Assignment 7
CS374

Harsh Patel(201701021)
Viraj Patel(201701439)

Assigned by :

Prof. Arnab Kumar

September 26, 2019

Contents

1 Piecewise, Spline Interpolation and $y = 1/x$

x	1	2	3	4
y	1	1/2	1/3	1/4

1.1 Equation

$$S"(x) = M_j \tag{1}$$

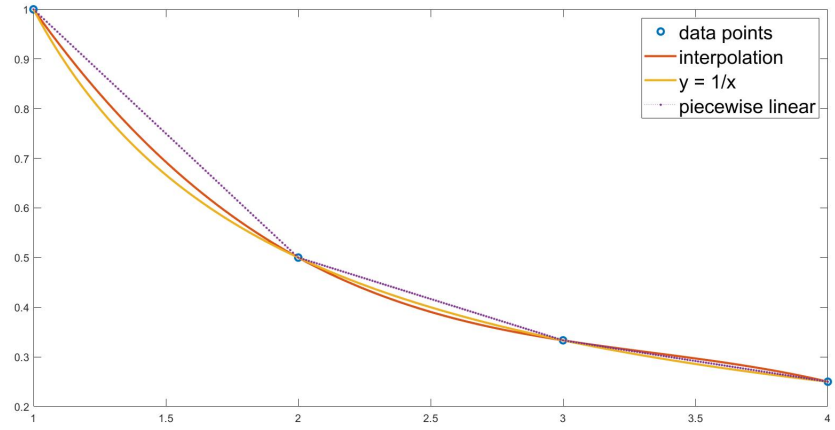
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}} \frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}} \frac{(x - x_{j-1})^3}{6} + Ax + B \tag{2}$$

$$A = D - C \tag{3}$$

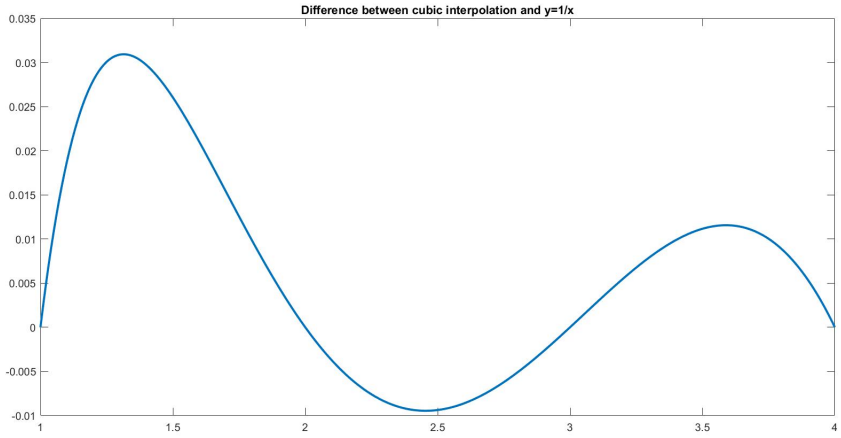
$$B = Cx_j - Dx_{j-1} \tag{4}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{5}$$

1.2 Graph



Plot of Piecewise Linear Interpolation, Cubic Spline and $y = 1/x$



Plot of Error between Cubic Spline and $y = 1/x$

2 Cubic Spline and Piecewise Linear Interpolation

x	0	1	2	2.5	3	3.5	4
y	2.5	0.5	0.5	1.5	1.5	1.125	0

2.1 Equation

$$S"(x) = M_j \tag{6}$$

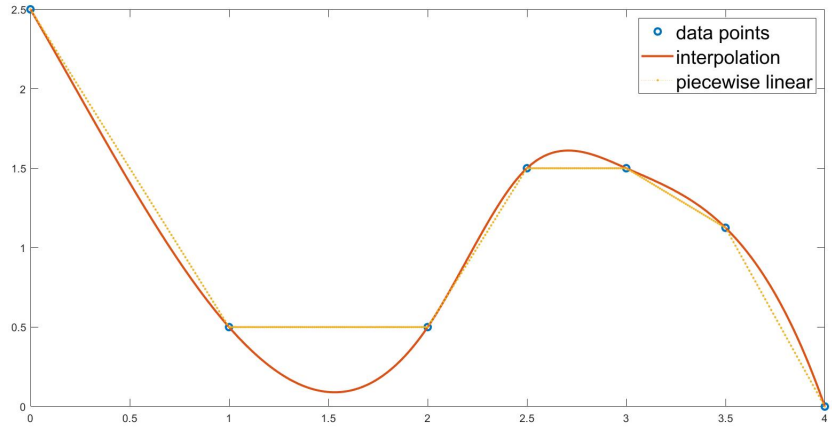
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{7}$$

$$A = D - C \tag{8}$$

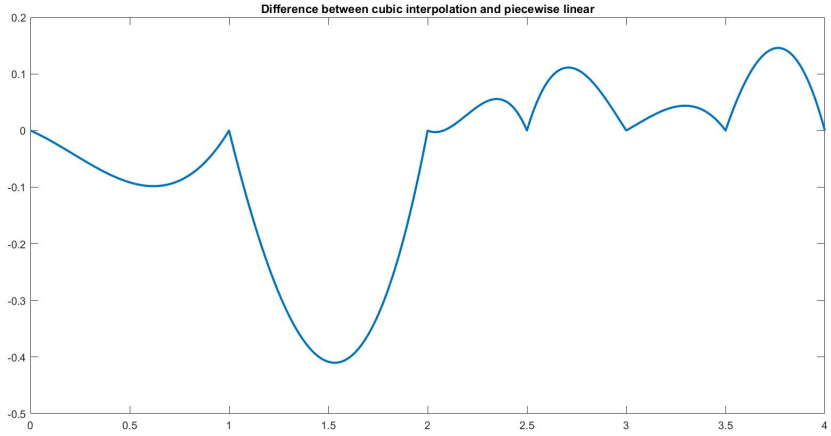
$$B = Cx_j - Dx_{j-1} \tag{9}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{10}$$

2.2 Graph



Plot of Piecewise Linear Interpolation and Cubic Spline Interpolation



Plot of Error between Piecewise Linear Interpolation and Cubic Spline Interpolation

3 Cubic Spline and Piecewise Linear Interpolation

x	-0.5	0	0.25	1
y	0.73151	1	1.2684	1.718282

3.1 Equation

$$S"(x) = M_j \tag{11}$$

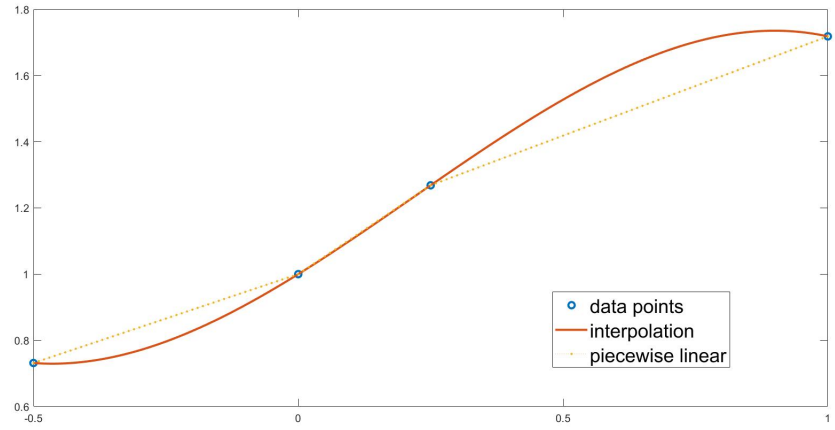
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}} \frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}} \frac{(x - x_{j-1})^3}{6} + Ax + B \tag{12}$$

$$A = D - C \tag{13}$$

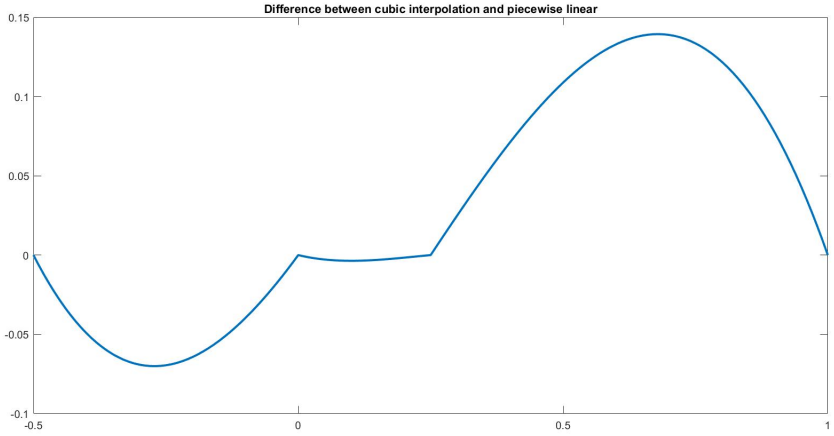
$$B = Cx_j - Dx_{j-1} \tag{14}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{15}$$

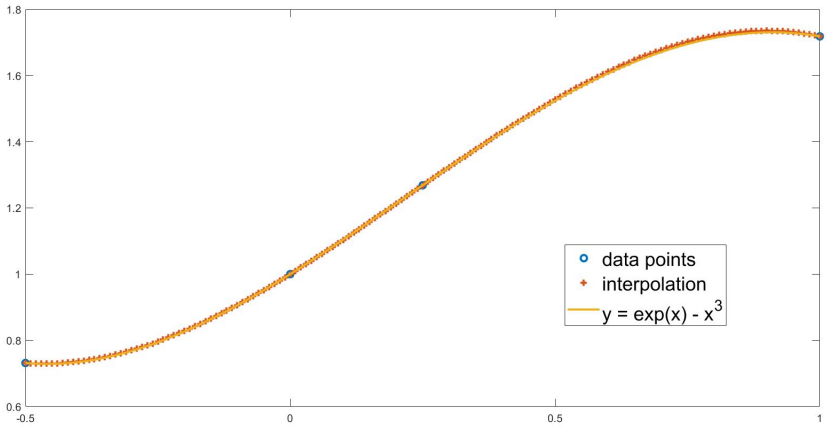
3.2 Graph



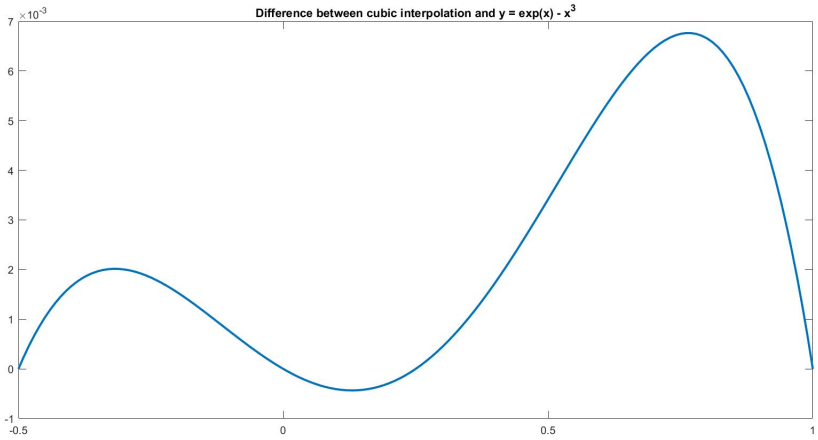
Plot of Piecewise Linear Interpolation and Cubic Spline Interpolation



Plot of Error between Cubic Spline and Piecewise Linear



Plot of Cubic Spline and $y = e^x - x^3$



Plot of Error between Cubic Spline and $y = e^x - x^3$

4 Theory Exercise Questions

4.1 Question 1

x	0	1	2
y	1	1	5

4.1.1 Equation

$$S"(x) = M_j \tag{16}$$

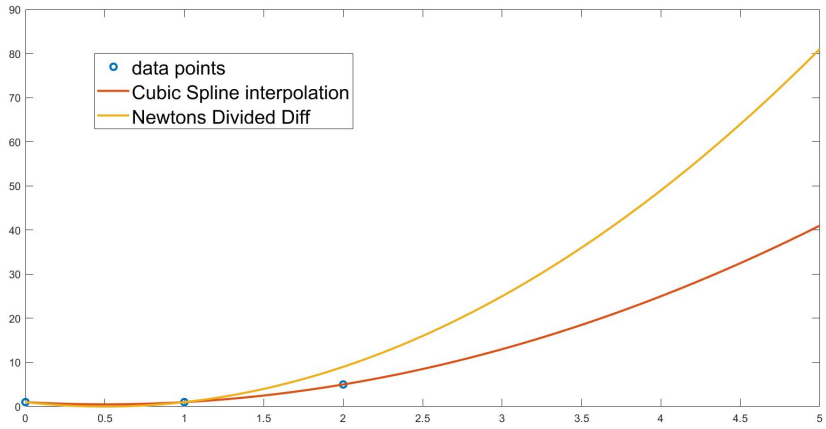
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{17}$$

$$A = D - C \tag{18}$$

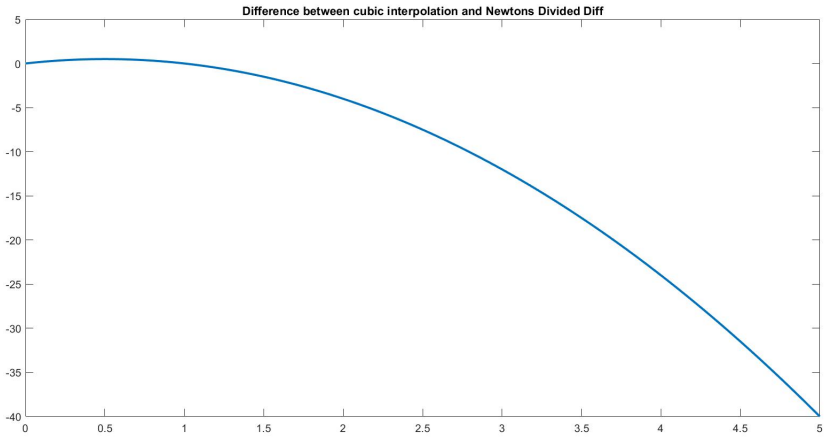
$$B = Cx_j - Dx_{j-1} \tag{19}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{20}$$

4.1.2 Graph



Plot of Newton Divided Difference and Cubic Spline Interpolation



Plot of Error between Cubic Spline and Newton Divided Difference

4.2 Question 2

x	1	2	3	4	5
y	3	1	2	3	2

4.2.1 Equation

$$S"(x) = M_j \tag{21}$$

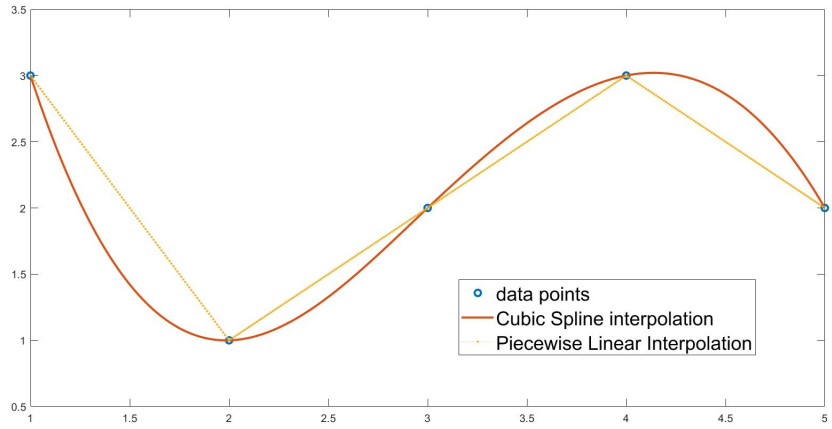
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{22}$$

$$A = D - C \tag{23}$$

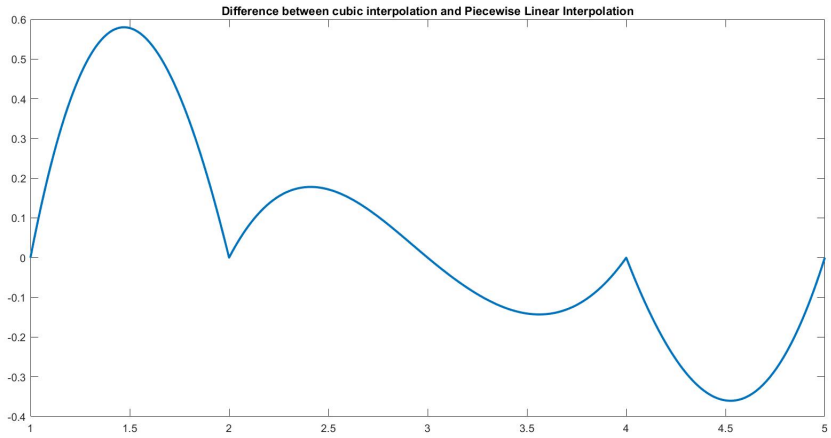
$$B = Cx_j - Dx_{j-1} \tag{24}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{25}$$

4.2.2 Graph



Plot of Piecewise Linear Interpolation and Cubic Spline Interpolation



Plot of Error between Cubic Spline and Piecewise Linear

4.3 Question 3

x	0	0.5	1	2	3
y	0	0.25	1	-1	-1

4.3.1 Equation

$$S"(x) = M_j \tag{26}$$

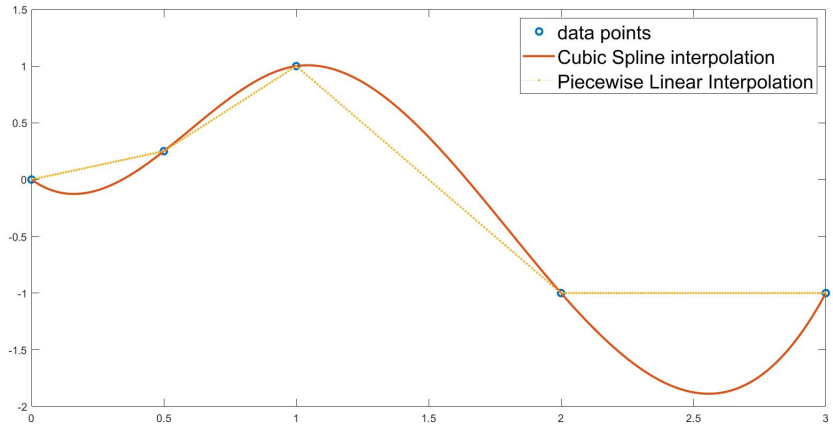
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{27}$$

$$A = D - C \tag{28}$$

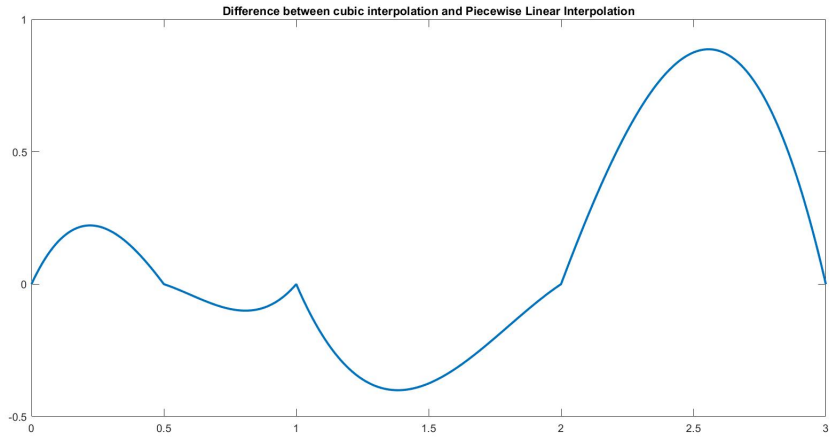
$$B = Cx_j - Dx_{j-1} \tag{29}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{30}$$

4.3.2 Graph



Plot of Piecewise Linear Interpolation and Cubic Spline Interpolation



Plot of Error between Cubic Spline and Piecewise Linear

4.4 Question 4

x	0	1	2	2.5	3	4
y	1.4	0.6	1	0.65	0.6	1

4.4.1 Equation

$$S"(x) = M_j \tag{31}$$

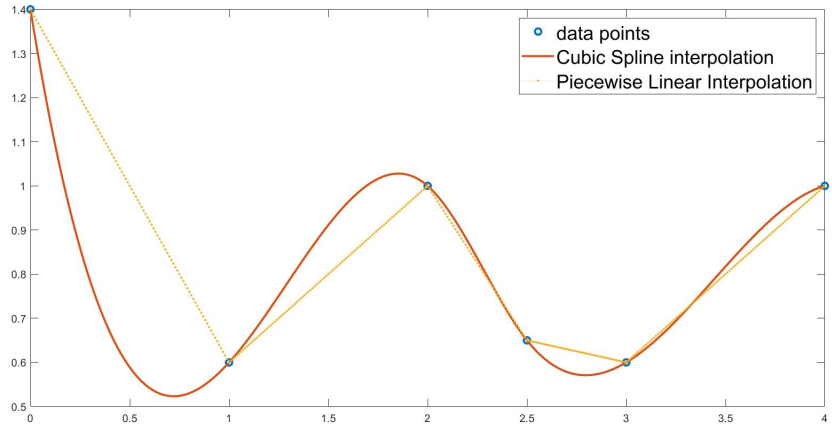
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{32}$$

$$A = D - C \tag{33}$$

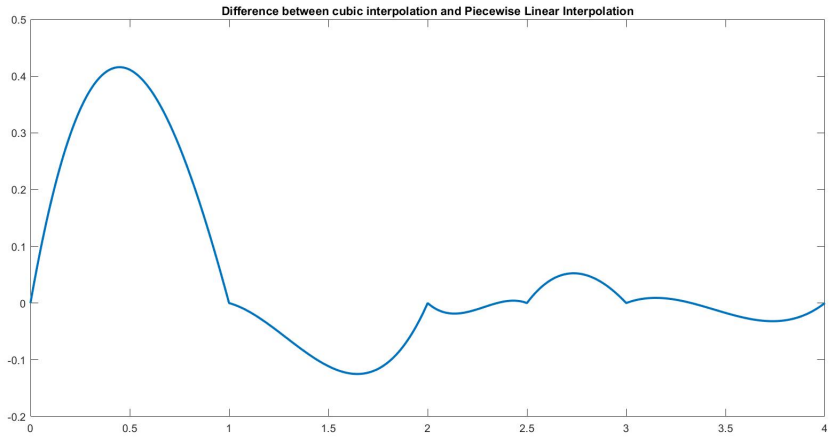
$$B = Cx_j - Dx_{j-1} \tag{34}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{35}$$

4.4.2 Graph



Plot of Piecewise Linear Interpolation and Cubic Spline Interpolation



Plot of Error between Cubic Spline and Piecewise Linear