Percolation transition of the E-R graph

Network Science

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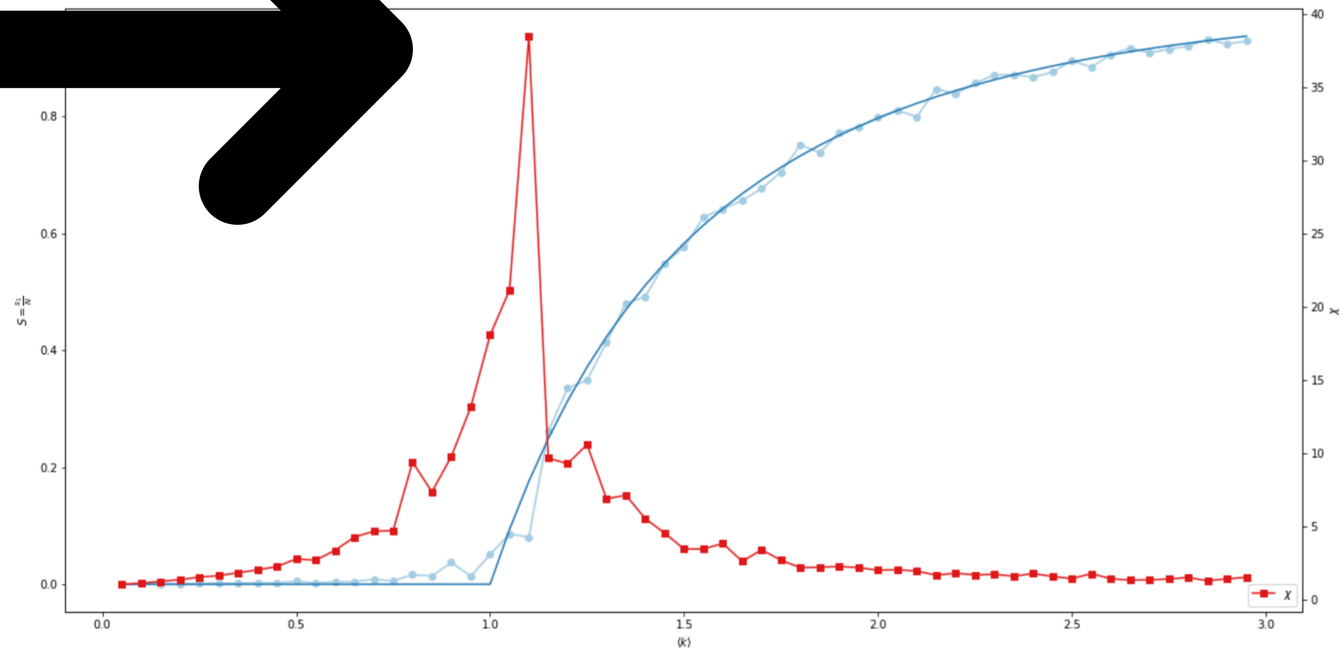
Percolation transition **- the qualitative change of structure of graphs when the connectivity varies from low values (small isolated trees) to higher ones (larger parts are connected together).**

Susceptibility

A quantity that measures how sensitive is the system with respect to small perturbations that try to drive the system from one phase to the other, and this quantity has a sharp peak at the transition point (critical point).

$$\chi = \sum_{s_i \neq s_{\max}} s_i^2 \, p(s),$$



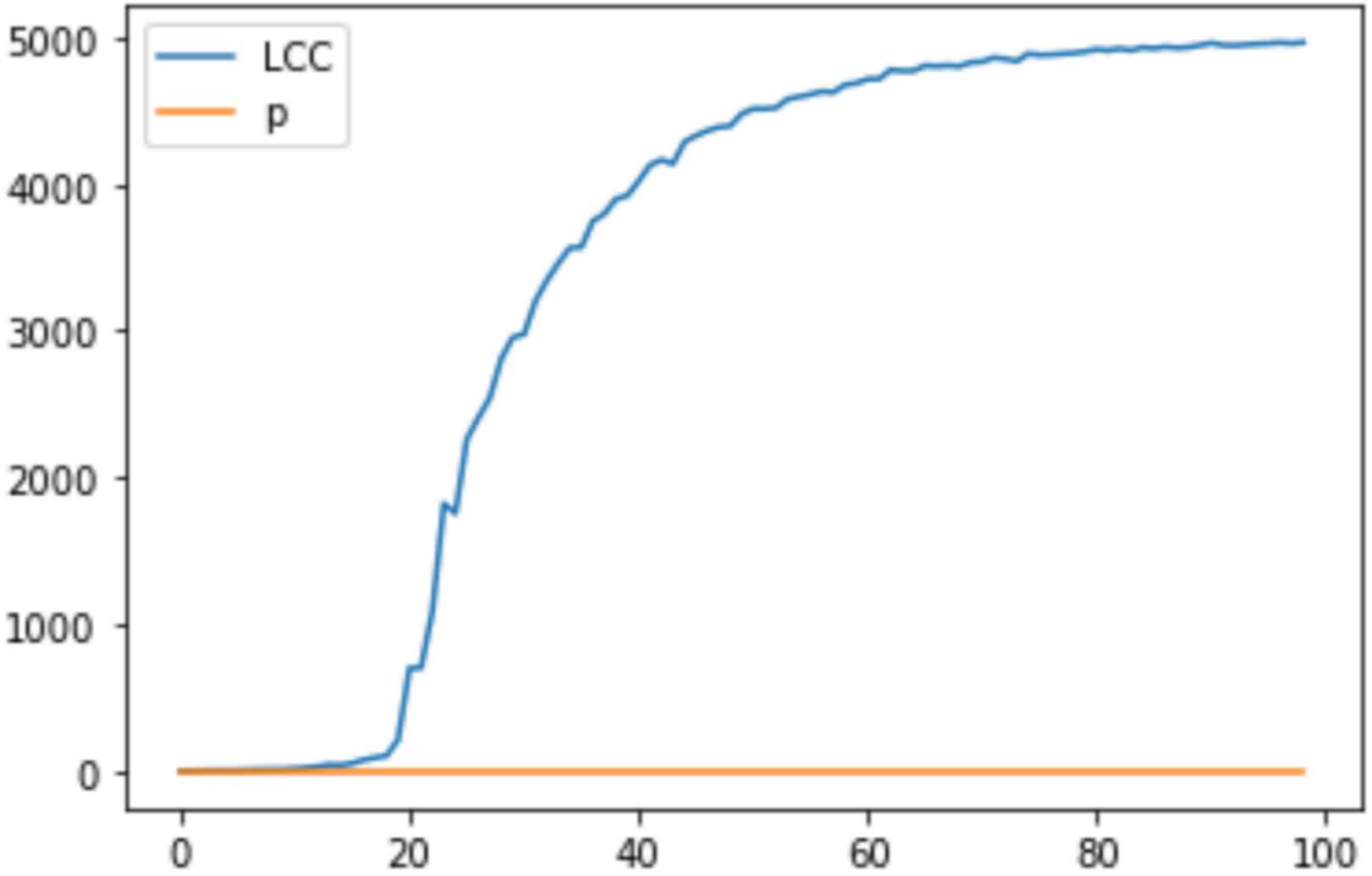
How does the size of the largest connected component grow with p at the critical point?

- Define a graph with 5000 nodes
- Number of samples set to 2
- Probability range is [0.00001, 0.001]
- Initial step is 0.00001
- E_R graph set with nx.generators.erdos_renyi_graph(5000, p)
- Max χ was calculated for each p and then LCC

	LCC	p
0	3.0	0.00001
1	5.0	0.00002
2	5.5	0.00003
3	7.0	0.00004
4	6.5	0.00005
...	...	...
94	4954.5	0.00095
95	4957.0	0.00096
96	4963.5	0.00097
97	4957.5	0.00098
98	4965.0	0.00099

99 rows × 2 columns

Growth of LCC along with p



How does the size of the largest connected component scale with the system size at the critical point?

- Independent of k
- Nodes range is [1000, 25000]
- Initial step is 500 nodes
- Number of samples set to 3
- Probability is 0.0001
- E_R graph set with nx.generators.erdos_renyi_graph(N, 0.0001)
- Max χ was calculated for each N and then LCC

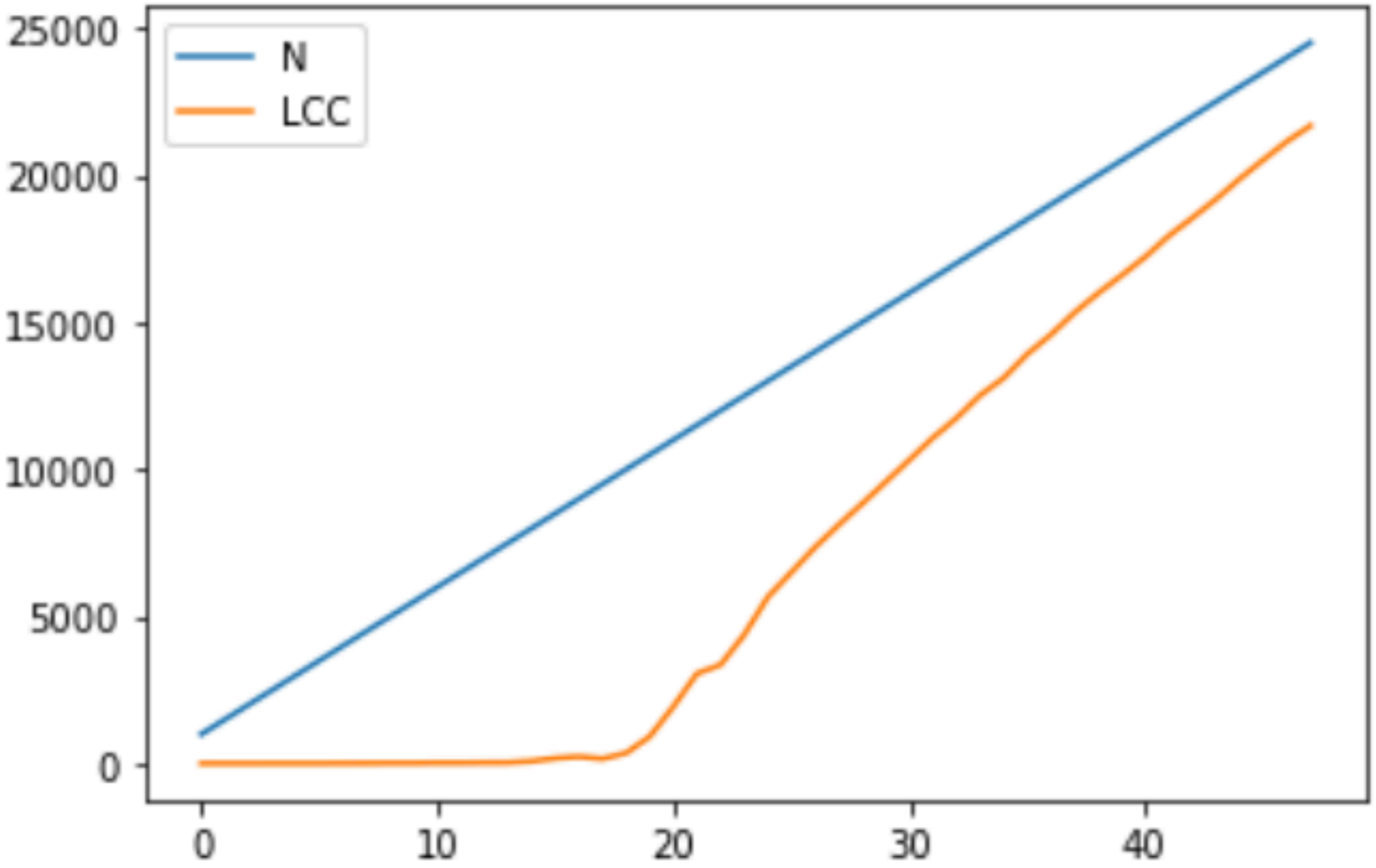
	N	LCC
0	1000.0	3.333333
1	1500.0	4.333333
2	2000.0	5.666667
3	2500.0	7.333333
4	3000.0	7.666667
5	3500.0	7.333333
6	4000.0	11.000000
7	4500.0	13.333333
8	5000.0	16.666667
9	5500.0	17.333333
10	6000.0	27.333333
36	19000.0	14576.333333
37	19500.0	15328.000000
38	20000.0	15984.000000
39	20500.0	16592.000000
40	21000.0	17222.333333
41	21500.0	17935.333333
42	22000.0	18549.000000
43	22500.0	19181.666667
44	23000.0	19872.333333
45	23500.0	20518.000000
46	24000.0	21143.666667
47	24500.0	21677.666667

How does the size of the largest connected component scale with the system size at the critical point?

- Dependent of k
- Nodes range is [1000, 10000]
- Initial step is 500 nodes
- Number of samples set to 2
- Range of k [0.05,3.0] with step 0.05
- E_R graph set with nx.generators.erdos_renyi_graph(N, k/(N-1.0))
- Max χ was calculated for each N, k and then LCC

	N	LCC
0	1000.0	58.0
1	1500.0	162.5
2	2000.0	187.5
3	2500.0	428.5
4	3000.0	170.0
5	3500.0	289.0
6	4000.0	833.0
7	4500.0	1179.0
8	5000.0	401.5
9	5500.0	641.0
10	6000.0	717.5
11	6500.0	781.5
12	7000.0	264.5
13	7500.0	502.0
14	8000.0	1096.0
15	8500.0	382.5
16	9000.0	1028.0
17	9500.0	1451.5
18	10000.0	1650.5

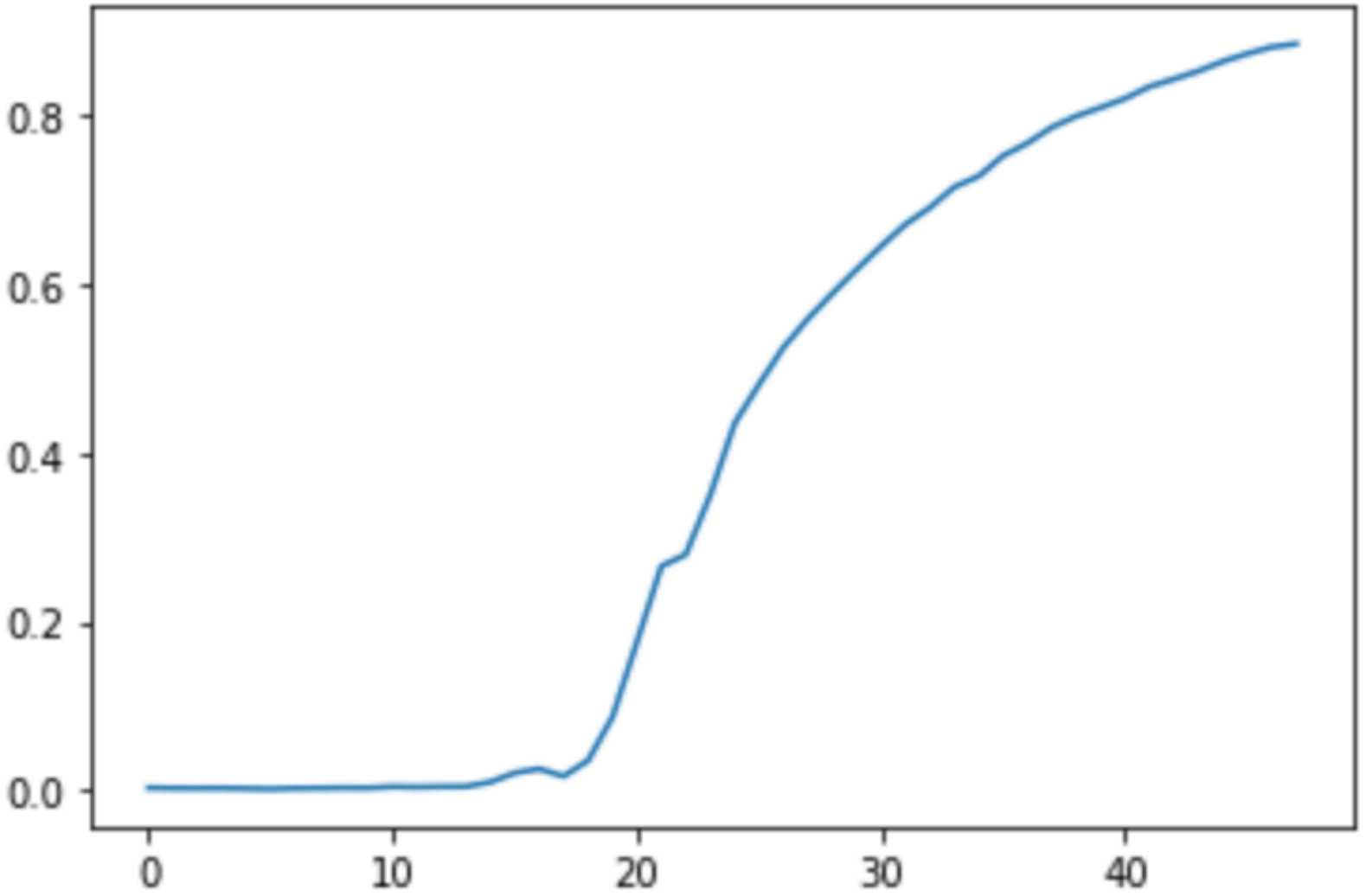
Growth of LCC along with N



Scaling numbers

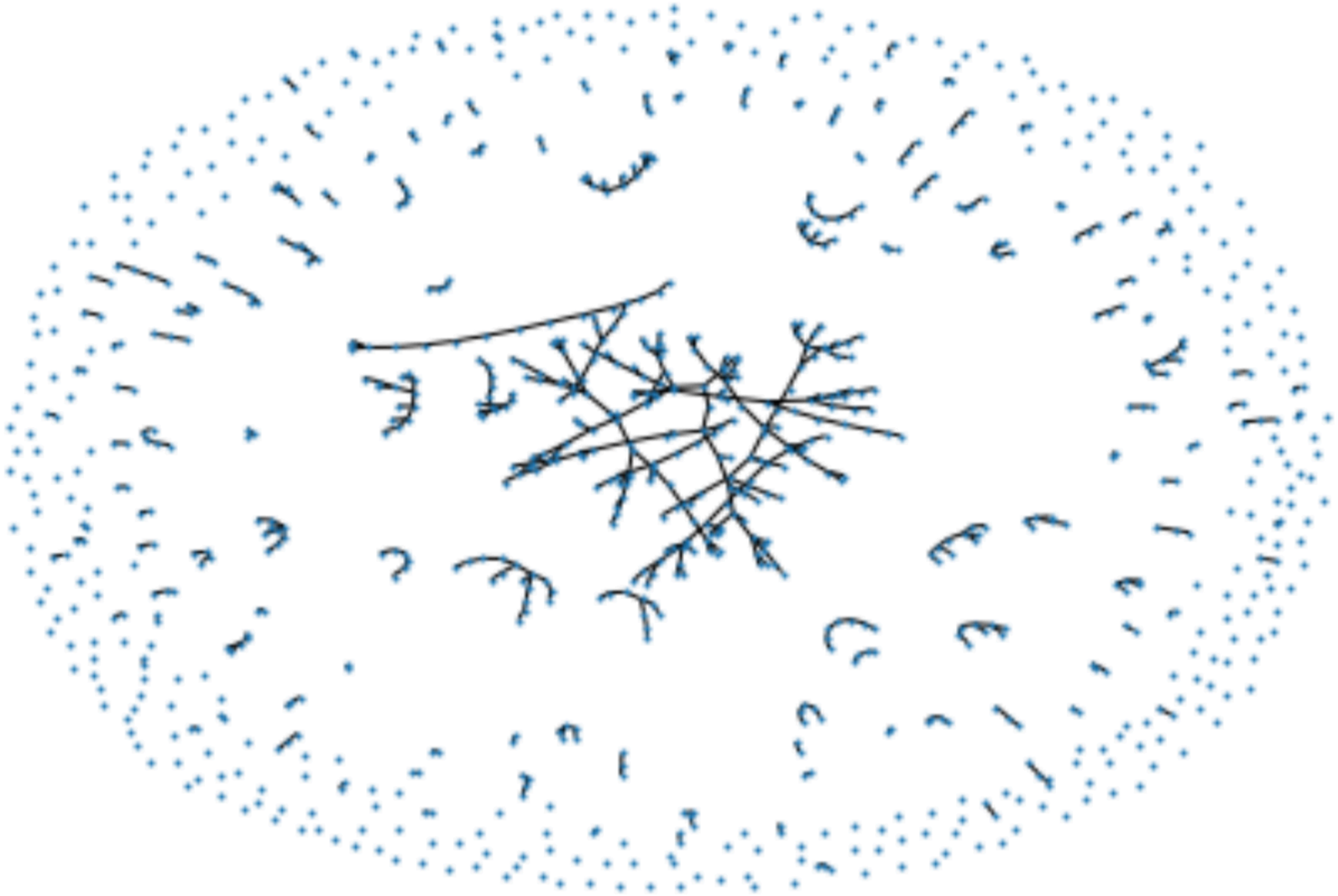
	N	LCC	Scale
0	1000.0	3.333333	0.003333
1	1500.0	4.333333	0.002889
2	2000.0	5.666667	0.002833
3	2500.0	7.333333	0.002933
4	3000.0	7.666667	0.002556
5	3500.0	7.333333	0.002095
6	4000.0	11.000000	0.002750
7	4500.0	13.333333	0.002963
8	5000.0	16.666667	0.003333
9	5500.0	17.333333	0.003152
10	6000.0	27.333333	0.004556
36	19000.0	14576.333333	0.767175
37	19500.0	15328.000000	0.786051
38	20000.0	15984.000000	0.799200
39	20500.0	16592.000000	0.809366
40	21000.0	17222.333333	0.820111
41	21500.0	17935.333333	0.834202
42	22000.0	18549.000000	0.843136
43	22500.0	19181.666667	0.852519
44	23000.0	19872.333333	0.864014
45	23500.0	20518.000000	0.873106
46	24000.0	21143.666667	0.880986
47	24500.0	21677.666667	0.884803

Scaling of LCC along with N

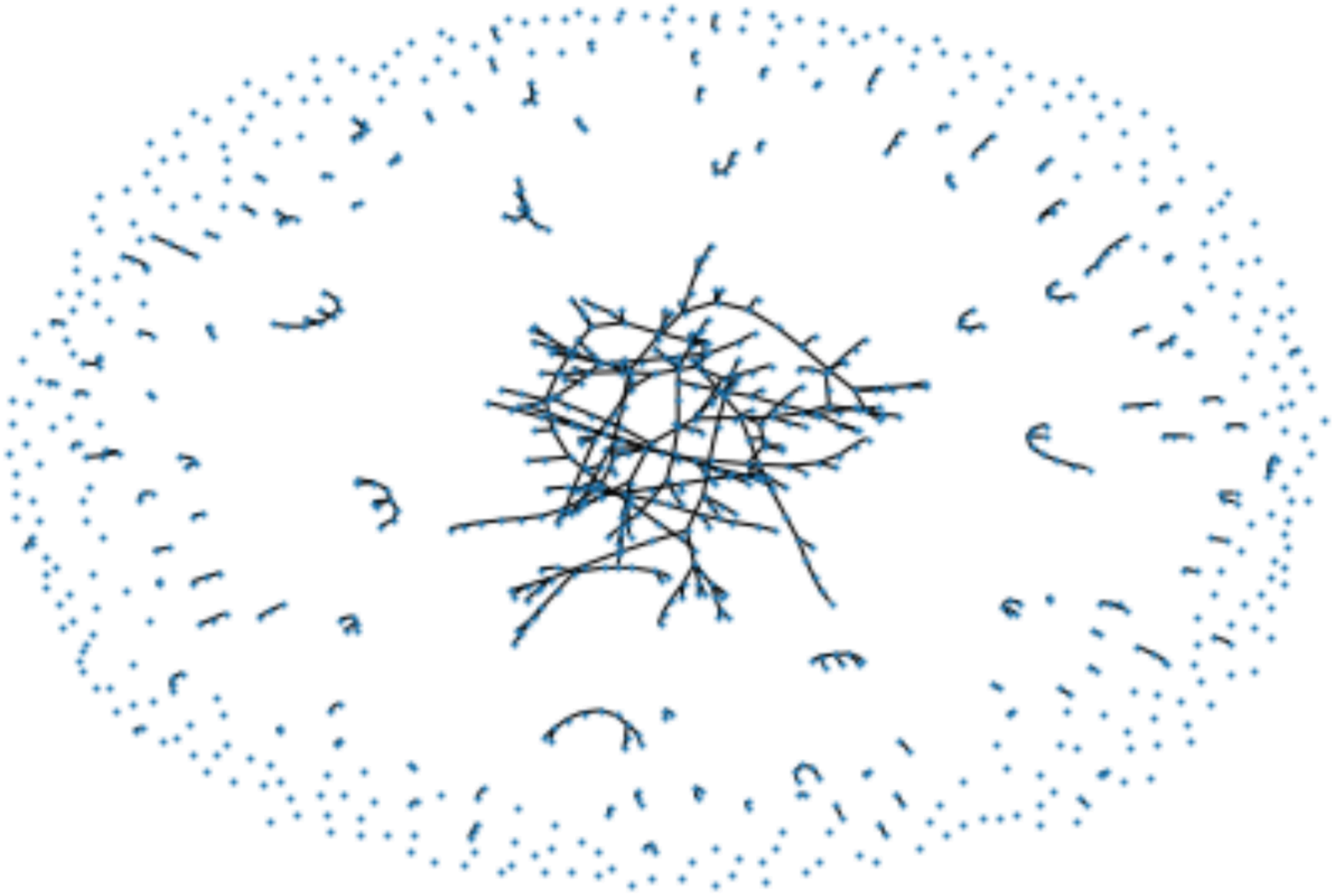


Generated graphs both below and above the critical point

Below the critical point, N=1000



Above the critical point, N=1000

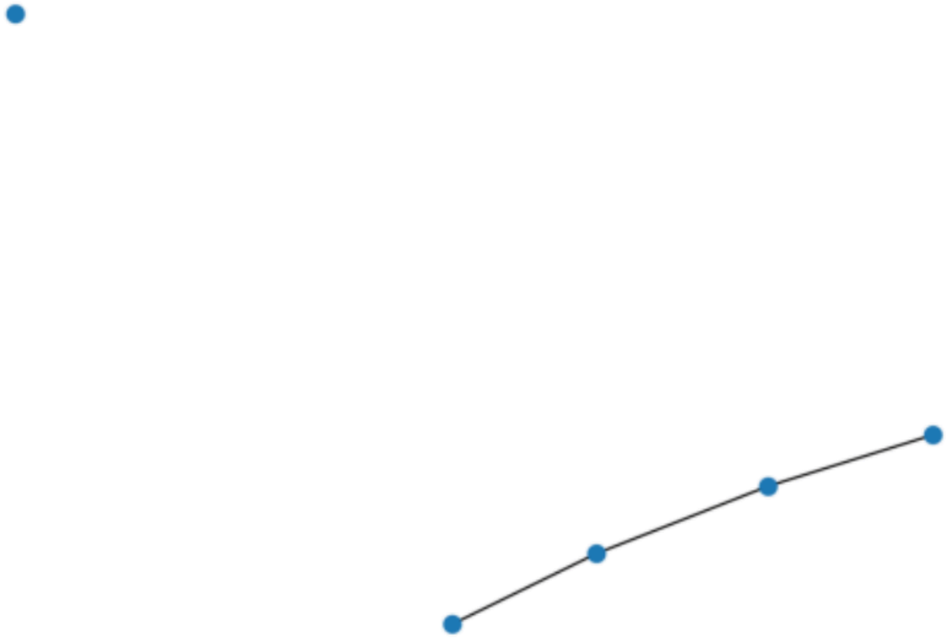


Low node number simulation

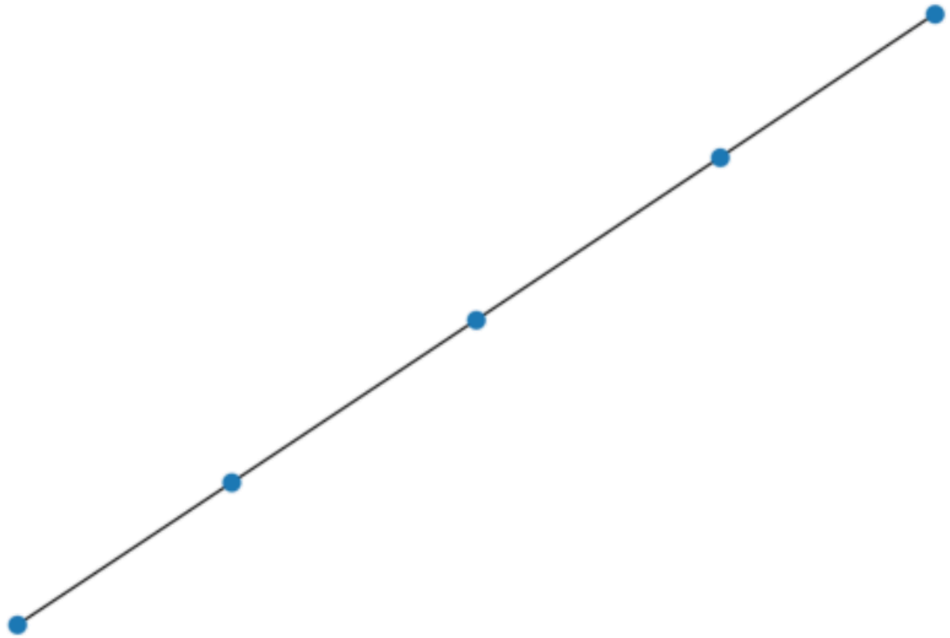
- Dependent of k
- Nodes range is [5, 55]
- Initial step is 5 nodes
- Number of samples set to 3
- Range of k [0.05,3.0] with step 0.05
- E_R graph set with nx.generators.erdos_renyi_graph(N, k/(N-1.0))
- Max χ was calculated for each N, k and then LCC

	N	LCC	k-1	k+1
0	5.0	2.666667	0.75	0.85
1	10.0	4.666667	1.25	1.35
2	15.0	9.666667	1.50	1.60
3	20.0	10.666667	1.55	1.65
4	25.0	18.666667	2.20	2.30
5	30.0	9.000000	1.05	1.15
6	35.0	17.333333	1.50	1.60
7	40.0	20.333333	1.40	1.50
8	45.0	26.666667	1.60	1.70
9	50.0	28.666667	1.55	1.65

Before CP



After CP



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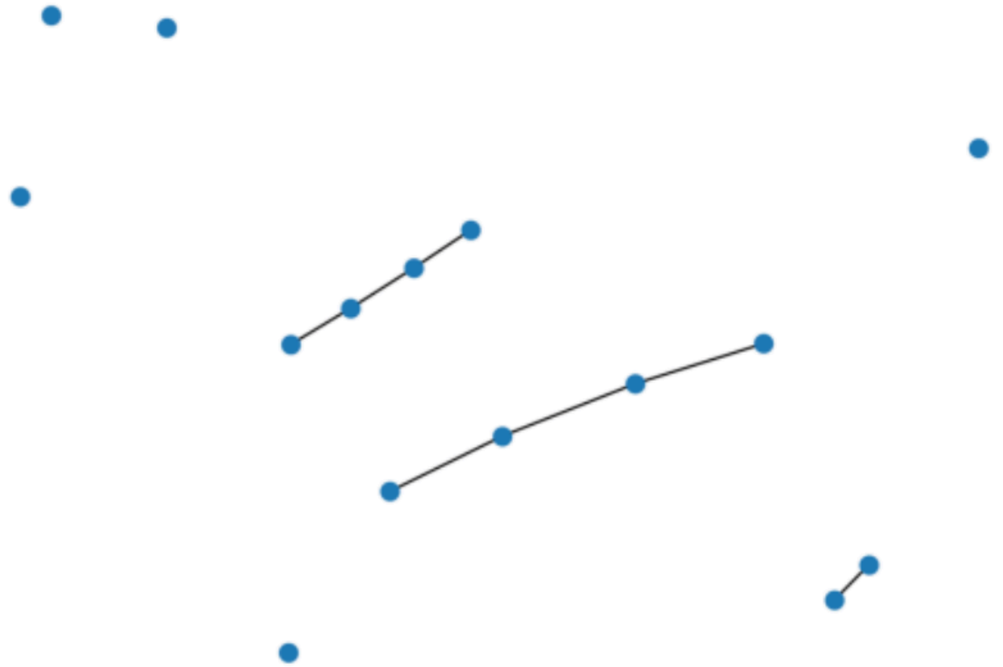

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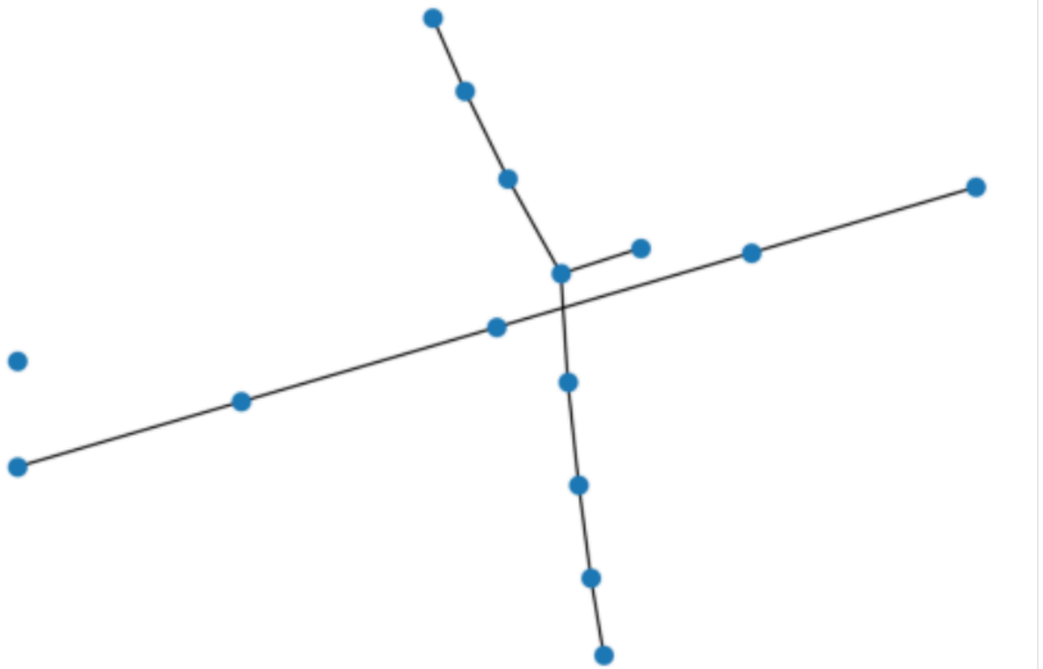

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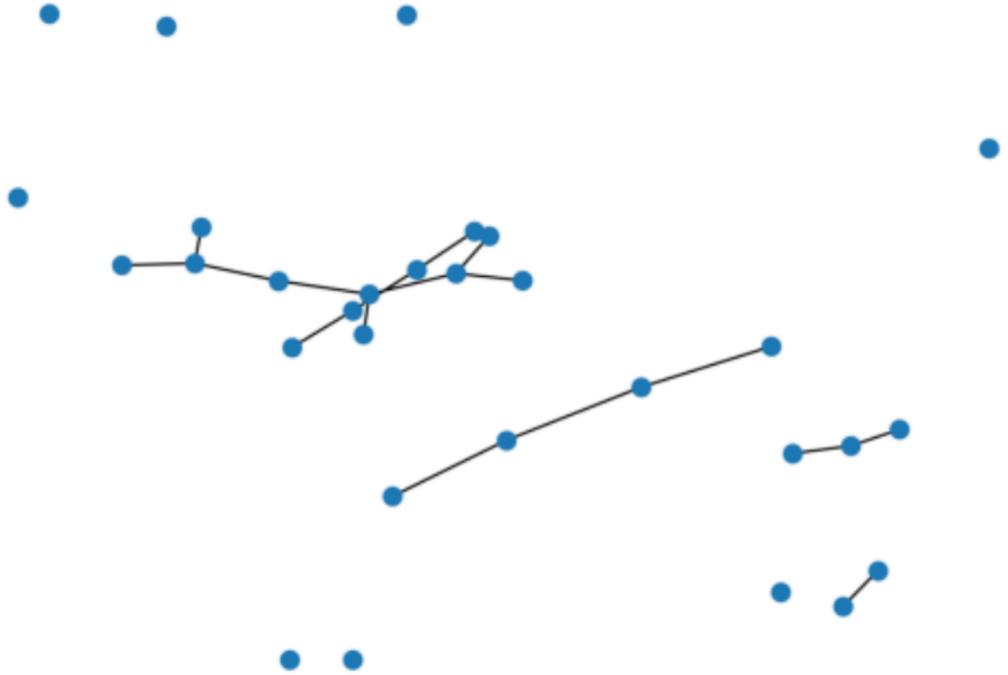

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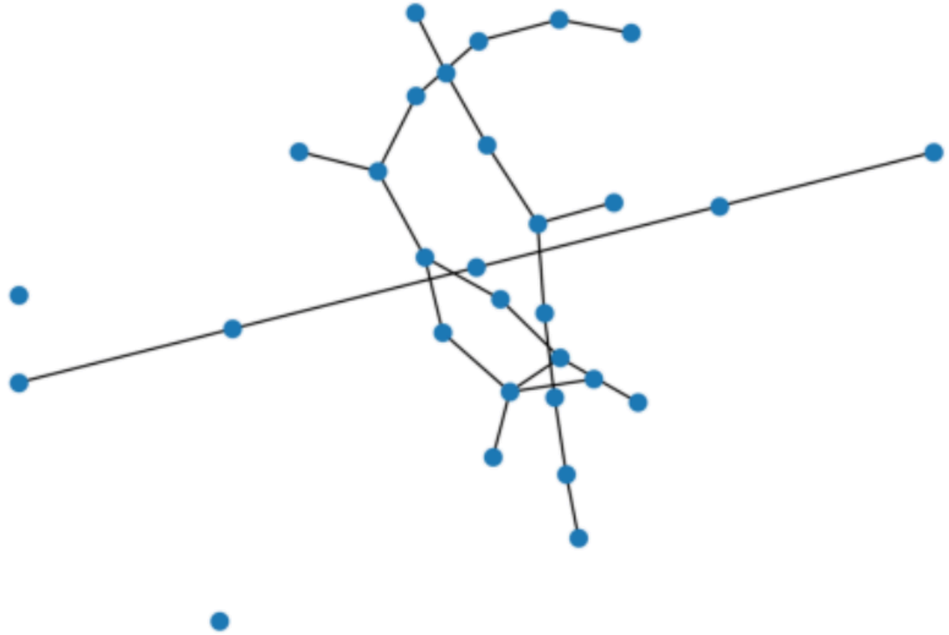

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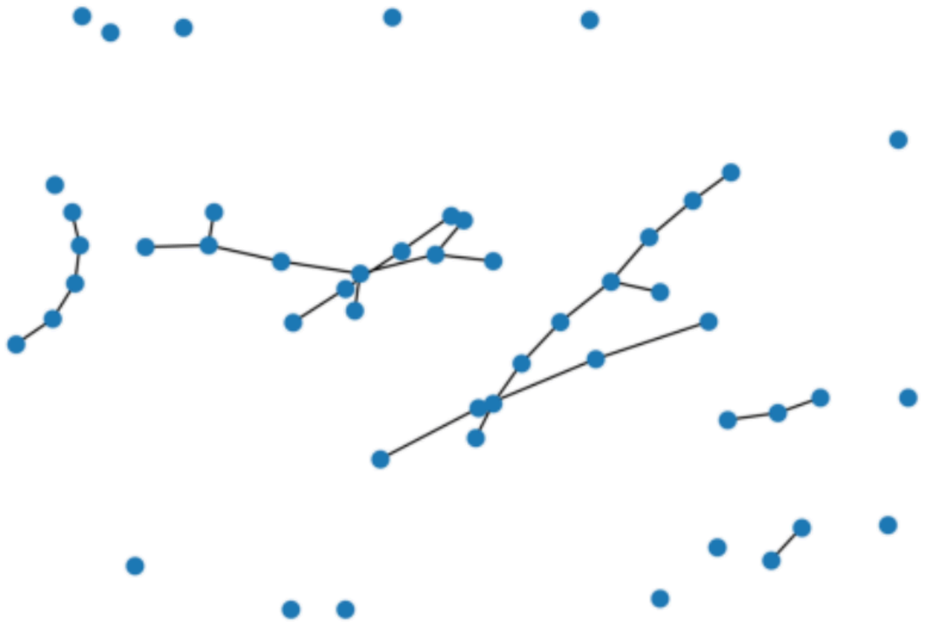

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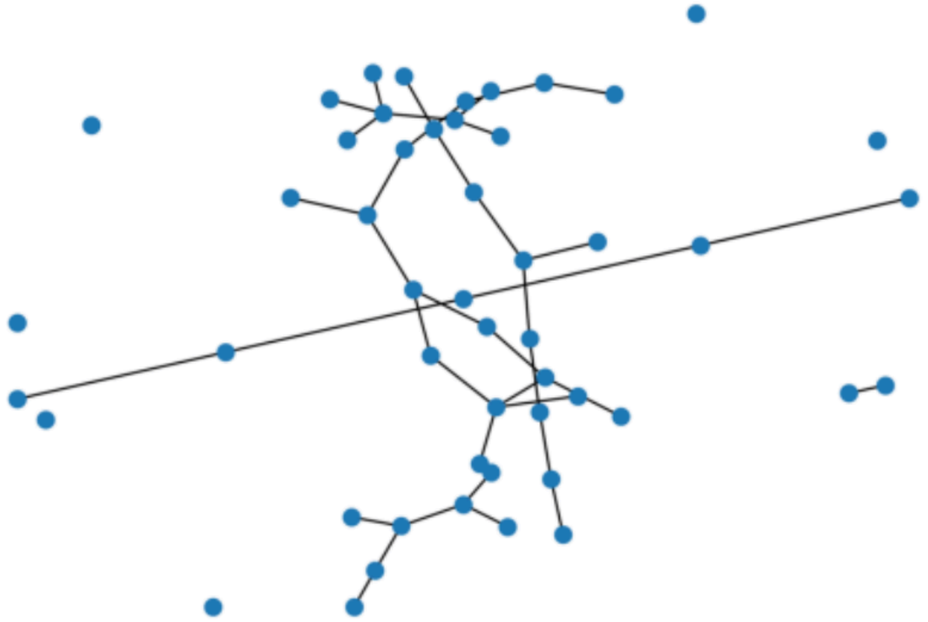

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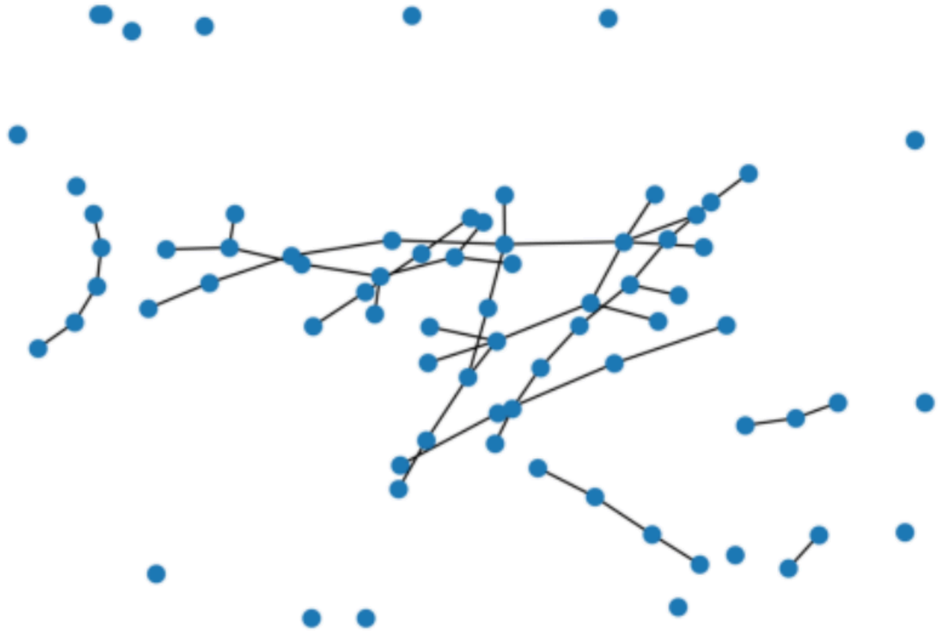

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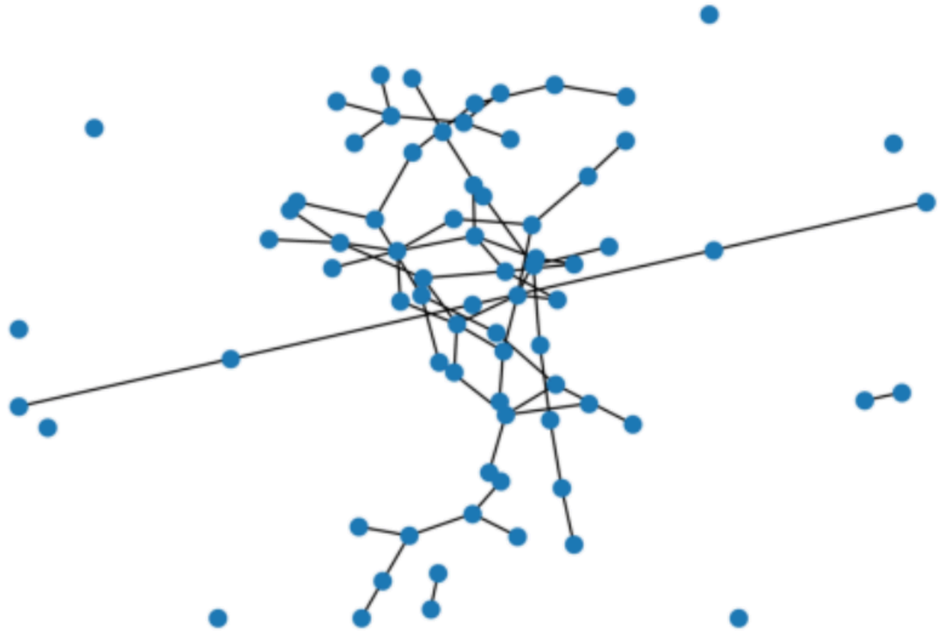

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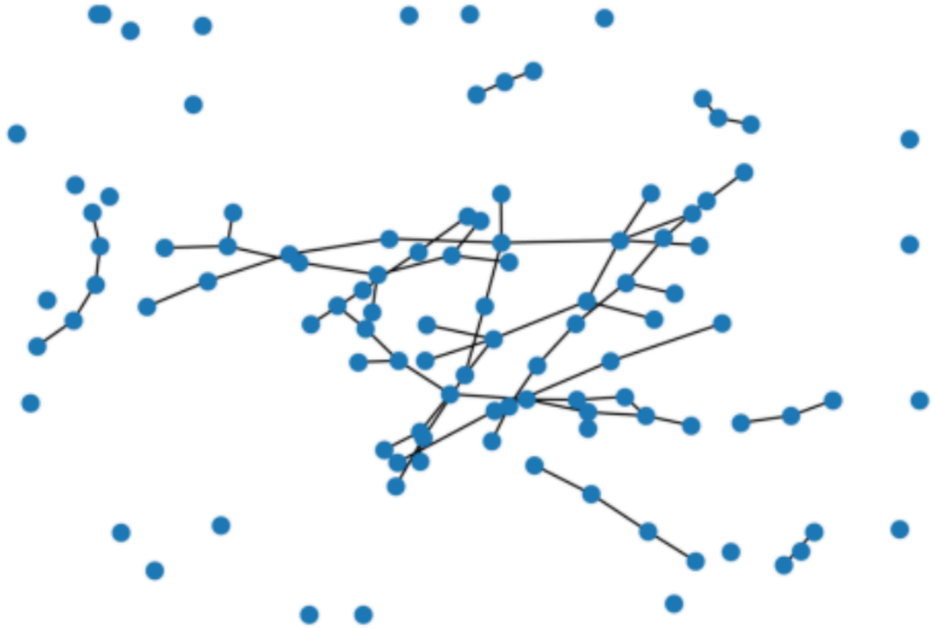

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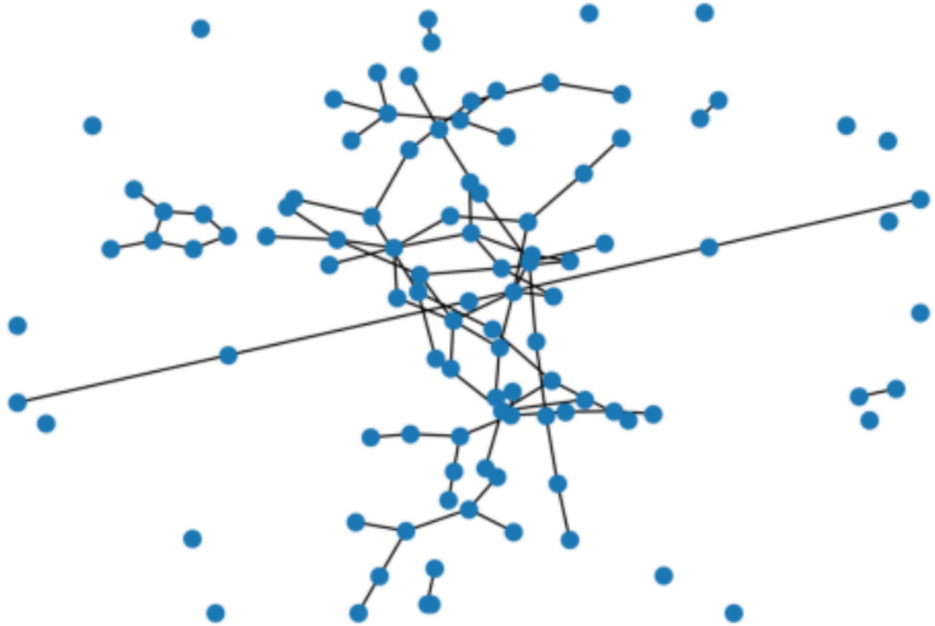

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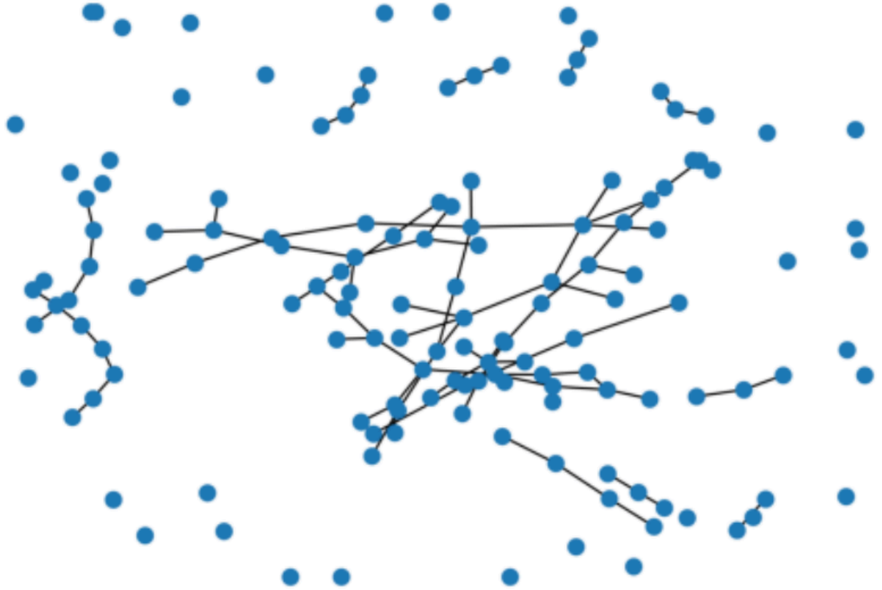

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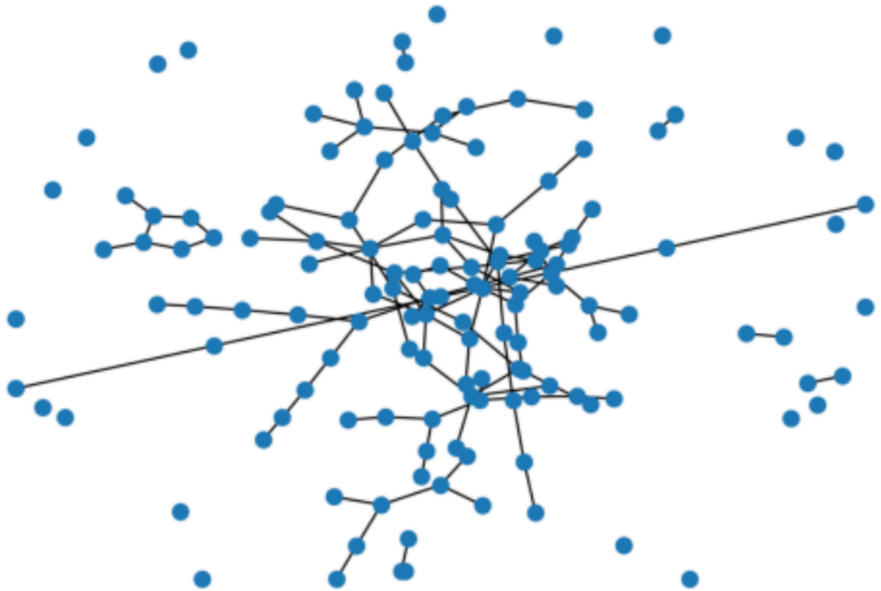

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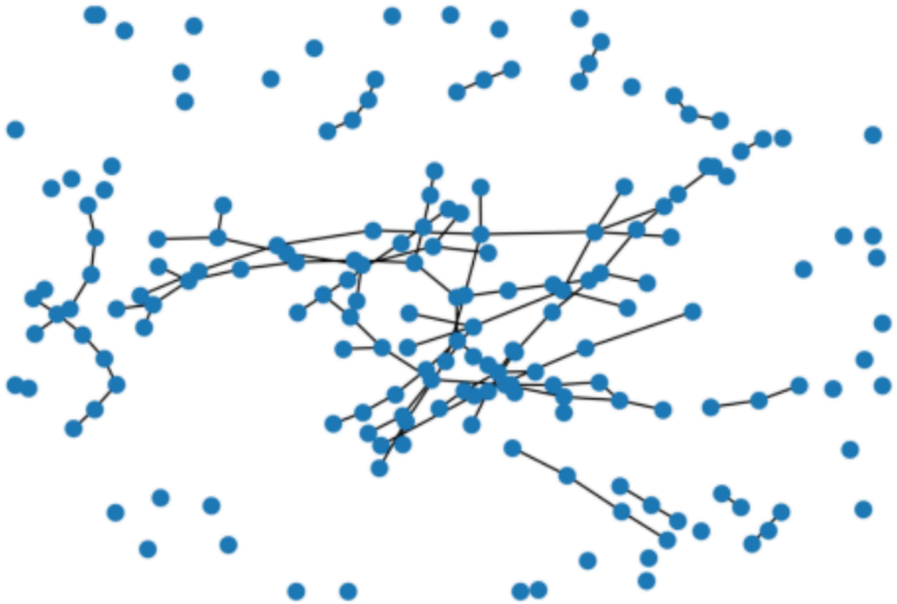

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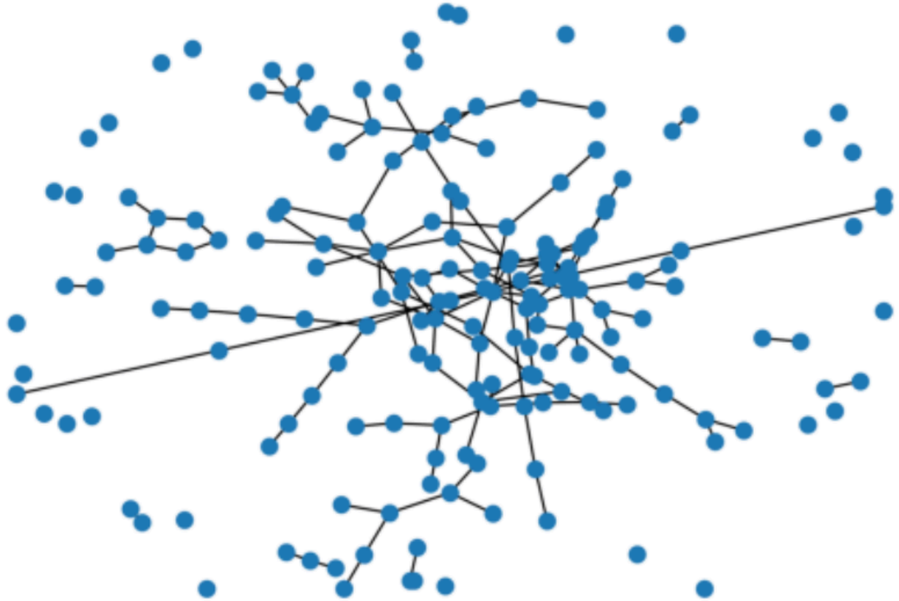
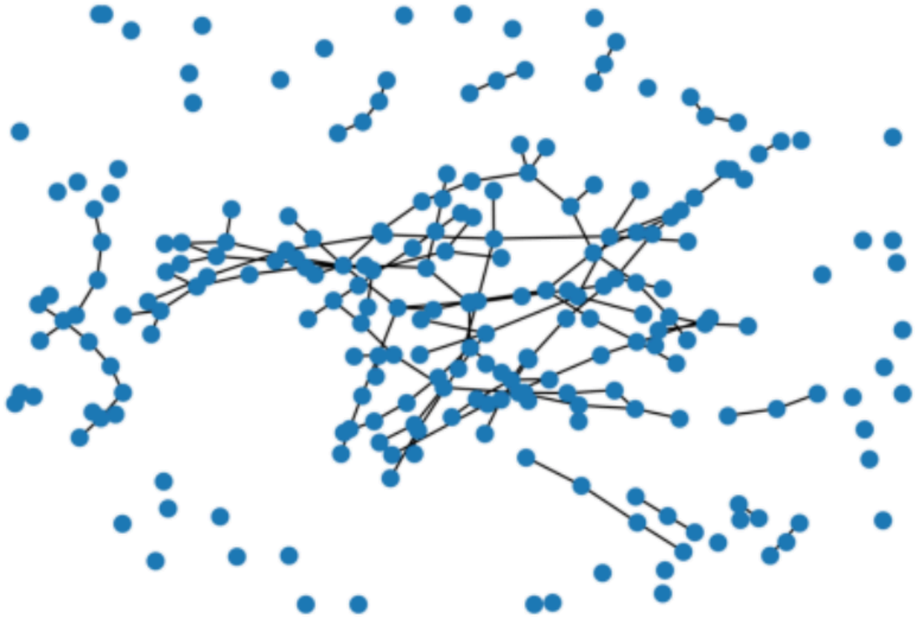


Before CP



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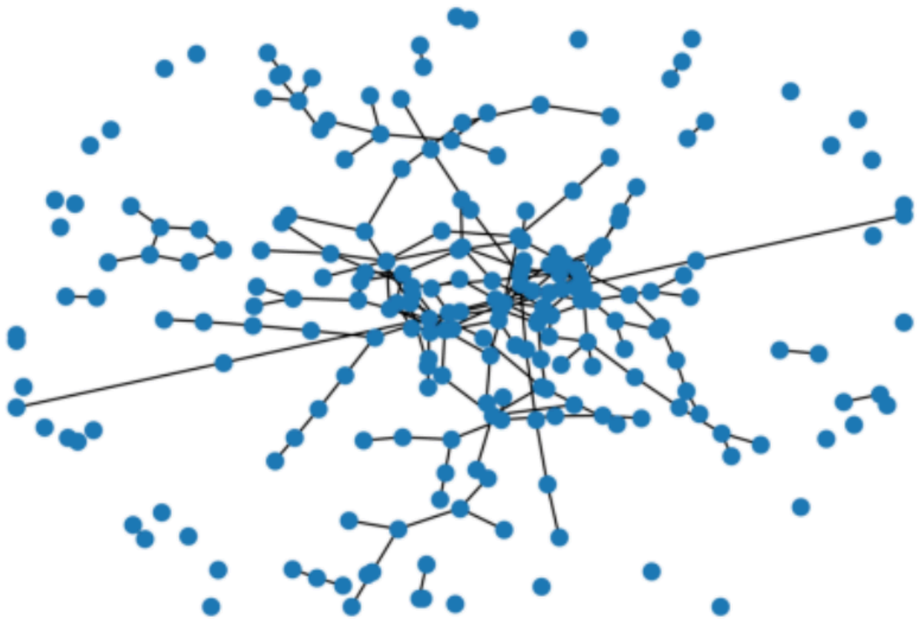

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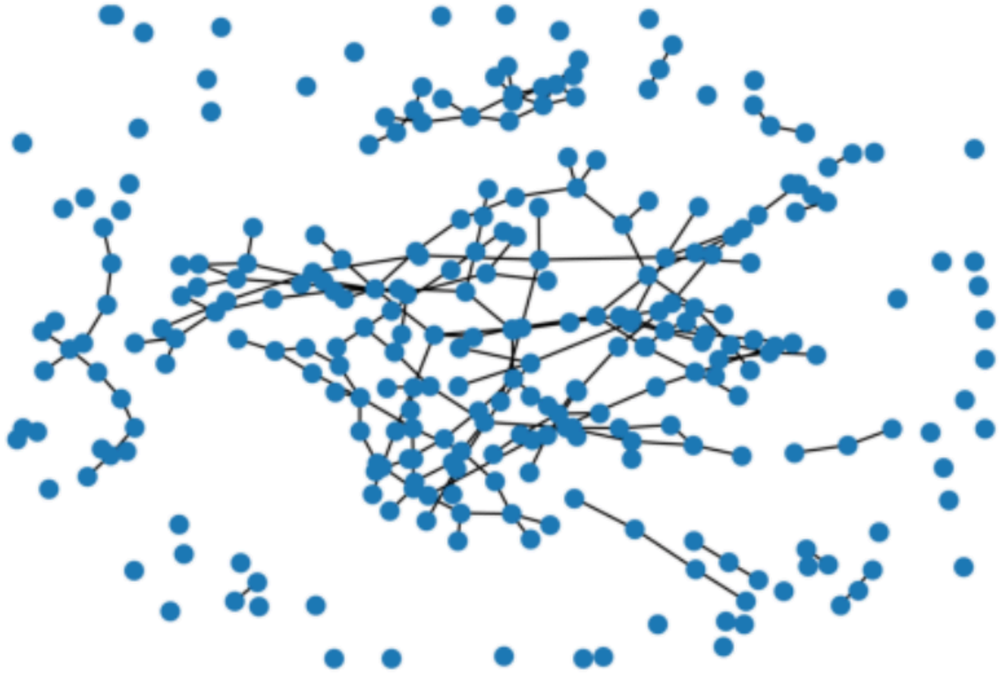

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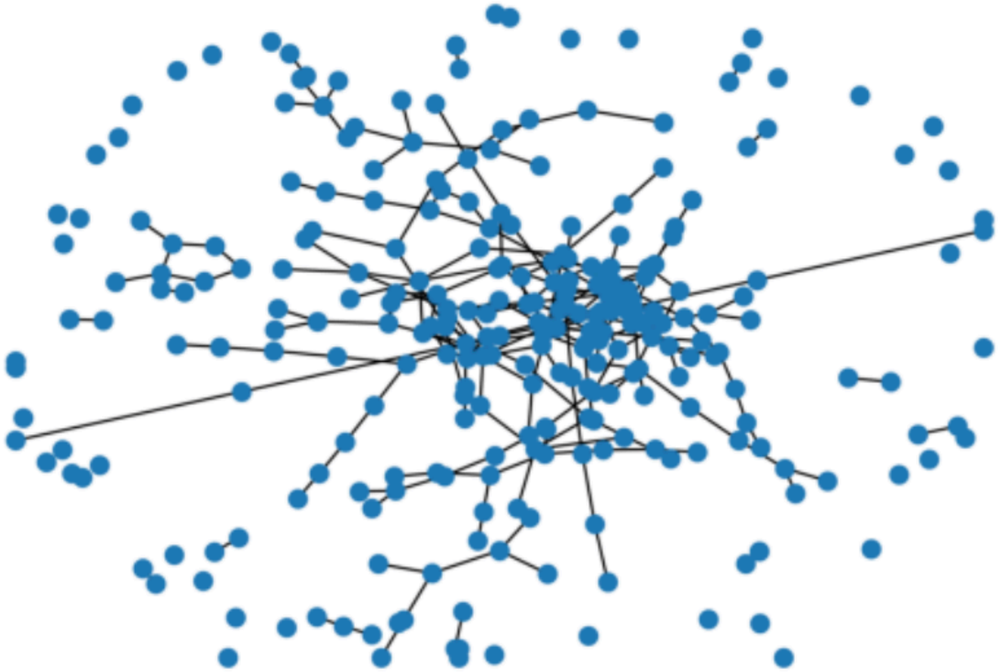

Thank you for your attention !

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