A Universal Statistical Simulator via Quantum Galton Boards

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Abstract

This document summarizes the principles and implementation of a quantum Galton board (QGB) as a universal statistical simulator. We outline the core algorithm for constructing a QGB circuit that reproduces the dynamics of a classical Galton board using quantum gates. Classical simulations scale exponentially that is $O(2^n)$ with the number of Galton board layers n , while the quantum implementation demonstrates a much more efficient polynomial scaling of $O(n^2)$. By manipulating the quantum peg operation, this framework can be adapted to generate a variety of statistical distributions, including Gaussian, exponential, and quantum Hadamard walk patterns. The approach demonstrates a versatile method for simulating complex probabilistic systems on a quantum computer.

1. The General QGB Algorithm

The Quantum Galton Board (QGB) provides an intuitive quantum-mechanical framework for simulating statistical processes. Building on the dynamics of a classical Galton board in Fig.1, where a ball's path is determined by a series of random deflections, the QGB uses quantum circuits to achieve a similar outcome. The core components of our generalized QGB algorithm for an arbitrary number of layers are as follows:

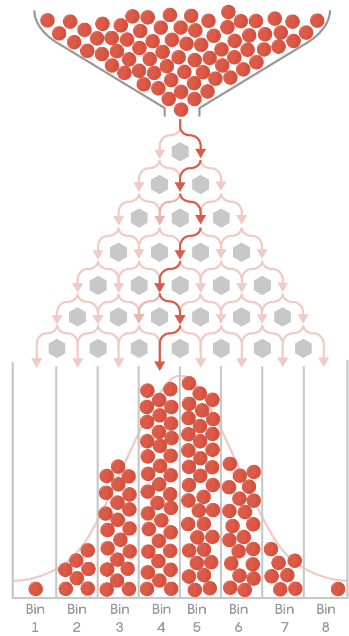

Figure 1: A 7-layer Galton board, the object shaded in grey represents a peg

1. **Initialization:** Qubits are initialized to represent the ball's possible positions. An ancilla qubit, is also prepared. An X gate on the central position qubit marks the ball's starting point.

2. **Controlled Movement:** At each layer, a quantum gate is applied to the ancilla, creating a superposition of states that determines whether the ball deflects left or right. A sequence of controlled-SWAP (CSWAP), Controlled-X (CX) and CSWAP gates then moves the ball's position, with the CX gates used to toggle the ancilla's state to control deflections in the either direction

(left/right). This complete operation represent one quantum peg in action.

3. **Layer Progression** A CX gate is used within each layer between two quantum pegs to undo the effect of the CX operation from the previous quantum peg. The process of controlled movement is then repeated for each subsequent peg. The entire sequence is repeated for every layer to simulate a quantum superposition of all possible paths. Before proceeding to a new layer, the ancilla qubit is reset, and the quantum gate for the ball's deflection is reapplied.

4. **Measurement:** After the final layer, the position qubits are measured. Measurement after many runs reveals patterns of probability distribution representing the ball's possible positions.

The flowchart in Fig.2 visually outlines the logic of our generalized QGB algorithm, from initialization to the final measurement.

2. Generating Diverse Distributions

By tuning the ancilla gate in the quantum peg operation, the QGB can produce a variety of statistical distributions beyond the standard Gaussian shape. This adaptability makes it a versatile tool for simulating a range of probabilistic behaviors. For example- A standard Hadamard gate in the ancilla provides an unbiased superposition, leading to a Gaussian-like distribution that mirrors the classical Galton board. A row-dependent R_y rotation gate can be used to bias the coin, making the probability of a right deflection decrease exponentially with each layer.

3. Conclusion

The QGB offers a flexible and powerful quantum framework for universal statistical simulation. By exploiting quantum superposition and carefully designing the circuit's gates, we can accurately simulate a diverse range of statistical distributions, providing a valuable platform for studying quantum analogs of classical random processes.

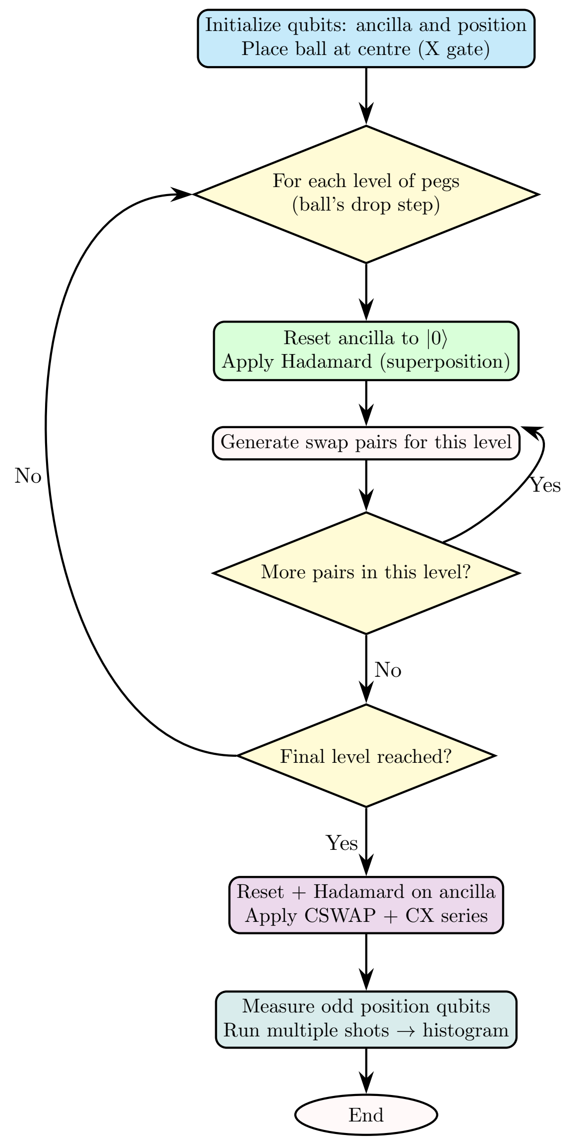

Figure 2: A flowchart representing the generalized quantum Galton board algorithm.