Assignment 5
CS374

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Assigned by :

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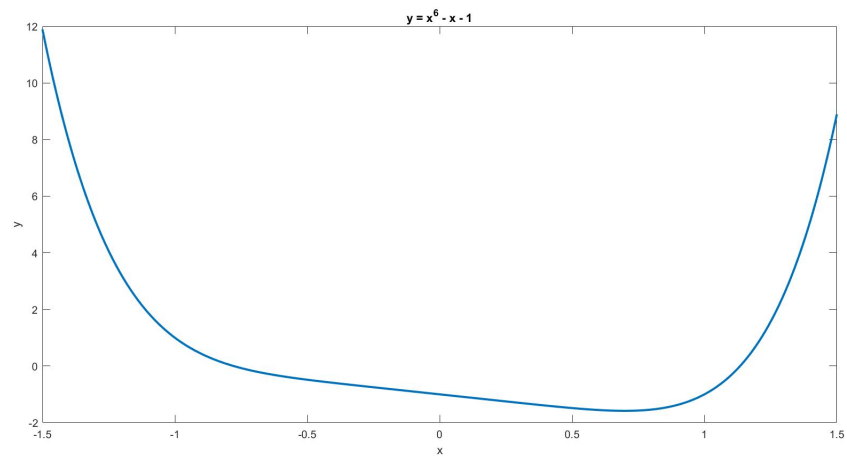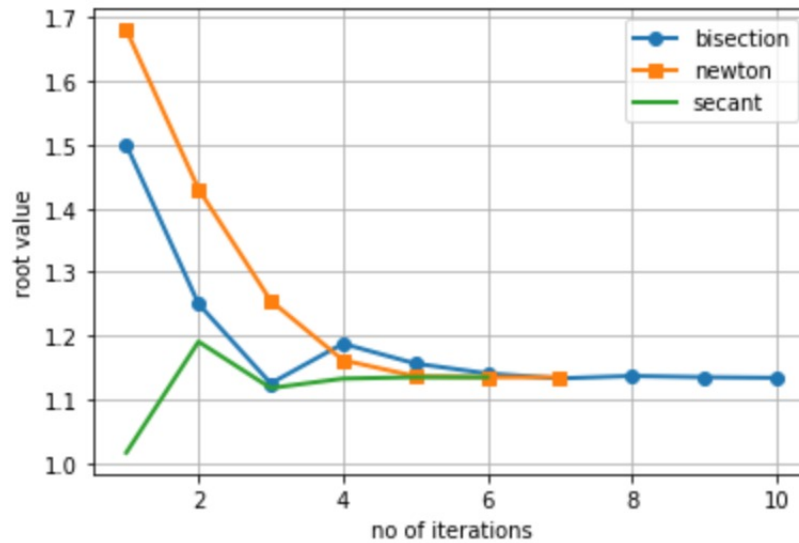
Contents

1 Part A

1.1 Equation

$$y = x^6 - x - 1 \tag{1}$$

1.2 Graph

- So, from the above graph we can observe that there are two real roots of these equations.
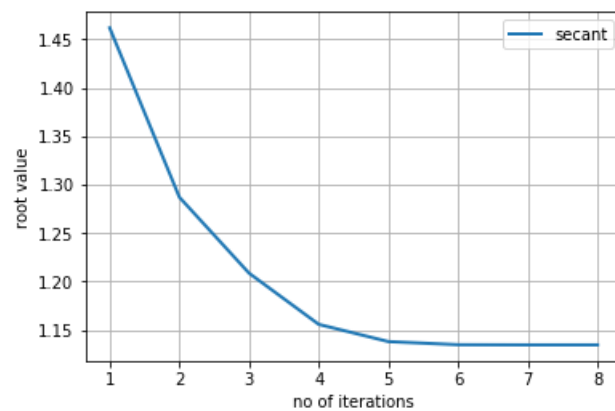
1.2.1 First Root

- Above shown plot is of convergence of first root vs iterations

- Value of first root is 1.13472.

- Bisection method - 10 iterations

- Newton Raphson method - 7 iterations

- Secant method - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	-1	-1	1.01613
1	1.01613	-0.915368	0.016129	1.19058
2	1.19058	0.657466	0.174449	1.11766
3	1.11766	-0.168491	-0.0729219	1.13253
4	1.13253	-0.0224373	0.0148757	1.13482
5	1.13482	0.000953564	0.00228526	1.13472
6	1.13472	-5.06617e-06	-9.31621e-05	1.13472

For a and b on the same side - 7 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1.5	8.89062	-0.5	1.41469
1	1.41469	5.60158	-0.0853073	1.26941
2	1.26941	1.91469	-0.145287	1.19395
3	1.19395	0.702893	-0.0754514	1.15019
4	1.15019	0.165156	-0.0437648	1.13675
5	1.13675	0.0209194	-0.0134415	1.1348
6	1.1348	0.000761739	-0.0019495	1.13472
7	1.13472	3.71574e-06	-7.36698e-05	1.13472

1.2.2   Second Root

- Above shown plot is of convergence of second root vs iterations.

- Value of second root is -0.7781.

- Bisection method - 10 iterations

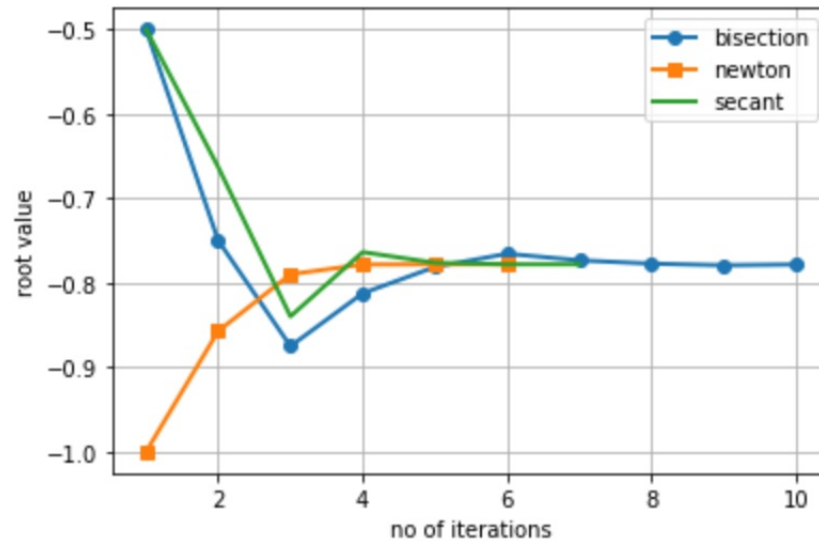- Newton Raphson method - 6 iterations

- Secant method - 7 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	-1	1	-1	-0.5
1	-0.5	-0.484375	0.5	-0.663158
2	-0.663158	-0.251787	-0.163158	-0.839783
3	-0.839783	0.190538	-0.176625	-0.763699
4	-0.763699	-0.0379041	0.076084	-0.776323
5	-0.776323	-0.00477122	-0.0126242	-0.778141
6	-0.778141	0.000140421	-0.00181792	-0.778089
7	-0.778089	-5.03141e-07	5.19732e-05	-0.778089

For a and b on the same side - 10 iterations

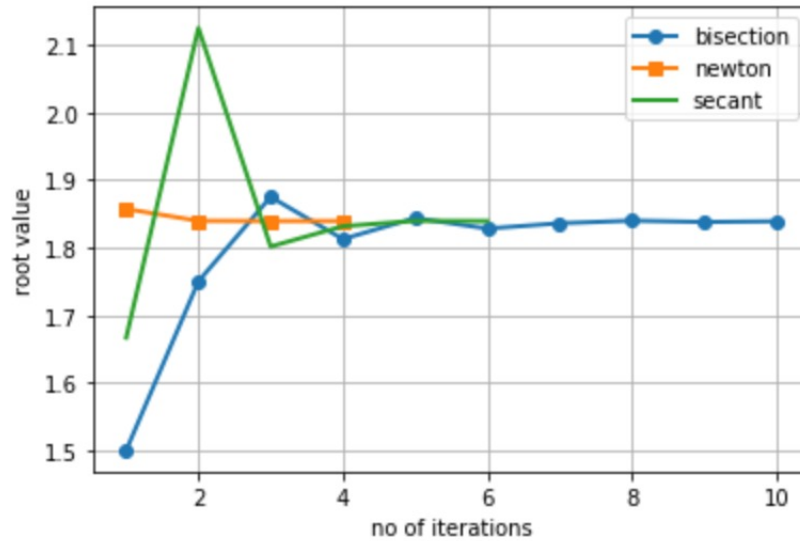|	xn	f(xn)	xn - xn-1	xn+1
0	-2	65	-0.5	-1.38806
1	-1.38806	7.54027	0.611945	-1.30775
2	-1.30775	5.30984	0.0803037	-1.11658
3	-1.11658	2.05449	0.191174	-0.995926
4	-0.995926	0.971728	0.120652	-0.887646
5	-0.887646	0.376793	0.10828	-0.819069
6	-0.819069	0.12101	0.0685772	-0.786625
7	-0.786625	0.023548	0.0324438	-0.778786
8	-0.778786	0.00189174	0.00783878	-0.778102
9	-0.778102	3.26015e-05	0.000684741	-0.77809
10	-0.77809	4.60539e-08	1.20075e-05	-0.77809
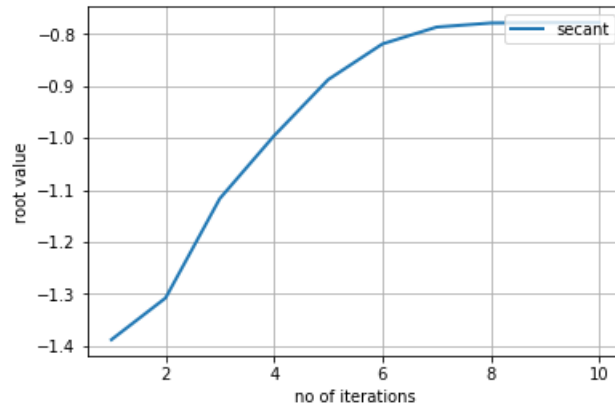
2 Part B

2.1 Question 1

2.1.1 (A)

Equation:

$$y = x^3 - x^2 - x - 1 \tag{2}$$

Graph:
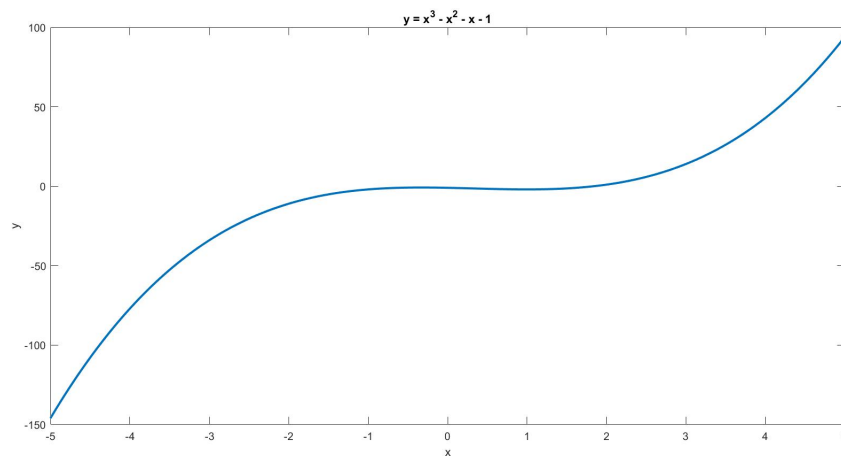
- So, from the above graph we can observe that there is one real root of these equation.
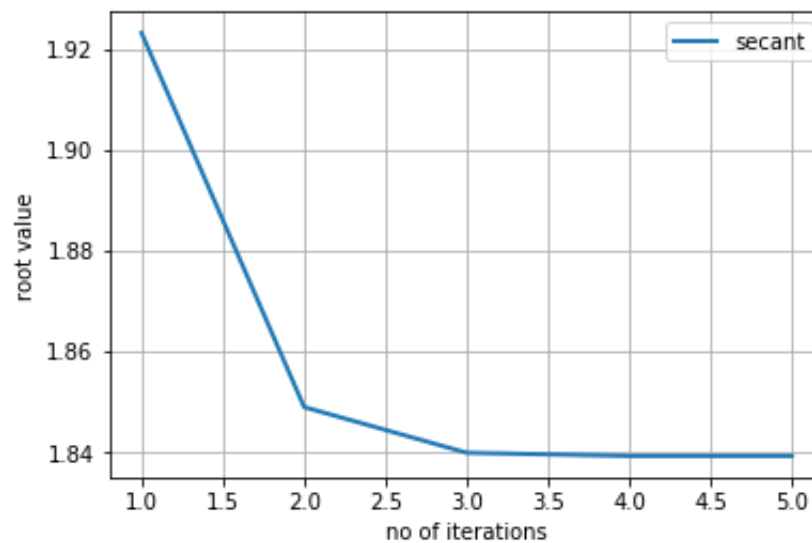
- Above shown plot is of convergence of first root vs iterations.

- Value of first root is 1.8393.

- Bisection method - 10 iterations

- Newton Raphson method - 4 iterations

- Secant method - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	-2	-1	1.66667
1	1.66667	-0.814815	0.666667	2.125
2	2.125	1.95508	0.458333	1.80149
3	1.80149	-0.200342	-0.323506	1.83156
4	1.83156	-0.0419823	0.0300692	1.83953
5	1.83953	0.00135629	0.00797159	1.83929
6	1.83929	-8.68907e-06	-0.000249473	1.83929

For a and b on the same side - 5 iterations

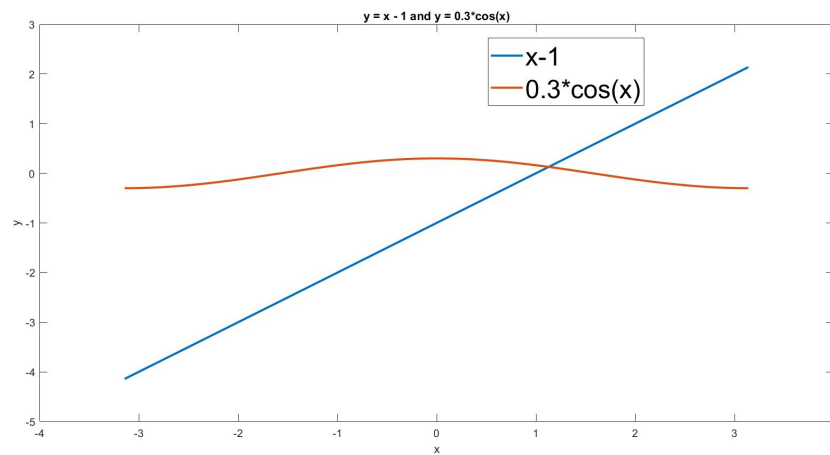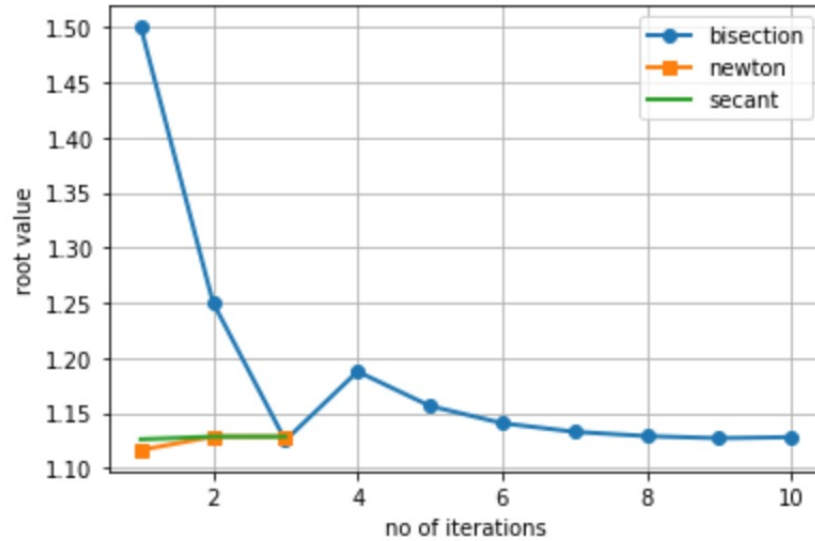|	xn	f(xn)	xn - xn-1	xn+1
0	2	1	-1	1.92308
1	1.92308	0.490669	-0.0769231	1.84897
2	1.84897	0.0534081	-0.0741046	1.83992
3	1.83992	0.00347129	-0.00905131	1.83929
4	1.83929	2.75806e-05	-0.00062919	1.83929
5	1.83929	1.44411e-08	-5.03917e-06	1.83929

2.1.2 (B)

Equation:

$$x = 1 + (0.3)\cos x \tag{3}$$

Graph:

- So, from the above graph we can observe that there is one real root of these equation between 1 and 2 as both curves intersect between these two points.
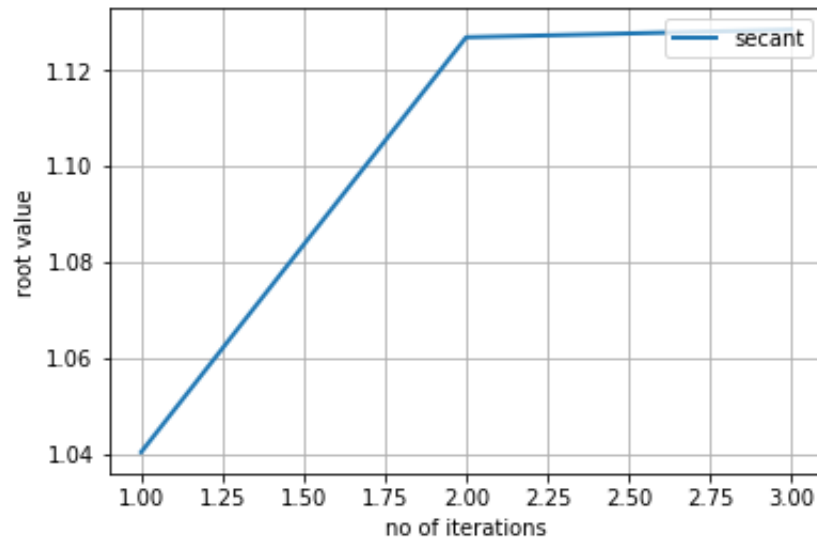
- Above shown plot is of convergence of first root vs iterations.

- Value of first root is 1.1284.

- Bisection method - 10 iterations

- Newton Raphson method - 3 iterations

- Secant method - 3 iterations.

	xn	f(xn)	xn - xn-1	xn+1
0	1	-0.162091	-1	1.12595
1	1.12595	-0.00314451	0.125951	1.12844
2	1.12844	2.24135e-05	0.00249175	1.12843
3	1.12843	-2.8065e-09	-1.7635e-05	1.12843

For a and b on the same side - 3 iterations

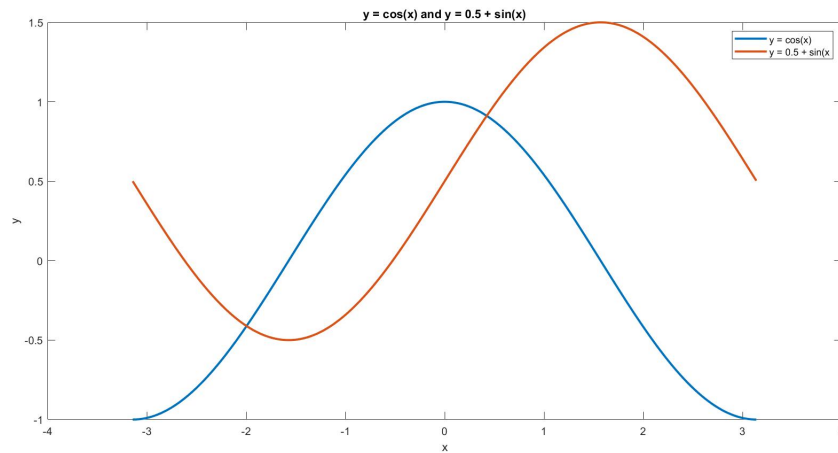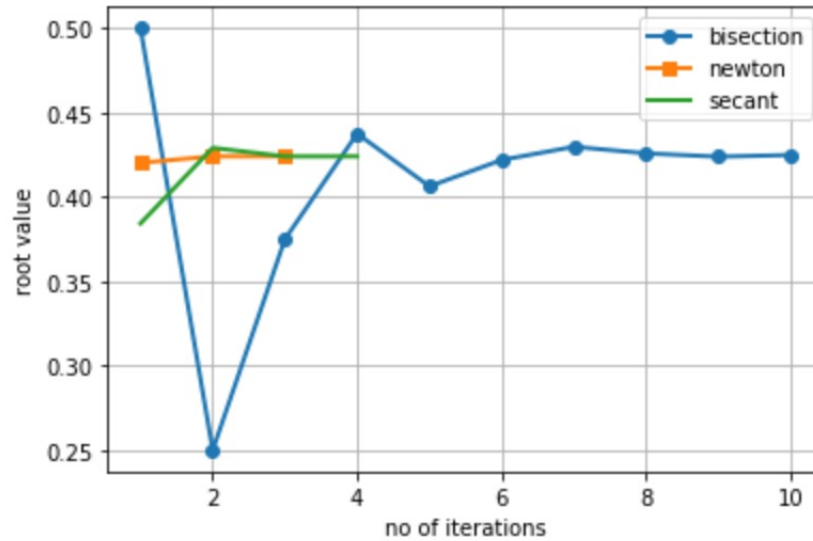|	xn	f(xn)	xn - xn-1	xn+1
0	2	1.12484	-1	1.04036
1	1.04036	-0.111411	-0.959639	1.12684
2	1.12684	-0.00200972	0.0864826	1.12843
3	1.12843	9.54511e-06	0.00158869	1.12843

2.1.3 (C)

Equation:

$$\cos x = 0.5 + \sin x \qquad (4)$$

Graph:

- So, from the above graph we can observe that there is one positive real root i.e the smallest positive root of these equation between 0 and 1 as both curves intersect between these two points.

Root Between : $a = 0$ and $b = 1$

- Above shown plot is of convergence of first root vs iterations.

- Value of first root is 0.4240.

- Bisection method - 10 iterations

- Newton Raphson method - 3 iterations

- Secant method - 4 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	0	-0.5	-1	0.38427
1	0.38427	-0.05219	0.38427	0.429055
2	0.429055	0.00665185	0.0447847	0.423992
3	0.423992	-5.18114e-05	-0.00506274	0.424031
4	0.424031	-4.89262e-08	3.91291e-05	0.424031

For a and b on the same side - 3 iterations
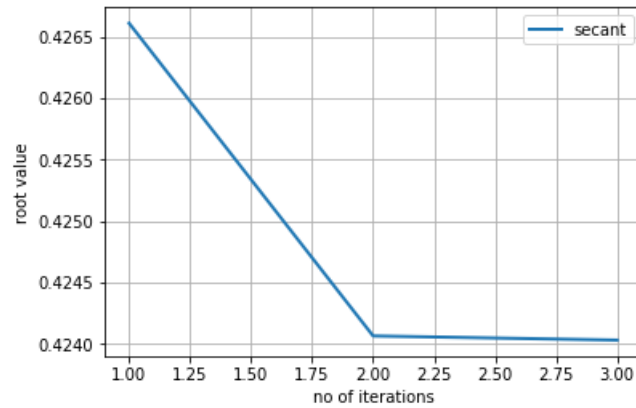
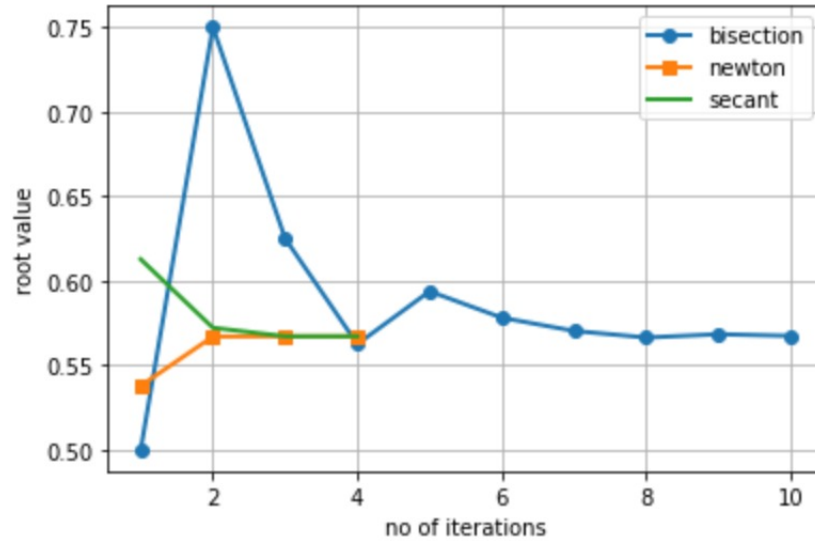|	xn	f(xn)	xn - xn-1	xn+1
0	0.5	-0.101843	-0.2	0.426608
1	0.426608	-0.003411	-0.0733917	0.424065
2	0.424065	-4.49118e-05	-0.00254327	0.424031
3	0.424031	-2.1813e-08	-3.39334e-05	0.424031

2.1.4 (D)

Equation:

$$x = e^{-x} \tag{5}$$

Graph:
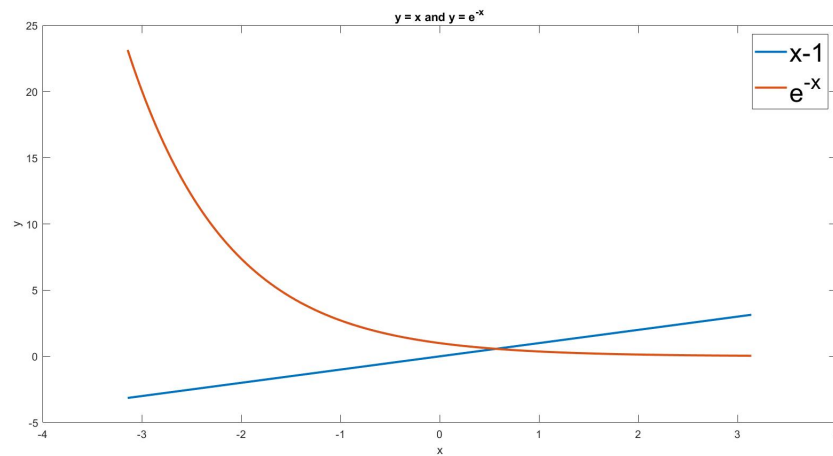
- So, from the above graph we can observe that there is one real root of these equation between 0 and 1 as both curves intersect between these two points.

- Above shown plot is of convergence of first root vs iterations.

- Value of the root is 0.5671.

- Bisection method - 10 iterations

- Newton Raphson method - 4 iterations

- Secant method - 4 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	0	-1	-1	0.6127
1	0.6127	0.0708139	0.6127	0.572181
2	0.572181	0.00788827	-0.0405184	0.567102
3	0.567102	-6.45828e-05	-0.00507933	0.567143
4	0.567143	5.88309e-08	4.12478e-05	0.567143

For a and b on the same side - 4 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	0.632121	-1	0.487142
1	0.487142	-0.127238	-0.512858	0.573076
2	0.573076	0.00928793	0.0859347	0.56723
3	0.56723	0.000136112	-0.00584616	0.567143
4	0.567143	-1.45994e-07	-8.69484e-05	0.567143

2.1.5   (E)

Equation:

$$e^{-x} = sin(x) \tag{6}$$

Graph:

- So, from the above graph we can observe that the two smallest real roots of these equation lie between 0 and 1 and another one between 3 and 4.
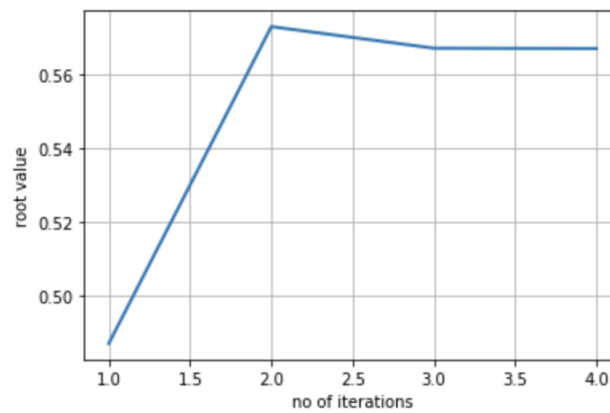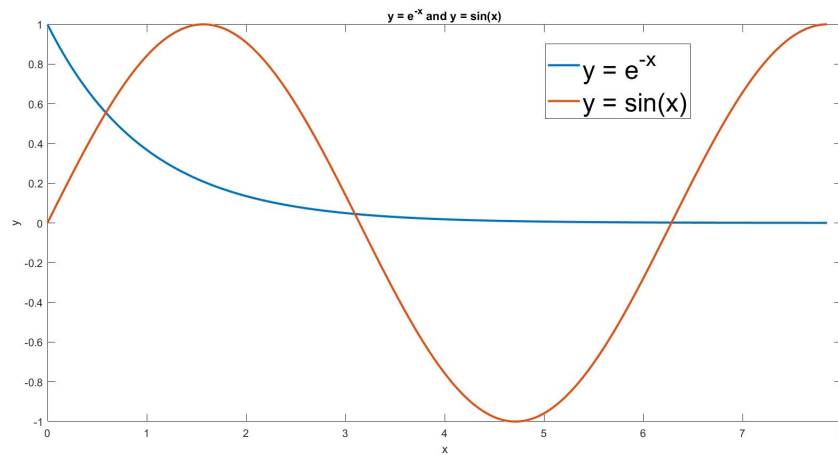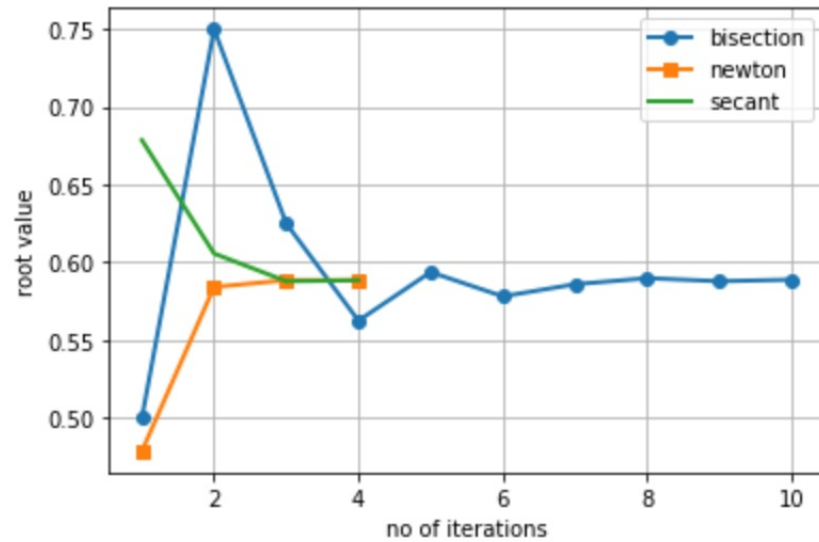
First Root(Between 0 and 1)

- Above shown plot is of convergence of first root vs iterations.

- Value of first root is 0.5885.

- Bisection method - 10 iterations

- Newton Raphson method - 4 iterations

- Secant method - 4 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	0	-1	-1	0.678614
1	0.678614	0.120395	0.678614	0.605692
2	0.605692	0.0236341	-0.0729224	0.58788
3	0.58788	-0.000905087	-0.0178114	0.588537
4	0.588537	6.26477e-06	0.000656945	0.588537

For a and b on the same side - 4 iterations

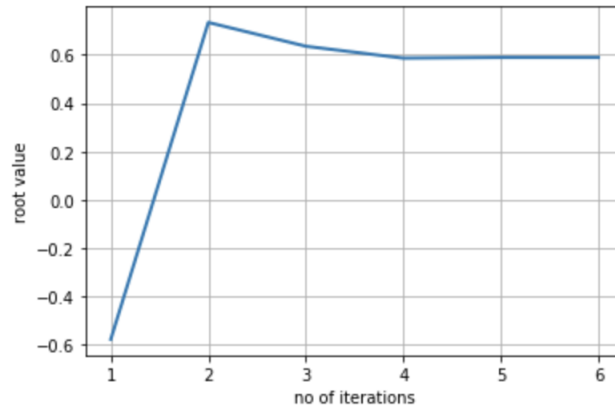|	xn	f(xn)	xn - xn-1	xn+1
0	1	-0.473592	-1	-0.576691
1	-0.576691	2.32539	-1.57669	0.733222
2	0.733222	-0.188908	1.30991	0.634803
3	0.634803	-0.0629796	-0.0984183	0.585582
4	0.585582	0.00409684	-0.0492212	0.588589
5	0.588589	-7.74065e-05	0.00300629	0.588533
6	0.588533	-9.12917e-08	-5.57481e-05	0.588533

Second Root(Between 3 and 4)

- Above shown plot is of convergence of first root vs iterations.

- Value of second root is 3.0964.

- Bisection method - 10 iterations

- Newton Raphson method - 4 iterations
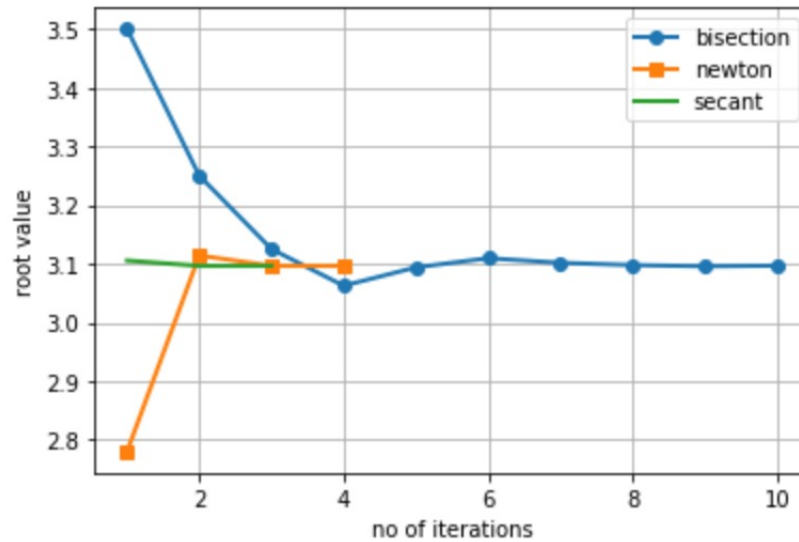
- Secant method - 3 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	3	0.0913329	-1	3.10541
1	3.10541	-0.00863175	0.10541	3.09631
2	3.09631	5.29574e-05	-0.00910198	3.09636
3	3.09636	2.19158e-08	5.55018e-05	3.09636

For a and b on the same side - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	2	-0.773962	-1	3.1338
1	3.1338	0.0357554	1.1338	3.08373
2	3.08373	-0.0120424	-0.050066	3.09634
3	3.09634	-1.93226e-05	0.0126139	3.09636
4	3.09636	1.21441e-08	2.0272e-05	3.09636

2.1.6 (F)

Equation:

$$x = x^3 - 2x - 2 \tag{7}$$

Graph:

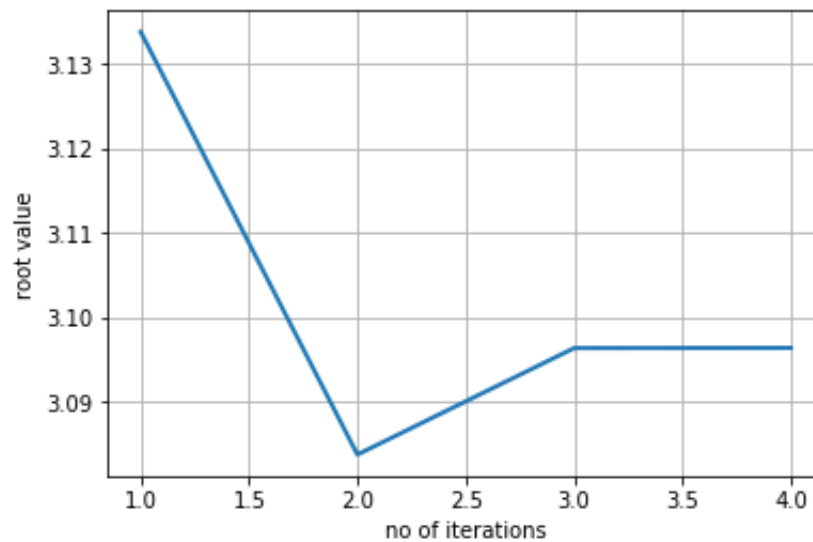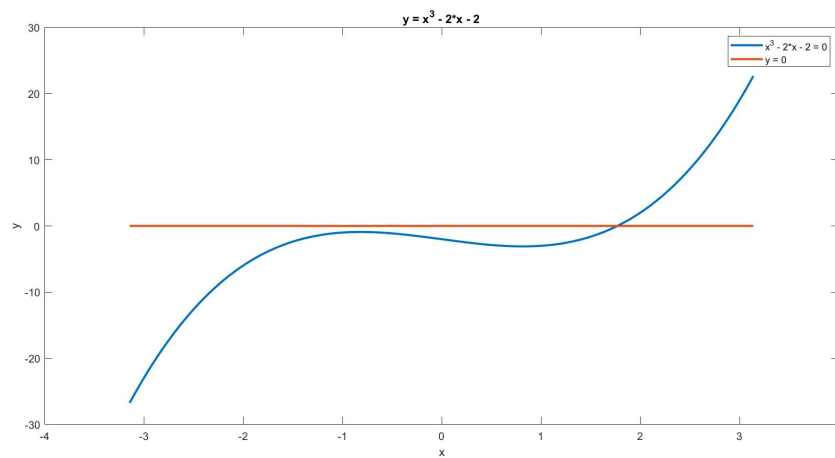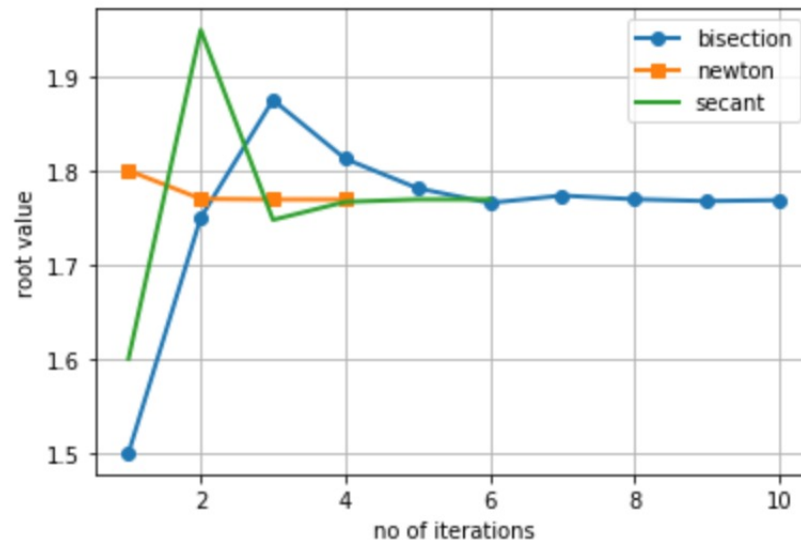- So, from the above graph we can observe that there is one real root of these equation between 1 and 2.

- Above shown plot is of convergence of first root vs iterations.

- Value of first root is 1.7693.

- Bisection method - 10 iterations

- Newton Raphson method - 4 iterations

- Secant method - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	-3	-1	1.6
1	1.6	-1.104	0.6	1.94937
2	1.94937	1.50892	0.349367	1.74761
3	1.74761	-0.157752	-0.201754	1.76671
4	1.76671	-0.0190577	0.0190962	1.76933
5	1.76933	0.000301132	0.00262397	1.76929
6	1.76929	-5.5937e-07	-4.08165e-05	1.76929

For a and b on the same side - 5 iterations
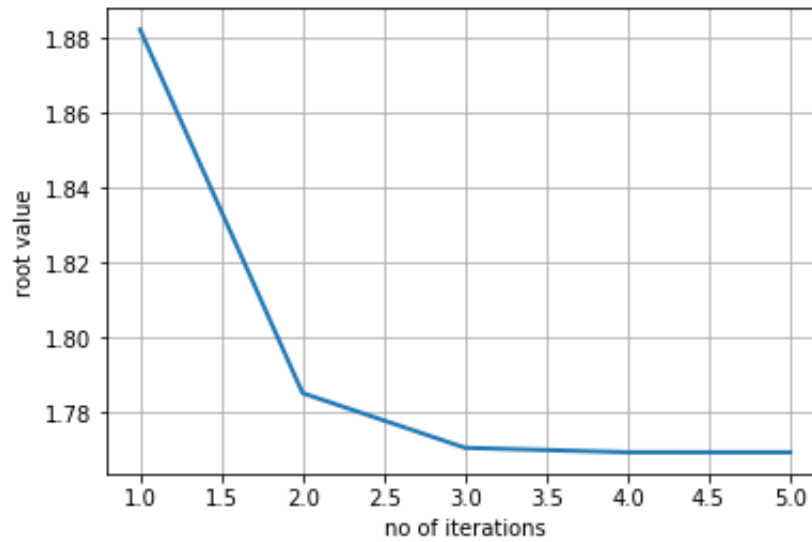
	xn	f(xn)	xn - xn-1	xn+1
0	2	2	-1	1.88235
1	1.88235	0.904946	-0.117647	1.78513
2	1.78513	0.118395	-0.0972228	1.7705
3	1.7705	0.00890179	-0.0146344	1.76931
4	1.76931	0.000100254	-0.00118977	1.76929
5	1.76929	8.65786e-08	-1.35521e-05	1.76929

2.1.7   (G)

Equation:

$$y = x^4 - x - 1 \tag{8}$$

Graph:
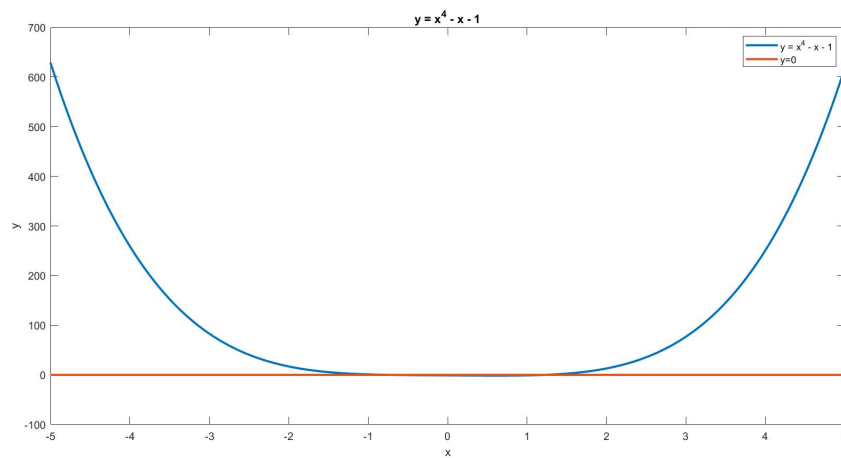
- So, from the above graph we can observe that there are two real roots of this equation between (-1,0) and (1,2).

Root Between $a = 1$ and $b = 1.5$
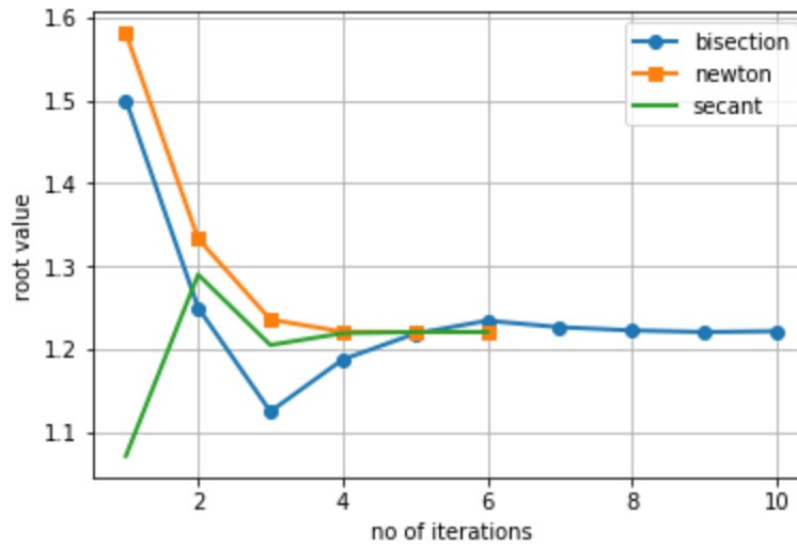

- Above shown plot is of convergence of positive root vs iterations.

- Value of first root is 1.2207.

- Bisection method - 10 iterations

- Newton Raphson method - 6 iterations

- Secant method - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	-1	-1	1.07143
1	1.07143	-0.753618	0.0714286	1.28991
2	1.28991	0.478548	0.218482	1.20506
3	1.20506	-0.0962846	-0.0848537	1.21927
4	1.21927	-0.00923606	0.014213	1.22078
5	1.22078	0.000210003	0.00150804	1.22074
6	1.22074	-4.41665e-07	-3.35264e-05	1.22074

For a and b on the same side - 8 iterations

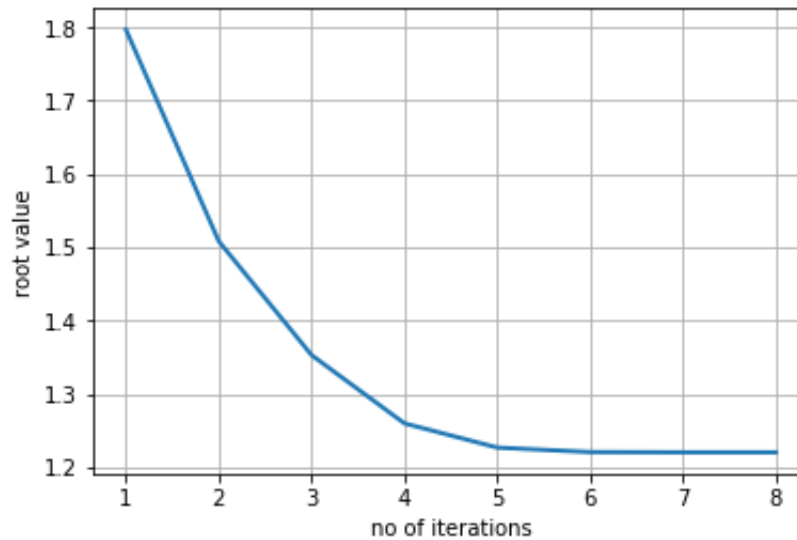|	xn	f(xn)	xn - xn-1	xn+1
0	2	13	-1	1.79688
1	1.79688	7.62801	-0.203125	1.50845
2	1.50845	2.66903	-0.28843	1.35321
3	1.35321	0.999965	-0.155239	1.2602
4	1.2602	0.261873	-0.0930064	1.2272
5	1.2272	0.0409049	-0.0329983	1.22109
6	1.22109	0.00219036	-0.00610855	1.22075
7	1.22075	2.00186e-05	-0.000345605	1.22074
8	1.22074	9.94345e-09	-3.18776e-06	1.22074

Root Between $a = -1$ and $b = -0.5$

- Above shown plot is of convergence of negative root vs iterations.

- Value of first root is -0.7245.

- Bisection method - 10 iterations

- Newton Raphson method - 5 iterations
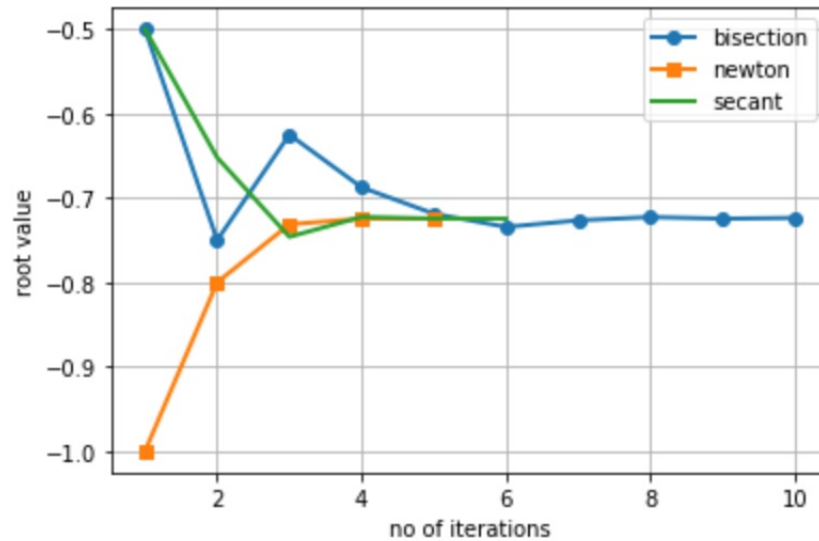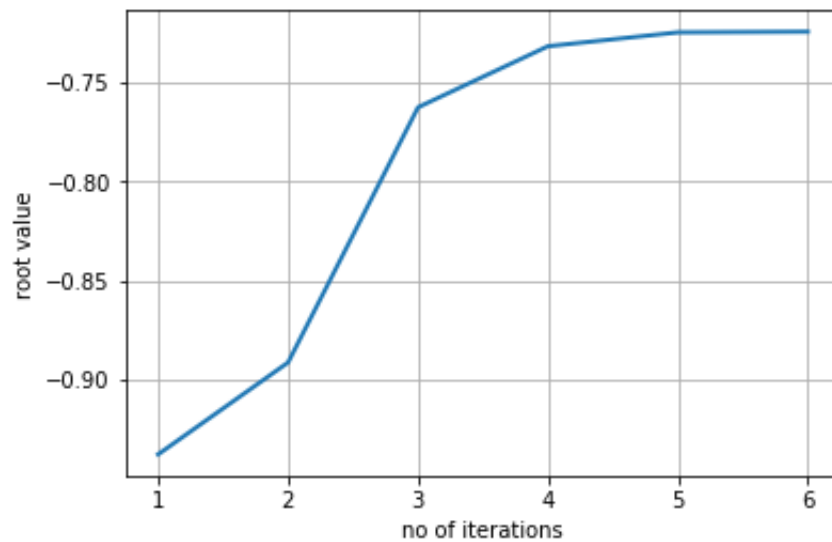
- Secant method - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	-1	1	-1	-0.5
1	-0.5	-0.4375	0.5	-0.652174
2	-0.652174	-0.16692	-0.152174	-0.746049
3	-0.746049	0.0558414	-0.0938754	-0.722517
4	-0.722517	-0.00496727	0.0235325	-0.724439
5	-0.724439	-0.000133196	-0.0019223	-0.724492
6	-0.724492	3.28877e-07	-5.29663e-05	-0.724492

For a and b on the same side - 6 iterations

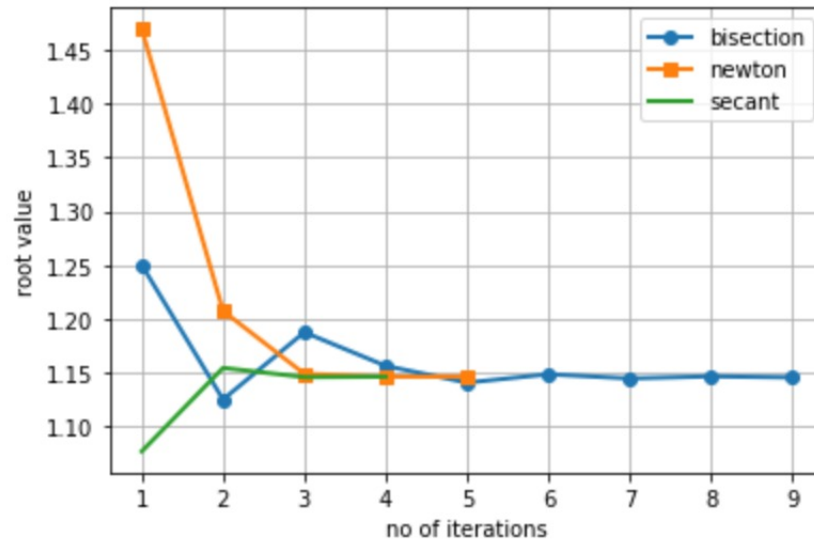|	xn	f(xn)	xn - xn-1	xn+1
0	-2	17	-1	-0.9375
1	-0.9375	0.709976	1.0625	-0.891193
2	-0.891193	0.521985	0.0463075	-0.762614
3	-0.762614	0.100848	0.128579	-0.731823
4	-0.731823	0.0186531	0.0307904	-0.724836
5	-0.724836	0.000866849	0.0069875	-0.724495
6	-0.724495	7.91501e-06	0.000340549	-0.724495

2.2   Question 2

2.2.1   Equation:

$$y = e^x - x - 2 \tag{9}$$

2.2.2   Graph:
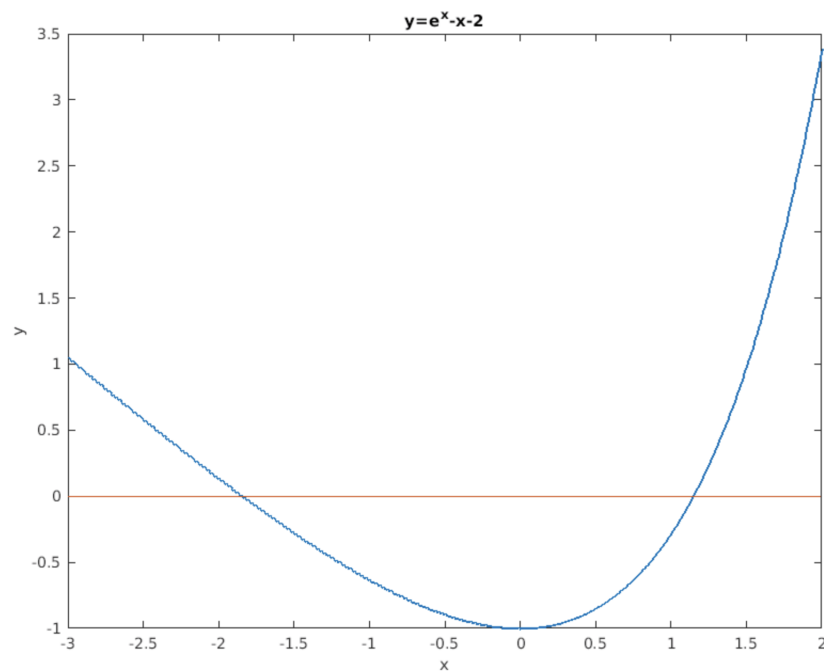
- So, from the above graph we can observe that there are two real roots of this equation between (-2,-1) and (1,2).
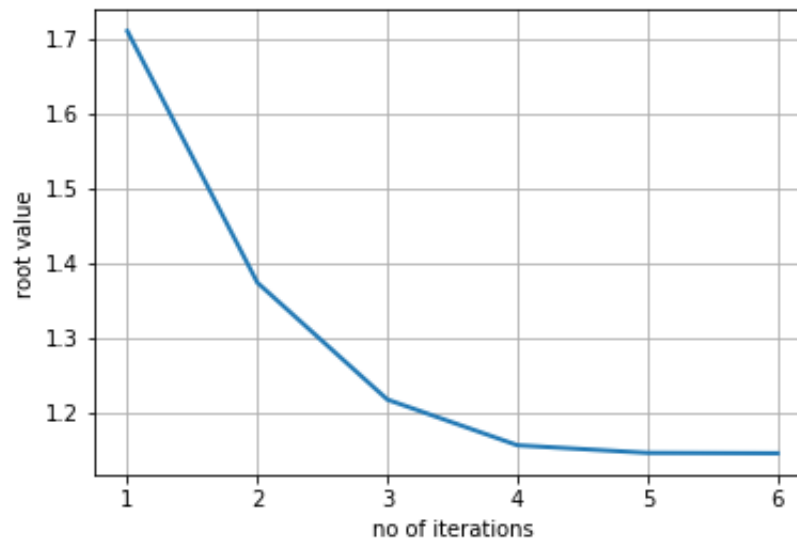
Root between : $a = 1$ and $b = 1.5$

- Above shown plot is of convergence of first root vs iterations.

- Value of positive root is 1.1462.

- Bisection method - 10 iterations

- Newton Raphson method - 5 iterations

- Secant method - 4 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	-0.281718	-1	1.07675
1	1.07675	-0.141632	0.0767463	1.15434
2	1.15434	0.0175888	0.0775935	1.14577
3	1.14577	-0.000911872	-0.00857159	1.14619
4	1.14619	-5.42989e-06	0.000422481	1.14619

For a and b on the same side - 6 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	2	3.38906	-1	1.71025
1	1.71025	1.82009	-0.28975	1.37412
2	1.37412	0.577483	-0.336128	1.21791
3	1.21791	0.16221	-0.15621	1.15689
4	1.15689	0.0231477	-0.0610173	1.14674
5	1.14674	0.00116921	-0.0101567	1.1462
6	1.1462	9.12622e-06	-0.000540312	1.1462

Root between : $a = -1$ and $b = -2$

- Above shown plot is of convergence of first root vs iterations.

- Value of negative root is -1.8414.

- Bisection method - 10 iterations

- Newton Raphson method - 4 iterations
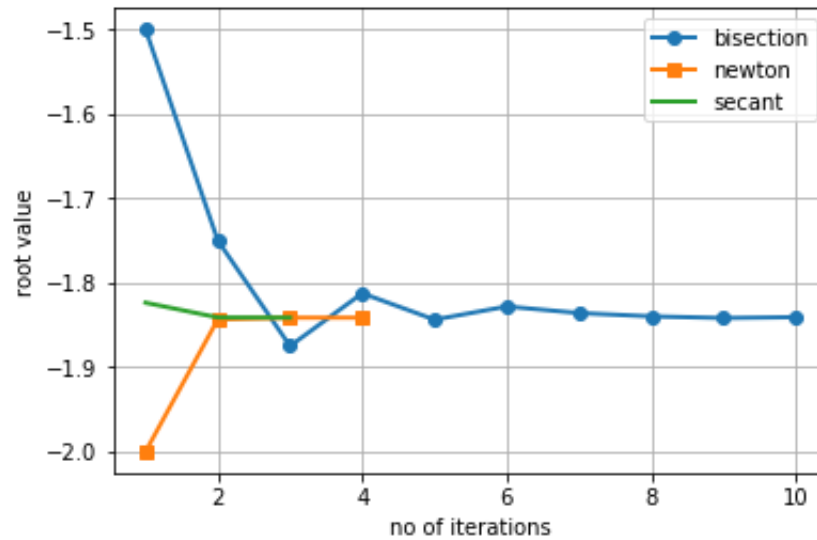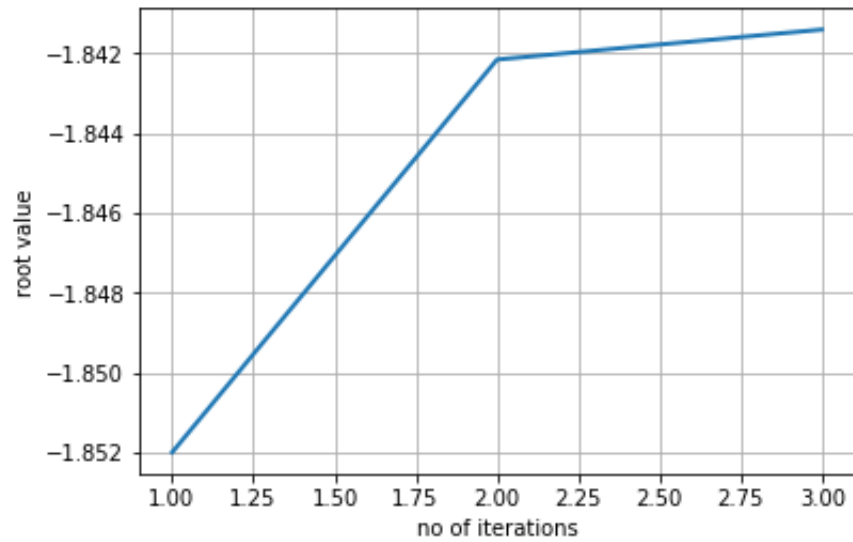
- Secant method - 3 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	-2	0.135335	-1	-1.82366
1	-1.82366	-0.0149085	0.176343	-1.84116
2	-1.84116	-0.00021048	-0.0174983	-1.84141
3	-1.84141	3.548e-07	-0.00025058	-1.84141

For a and b on the same side - 3 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	-3	1.04979	-1	-1.852
1	-1.852	0.00892631	1.148	-1.84216
2	-1.84216	0.000633764	0.00984509	-1.84141
3	-1.84141	6.2991e-07	0.000752418	-1.84141

2.3   Question 3
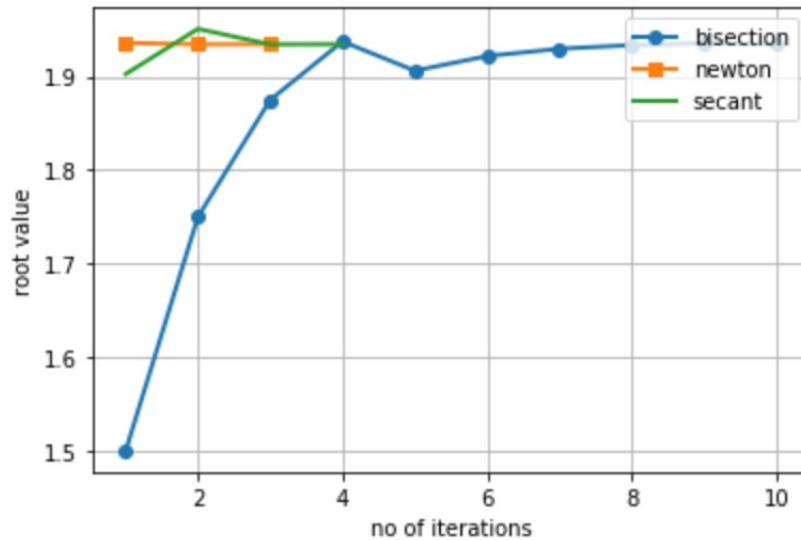
2.3.1   Equation:

$$y = 1 - x + sin(x) \tag{10}$$
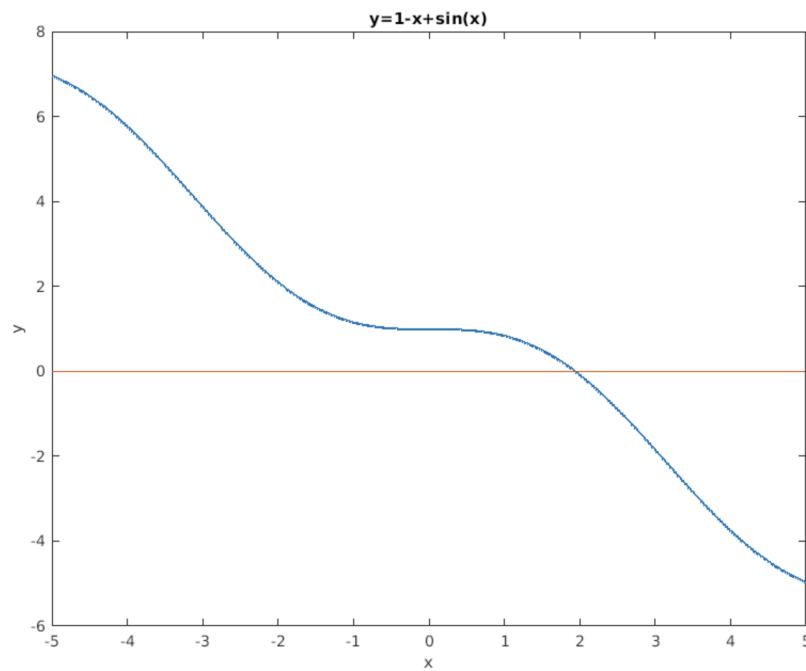
2.3.2   Graph:

- So, from the above graph we can observe that there is one real root of these equation between 1 and 3.

Root Between : $a = 1.8$ and $b = 2.1$

- Above shown plot is of convergence of first root vs iterations.

- Value of smallest positive root is 1.9346.

- Bisection method - 10 iterations

- Newton Raphson method - 3 iterations

- Secant method - 4 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	1	0.841471	-1	1.9027
1	1.9027	0.0427267	0.902698	1.95099
2	1.95099	-0.0223906	0.0482874	1.93438
3	1.93438	0.000246293	-0.0166037	1.93456
4	1.93456	1.38345e-06	0.00018065	1.93456

For a and b on the same side - 4 iterations
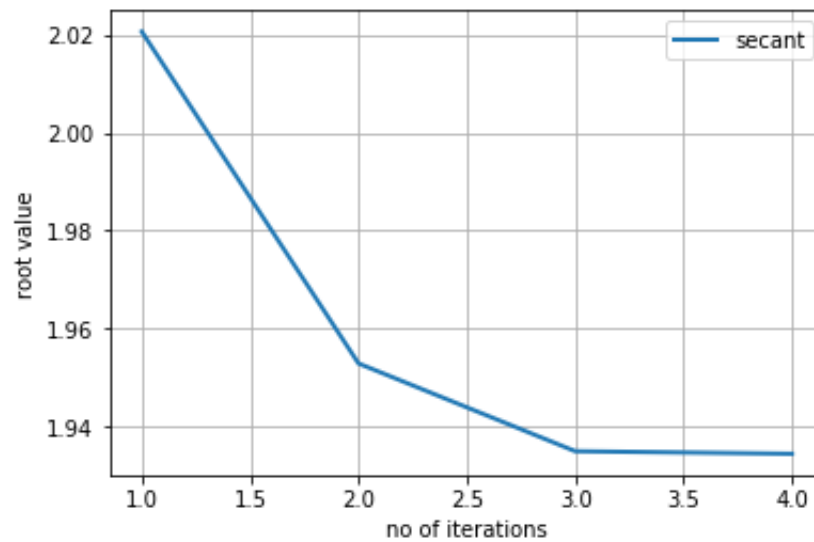
	xn	f(xn)	xn - xn-1	xn+1
0	3	-1.85888	-1	2.02057
1	2.02057	-0.120026	-0.979429	1.95297
2	1.95297	-0.0251071	-0.0676061	1.93508
3	1.93508	-0.000704291	-0.0178825	1.93457
4	1.93457	-4.42638e-06	-0.000516109	1.93457

2.4   Question 4

2.4.1   Equation:
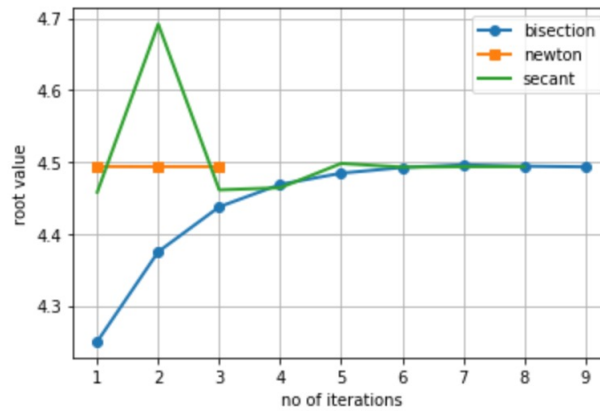
$$y = tan(x) - x \tag{11}$$

2.4.2   Graph:
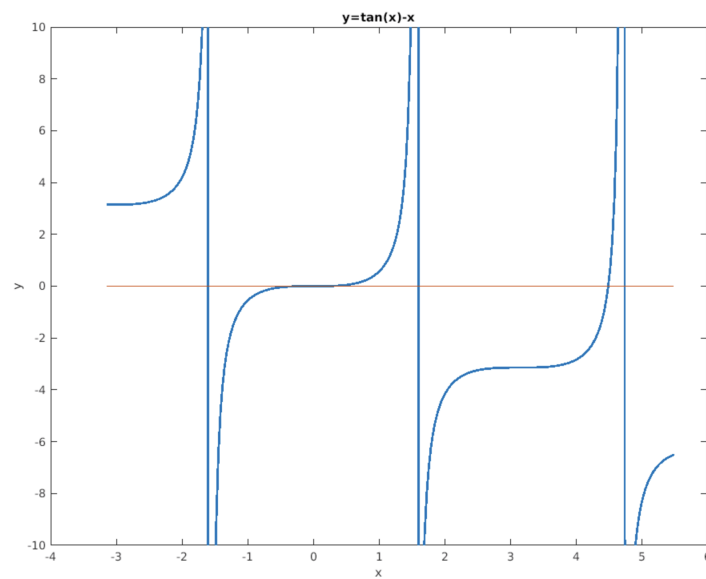
- So, from the above graph we can observe that there are infinitely many positive real roots possible for this equation, smallest of which lies between 4 and 5.

2.4.3   Smallest positive real root

Assumed values : $a = 4$ and $b = 4.5$

- Value of smallest positive root is 4.4934.

- Bisection method - 9 iterations

- Newton Raphson method - 3 iterations

- Secant method - 8 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	3.5	-3.12541	-1	4.45791
1	4.45791	-0.613521	0.957909	4.69187
2	4.69187	44.0484	0.233966	4.46112
3	4.46112	-0.565386	-0.230752	4.46405
4	4.46405	-0.520459	0.00292429	4.49792
5	4.49792	0.0931395	0.0338769	4.49278
6	4.49278	-0.0126308	-0.00514225	4.4934
7	4.4934	-0.000269554	0.000614073	4.49341
8	4.49341	7.97712e-07	1.33907e-05	4.49341

For a and b on the same side - 8 iterations
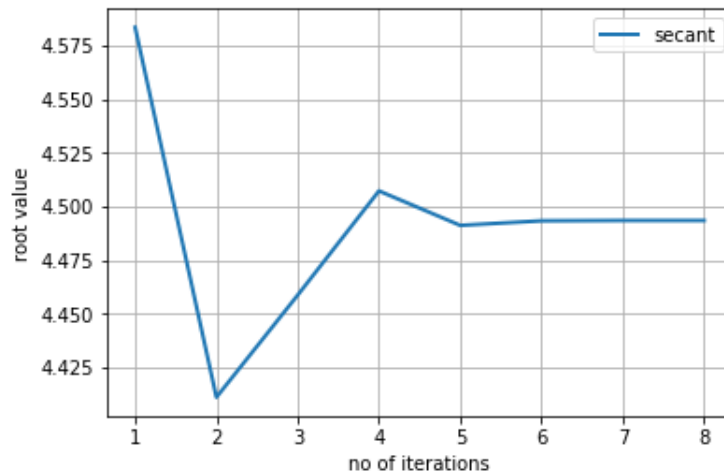
	xn	f(xn)	xn - xn-1	xn+1
0	4.3	-2.01415	-0.1	4.58349
1	4.58349	3.13171	0.283493	4.41096
2	4.41096	-1.19449	-0.172531	4.4586
3	4.4586	-0.603288	0.0476368	4.50721
4	4.50721	0.297998	0.0486105	4.49114
5	4.49114	-0.0453841	-0.0160724	4.49326
6	4.49326	-0.00297815	0.00212426	4.49341
7	4.49341	3.19418e-05	0.000149185	4.49341
8	4.49341	-2.22347e-08	-1.58309e-06	4.49341

2.4.4   Positive real root near x = 100

Assumed values : $a = 98.5$ and $b = 99.5$
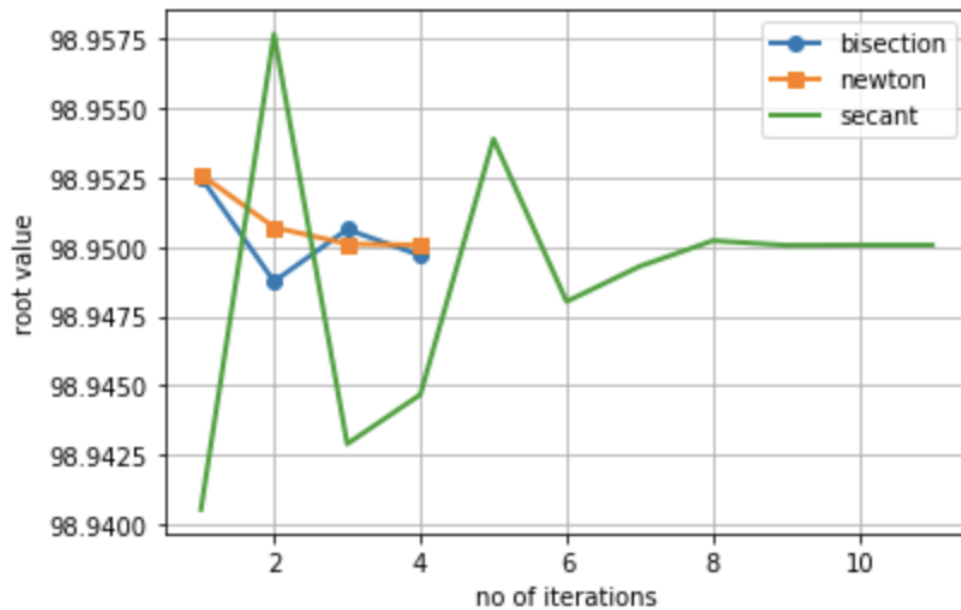
- Above shown plot is of convergence of approximate root vs iterations.

- The value of $tan(x)$ is infinite at 97.38 which is in the range of assumed values hence we can't find the exact root.

- Bisection method - 4 iterations

- Newton Raphson method - 4 iterations

- Secant method - 11 iterations

	xn	f(xn)	xn - xn-1	xn+1
0	98.942	-43.908	-0.02	98.9405
1	98.9405	-48.0035	-0.00146097	98.9577
2	98.9577	300.171	0.0171241	98.9429
3	98.9429	-41.0402	-0.0147632	98.9447
4	98.9447	-34.4043	0.00177568	98.9539
5	98.9539	60.1098	0.00920623	98.948
6	98.948	-16.5917	-0.00585505	98.9493
7	98.9493	-7.00077	0.00126653	98.9502
8	98.9502	1.54162	0.000924493	98.9501
9	98.9501	-0.115452	-0.00016684	98.9501
10	98.9501	-0.00177334	1.16242e-05	98.9501