Bandit Learning with Positive Externalities

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Positive Externalities in Online Platforms

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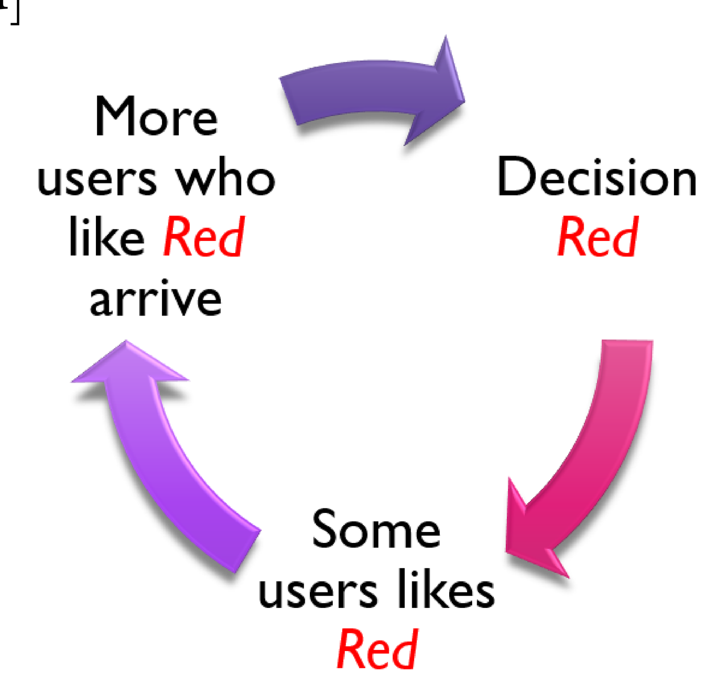
- Learning algorithms are often used to recommend alternatives to users.
- *Common assumption*: arrivals not influenced by decisions.
- *Reality*: **Positive Externalities / Self-Reinforcement / Network Effects** – a positive experience attracts more users of the same type

What could go wrong?

- Suppose Blue users like Blue items (but not Red items)
- Suppose Red users like Red items (but not Blue items)
- Suppose $\mathbb{E}$[Blue-Blue match reward] > $\mathbb{E}$[Red-Red match reward]



- Successful Red-Red matches made early on may trigger more Red user arrivals.
- So the platform might learn to prefer Red-Red matches even if it is suboptimal!

Main Insights from our Results

- There is a cost to being optimistic in the face of uncertainty as initial mistakes are amplified. UCB algorithm, in particular, fails miserably.
- It is possible to prevent the snowballing arising from positive externalities by structuring the exploration procedure well.
- Once enough evidence is gathered, one may even use the externalities to shift the arrivals to a profit-maximizing population.

Model

Standard bandit setting:

- m: Number of arms (items)
- T : Time horizon; one user arrives per time step
- μ_a: Expected reward when arm a pulled (Bernoulli)
- a^*: best arm
- $T_a(t)$: number of times arm a pulled up to time t
- $S_a(t)$: total reward at arm a up to time t
- Goal: maximize expected total reward (R_T).

Positive externalities:

- Let θ_a be initial "bias" of arm a.
- We assume the user arriving at time t *likes* arm a independently with probability:

$$\lambda_a(t) = \frac{f(\theta_a + S_a(t))}{\Sigma_b f(\theta_b + S_b(t))}.$$

- f is the *externality function*. We consider $f(x) = x^\alpha$, $\alpha > 0$. α determines the strength of the positive externality.
- $\mathbb{P}$(reward at $t|$ arm a pulled) $= \mu_a$ if user t likes a, otherwise zero.

Main Results

Suboptimality of UCB Algorithm:

Definition: UCB(γ) algorithm, at time t, pulls the arm a with largest:

$$\text{empirical mean reward up to } t + \sqrt{\frac{\gamma \ln t}{T_a(t-1)}}.$$

Theorem

UCB(γ) exhibits $O(T)$ regret for each $\gamma > 0$. In fact,

$$\lim_{T\to\infty} \mathbb{P}(S_{a^*}(T\ ) = 0) > 0.$$

Optimal Algorithm:

Definition: The *Balanced-Exploration* (BE) algorithm is as follows:

Suppose $w_k = \ln\ln k$ for each $k \geq 1$. Fix $\tau = w_T \ln T$.

- For $t \leq \tau$, pull the arm with lowest cumulative reward $S_a(t-1)$ (ties broken at random).
- For $t > \tau$, pull the arm with highest mean reward $S_a(\tau)/T_a(\tau)$ at time τ.

Theorem

The regret of the BE algorithm is as follows:

1. If $0 < \alpha < 1$ then $\mathbb{E}[R_T] = \tilde{O}(T^{1-\alpha} \ln^\alpha T)$.
2. If $\alpha = 1$ then $\mathbb{E}[R_T] = \tilde{O}(\ln^2 T)$.
3. If $\alpha > 1$ then $\mathbb{E}[R_T] = \tilde{O}(\ln^\alpha T)$.

Further, there exists no algorithm which can achieve better asymptotic performance than above.

- We also provide an adaptive variant of BE, called BE-AE, which successively drops suboptimal arms.