Machine Learning 4: Neural Networks

Consider a face recognition problem

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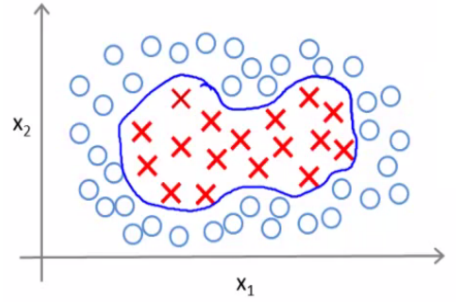
For a 50×50 grayscale image, we have 2500 basic features: $x_{1,\ldots,2500} \in \{0, 255\}$

As before, we can map in feature space the 'face' and 'non-face' labels



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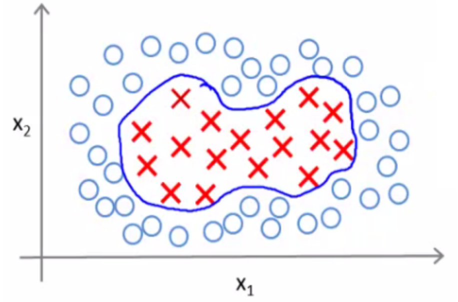

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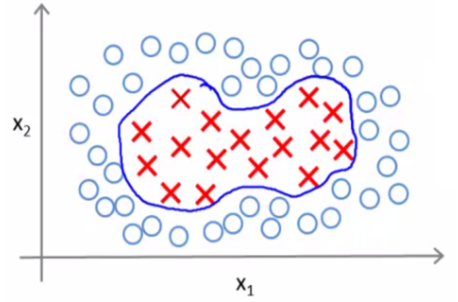

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Up to quadratic order this is already $\sim 3,000,000$ features ....

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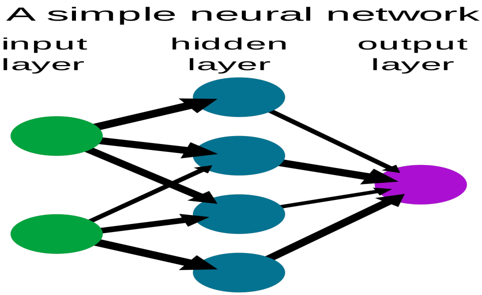
... you get the picture

Neural Networks



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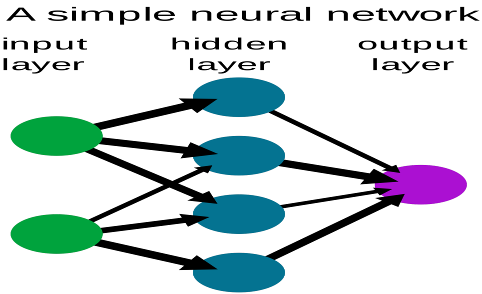

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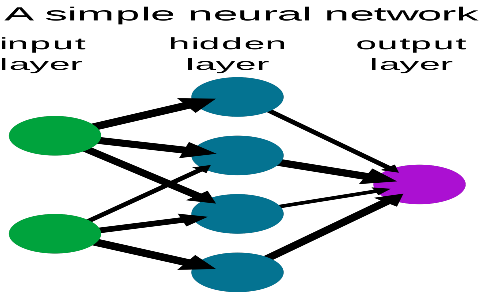

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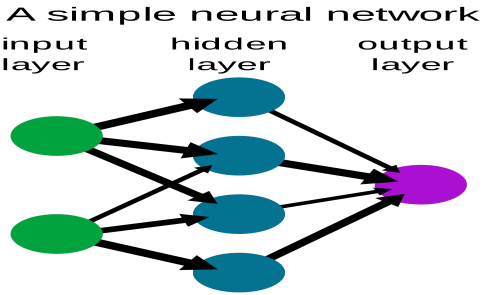

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Let's see what this means more concretely

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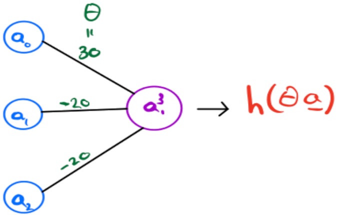

a_1	a_2	h = NAND
0	0	1
0	1	1
1	0	1
1	1	0

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$$= \mathrm{sig}(30 - 20 a_1 - 20 a_2)$$

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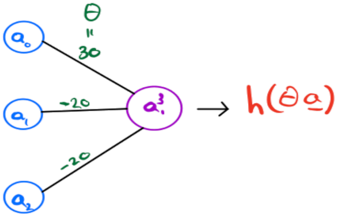

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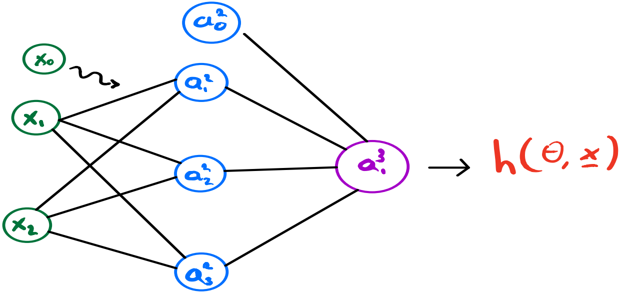
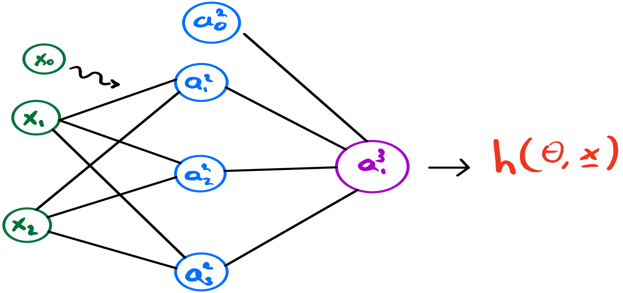
Let's now consider some binary classification as our goal eg. face or not face.

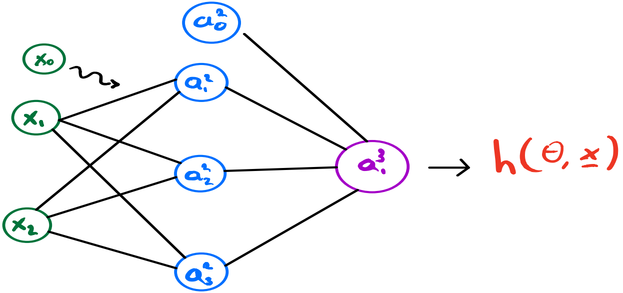


How is the hypothesis defined?



How is the hypothesis defined?

$$h(\Theta, \boldsymbol{x}) = a_1^3 = \mathrm{sig}((\theta^{(2)})^{\mathrm{T}} \boldsymbol{a}),$$
$$(\theta^{(2)})^T \boldsymbol{a} = \theta_{10}^{(2)} a_0^2 + \theta_{11}^{(2)} a_1^2 + \theta_{12}^{(2)} a_2^2 + \theta_{13}^{(2)} a_3^2 \tag{1}$$



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where

$$
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\tilde{\boldsymbol{a}} =& [a_1^2, a_2^2, a_3^2] = \mathrm{sig}((\theta^{(1)})^{\mathrm{T}} \boldsymbol{x}), \\
a_i^2 =& \mathrm{sig}(\theta_{\mathrm{i}0}^{(1)} \mathrm{x}_0 + \theta_{\mathrm{i}1}^{(1)} \mathrm{x}_1 + \theta_{\mathrm{i}2}^{(1)} \mathrm{x}_2)
\end{aligned} \tag{2}
$$

Some points

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- We want to find Θ which is a collection of weight vectors/matrices θ^j where 'j' is the number of layers. Note that the dimension of θ^i is $s_{j+1} \times (s_j + 1)$ where s_j is the number of units in layer j excluding the bias unit.

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So what is our cost function ?

For a neural network with **L** layers, looking to classify into **K** different classes, and a training set of **m** examples, we can extend our old logistic cost function as follows

$$J(\Theta) = -\frac{1}{m}\sum_{i=1}^{m}\sum_{k=1}^{K}\left[y_k^i \log[h_k(x^i)] + (1-y_k^i)\log[1-h_k(x^i)]\right] \\ + \frac{\lambda}{2m}\sum_{\ell=1}^{L-1}\sum_{i=1}^{s_\ell}\sum_{j=1}^{s_{\ell+1}}(\theta_{ji}^{\ell})^2, \tag{3}$$

where λ is our regularisation parameter and s_ℓ is the number of units in the ℓ^{th} layer **not** including the bias unit.

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We want to minimise the cost by varying the components of *Theta*. To do this, we can use the **back propagation** algorithm. This is a generalisation of gradient descent which requires the partial derivatives $\frac{\partial J(\Theta)}{\partial \theta_{ij}^{\ell}}$.

Autonomous driving as an example:

https://www.coursera.org/learn/machine-learning/lecture/zYS8T/autonomous-driving

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Take a look at Python tutorial 4