Quantum Computing
Spring’24, Plaksha University

Assignment 3
Due Date: 01 Apr’24

---

1. **No Cloning Theorem**
Quantum bits have one interesting ”shortcoming”: unlike classical bits which we can perfectly copy and do all sorts of operations with, an arbitrary quantum bit can not be perfectly copied. This is formally referred to as the No-cloning theorem. As you will show next, no-cloning follows directly from the linearity of quantum mechanics and the Unitarity of quantum gates.

   A cloning gate should realize the operation: $\hat{U}|b_1\rangle|b_2\rangle = |b_1\rangle|b_1\rangle$. Choose $|b_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$ and $|b_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$ and use the linearity of $\hat{U}$ to show that this is not possible. Where in this derivation/problem statement have we implicitly assumed that $\hat{U}$ is unitary?

   Note: what we loose in terms of the No-cloning theorem, we try to make up via entanglements!

2. **Quantifying Entanglement**
Consider the states:

$$|\zeta_1\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right),$$
$$|\zeta_2\rangle = \frac{1}{\sqrt{3}}\left(|00\rangle + |01\rangle + |10\rangle\right).$$

   (a) Compute the density matrices ρ for $|\zeta_{1,2}\rangle$ respectively.

   (b) Compute the partial trace of ρ over Qubit-1 to obtain the reduced density matrix $\rho_1 = \text{Trace}_1\rho$ for both $\zeta_{1,2}$. Interpret the resultant 1-Qubit states.

   (c) As a quantifiable measure of degree of entanglement, define and evaluate

$$S = \frac{d}{d-1}\left[1 - \text{Trace}\rho_1^2\right],$$

   for the reduced density matrices of both the states.

3. **3 Qubit States**
Consider the three qubit states:

$$|G\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right),$$
$$|W\rangle = \frac{1}{\sqrt{3}}\left(|001\rangle + |010\rangle + |100\rangle\right).$$

   (a) Write the density matrix ρ for these states and find the trace and eigenvalues of ρ.

   (b) Compute the partial trace over qubit-3 and interpret the resultant 2-qubit states in terms of their entanglement properties.

4. **BB-84 Protocol** Do the lab exercise.