Assignment 2
CS374

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Assigned by :

Prof. Arnab Kumar

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Contents

1 Part A

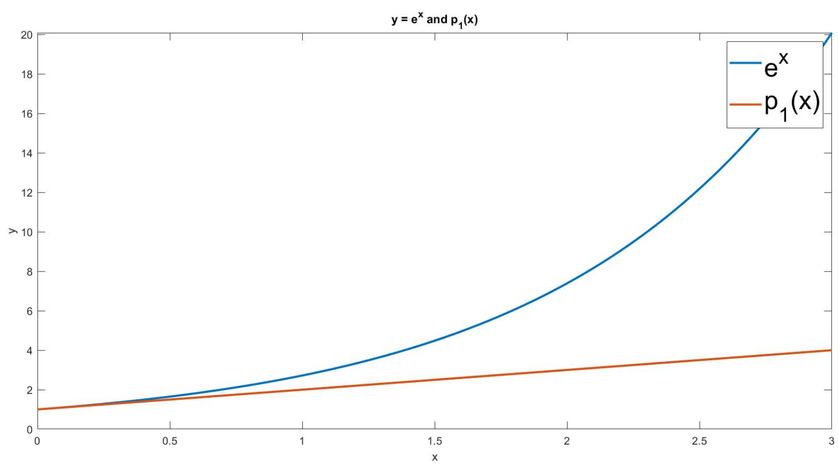
1.1 Equation

$$y = e^x \tag{1}$$

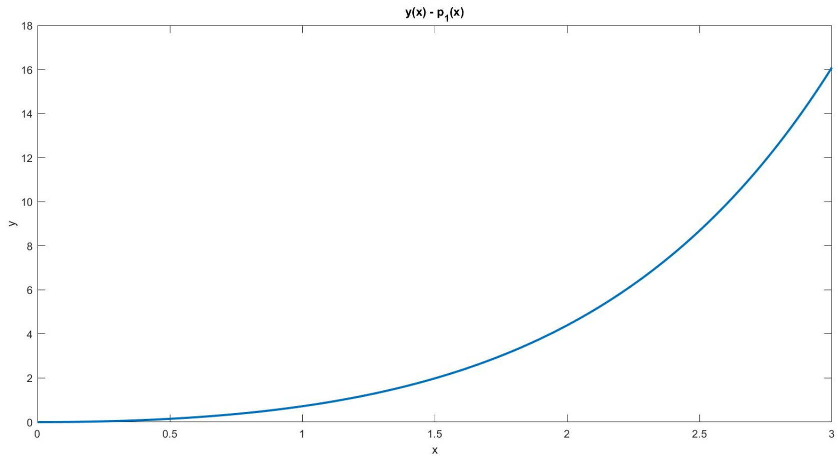
$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... \tag{2}$$

1.2 Graphs

1.2.1 First Order Polynomial

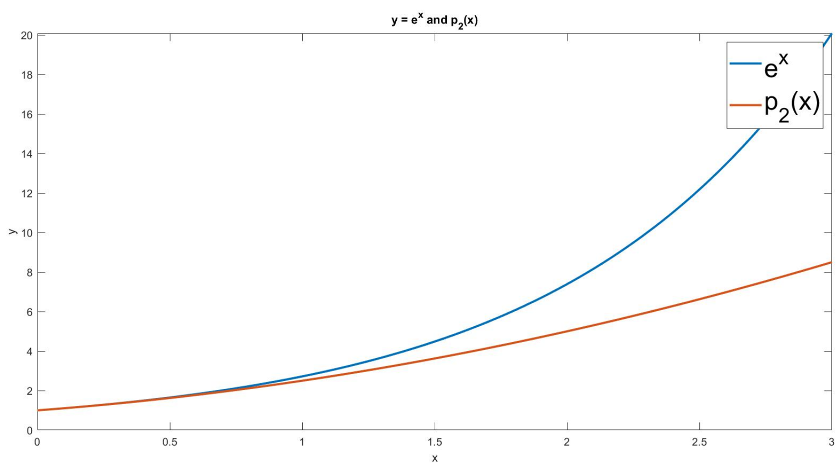


$y = e^x$ and First order Taylor Polynomial

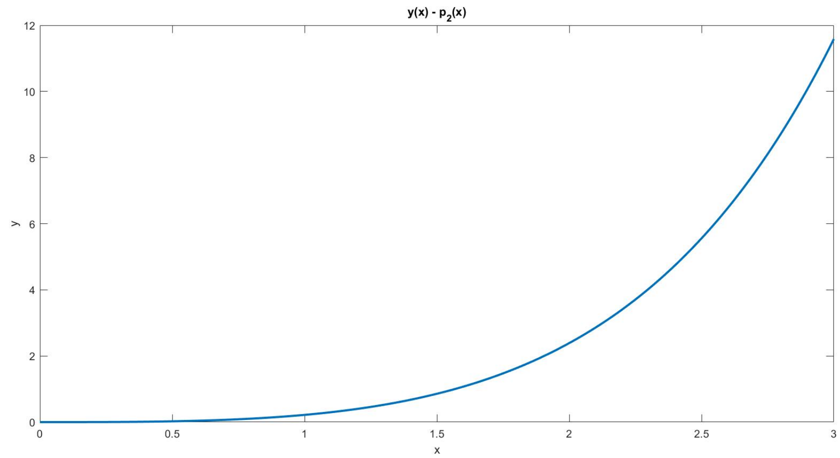


Error

1.2.2 Second Order Polynomial

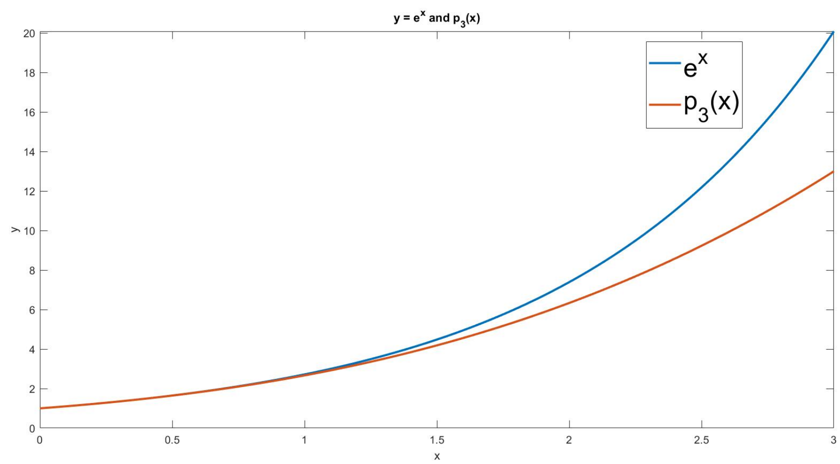


$y = e^x$ and Second order Taylor Polynomial

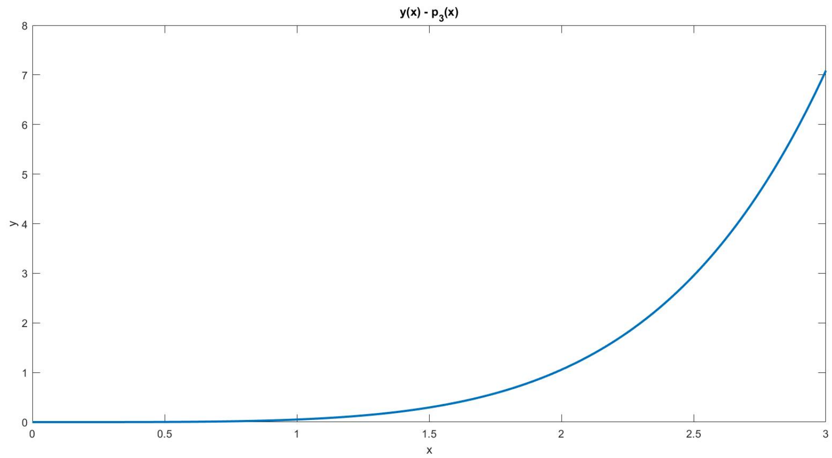


Error

1.2.3 Third Order Polynomial



$y = e^x$ and Third order Taylor Polynomial



Error

1.3 Observations

1. The first order Taylor series expansion of $y = e^x$ is equal to 1. Hence, a straight line y = 1 is observed. Therefore, the error function is $y = e^x$ shifted by a single unit towards negative Y axis as demonstrated in the graph.

2. The second order Taylor series expansion of $y = e^x$ is equal to $1 + x$. Hence, a straight line $y = x$ is observed. Therefore, for values in the range [0, 1], the error count decreases to almost zero whereas, increase in error can be observed for values greater than 1.

3. The third order Taylor series expansion of $y = e^x$ is $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$. Thus, the effect of quadratic term can be observed in the demonstrated results. The Taylor series expansion now starts to inhibit the properties of the function $y = e^x$ and so the error is decreased to zero for small values of x and for larger values of x it deviates.

2 Part B

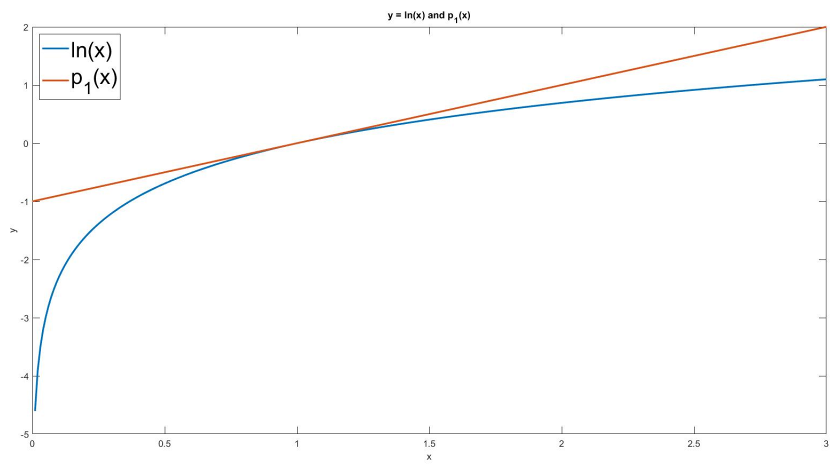
2.1 Equation

$$y = \ln x \tag{3}$$

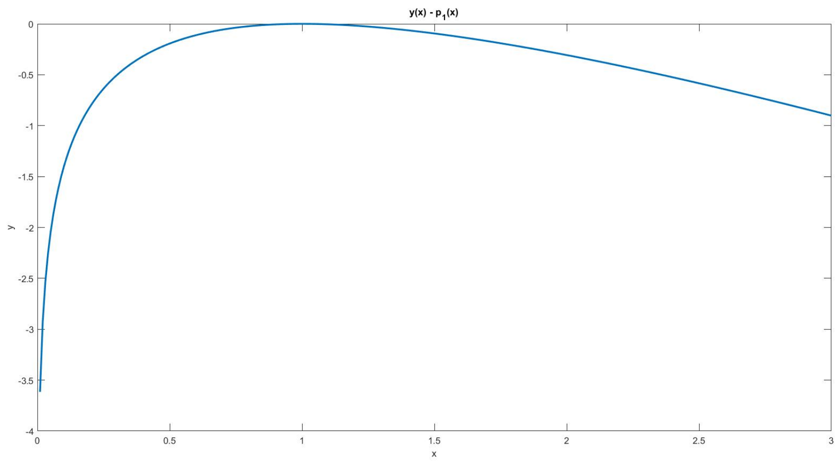
$$y = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - ... \tag{4}$$

2.2 Graphs

2.2.1 First Order Polynomial

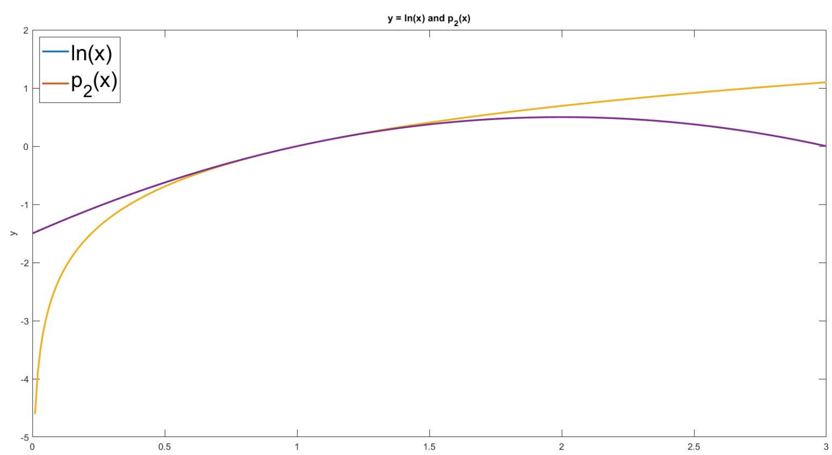


$y = \ln x$ and First order Taylor Polynomial

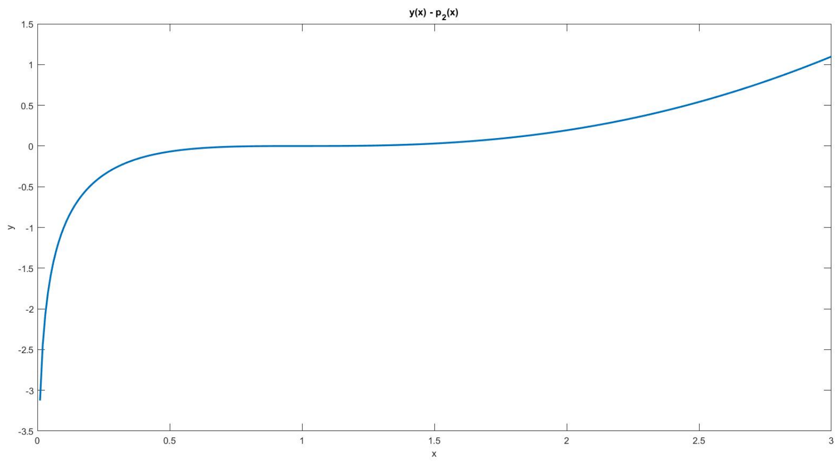


Error

2.2.2 Second Order Polynomial

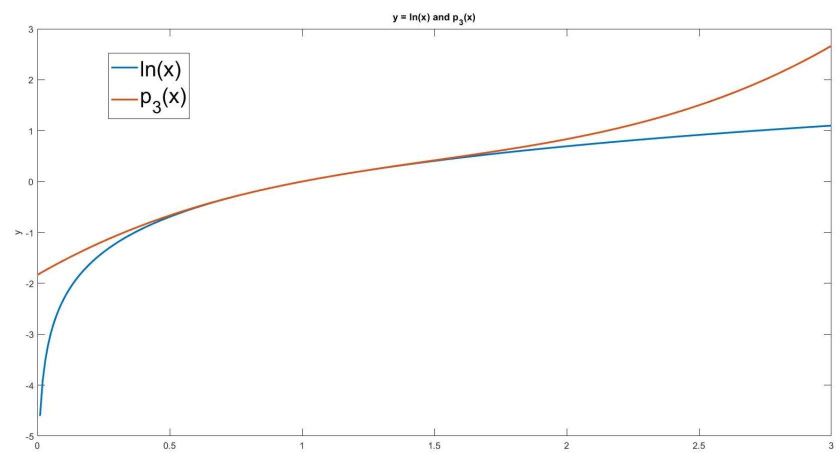


$y = \ln x$ and Second order Taylor Polynomial

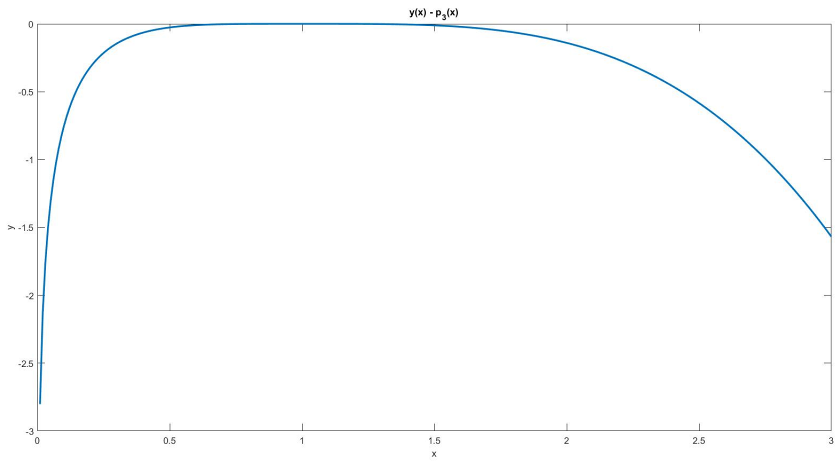


Error

2.2.3 Third Order Polynomial



$y = \ln x$ and Third order Taylor Polynomial



Error

2.3 Observations

1. The first order Taylor series expansion of $\ln x$ is equal to $x - 1$. Hence, a straight line $y = x - 1$ is observed. Therefore, for values near $x = 1$, the error count is very low whereas for values far from $x = 1$, increase in error can be observed.

2. The second order Taylor series expansion of $\ln x$ is equal to $y = (x - 1) - \frac{(x-1)^2}{2}$.For $x \epsilon [0, 1]$, $x - 1$ term dominates but for larger values of x i.e. $x > 1$ the negative quadratic term dominates and so the curve of $p_2 x$ shifts "downwards" and error increases continuously.

3. The third order Taylor series expansion of $\ln x$ is $y = (x - 1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$.For $x \epsilon [0, 1]$, $x - 1$ term dominates but for larger values of x now the positive cubic term dominates over negative quadratic term and so the curve of $p_2(x)$ shifts "upwards" and error increases.

3 Part C

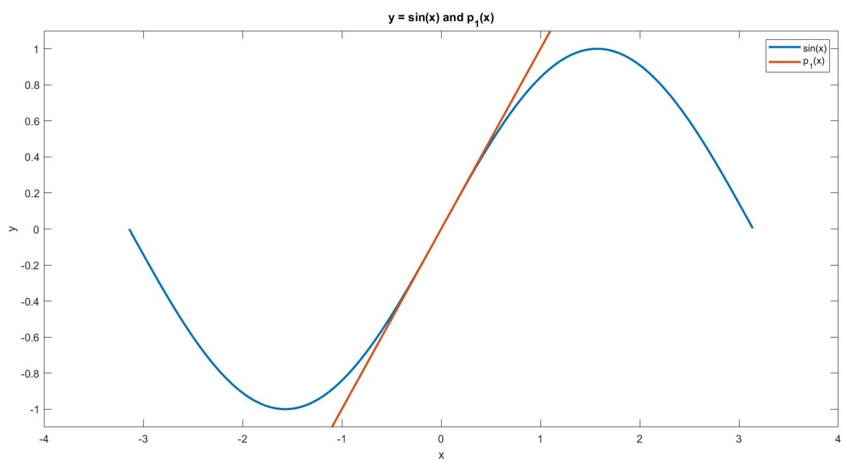
3.1 Equation

$$y = \sin x \tag{5}$$

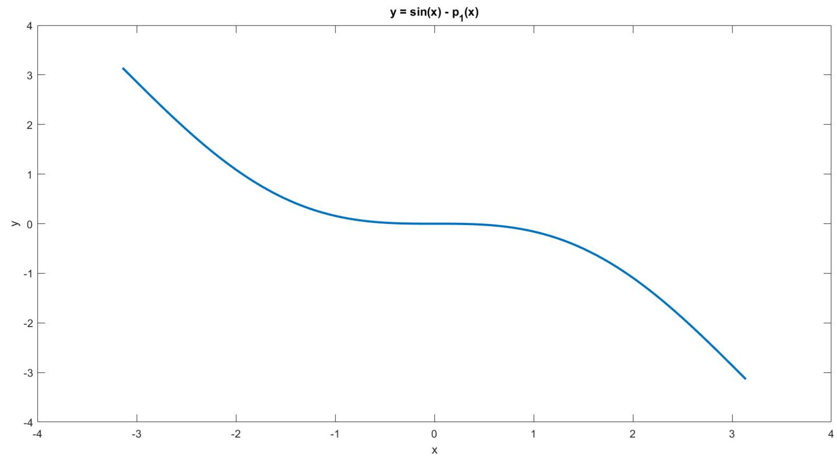
$$y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... \tag{6}$$

3.2 Graphs

3.2.1 First Order Polynomial

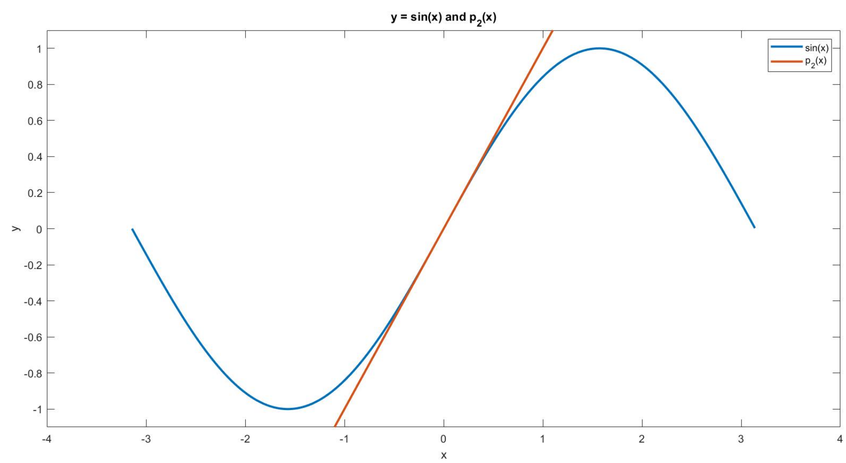


$y = \sin x$ and First order Taylor Polynomial

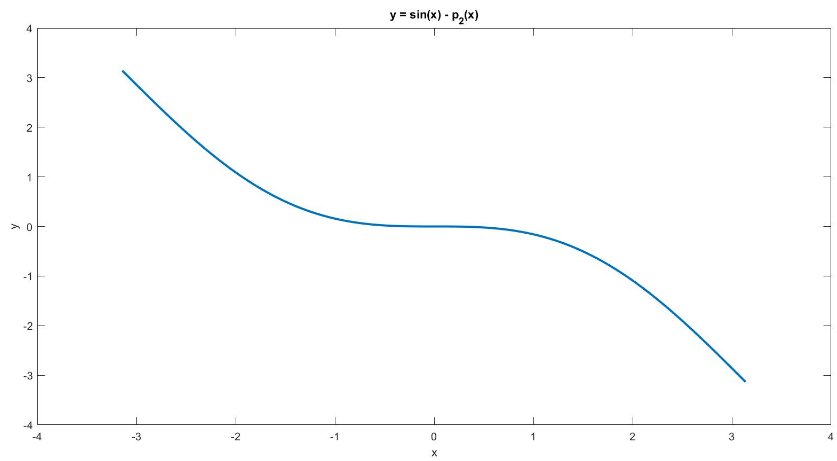


Error

3.2.2 Second Order Polynomial

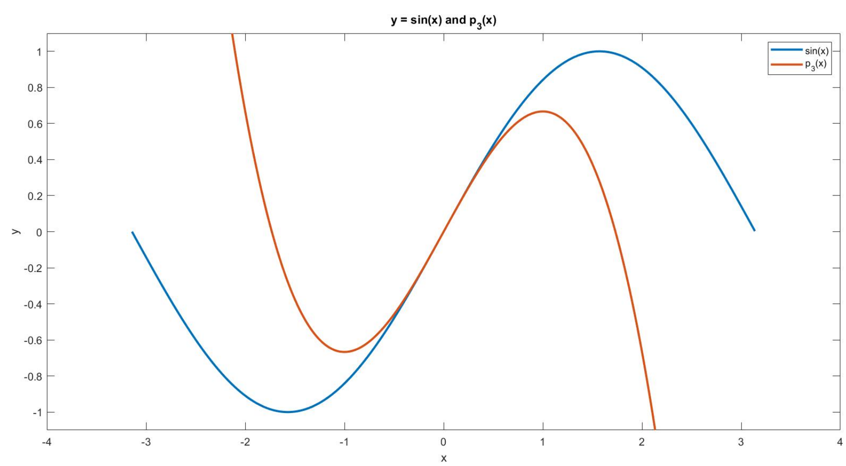


$y = \sin x$ and Second order Taylor Polynomial

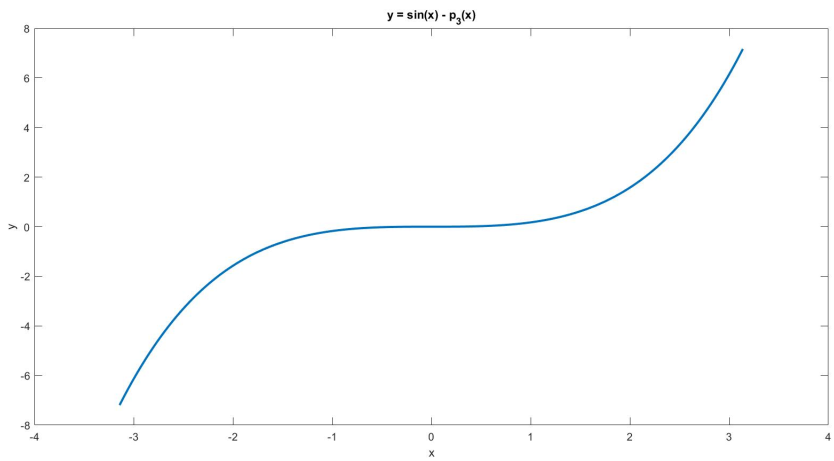


Error

3.2.3 Third Order Polynomial



$y = \sin x$ and Third order Taylor Polynomial



Error

3.3 Observations

1. The first order Taylor series expansion of $\sin x$ is equal to x. Hence, a straight line $y = x$ is observed. Therefore, for values near $x = 0$, the error count is very low whereas for values of $x > 1$,larger positive or negative values of error can be observed.

2. The second order Taylor series expansion of $\sin x$ is equal to x. Hence, same results as in the case of first order Taylor series can be observed.

3. The third order Taylor series expansion of $\sin x$ is equal to $x - \frac{x^3}{3!}$. Thus, the effect of cubic term being subtracted from x can be observed in the shown results. The Taylor series expansion now starts to inhibit the properties of $\sin x$ and so the error is decreased in the $x \epsilon [-\pi, \pi]$ drastically as compared to the first two cases but after that the error increases as the Taylor Polynomial deviates from $\sin x$.

4 Part D

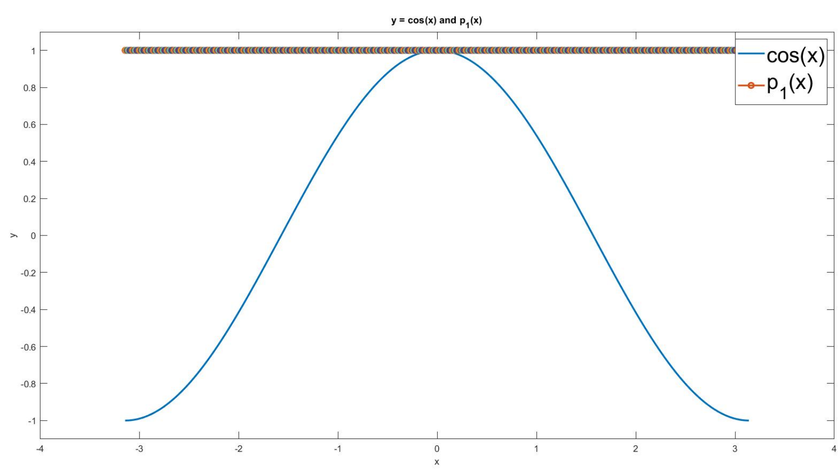
4.1 Equation

$$y = \cos x \tag{7}$$

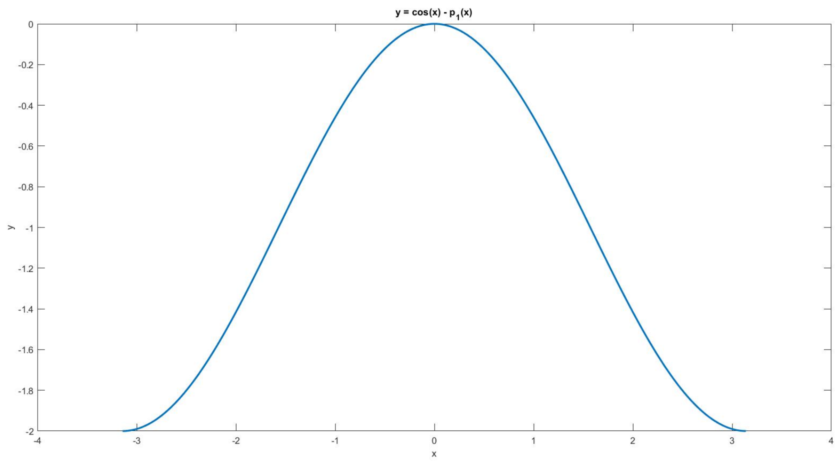
$$y = 1 - \frac{x^2}{2!} + \frac{x^3}{3!} - ... \tag{8}$$

4.2 Graphs

4.2.1 First Order Polynomial

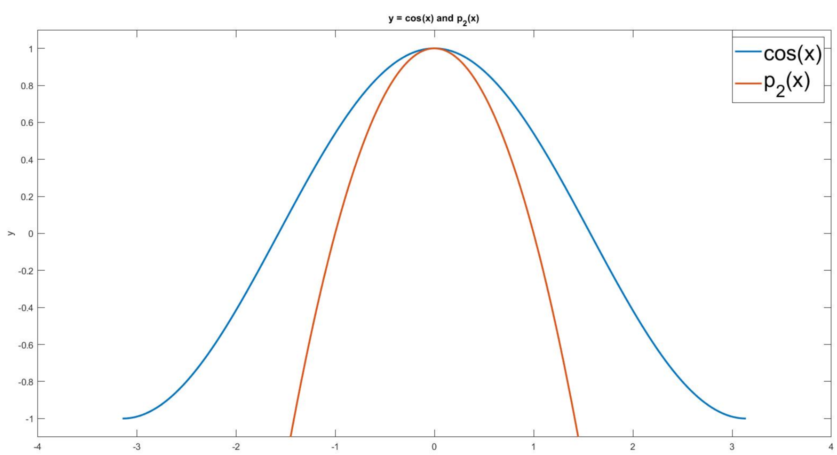


$y = \cos x$ and First order Taylor Polynomial

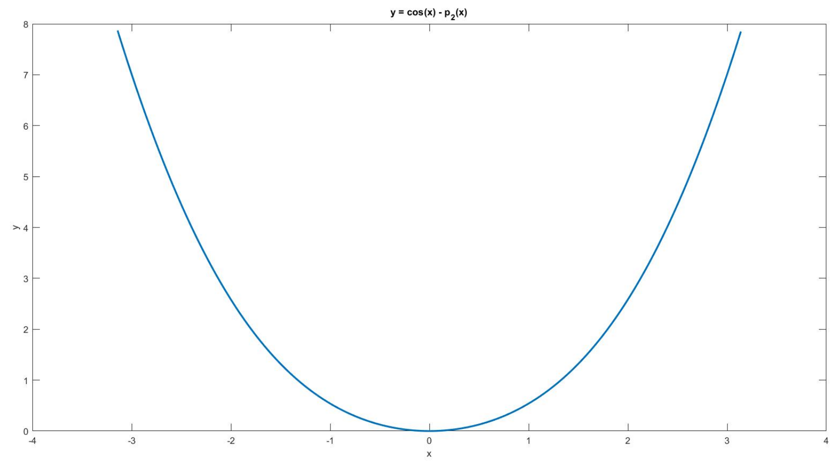


Error

4.2.2 Second Order Polynomial

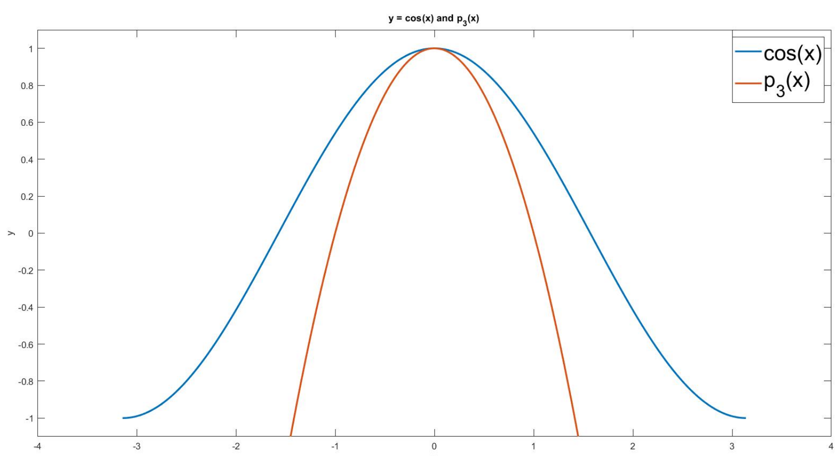


$y = \cos x$ and Second order Taylor Polynomial

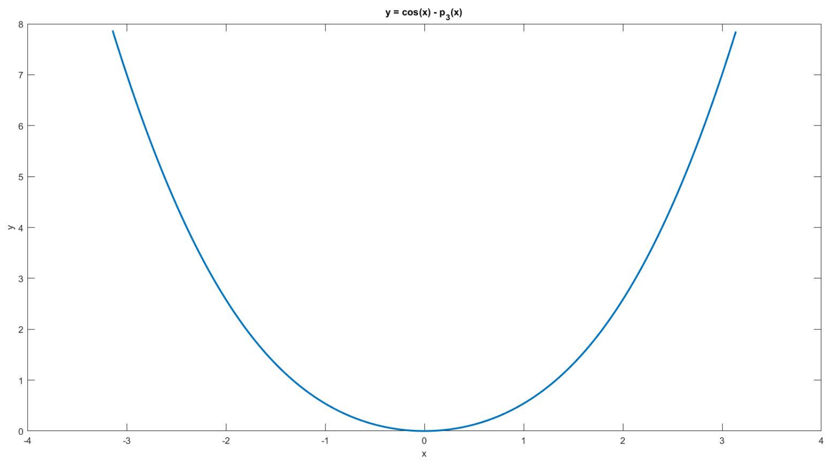


Error

4.2.3 Third Order Polynomial



$y = \cos x$ and Third order Taylor Polynomial



Error

4.3 Observations

1. The first order Taylor series expansion of $\cos x$ is equal to 1. Hence, a straight line $y = 1$ is observed. Therefore, the error function is the $\cos x$ curve shifted by a single unit towards negative Y axis as demonstrated in the graph.

2. The second order Taylor series expansion of $\cos x$ is equal to $1 + \frac{x^2}{2!}$. Thus, the effect of quadratic term can be observed in the demonstrated results. The Taylor series expansion now starts to inhibit the properties of $\cos x$ for values of x near to $x = 0$ and so the error is decreased near the values of x close to zero.

3. The third order Taylor series expansion of $\cos x$ is also equal to $1 + \frac{x^2}{2!}$. Hence, same results as in the case of second order Taylor series can be observed.