping the excitation force constant. The response is unique at all the frequencies of excitation, irrespective of the case whether the frequency is increasing or decreasing. However, in a nonlinear system, as the frequency of excitation is increased for a constant force, the response increases gradually and at a certain frequency sudden downward jump occurs and when the frequency is gradually decreased a sudden upward jump occurs at a particular frequency.

- Some solutions theoretically possible may not be realised for nonlinear systems in practice. Also, some solutions in nonlinear systems may be unstable.

- A linear system analysis is useful to predict the natural frequencies, where large amplitudes of vibration will occur. The designer can then avoid frequencies of excitation around the natural frequencies. Linear system analysis may therefore be of limited value, if large deformations are to be considered in a system.

From 5.1 it can be seen that there are three solutions to the given problem.

5.1 Harmonic balance method applied to a DVA

5.1.1 First order Harmonic balance

To investigate the dynamic behaviour of the primary system with the nonlinear spring and an absorber attached, the HBM is used, as this enables mathematical expressions to be derived and the analysis to be conducted relatively easily. The fundamental assumption in the HBM approach used for the first order solution is that the response of the primary system and the absorber is predominantly harmonic at the harmonic excitation frequency. Applying the HBM, it is assumed that a solution exists of the form:

$$x = Asin(\omega t) + Bcos(\omega t) z = Csin(\omega t) + Dcos(\omega t)$$

where $z = x - x_2$,this two equations are substituted into equation of motion.

$$m_1\ddot{x}_1 + k_1x_1 + k_2z + c_2(\dot{z}) + c_2(\dot{x}) + k_{1N}x^3 = F_0sin(\omega * t)$$
$$m_2\ddot{z} + k_2z + c_2\dot{z} = m_2\ddot{x}$$

By neglecting higher order harmonics and equating the co-efficient of cosine and sine in both the equations,we get 4 equations in 4 variables A,B,C,D as given below:

$$-m2 * C * \omega^2 + k2 * C - c2 * D * \omega + m2 * A * \omega^2 = 0 - m2 * D * \omega^2 + k2 * D - c2 * C * \omega + B * m2 * \omega$$

The response of primary mass is given by:

$$\sqrt{A^2 + B^2}$$

Chapter 6

Conclusion

Many researchers are interested in and have investigated the linear vibration absorber. This thesis has been concerned with the way in which other parameters such as coulomb damping, configuration or attachment of damping and nonlinearity produced by the nonlinear absorber stiffness can be put to good use in a vibration absorber. The reasons for the preference of this passive approach are to be found in the cost, simplicity, and reliability of this type of absorbers. The effect of the nonlinear vibration absorber parameters on the vibration response was investigated and the effects of such a change were noted and tabulated. The results of this research are summarized in the following.

- We started of with optimum solution proposed by Den-hartog using two point theory for a DVA without a primary mass damping.Then we had presented a max-min search when damping is introduced,as closed form solution becomes difficult to obtain,even though researchers such as Ghosh et al have extended the two point theory to this type of such systems with limitations that it can be applied to low-moderate damping only.

- For a hardening stiffness nonlinear absorber design, the limitation on the value of the nonlinear stiffness parameter should be identified first. In order to produce an effective vibration bandwidth, the limitation on the value of the damping and the mass were determined. The larger the damping and the heavier the mass in the nonlinear absorber, a much wider effective vibration bandwidth will be produced compared to using a linear absorber with the same damping and mass levels.

- Most of the research in the field of DVA is concerned with the viscous damping,in this thesis we have evaluated the effects when the viscous damping is replaced with coulomb damping.It was found that only over a certain range of frequency (in the lower and upper band) coulomb damping offered a better vibration reduction than viscous damping.Overall the maximum vibration response of viscous is lower than the coulomb damping.Also,sometimes very high frictional force is required to achieve the required reduction,which in some cases is practically impossible to achieve and large frictional force leads to wear and tear of mechanical system.

- The nonlinear absorber has a much wider effective bandwidth compared to a conventional linear absorber. Compared to the linear absorber, the nonlinearity has the effect of shifting the second resonance peak to a higher frequency away from the effective tuned frequency, improving the robustness of the device to mistune and on adding the cubic hardening spring in the primary system,the second peak goes up and first peak comes down and both these peaks shift towards right.

- The lower the damping in the nonlinear absorber, the effective bandwidth is slightly increased compared to the linear absorber with the same level of damping. However, larger damping in the nonlinear absorber generally produces a wider frequency vibration reduction bandwidth compared to the linear absorber. When damping in the nonlinear absorber is further increased above a certain value though, there appears to be no effective vibration bandwidth. In engineering applications, it is desirable to have a large vibration reduction bandwidth, so that the damping in the absorber needs to be quite small.

- It was seen that the parameters such as stiffness(k_2)and secondary damping(c_2) obtained for optimum linear system changes especially k_2 when a cubic non-linear spring is introduced but the damping c_2 almost remains same.

- By introducing non-linearity,we can reduce the vibration of primary system.The optimum non-linear FRF lies below the linear FRF.But as said earlier the spring stiffness to obtain optimum FRF is different.Also the FRF as a whole shifts towards right.

- For a low nonlinear stiffness in the primary system, the absorber parameters can be chosen such that it produces an FRF that is better than linear optimum FRF case. But for high nonlinear stiffness, the nonlinear absorber would have higher mean square primary system displacement compared to a linear absorber. The use of a high stiffness nonlinearity does not improve the vibration reduction compared to the linear absorber case.

6.1 Future Work

- The numerical results for the vibration reduction bandwidth and effective tuned frequency have been obtained . However, the mathematical expressions for the vibration reduction bandwidth and effective tuned frequency have not been investigated. A recommendation is to determine the effect of NVDA parameters on the vibration reduction bandwidth and effective tuned frequency with, if possible, analytical expressions.

- Analytical work,when a non-linearity is added to the system has tried to derive the mathematical expression for a vibration response at a particular frequency but more could be done to derive the optimum parameters for a non-linear system.