

# **User manual**

## **Integral\_equ**

**Integral equations for source functions inside homogeneous  
Rayleigh-scattering sphere with star in its centre**

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The code described here has been developed by Juris Freimanis and Romāns Peženkovs at the Engineering Research Institute “Ventspils International Radio Astronomy Centre” (ERI VIRAC) of Ventspils University of Applied Sciences (VUAS) as part of the European Regional Development Fund (ERDF) project No. 1.1.1.1/16/A/213 “Physical and chemical processes in the interstellar medium” (“ASTRA”).

## Program using guide

In this manual we describe how to use `Integral_equ`. The result of our program is virtual astronomical CCD matrix images, each pixel of this matrix containing three Stokes vector parameters (I,Q,U). The CCD matrix images could be compared with the results of the other programs (for example, Ventspils RTMC [Radiative Transfer Monte Carlo] also created by J.Freimanis and R.Peženkovs).

Program is written using **Python 3.7** language, several libraries were imported ( **math** , **numpy** , from **scipy** import **special** ). The program was tested on **Dell G3 15** laptop ( Intel Core i5-8300H CPU 2.3 GHz, 16 GB RAM) using **Windows 10**. `Integral_equ` uses a lot of RAM memory (by testing it was about 5 GB), it is necessary to solve system of linear algebraic equations (in our tests it was about 8000 equations [matrix of  $8000 \times 8000 = 64\,000\,000$  elements] ).

## Variables of `Integral_equ`

Variable **optical\_radius** is the optical radius of the cloud. The cloud is a sphere, and the star is a spherical light source, located in the centre of the dust cloud. Variable **full\_opt\_rad** is the full optical radius; it is the sum of optical radii of cloud and star.

Variable **albedo** is the single scattering albedo. Variables **tau** , **tau\_prime** are optical radius parameters which are used in integral equations. Variable **tau\_star** is the formal optical radius of the star (calculated as the product of star's geometrical radius and dust extinction coefficient).

The linear polarization reference plane for the radiation propagating in the sphere is the plane going through the direction of propagation of radiation and the radial direction; only two Stokes parameters (I,Q) are nonzero. The angle between the radial direction and the direction of propagation of radiation is “theta” , variable **mu** is the cosine of angle “theta”.

Variable **Luminosity** is the luminosity of the star in watts. Variable **parsec** is equal to value of one parsec in meters. Variable **astron\_unit** is equal to value of one astronomical unit in meters. Variable **cloud\_radius** is the geometrical radius of dust cloud in meters. Variable **dist\_cloud\_tel** is the distance between cloud surface and telescope in meters. Variable **dist\_star\_cent\_tel** is the distance between star centre and telescope. Variable **a** is the dust extinction coefficient, to convert geometrical radius to optical.

Variables **width** and **height** are number of points for **tau\_prime** (for integral calculations). Variables **image\_x\_dim** and **image\_y\_dim** are the number of pixels in X and Y direction for image of light scattered by dust cloud (dimensions of CCD matrix).

Arrays **K00\_array** ; **K02\_array** ; **K20\_array** ; **K22\_array** ; **B00\_array** ; **B20\_array** ; **f02\_array** ; **f20\_array** ; **f22\_array** ; **I00\_array** ; **I02\_array** ; **I20\_array** ; **I22\_array** ; **B0\_array** ; **B2\_array** are used to numerically solve the system of integral equations. Arrays **I\_array** and **Q\_array** are first and second Stokes parameters (I;Q).

Two-dimensional arrays **data\_gnu\_I\_2** ; **data\_gnu\_Q\_2** and **data\_gnu\_U\_2** are cells of CCD matrix image (each cell contains three Stokes parameters [I,Q,U] ).

Variables **delta\_1** and **delta\_2** are used to calculate expressions **I00\_array** ; **I02\_array** ; **I20\_array** and **I22\_array** .

System of linear algebraic equations is created to calculate **B0\_array** and **B2\_array** (we also use Simpson's method to calculate integral). We use **numpy.linalg.solve()** function to solve the system of linear equations. After **B0\_array** and **B2\_array** calculations we check if values are physically correct.

After calculations of arrays **I\_array** and **Q\_array** , we rotate the two values of Stokes vector (I,Q) to get image of sphere (to get two-dimensional arrays **data\_gnu\_I\_2** ; **data\_gnu\_Q\_2** and **data\_gnu\_U\_2**). If there are some pixels the values of which were not calculated, linear interpolation could be used.

After calculations the results should be saved in files **data\_gnu\_I\_2.txt** ; **data\_gnu\_Q\_2.txt** and **data\_gnu\_U\_2.txt** . To create image the **Python** program **p02\_Integral\_equ.py** could be used.

## Input variables

It is necessary to set physical parameters of light scattering simulation. Use the following parameters to do so:

**optical\_radius** – optical radius of the dust cloud;

**albedo** – the single scattering albedo;

**tau\_star** – optical radius of the star;

**Luminosity** – luminosity of the star;

**cloud\_radius** – geometrical radius of the cloud;

**dist\_cloud\_tel** – distance between dust cloud and telescope;

**width** – the number of integration points;

**image\_x\_dim** – number of pixels of virtual CCD matrix (X – direction);

**image\_y\_dim** – number of pixels of virtual CCD matrix (Y – direction).