Quantum Computing
Spring'24, Plaksha University

Assignment 2
Due Date: 27 Feb'24

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1. **Riemann sphere and Bloch Sphere**

   (a) Using stereographic projection, show that any arbitrary 1-Qubit state can be written as a vector on the unit Riemann sphere of the form discussed in Assignment 1:

   $$|+\psi\rangle = \cos\frac{\theta}{2}|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}|-z\rangle. \tag{1}$$

   (b) Starting with the state $|0\rangle$, show that the state in Eq. (1) can be obtained through a cascade of 4 gates: Hadamard gate followed by Phase-Shift gate of $-\theta$, followed by another Hadamard, followed by yet another Phase-shift gate of $-\frac{\pi}{2}+\phi$.

2. **Product States vs Entangled States**

   (a) Show that $|\xi\rangle_\pm = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ cannot be written as product of two single Qubit states, i.e., as Product states.

   (b) For $|\xi\rangle_\pm$ find the expectation values of $\sigma_x^A\sigma_x^B, \sigma_y^A\sigma_y^B, \sigma_z^A\sigma_z^B$, where A, B superscripts refer to the first (left) and second (right) Qubits respectively.

   (c) Define a new observable (Hermitian operator) $\sigma^A \cdot \sigma^B = \sigma_x^A\sigma_x^B + \sigma_y^A\sigma_y^B + \sigma_z^A\sigma_z^B$ and show that $|\xi\rangle_\pm$ is an eigenvector of this observable.

3. **Quantum C-Not gate**

   (a) Write the matrix representation of Quantum C-Not gate.

   (b) Starting with the state $|00\rangle$, use the 1-Qubit Hadamard gate and the C-Not gate to construct an entangled state.

4. **Hermitian and Unitary matrices**

   (a) Show that for a Hermitian operator ($H = H^\dagger$), the eigenvalues are real and eigenvectors are orthogonal.

   (b) Show that for a Unitary operator ($UU^\dagger = I$), the eigenvalues have modulus 1 and the eigenvectors are orthogonal.

5. **Quantum cryptography**

   This is a coding based question. Download this notebook. You are expected to complete the tasks in it and submit the notebook (with all cells executed) as a .pdf file. Thus your solution file should have the resulting quantum circuits and all the outputs as required by the exercise.