Naumov-FY46IN_Hajieva-DKXCU0

December 9, 2020

1 Vitalii Naumov (FY46IN)

2 Sakina Hajieva (DKXCU0)

```
In [1]: # Some useful imports

        import networkx as nx
        import numpy as np
        import matplotlib.pyplot as plt
        import math
        import pandas as pd
        import os
```

```
In [2]: # Creating some tables to store the measured numbers

        table_1 = pd.DataFrame(columns=['N','LCC'])
        table_1a = pd.DataFrame(columns=['N','LCC','k-1','k+1'])
        table_1b = pd.DataFrame(columns=['N','LCC','k-1','k+1'])
        table_2 = pd.DataFrame(columns=['LCC','p'])
```

```
In [3]: # A function which calculates Susceptibility

        def Calc_chi_and_S(network):
            _network_N = network.number_of_nodes();
            _comps = nx.connected_components(network);
            _comp_sizes = [len(_comp) for _comp in _comps];
            _sort_c_sizes = sorted(_comp_sizes,reverse=True);
            _lcs = _sort_c_sizes[0]/_network_N;
            _chi = 0;
            if len(_sort_c_sizes) > 1:
                _chi = sum([_sort_c_sizes[i]*_sort_c_sizes[i] for i in range(1,len(_sort_c_sizes
                return _chi/(len(_sort_c_sizes)),_lcs;
            else:
                return _chi,_lcs;
```

3 How does the size of the largest connected component scale with the system size (N) at the critical point?

3.0.1 First simulation is dependent of k, number of samples = 2, range of network size is [1000,10000] with a step of 500 nodes. k parameter varies from 0.05 to 3 with a step of 0.05, so for each network size N there will be generated 59 different networks with different probability p. The average size of a largest connected component (LCC) will be calculated for each network size as well.

In [ ]:
```
for N in range(1000,10500,500):
    num_samp = 2
    k_list = np.arange(0.05,3.0,0.05)
    av_chi, av_lcc_size = [],[]
    av_chi.clear()
    av_lcc_size.clear()

    print('Computing LCC for {} nodes...'.format(N))
    for k in k_list:

        chi_values, comp_sizes = [],[]
        comp_sizes.clear()
        chi_values.clear()

        for i in range(0,num_samp):
            ER_graph = nx.generators.erdos_renyi_graph(N,k/(N-1.0))

            chi,S = Calc_chi_and_S(ER_graph)
            chi_values.append(chi)

            largest_comp_size = len(max(nx.connected_components(ER_graph), key = len))
            comp_sizes.append(largest_comp_size)

        av_chi.append(np.mean(chi_values))
        av_lcc_size.append(np.mean(comp_sizes))

    table_1a = table_1a.append({'LCC':av_lcc_size[av_chi.index(max(av_chi))],
                                'N':N, 'k-1':k_list[av_chi.index(max(av_chi))-1],
                                'k+1':k_list[av_chi.index(max(av_chi))+1]},ignore_index=Tr
```

In [5]:
```
table_1a
```

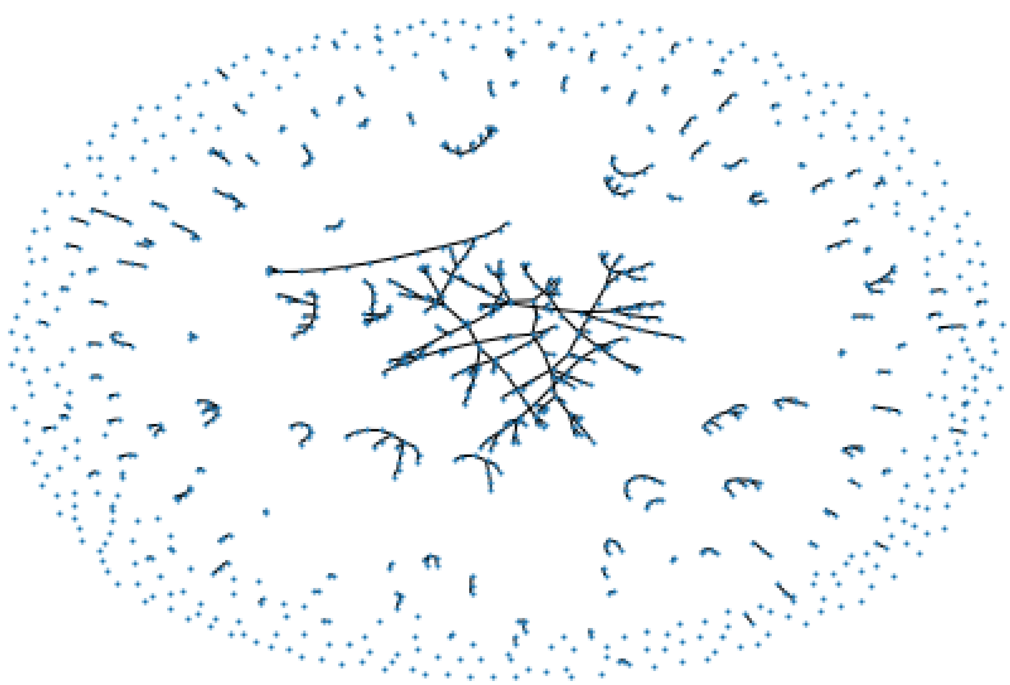
Out[5]:

	N	LCC	k-1	k+1
0	1000.0	58.0	1.00	1.10
1	1500.0	162.5	1.05	1.15
2	2000.0	187.5	1.05	1.15
3	2500.0	428.5	1.05	1.15
4	3000.0	170.0	1.00	1.10
5	3500.0	289.0	1.00	1.10

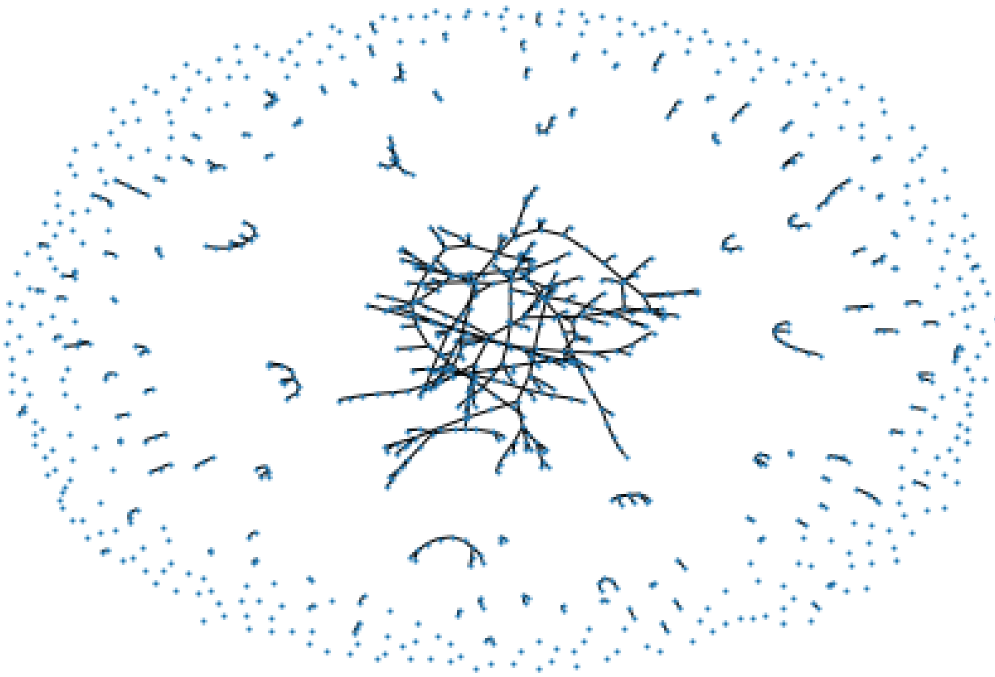
6	4000.0	833.0	1.15	1.25
7	4500.0	1179.0	1.10	1.20
8	5000.0	401.5	1.00	1.10
9	5500.0	641.0	1.05	1.15
10	6000.0	717.5	1.00	1.10
11	6500.0	781.5	1.00	1.10
12	7000.0	264.5	0.95	1.05
13	7500.0	502.0	1.00	1.10
14	8000.0	1096.0	1.05	1.15
15	8500.0	382.5	0.95	1.05
16	9000.0	1028.0	1.00	1.10
17	9500.0	1451.5	1.05	1.15
18	10000.0	1650.5	1.05	1.15

3.0.2 Some pictures of the generated graphs with a 1000 links.

In [44]:
```
# Below the critical point
ER_graph = nx.generators.erdos_renyi_graph(1000,1.0/(1000.0-1.0))
nx.draw(ER_graph, pos=nx.spring_layout(ER_graph),node_size=1)
```



In [45]:
```
# Above the critical point
ER_graph = nx.generators.erdos_renyi_graph(1000,1.10/(1000.0-1.10))
nx.draw(ER_graph, pos=nx.spring_layout(ER_graph),node_size=1)
```



3.1 It is not clearly visible the actual difference on the pictures above, so we decided to create a different type of simulation which will show how networks below critical point differ from ones above it.

3.1.1 Low node number simulation of 10 networks with size from 5 to 50, 3 networks will be generated for each number of nodes, k parameter varies from 0.05 to 3 with a step of 0.05, so for each network size N there will be generated 59 different networks with different probability p. The average size of a LCC will be calculated for each network size as well.

```
In [4]: for N in range(5,55,5):
            num_samp = 3
            k_list = np.arange(0.05,3.0,0.05);
            av_chi, av_lcc_size = [],[]
            av_chi.clear()
            av_lcc_size.clear()

            print('Computing LCC for {} nodes...'.format(N))
            for k in k_list:

                chi_values, comp_sizes = [],[]
                comp_sizes.clear()
                chi_values.clear()
```

```
            for i in range(0,num_samp):
                ER_graph = nx.generators.erdos_renyi_graph(N,k/(N-1.0))

                chi,S = Calc_chi_and_S(ER_graph)
                chi_values.append(chi)

                largest_comp_size = len(max(nx.connected_components(ER_graph), key = len))
                comp_sizes.append(largest_comp_size)

            av_chi.append(np.mean(chi_values))
            av_lcc_size.append(np.mean(comp_sizes))

        table_1b = table_1b.append({'LCC':av_lcc_size[av_chi.index(max(av_chi))],
                                    'N':N, 'k-1':k_list[av_chi.index(max(av_chi))-1],
                                    'k+1':k_list[av_chi.index(max(av_chi))+1]},ignore_index=Tr
```

```
Computing LCC for 5 nodes...
Computing LCC for 10 nodes...
Computing LCC for 15 nodes...
Computing LCC for 20 nodes...
Computing LCC for 25 nodes...
Computing LCC for 30 nodes...
Computing LCC for 35 nodes...
Computing LCC for 40 nodes...
Computing LCC for 45 nodes...
Computing LCC for 50 nodes...
```

In [5]:
```
table_1b
```

Out[5]:

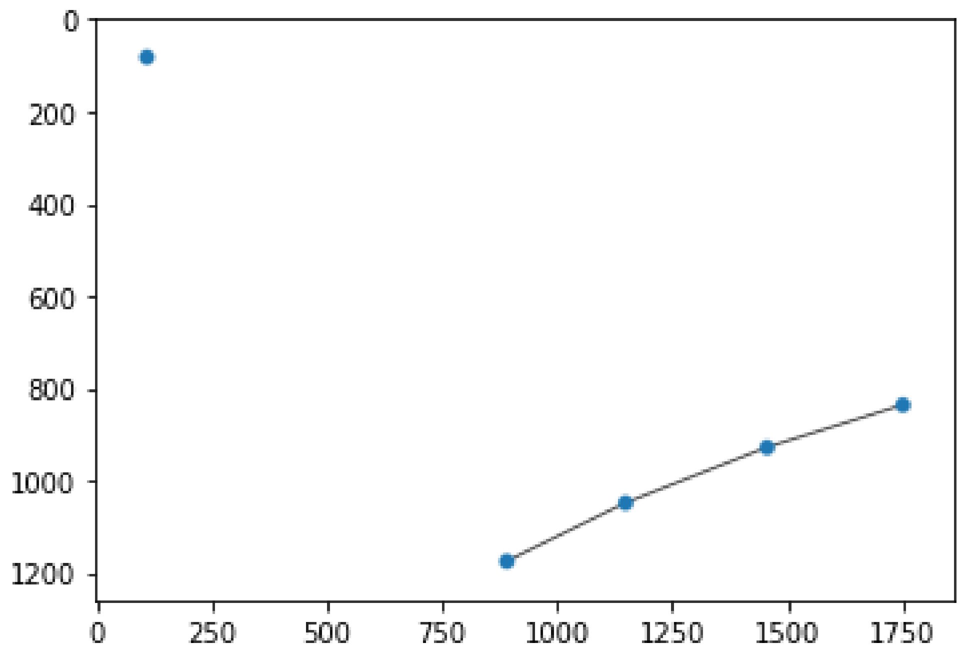
	N	LCC	k-1	k+1
0	5.0	2.666667	0.75	0.85
1	10.0	4.666667	1.25	1.35
2	15.0	9.666667	1.50	1.60
3	20.0	10.666667	1.55	1.65
4	25.0	18.666667	2.20	2.30
5	30.0	9.000000	1.05	1.15
6	35.0	17.333333	1.50	1.60
7	40.0	20.333333	1.40	1.50
8	45.0	26.666667	1.60	1.70
9	50.0	28.666667	1.55	1.65

In [ ]:
```
# Generating images below the critical point

os.mkdir('/content/-')

for i in range(len(table_1b)):
  graph = nx.generators.erdos_renyi_graph(int(table_1b['N'][i]), table_1b['k-1'][i] / (t
```

```
  nx.draw(graph, node_size=50)
  plt.savefig('/content/-/{}-.png'.format(i), dpi=300, bbox_inches='tight')
```

In [8]:
```
img = plt.imread('/content/-/0-.png')
plt.imshow(img)
```

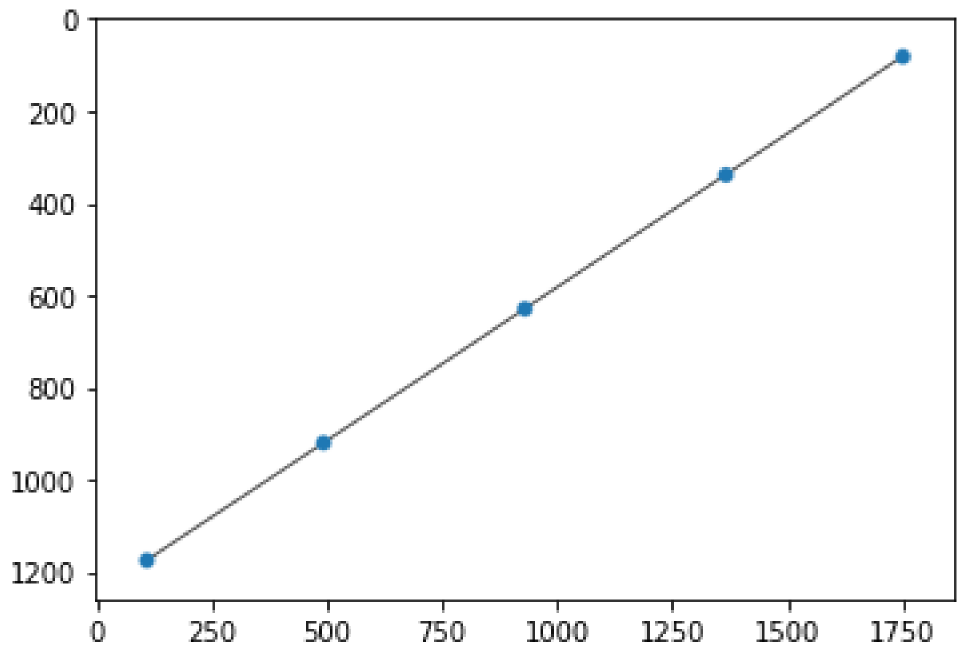
Out[8]: <matplotlib.image.AxesImage at 0x7f3c0c0f4be0>



In [ ]:
```
# Generating images above the critical point

os.mkdir('/content/+')

for i in range(len(table_1b)):
  graph = nx.generators.erdos_renyi_graph(int(table_1b['N'][i]), table_1b['k+1'][i] / (t
  nx.draw(graph, node_size=50)
  plt.savefig('/content/+/{}+.png'.format(i), dpi=300, bbox_inches='tight')
```

In [10]:
```
img = plt.imread('/content/+/0+.png')
plt.imshow(img)
```

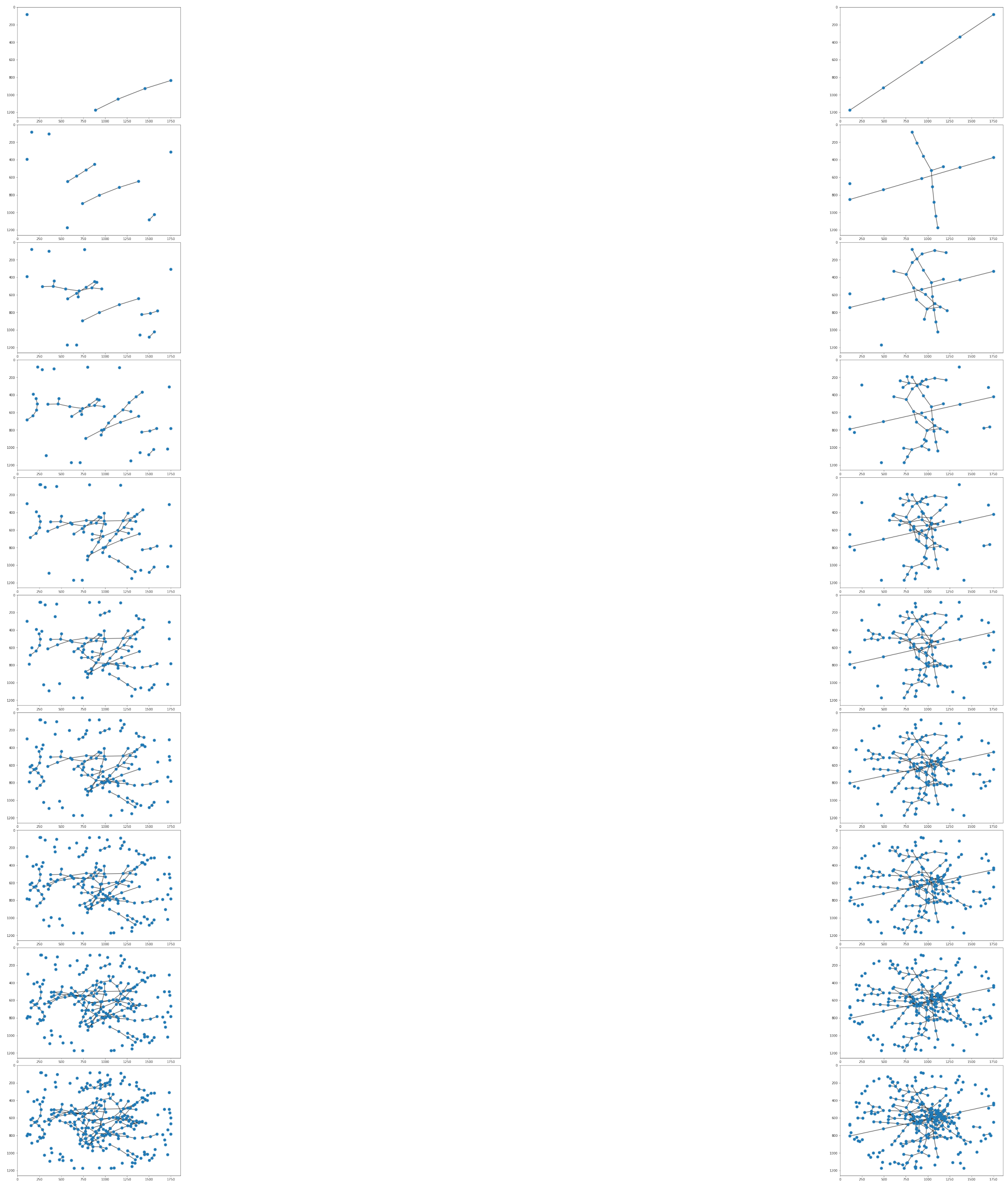
Out[10]: <matplotlib.image.AxesImage at 0x7f3c0bf8bf28>



In [11]:
```
# Loading a list of images

bel = []
for pic in np.sort(os.listdir('/content/-')):
  img = plt.imread('/content/-/'+pic)
  bel.append(img)

ab = []
for pic in np.sort(os.listdir('/content/+')):
  img = plt.imread('/content/+/'+pic)
  ab.append(img)
```

In [21]:
```
# Plotting images to compare

fig=plt.figure(figsize=(100, 50))

columns = 2
rows = 10

for i in range(10):
    img = bel[i]
    fig.add_subplot(rows, columns, i*2+1)
    plt.imshow(img)

for i in range(10):
    img = ab[i]
```

```
        fig.add_subplot(rows, columns, i*2+2)
        plt.imshow(img)


    fig.tight_layout(pad=0.1)

    plt.show()
```



3.1.2 Independent of k, it is deprecated since the probability is now set to 0001, 3 networks will be generated for each number of nodes. The average size of LCC will be calculated for each network size as well.

```
In [ ]: for N in range(1000,25000,500):
            num_samp = 3
            p = 0.0001
            av_chi, av_lcc_size = [],[]
            av_chi.clear()
            av_lcc_size.clear()

            print('Computing LCC for {} nodes...'.format(N))

            chi_values, comp_sizes = [],[]
            comp_sizes.clear()
            chi_values.clear()

            for i in range(0,num_samp):
                ER_graph = nx.generators.erdos_renyi_graph(N,p)

                chi,S = Calc_chi_and_S(ER_graph)
                chi_values.append(chi)

                largest_comp_size = len(max(nx.connected_components(ER_graph), key = len))
                comp_sizes.append(largest_comp_size)

            av_chi.append(np.mean(chi_values))
            av_lcc_size.append(np.mean(comp_sizes))

            table_1 = table_1.append({'LCC':av_lcc_size[av_chi.index(max(av_chi))], 'N':N},ignor
```

```
Computing LCC for 1000 nodes...
Computing LCC for 1500 nodes...
Computing LCC for 2000 nodes...
Computing LCC for 2500 nodes...
Computing LCC for 3000 nodes...
Computing LCC for 3500 nodes...
Computing LCC for 4000 nodes...
Computing LCC for 4500 nodes...
Computing LCC for 5000 nodes...
Computing LCC for 5500 nodes...
Computing LCC for 6000 nodes...
Computing LCC for 6500 nodes...
Computing LCC for 7000 nodes...
Computing LCC for 7500 nodes...
Computing LCC for 8000 nodes...
Computing LCC for 8500 nodes...
Computing LCC for 9000 nodes...
Computing LCC for 9500 nodes...
```

```
Computing LCC for 10000 nodes...
Computing LCC for 10500 nodes...
Computing LCC for 11000 nodes...
Computing LCC for 11500 nodes...
Computing LCC for 12000 nodes...
Computing LCC for 12500 nodes...
Computing LCC for 13000 nodes...
Computing LCC for 13500 nodes...
Computing LCC for 14000 nodes...
Computing LCC for 14500 nodes...
Computing LCC for 15000 nodes...
Computing LCC for 15500 nodes...
Computing LCC for 16000 nodes...
Computing LCC for 16500 nodes...
Computing LCC for 17000 nodes...
Computing LCC for 17500 nodes...
Computing LCC for 18000 nodes...
Computing LCC for 18500 nodes...
Computing LCC for 19000 nodes...
Computing LCC for 19500 nodes...
Computing LCC for 20000 nodes...
Computing LCC for 20500 nodes...
Computing LCC for 21000 nodes...
Computing LCC for 21500 nodes...
Computing LCC for 22000 nodes...
Computing LCC for 22500 nodes...
Computing LCC for 23000 nodes...
Computing LCC for 23500 nodes...
Computing LCC for 24000 nodes...
Computing LCC for 24500 nodes...
```

In [ ]: `table_1`

Out[ ]:

	N	LCC
0	1000.0	3.333333
1	1500.0	4.333333
2	2000.0	5.666667
3	2500.0	7.333333
4	3000.0	7.666667
5	3500.0	7.333333
6	4000.0	11.000000
7	4500.0	13.333333
8	5000.0	16.666667
9	5500.0	17.333333
10	6000.0	27.333333
11	6500.0	26.666667
12	7000.0	32.666667

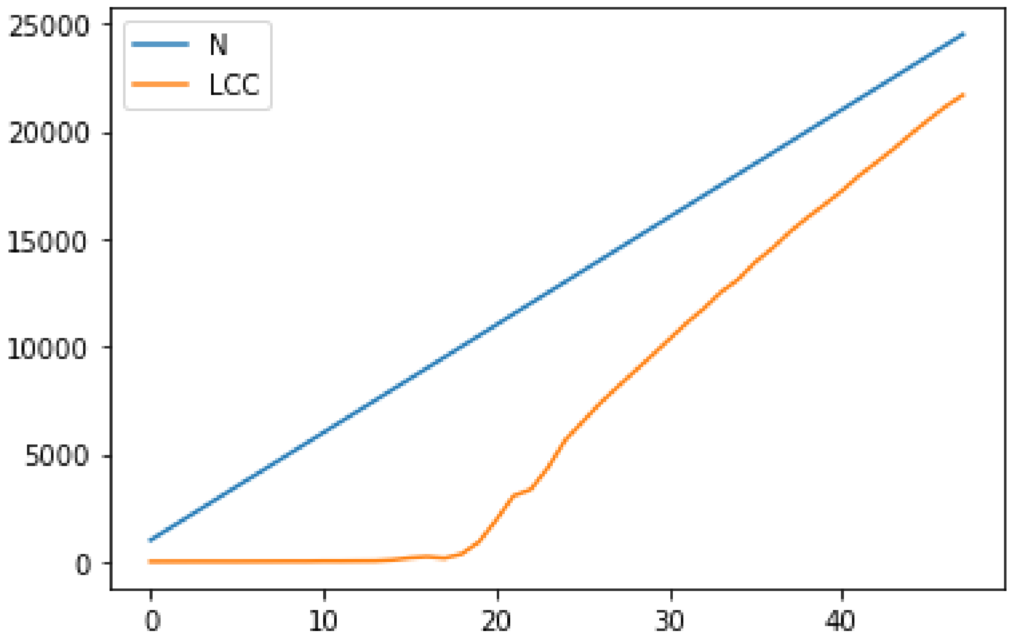
```
13   7500.0     36.333333
14   8000.0     81.666667
15   8500.0    178.666667
16   9000.0    232.666667
17   9500.0    162.000000
18  10000.0    357.666667
19  10500.0    917.000000
20  11000.0   1936.333333
21  11500.0   3057.333333
22  12000.0   3356.666667
23  12500.0   4374.666667
24  13000.0   5655.666667
25  13500.0   6496.666667
26  14000.0   7351.000000
27  14500.0   8104.666667
28  15000.0   8834.666667
29  15500.0   9572.000000
30  16000.0  10320.333333
31  16500.0  11072.000000
32  17000.0  11742.333333
33  17500.0  12515.333333
34  18000.0  13104.000000
35  18500.0  13917.000000
36  19000.0  14576.333333
37  19500.0  15328.000000
38  20000.0  15984.000000
39  20500.0  16592.000000
40  21000.0  17222.333333
41  21500.0  17935.333333
42  22000.0  18549.000000
43  22500.0  19181.666667
44  23000.0  19872.333333
45  23500.0  20518.000000
46  24000.0  21143.666667
47  24500.0  21677.666667
```

3.1.3 Scaling of LCC

In [ ]:
```
# On this graph we might see how the average size of LCC grows
# along with a number of nodes in the system

plt.figure()
table_1.plot()
```

Out[ ]:
```
<AxesSubplot:>
```

```
<Figure size 432x288 with 0 Axes>
```



In [ ]:
```
scale = []
for i in range(len(table_1['LCC'])):
    scale.append(table_1['LCC'][i] / table_1['N'][i])
scale, len(scale)
```

Out[ ]:
```
([0.0033333333333333335,
  0.0028888888888888888,
  0.0028333333333333335,
  0.0029333333333333334,
  0.0025555555555555557,
  0.0020952380952380953,
  0.00275,
  0.0029629629629629632,
  0.0033333333333333335,
  0.0031515151515151513,
  0.004555555555555556,
  0.0041025641025641026,
  0.004666666666666666,
  0.004844444444444445,
  0.010208333333333333,
  0.021019607843137254,
  0.02585185185185185,
  0.017052631578947368,
  0.035766666666666667,
  0.08733333333333333,
  0.17603030303030304,
```

```
  0.2658550724637681,
  0.2797222222222222,
  0.34997333333333336,
  0.4350512820512821,
  0.4812345679012346,
  0.5250714285714285,
  0.5589425287356322,
  0.5889777777777777,
  0.6175483870967742,
  0.6450208333333334,
  0.671030303030303,
  0.6907254901960784,
  0.7151619047619048,
  0.728,
  0.7522702702702703,
  0.7671754385964913,
  0.786051282051282,
  0.7992,
  0.8093658536585366,
  0.820111111111111,
  0.8342015503875968,
  0.8431363636363637,
  0.8525185185185186,
  0.8640144927536232,
  0.8731063829787234,
  0.8809861111111111,
  0.8848027210884354],
 48)
```

In [ ]:
```
table_1['Scale'] = scale
table_1
```

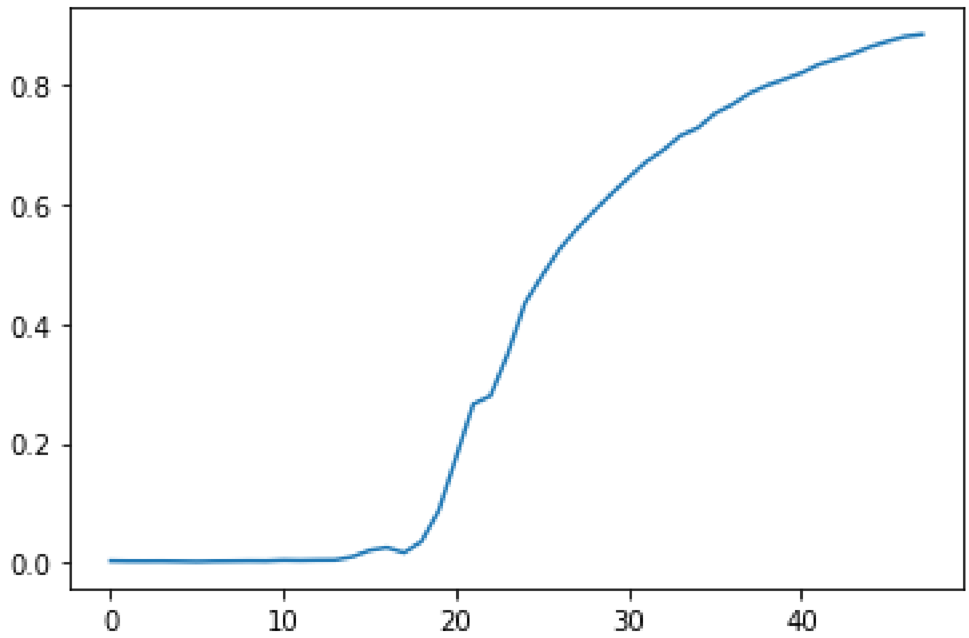
Out[ ]:

	N	LCC	Scale
0	1000.0	3.333333	0.003333
1	1500.0	4.333333	0.002889
2	2000.0	5.666667	0.002833
3	2500.0	7.333333	0.002933
4	3000.0	7.666667	0.002556
5	3500.0	7.333333	0.002095
6	4000.0	11.000000	0.002750
7	4500.0	13.333333	0.002963
8	5000.0	16.666667	0.003333
9	5500.0	17.333333	0.003152
10	6000.0	27.333333	0.004556
11	6500.0	26.666667	0.004103
12	7000.0	32.666667	0.004667
13	7500.0	36.333333	0.004844
14	8000.0	81.666667	0.010208

```
15   8500.0    178.666667  0.021020
16   9000.0    232.666667  0.025852
17   9500.0    162.000000  0.017053
18  10000.0    357.666667  0.035767
19  10500.0    917.000000  0.087333
20  11000.0   1936.333333  0.176030
21  11500.0   3057.333333  0.265855
22  12000.0   3356.666667  0.279722
23  12500.0   4374.666667  0.349973
24  13000.0   5655.666667  0.435051
25  13500.0   6496.666667  0.481235
26  14000.0   7351.000000  0.525071
27  14500.0   8104.666667  0.558943
28  15000.0   8834.666667  0.588978
29  15500.0   9572.000000  0.617548
30  16000.0  10320.333333  0.645021
31  16500.0  11072.000000  0.671030
32  17000.0  11742.333333  0.690725
33  17500.0  12515.333333  0.715162
34  18000.0  13104.000000  0.728000
35  18500.0  13917.000000  0.752270
36  19000.0  14576.333333  0.767175
37  19500.0  15328.000000  0.786051
38  20000.0  15984.000000  0.799200
39  20500.0  16592.000000  0.809366
40  21000.0  17222.333333  0.820111
41  21500.0  17935.333333  0.834202
42  22000.0  18549.000000  0.843136
43  22500.0  19181.666667  0.852519
44  23000.0  19872.333333  0.864014
45  23500.0  20518.000000  0.873106
46  24000.0  21143.666667  0.880986
47  24500.0  21677.666667  0.884803
```

In [ ]:
```
# This graph represents the scaling itself ,
# so we might see that after 40 iterations
# with the given above parameters the scale goes above 0.8

table_1['Scale'].plot()
```

Out[ ]:
```
<AxesSubplot:>
```



4 How does the size of the largest connected component grow with p at the critical point?

4.0.1 To answer this question we generated 99 networks with 5000 nodes, number of samples set to 2, probability range [0.00001, 0.001] with a step of 0.00001. Then average number of LCC was calculated with respect to each p.

```
In [ ]: for p in np.arange(0.00001, 0.001, 0.00001):

            N = 5000
            num_samp = 2

            av_chi, av_lcc_size = [],[]
            av_chi.clear()
            av_lcc_size.clear()

            print('Computing LCC with p = {}'.format(p))

            chi_values, comp_sizes = [],[]
            comp_sizes.clear()
            chi_values.clear()
            for i in range(0,num_samp):
                ER_graph = nx.generators.erdos_renyi_graph(N, p)

                chi,S = Calc_chi_and_S(ER_graph)
```

```python
                chi_values.append(chi)

                largest_comp_size = len(max(nx.connected_components(ER_graph), key = len))
                comp_sizes.append(largest_comp_size)

            av_chi.append(np.mean(chi_values))
            av_lcc_size.append(np.mean(comp_sizes))

            table_2 = table_2.append({'LCC':av_lcc_size[av_chi.index(max(av_chi))], 'p':p},ignor
```

```
Computing LCC with p = 1e-05
Computing LCC with p = 2e-05
Computing LCC with p = 3.0000000000000004e-05
Computing LCC with p = 4e-05
Computing LCC with p = 5e-05
Computing LCC with p = 6e-05
Computing LCC with p = 7.000000000000001e-05
Computing LCC with p = 8e-05
Computing LCC with p = 9e-05
Computing LCC with p = 0.0001
Computing LCC with p = 0.00011
Computing LCC with p = 0.00012
Computing LCC with p = 0.00013000000000000002
Computing LCC with p = 0.00014000000000000001
Computing LCC with p = 0.00015000000000000001
Computing LCC with p = 0.00016
Computing LCC with p = 0.00017
Computing LCC with p = 0.00018
Computing LCC with p = 0.00019
Computing LCC with p = 0.0002
Computing LCC with p = 0.00021
Computing LCC with p = 0.00022
Computing LCC with p = 0.00023
Computing LCC with p = 0.00024
Computing LCC with p = 0.00025000000000000006
Computing LCC with p = 0.00026000000000000003
Computing LCC with p = 0.00027000000000000006
Computing LCC with p = 0.00028000000000000003
Computing LCC with p = 0.00029000000000000006
Computing LCC with p = 0.00030000000000000003
Computing LCC with p = 0.00031000000000000005
Computing LCC with p = 0.00032
Computing LCC with p = 0.00033000000000000005
Computing LCC with p = 0.0003400000000000001
Computing LCC with p = 0.00035000000000000005
Computing LCC with p = 0.0003600000000000001
Computing LCC with p = 0.00037000000000000005
```

```
Computing LCC with p = 0.0003800000000000001
Computing LCC with p = 0.00039000000000000005
Computing LCC with p = 0.0004000000000000001
Computing LCC with p = 0.00041000000000000005
Computing LCC with p = 0.00042000000000000007
Computing LCC with p = 0.00043000000000000004
Computing LCC with p = 0.00044000000000000007
Computing LCC with p = 0.00045000000000000004
Computing LCC with p = 0.00046000000000000007
Computing LCC with p = 0.00047000000000000004
Computing LCC with p = 0.00048000000000000007
Computing LCC with p = 0.0004900000000000001
Computing LCC with p = 0.0005000000000000001
Computing LCC with p = 0.00051
Computing LCC with p = 0.0005200000000000001
Computing LCC with p = 0.0005300000000000001
Computing LCC with p = 0.0005400000000000001
Computing LCC with p = 0.00055
Computing LCC with p = 0.0005600000000000001
Computing LCC with p = 0.0005700000000000001
Computing LCC with p = 0.0005800000000000001
Computing LCC with p = 0.00059
Computing LCC with p = 0.0006000000000000001
Computing LCC with p = 0.0006100000000000001
Computing LCC with p = 0.0006200000000000001
Computing LCC with p = 0.00063
Computing LCC with p = 0.00064
Computing LCC with p = 0.0006500000000000001
Computing LCC with p = 0.0006600000000000001
Computing LCC with p = 0.0006700000000000001
Computing LCC with p = 0.00068
Computing LCC with p = 0.0006900000000000001
Computing LCC with p = 0.0007000000000000001
Computing LCC with p = 0.0007100000000000001
Computing LCC with p = 0.00072
Computing LCC with p = 0.0007300000000000001
Computing LCC with p = 0.0007400000000000001
Computing LCC with p = 0.0007500000000000001
Computing LCC with p = 0.00076
Computing LCC with p = 0.0007700000000000001
Computing LCC with p = 0.0007800000000000001
Computing LCC with p = 0.0007900000000000001
Computing LCC with p = 0.0008
Computing LCC with p = 0.0008100000000000001
Computing LCC with p = 0.0008200000000000001
Computing LCC with p = 0.0008300000000000001
Computing LCC with p = 0.0008400000000000001
Computing LCC with p = 0.0008500000000000001
```

```
Computing LCC with p = 0.0008600000000000001
Computing LCC with p = 0.0008700000000000001
Computing LCC with p = 0.0008800000000000001
Computing LCC with p = 0.0008900000000000001
Computing LCC with p = 0.0009000000000000001
Computing LCC with p = 0.0009100000000000001
Computing LCC with p = 0.0009200000000000001
Computing LCC with p = 0.00093
Computing LCC with p = 0.0009400000000000001
Computing LCC with p = 0.0009500000000000001
Computing LCC with p = 0.0009600000000000001
Computing LCC with p = 0.0009700000000000002
Computing LCC with p = 0.00098
Computing LCC with p = 0.0009900000000000002
```

In [ ]:
```
table_2
```

Out[ ]:

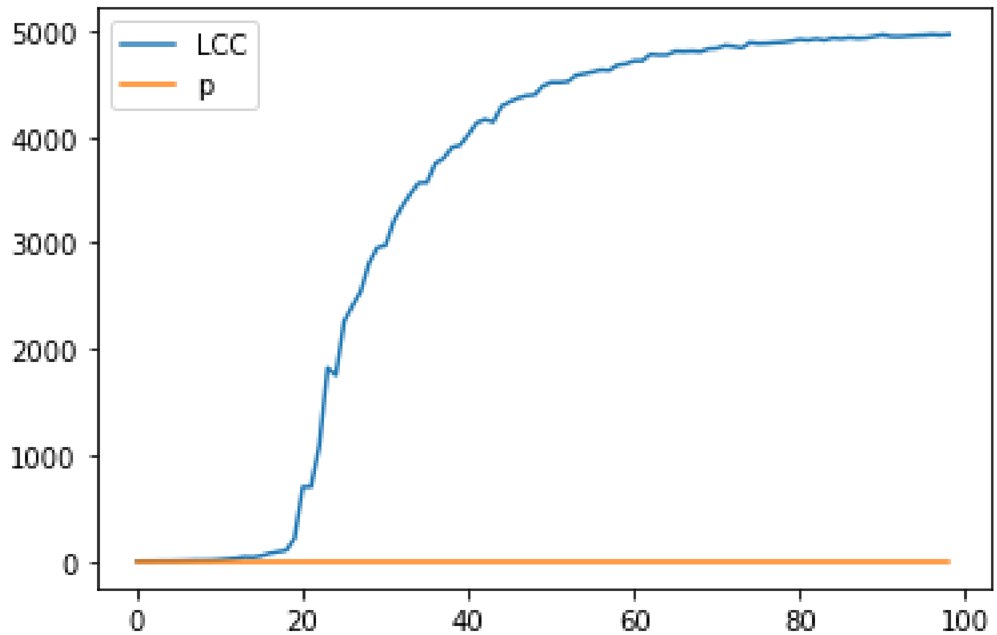
	LCC	p
0	3.0	0.00001
1	5.0	0.00002
2	5.5	0.00003
3	7.0	0.00004
4	6.5	0.00005
..	...	...
94	4954.5	0.00095
95	4957.0	0.00096
96	4963.5	0.00097
97	4957.5	0.00098
98	4965.0	0.00099

[99 rows x 2 columns]

In [ ]:
```
# On this graph we can see how the size of the largest connected component
# grow with p at the critical point

plt.figure()
table_2.plot()
```

Out[ ]: <AxesSubplot:>

```
<Figure size 432x288 with 0 Axes>
```



LCC
p
5000
4000
3000
2000
1000
0
0
20
40
60
80
100