Quantum Walks and Monte Carlo (Universal Statistical Simulator)

1 Summary

1.1 Introduction

The Galton Board, also known as the Bean Machine, is a mechanical device designed to demonstrate statistical distributions, particularly the formation of the normal (Gaussian) distribution.

Its quantum counterpart, the Quantum Galton Board (QGB), replicates this behavior through quantum circuits. Unlike the classical board where each path is determined randomly, the QGB leverages quantum superposition and interference to explore all possible trajectories simultaneously. This quantum parallelism allows for an exponential speed-up in sampling distributions using significantly fewer resources. While the classical version produces binomial distributions, QGBs can generate uniquely quantum distributions. They are typically built using fundamental quantum gates such as the Hadamard, X, and controlled-SWAP (CSWAP) gates. As a result, QGBs provide valuable insights into the nature of quantum randomness and offer powerful tools for simulating statistical processes that extend beyond classical capabilities.

1.2 Related work

Previous implementations of QGBs often involved quantum walks or optical analogs to model classical distributions. These typically required complex setups or deep circuits. Earlier works aimed to replicate the binomial output of classical Galton Boards, but lacked efficiency and generality. This paper improves upon those efforts by presenting a minimal-depth circuit using only three Clifford gate types: Hadamard, controlled-SWAP (Fredkin), and X gates. The authors present a modular approach to building a Quantum Galton Board (QGB) using quantum circuits. At its core is a quantum peg a circuit module comprising Hadamard, X, and controlled-SWAP gates that simulates the binary decision a classical ball makes at a peg (left or right). These pegs are composed into layers to form a complete QGB. By initializing the control qubit in superposition and propagating the ball through successive pegs, the circuit generates a coherent superposition of all possible paths, mimicking a classical Galton Board but exponentially faster.

The paper introduces three QGB variants:

- **Standard QGB:** Uses Hadamard gates to produce a balanced (unbiased) quantum walk, generating binomial or near-Gaussian distributions.
- **Biased QGB (B-QGB):** Replaces Hadamard with θ gates to skew the left-right decision at each peg, enabling the simulation of asymmetric distributions.
- **Fine-Grained Biased QGB:** Extends the biasing by assigning a unique angle to each peg, allowing precise, localized control over the output distribution shape.

This hierarchical, circuit-efficient design built using only Clifford gates and a recycled control qubit achieves both minimal depth and adaptability, showcasing how quantum logic can naturally generalize classical statistical processes.

1.3 Working

By varying peg biases using rotation gates and adjusting control qubits, the QGB can replicate a wide variety of statistical distributions not just binomial or Gaussian. This tunability makes it a universal statistical simulator. For example, replacing Hadamard gates with Rx (θ) enables skewed outcomes, and varying at each peg achieves fine-grained distribution shaping. Post-processing of measurements allows transformation into meaningful statistics. As such, QGBs can model randomness in financial data, cryptography, and machine learning. The quantum nature of the device ensures high entropy and speed, offering powerful advantages for tasks where classical sampling is computationally intensive.

1.4 Conclusion/future scope

The paper proposes a QGB architecture that is more intuitive and hardware-efficient than earlier models. Using a small set of quantum gates and ancilla qubits, it simulates a superposition of all classical trajectories, enabling rich statistical behavior. It also introduces biased and fine-grained QGB variants that can mimic non-standard distributions. While hardware noise and mid-circuit resets present challenges, the circuit's shallow depth reduces susceptibility to decoherence. The authors emphasize the potential of QGBs in building quantum-native statistical tools, especially as quantum hardware evolves. Overall, the approach offers a promising path toward practical, flexible, and scalable quantum statistical simulation. Despite its innovation, the QGB circuit faces practical limitations. First, it requires many ancilla qubits and mid-circuit resets, which are not fully supported on current NISQ devices. Second, hardware noise, particularly from transpiled controlled-SWAP gates, leads to significant errors. Third, the output encoding—based on 1-hot position—necessitates additional post-processing to yield interpretable distributions. The circuit also scales quadratically in resources, limiting feasibility for large n. Future improvements could focus on native support for complex gates, better noise mitigation, and optimizations in gate efficiency. Further research is also needed to expand QGB use cases and apply them in real-world quantum workloads.