Assignment 1
CS374

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Assigned by :

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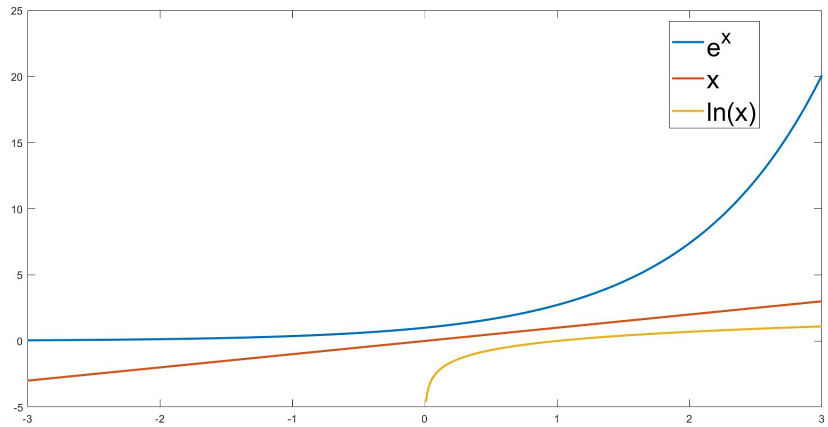
1 Question 1

1.1 Part A :

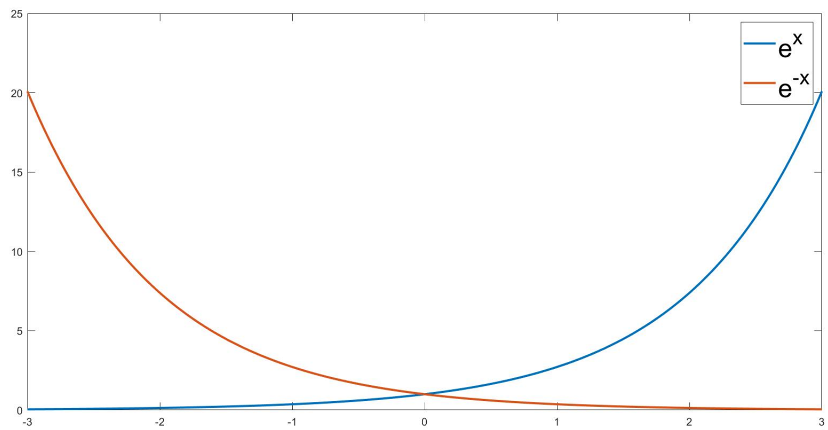


$$y = e^x, \quad y = x \quad and \quad y = \ln x$$

1.1.1 Observations

1. As shown in the figure, when x is positive, $e^x > x > \ln x$.
2. After x = 1, e^x increases rapidly than other two functions and goes to infinity.
3. In $0 < x < 1$ $\ln x$ is negative and has very steep curve which approaches to negative infinity at $x = 0$.
4. Also, $y = x$ is negative for negative values of 'x' while e^x is always positive.
5. The graphs of $y = e^x$ and $y = \ln x$ are mirror images of each other with $y = x$ line being the mirror.

1.2 Part B :



$y = e^x$ *and* $y = e^{-x}$

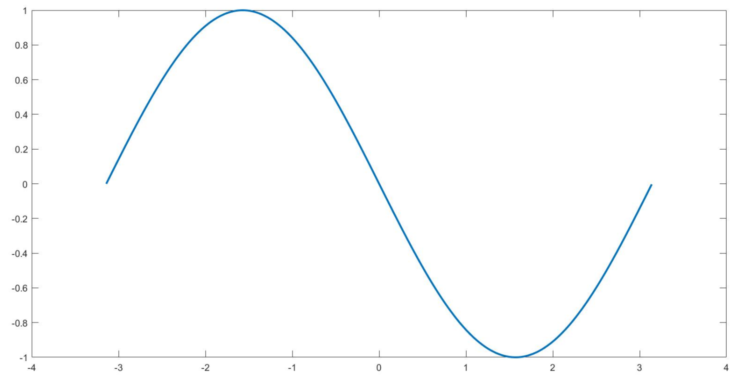
1.2.1 Observations

1. As shown in the figure, the graphs of $y = e^x$ and $y = e^{-x}$ are mirror images of each other with $x = 0$ line being the mirror.

2. These curves intersect only at $x = 0$ both giving $y = 1$.

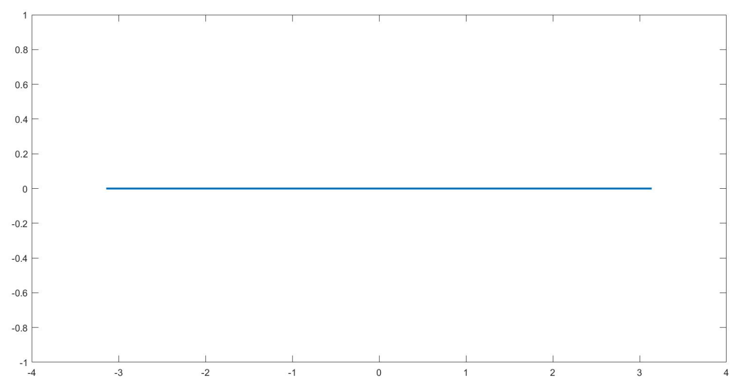
2 Question 2

2.1 Part A :



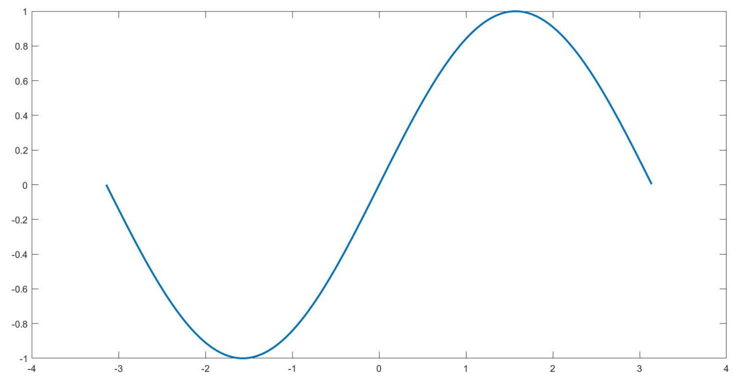
$$y = \sin k * x$$

For k = -1



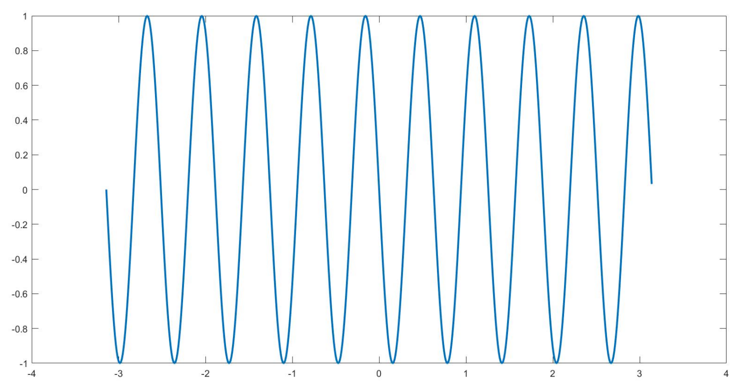
$$y = \sin k * x$$

For k = 0



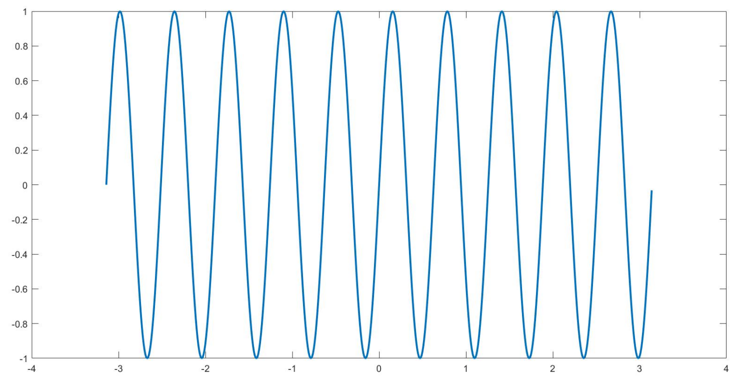
$$y = \sin k * x$$

For k = 1



$$y = \sin k * x$$

For k = -10



$$y = \sin k * x \tag{1}$$

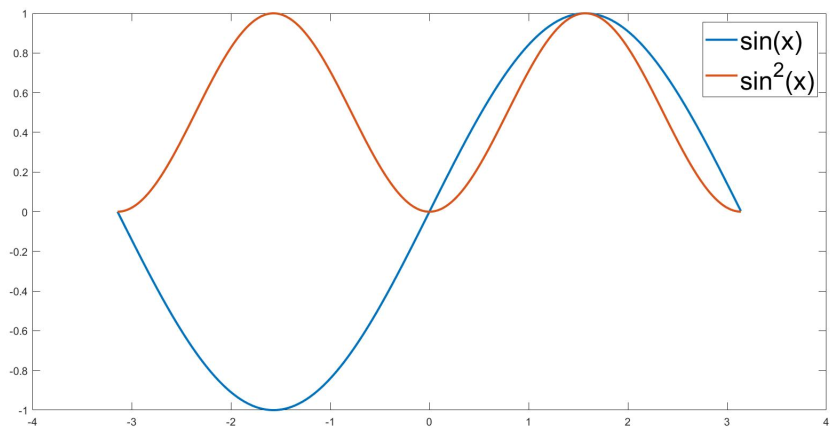
For k = 10

2.1.1 Observations

1. The value of 'k' determines the frequency i.e. the number of cycles completed by the function in a given range of 'x'. For large values of 'k' the graph has high frequency and so completes more cycles while for small values of 'k' graph completes less cycles in given range.

2. Negative values of 'k' just rotates the graph around the X - axis by 180 degree and it's effect on number of cycles remain the same.

2.2 Part B :



$$y = \sin x \quad and \quad y = \sin^2 x \tag{2}$$

2.2.1 Observations

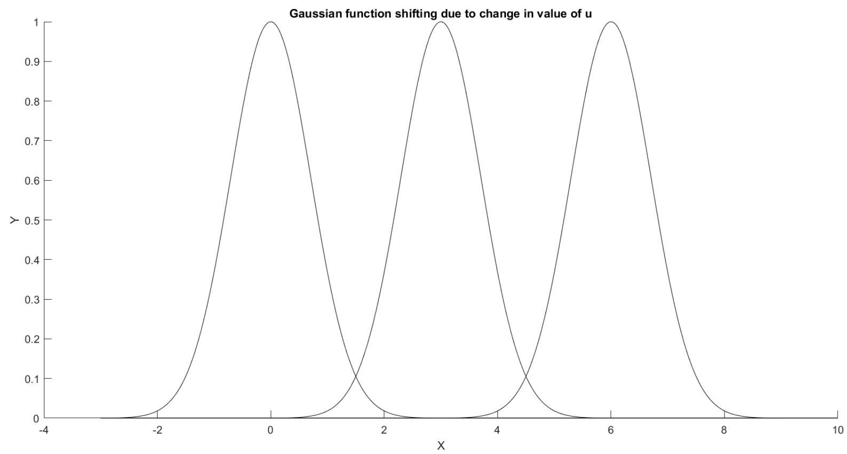
1. As shown in the graph, $\sin x$ will be negative in $-\pi < x < \pi$ while $\sin^2 x$ will always be positive as it has even power.

2. The range of $\sin x$ will be $-1 <= y <= 1$ while the range of $\sin^2 x$ will be $0 <= y <= 1$.

3. Both $\sin x$ and $\sin^2 x$ will have same values i.e they will intersect at at $x = 0, \quad x = \pi \quad$ and $\quad x = -\pi$.

3 Question 3

3.1 Part A :



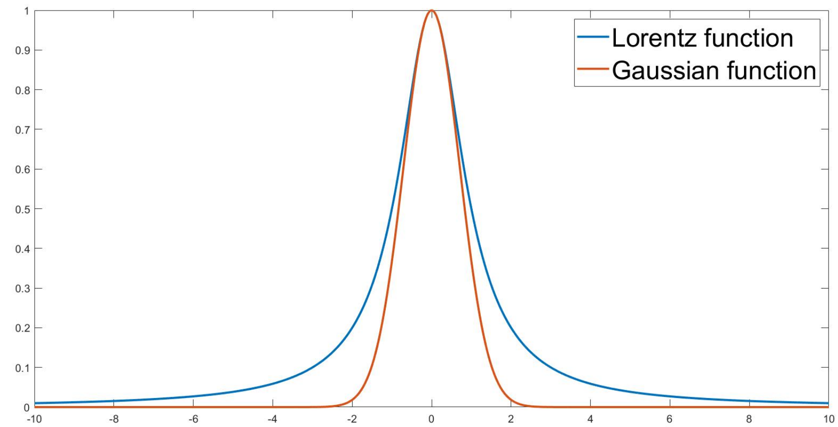
$$y = y_0 * e^{-a*(x-u)^2}$$

3.1.1 Observations

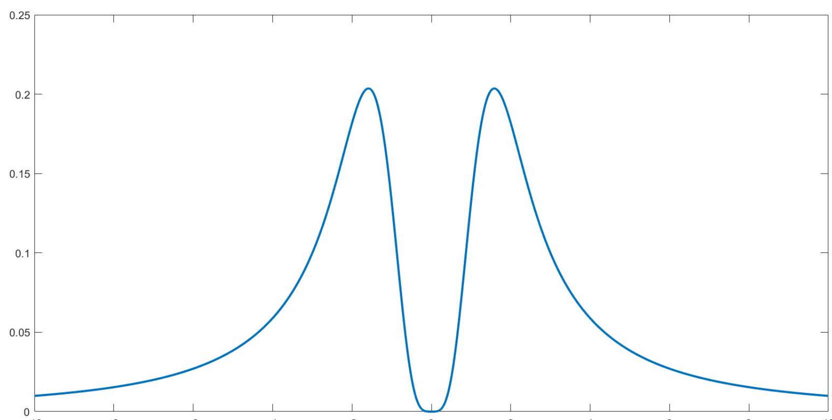
1. As shown in the graph, u determines the shift in the center of the Gaussian curve. So, on changing the value of u the center shifts according to the change. So Gaussian curves with different centers are shown.

2. In the graph shown u varies as $u = v * t$ for a constant value v and t changing from '0' to '4'.So, the center of the Gaussian curve shifts in the positive direction of X axis.

3.2 Part B :



$$y = y_0 * e^{-x^2} \quad and \quad y = \frac{1}{1+x^2}$$



$$y = \frac{1}{1+x^2} - y_0 * e^{-x^2}$$

3.2.1 Observations

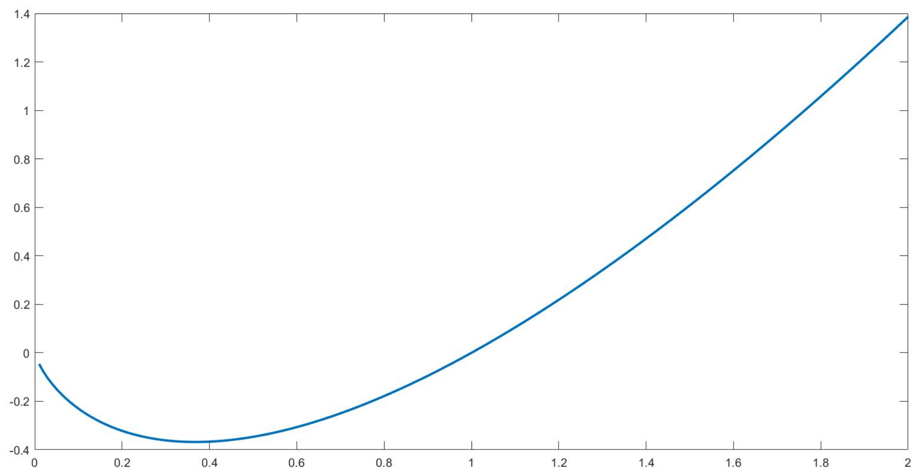
1. The Lorentz function's curve matches to the Gaussian curve at $x = 0$ and values in the near neighbourhood of $x = 0$ while at other values on both the sides it is greater than Gaussian function as shown in figure.

2. Again at infinity they match again as they both tend to $y = 0$.

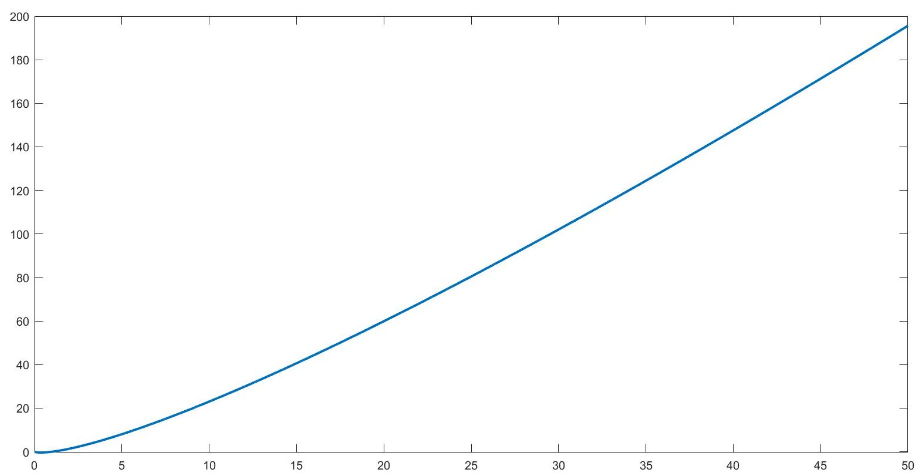
3. So, the difference curve starts from bottom then increases attains Maximum and starts decreasing and then becomes minimum at x=0.Again it starts to increase then becomes maximum and then again decreases for farther values of x.

4. Both these maximum occurs at $x = -0.2036$ and $x = 0.2036$.

4 Question 4



$y = x * \ln x$ for small values of x



$y = x * \ln x$ for large values of x

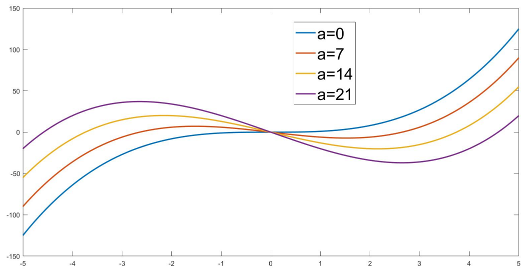
4.1 Observations

1. For large values of 'x' the graph of the function $y = x * \ln x$ will tend to $y = x$ because $x > \ln x$ will become more significant for very large 'x'.

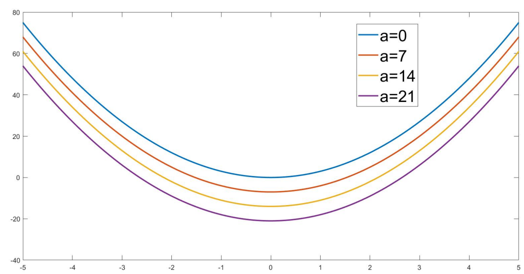
2. For small values of 'x',the starting point eill be $y = 0$ which is the value of function at $x = 0$ and there will be a turning point at $x = \frac{1}{e}$ and this will be a minima as the second derivative will be positive at $x = \frac{1}{e}$ and then the function will continuously increase.

5 Question 5

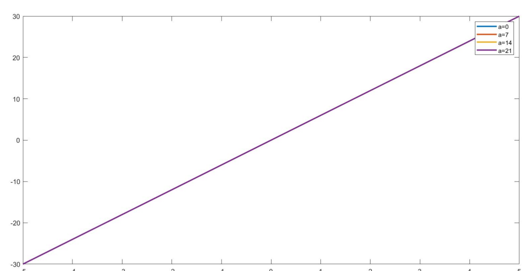
5.1 Part A.



$y(x)$



$y'(x)$



$y''(x)$

5.1.1 Observations

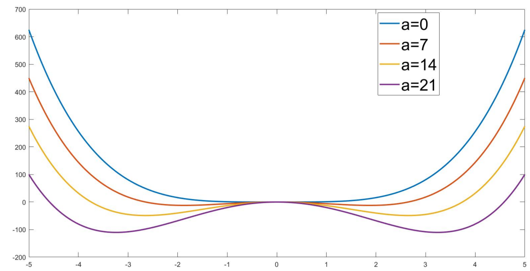
1. For $a = 0$, $x = 0$ is the inflection point i.e. it is the point where the curve changes the direction of its curvature. $y = x^3$, which is a cubic equation but there's only one root at $x = 0$ visible in the graph instead of three distinct roots. The other two real roots gets merged at the inflection point due to the property of the curve and thus only a single root is observed.

2. As shown in graph of $y(x)$, on increasing the value of a, the graph bend more towards negative values of y in the range $x > 0$. Conversely, in the range $x < 0$, the graph tends to move outwards towards positive values of y.

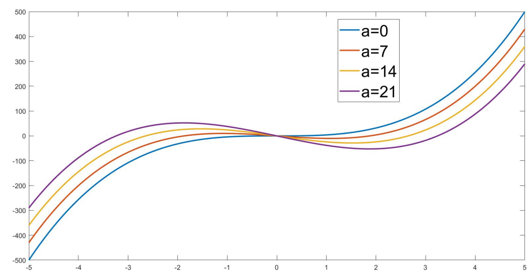
3. We can see that as we increase the value of a, the curve of the function drops along Y-axis for the graph of $y'(x)$.

4. The second order derivative for any value of 'a' will remain the same because the term $-a * x$ turns to 0 on double differentiation. So, we obtain a straight line.

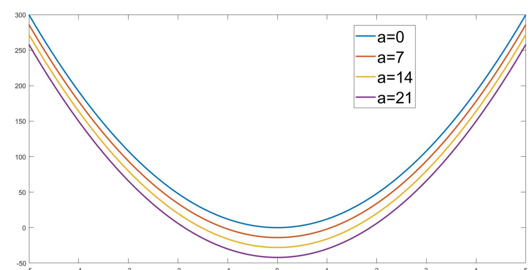
5.2 Part B.



$y(x)$



$y'(x)$



$y''(x)$

5.2.1 Observations

1. For a = 0, $y = x^4$ which is a quadratic equation and hence should have four roots. But from the graph only one root is visible at $x = 0$. In this case also all the roots gets merged at the inflection point($x = 0$) and hence only a single root is visible.

2. As the value of 'a' increases, the curve of the function $y(x)$ becomes less steeper, that is, it's magnitude becomes more negative or positive as x shifts away from 0.

3. In the graph of $y'(x)$, on increasing the value of a, the graph bend more towards negative values of y in the range $x > 0$. Conversely, in the range $x < 0$, the graph tends to move outwards towards positive values of y.

4. The second order derivative i.e $y''(x)$ shifts lower as the value of a is increased for the second equation because of the constant term -2a in the equation