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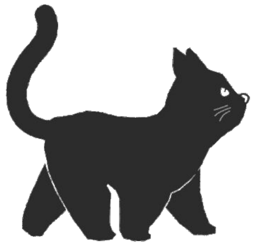
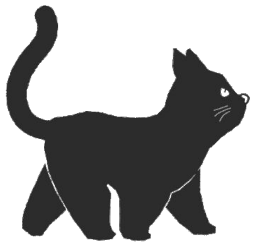
Project 1 : Quantum Walks and Monte Carlo

Quantum Galton Board (QGB) as a Universal Statistical Simulator



TEAM: Q-Plinkers



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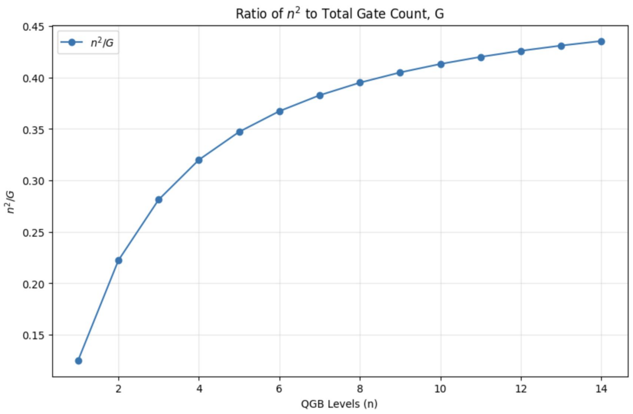
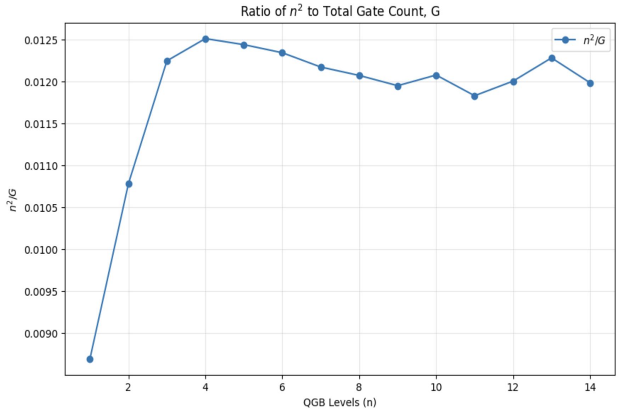
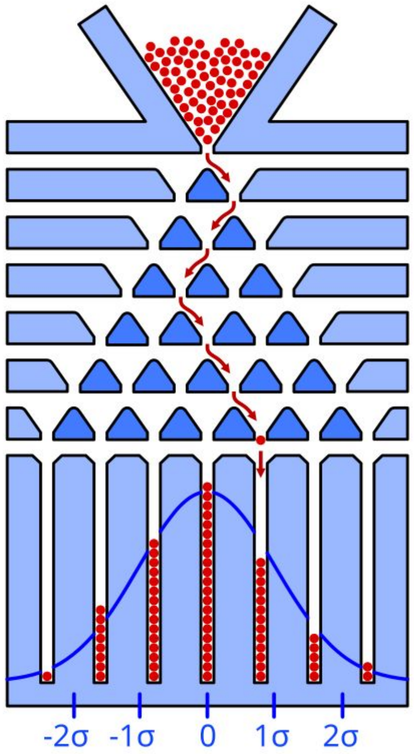
Problem Statement

Simulating a Galton Box using quantum circuits and creating a quantum statistical simulator with exponential speedup.



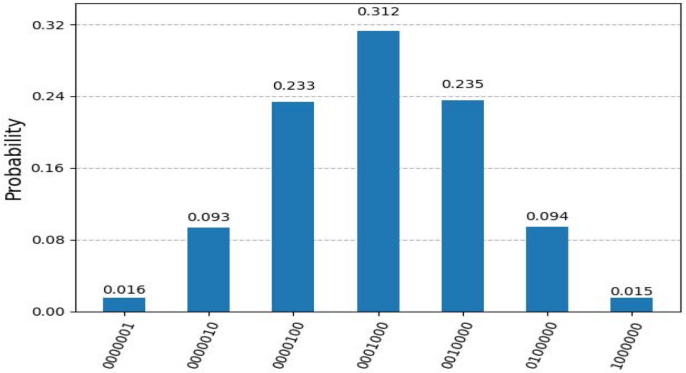




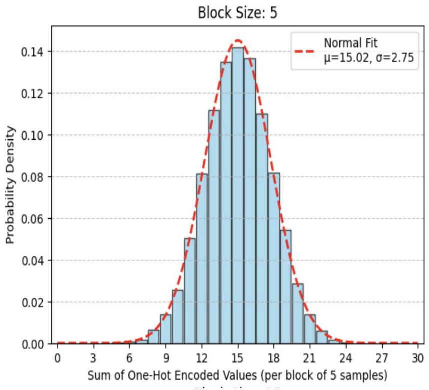
*A quantum Galton board provides an **exponential speedup** over classical methods ($O(2^n)$ vs $O(n^2)$), enabling a practical quantum statistical simulator that simulates*

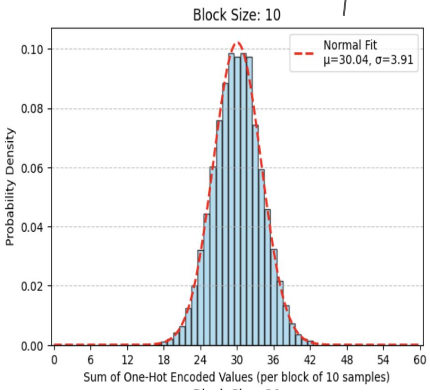
Source: Fichier:Galton-Brett.svg

Task 1: Generalized QGB



n=6





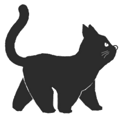
As block size ↑

mean ∝ size

variance ↑

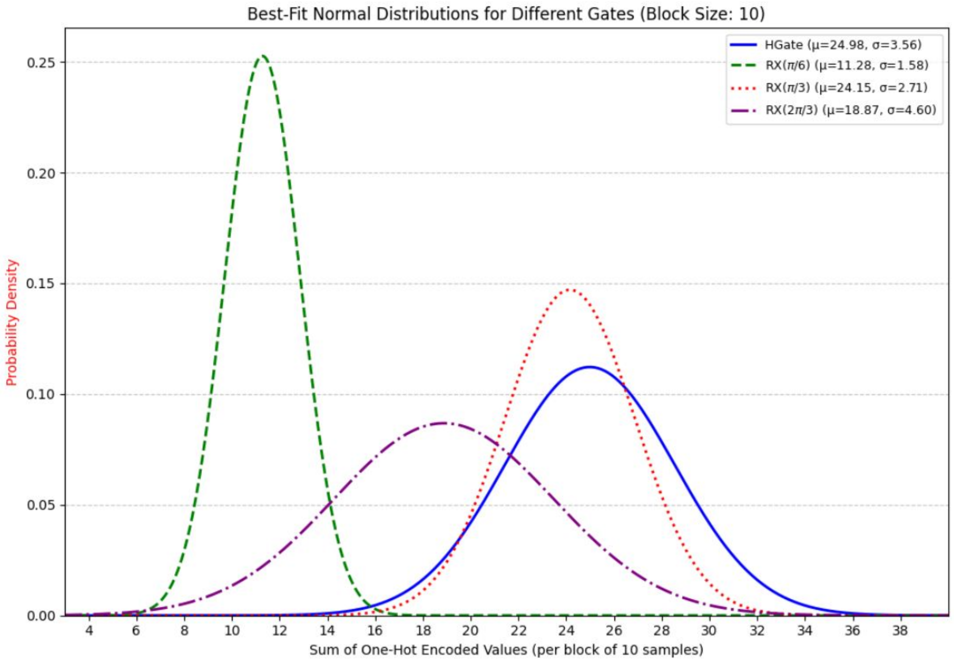
Varied characteristics via post-processing

- ➔ Outputs processed into one-hot encodings, over block sizes: 5, 10, ...
- ➔ Fitted with normal distributions

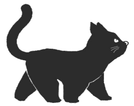


Task 1

Replacing the H gate with Rx(θ) introduces bias, creating asymmetric probability distributions.



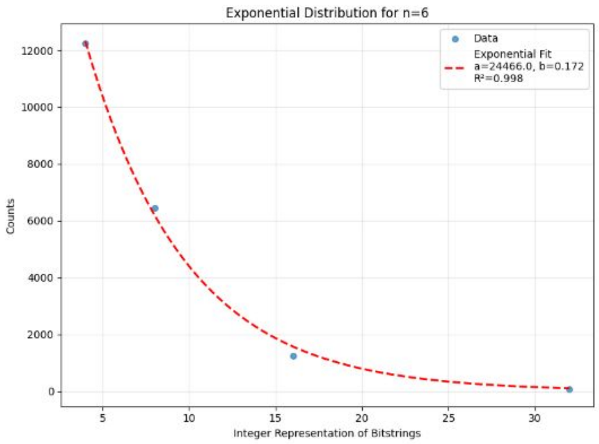
- Tuning θ → custom mean & spread for specific sampling needs.
- Biased pegs → varied normal distributions
- Tuning block size while post-processing

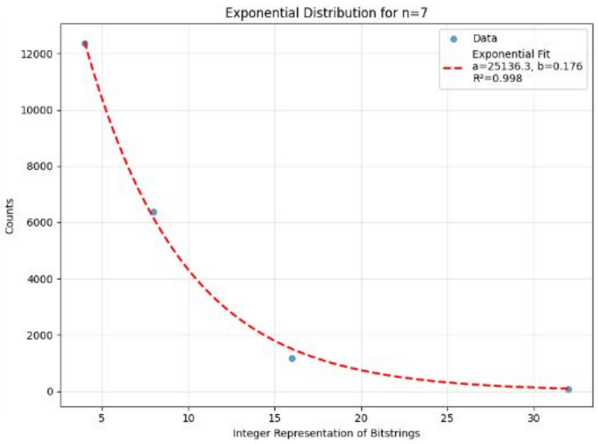


Task 2a: Exponential Distribution

Type A : Introduce bias decay at each level using a row-dependent Rx(θ) gate. Probability of rightward movement decreases with step r :

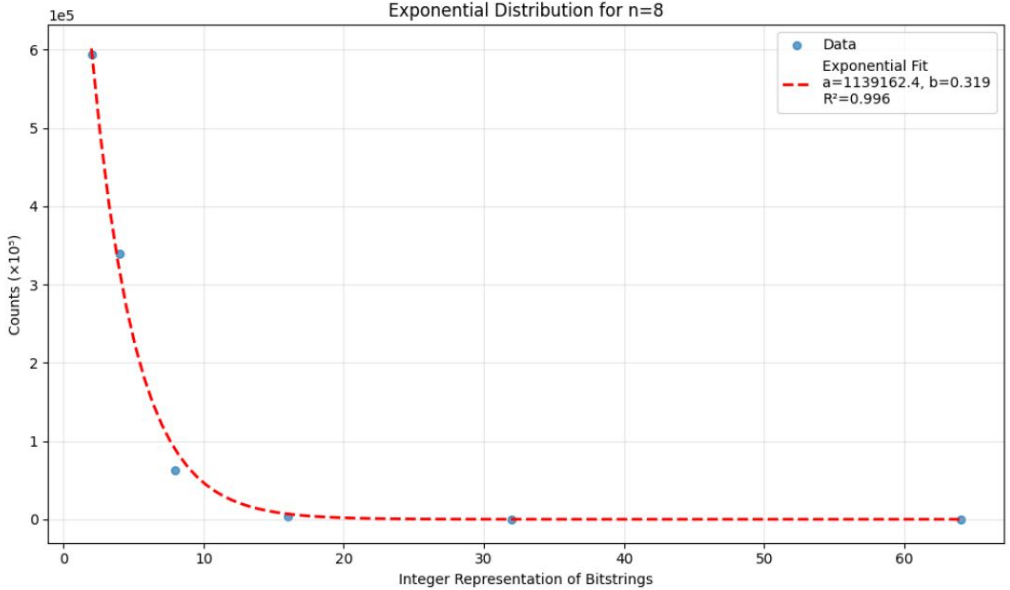
$$p(r) = 0.5 + 0.1 \cdot r \Rightarrow \theta = 2\cos^{-1}(\sqrt{p})$$

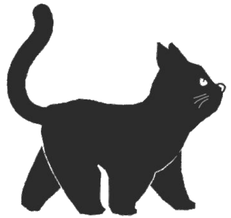




Task 2a

Type B : Modified the board to act like a 1D quantum walk with absorption. More control over decay rate possible





Task 2b: Hadamard Quantum Walk

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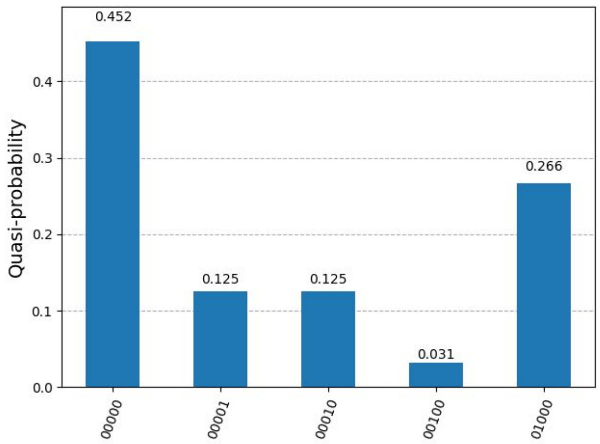
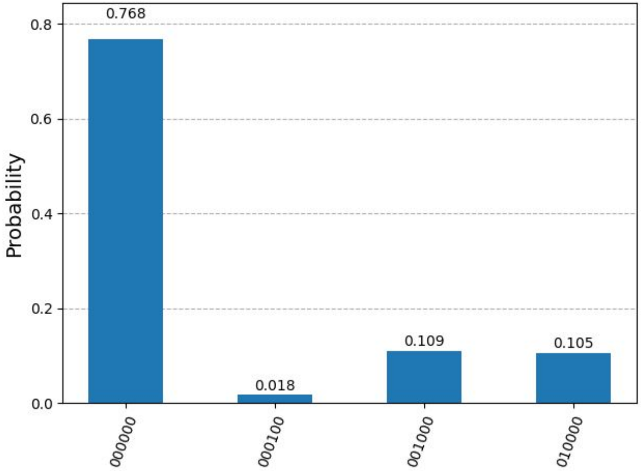
Produces **bimodal** probability distributions with **interference patterns** → baseline reference model.

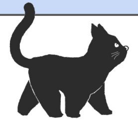


2 working cases



4-layers → 10 qubits (measure 5 positions)

5-layers → 12 qubits (measure 6 positions)



Task 3: Results on a noisy simulator

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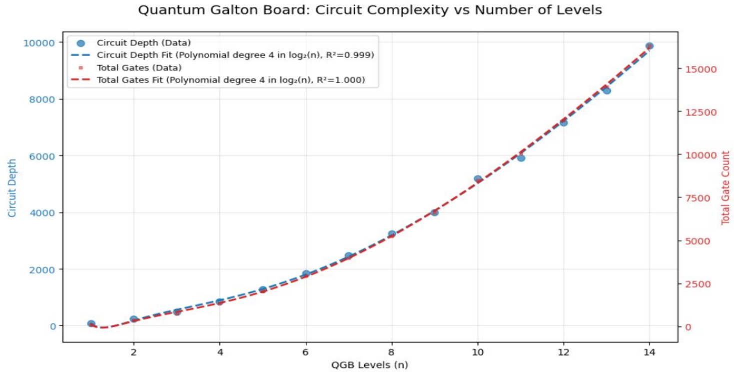
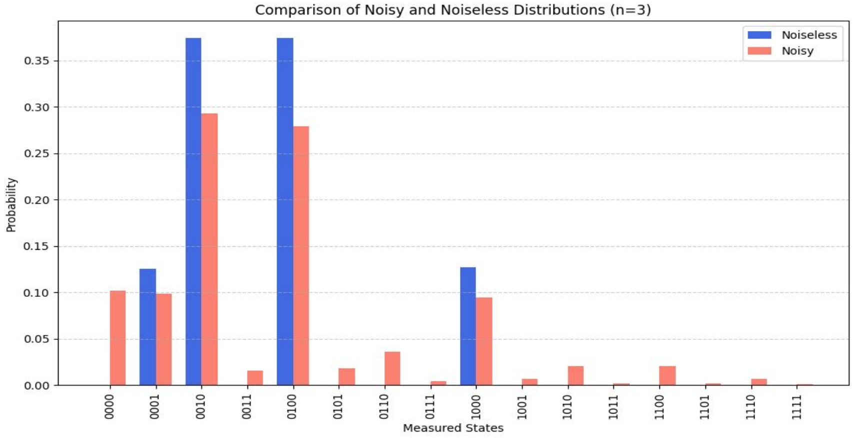
- ➔ **Noisy results** showed samples that were not expected as per ideal (noiseless) runs.
- ➔ Interference patterns are **partially destroyed**, reducing symmetry.
- ➔ Higher noise → more deviation from theoretical prediction.



Efficient transpiled circuit on noisy simulator




```
Backend aer_simulator_from(ibm_brisbane)
Accumulated two-qubit error of 169 gates: 0.561
Accumulated one-qubit error of 870 gates: 0.072
Accumulated readout error: 0.094
Accumulated total error: 0.727

Backend aer_simulator_from(ibm_torino)
Accumulated two-qubit error of 171 gates: 0.359
Accumulated one-qubit error of 570 gates: 0.093
Accumulated readout error: 0.096
Accumulated total error: 0.547
```

For n=3: Before finding optimal layout;
Note:Optimization level=3 (Highest);

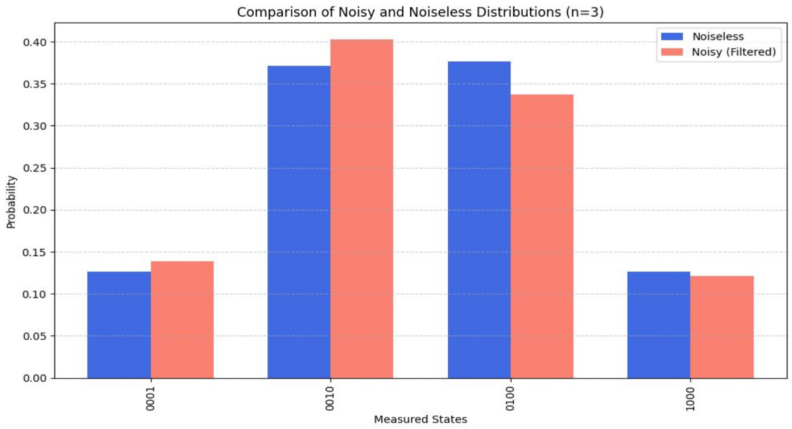
Best total error: 54.7%

Task 3:

- **Post-processing:** Keeping only samples with Hamming weight= 1 (known from the ideal case)
- Finding best optimal layout- mapping to physical qubits: Lowest total error (metric used)

```
Optimizing for Backend: aer_simulator_from(ibm_brisbane)
  Seed: 371
  Layout: [0, 18, 14, 2, 1, 3, 4]
  TOTAL error: 0.250
Optimizing for Backend: aer_simulator_from(ibm_torino)
  Seed: 148
  Layout: [59, 62, 54, 61, 60, 72, 58]
  TOTAL error: 0.520
=== Best (by total accumulated error) ===
Backend: AerSimulator('aer_simulator_from(ibm_brisbane)'
           noise_model=<NoiseModel on ['reset', 'measure', 'x', 'id', 'sx', 'ecr']>)
Seed: 371
Total error: 0.250
Layout: [0, 18, 14, 2, 1, 3, 4]
```

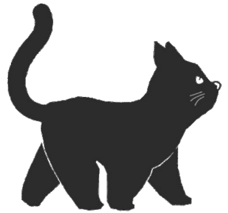
Best total error: 25% (ibm_brisbane)



Total Variation Distance(TVD): 0.0438
Kullback-Leibler Divergence (KL): 0.0041

Future Scope

- While other distributions were generalized, the **Hadamard walk still remains a future scope** as we were unable to generalize it to an arbitrary number of Galton board levels.
- Characterizing distribution type with Galton board structure
- Robust optimization strategies for noisy simulation
- Rigorous benchmarking of quantum implementation with classical methods



Acknowledgement

1. Universal Statistical Simulator, Mark Carney, Ben Varcoe, arXiv:2202.01735 [quant-ph]
2. A heartfelt gratitude to Womanium and Wiser team for designing & organizing this program, and offering scholarships.
3. Grateful to Brian McDermott and the NNL team for their guidance and support on this project.
4. Thanks to the Discord community for active discussion and timely support.
5. This project majorly uses Qiskit by IBM Quantum.

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THANK YOU