Assignment 8
CS374

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Contents

1 Cubic Spline Interpolation

x	0	1	2	3	4	5	6
y	2	2.1592	3.1697	5.4332	9.1411	14.406	21.303

1.1 Equation

$$S"(x) = M_j \tag{1}$$

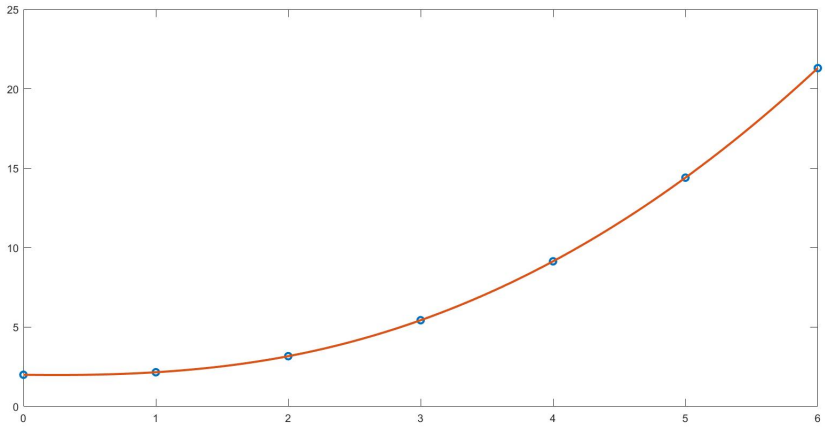
$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{2}$$

$$A = D - C \tag{3}$$

$$B = Cx_j - Dx_{j-1} \tag{4}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{5}$$

1.1.1 Graph



Plot of Cubic Spline Interpolation

2 Cubic Spline Interpolation and Quadratic Lagrange Interpolation

2.1 Equation

$$S"(x) = M_j \tag{6}$$

$$S(x) = \frac{M_{j-1}}{x_j - x_{j-1}}\frac{(x_j - x)^3}{6} + \frac{M_j}{x_j - x_{j-1}}\frac{(x - x_{j-1})^3}{6} + Ax + B \tag{7}$$

$$A = D - C \tag{8}$$

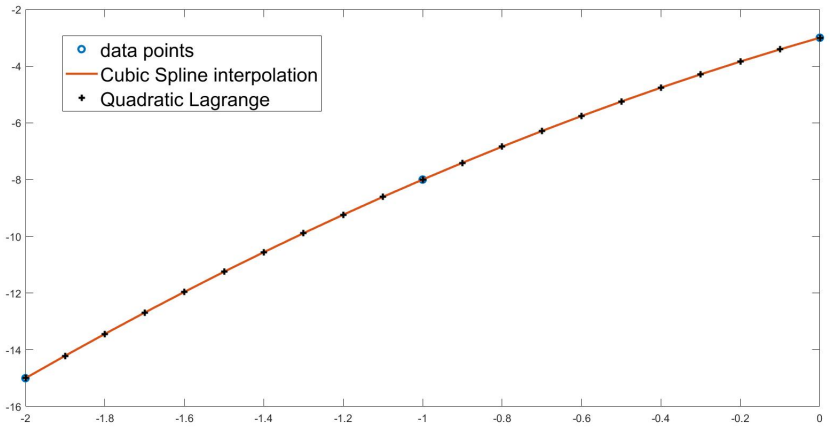
$$B = Cx_j - Dx_{j-1} \tag{9}$$

$$\frac{M_{j-1}}{6}(x_j - x_{j-1}) + \frac{M_j}{3}(x_{j+1} - x_{j-1}) + \frac{M_{j+1}}{6}(x_{j+1} - x_j) = \frac{y_{j+1} - y_j}{x_{j+1} - x_j} + \frac{y_j - y_{j-1}}{x_j - x_{j-1}} \tag{10}$$

$$y = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x2)}y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)}y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}y_2 \tag{11}$$

2.2 Graph

2.2.1 Graph



Plot of Cubic Spline Interpolation and Quadratic Lagrange Interpolation

3 Numerical Integration and Differentiation

3.1 Q1

3.1.1 Equation

$$\int_0^{\pi} \mathrm{e}^x \cos 4x = \frac{\mathrm{e}^{\pi} - 1}{17} \tag{12}$$

3.1.2 Table

n	h	Tn	Sn	TnError	SnError
2	1.570796	26.516336	22.715077	25.213942	2.141268e+01
4	0.785398	3.249050	-4.506711	1.946657	5.809105e+00
8	0.392699	1.624525	1.083017	0.322132	2.193769e-01
16	0.196350	1.375723	1.292788	0.073329	9.605410e-03
32	0.098175	1.320312	1.301842	0.017918	5.520189e-04
64	0.049087	1.306848	1.302360	0.004454	3.379643e-05
128	0.024544	1.303506	1.302392	0.001112	2.101451e-06
256	0.012272	1.302672	1.302394	0.000278	1.311722e-07
512	0.006136	1.302463	1.302394	0.000069	8.195633e-09

3.2 Q2

3.2.1 Equation

$$\int_0^1 x^{5/2} = 2/7 \tag{13}$$

3.2.2 Table

n	h	Tn	Sn	TnError	SnError
2	0.500000	0.338388	0.284518	5.267406e-02	1.196489e-03
4	0.250000	0.298791	0.285593	1.307721e-02	1.217400e-04
8	0.125000	0.288975	0.285702	3.260454e-03	1.179823e-05
16	0.062500	0.286529	0.285713	8.142822e-04	1.108410e-06
32	0.031250	0.285918	0.285714	2.034940e-04	1.020813e-07
64	0.015625	0.285765	0.285714	5.086654e-05	9.279914e-09
128	0.007812	0.285727	0.285714	1.271601e-05	8.363088e-10
256	0.003906	0.285717	0.285714	3.178946e-06	7.492457e-11
512	0.001953	0.285715	0.285714	7.947314e-07	6.685430e-12

3.3 Q3

3.3.1 Equation

$$\int_0^5 \frac{1}{1+(x-\pi)^2} = \arctan(5-\pi) + \arctan(\pi) \tag{14}$$

3.3.2 Table

n	h	Tn	Sn	TnError	SnError
2	2.500000	2.166655	2.625095	0.173111	2.853292e-01
4	1.250000	2.268668	2.302672	0.071099	3.709437e-02
8	0.625000	2.332270	2.353471	0.007496	1.370512e-02
16	0.312500	2.337813	2.339660	0.001953	1.059309e-04
32	0.156250	2.339277	2.339765	0.000489	1.079988e-06
64	0.078125	2.339644	2.339766	0.000122	6.743238e-08
128	0.039062	2.339736	2.339766	0.000031	4.216919e-09
256	0.019531	2.339759	2.339766	0.000008	2.635929e-10
512	0.009766	2.339764	2.339766	0.000002	1.647504e-11

3.4 Q4

$$\exp\left(-x^2\right) \qquad x\epsilon[0,10] \tag{15}$$

3.4.1 Table

n	h	Tn	Sn	TnError	SnError
4	2.500000	1.254826	0.839768	3.685992e-01	4.645874e-02
8	1.250000	0.889428	0.767629	3.201353e-03	1.185979e-01
16	0.625000	0.886227	0.885160	1.886302e-11	1.067118e-03
32	0.312500	0.886227	0.886227	1.110223e-16	6.287748e-12
64	0.156250	0.886227	0.886227	2.220446e-16	2.220446e-16
128	0.078125	0.886227	0.886227	2.220446e-16	0.000000e+00
256	0.039062	0.886227	0.886227	5.551115e-16	4.440892e-16
512	0.019531	0.886227	0.886227	7.771561e-16	2.220446e-16

3.5 Q5

3.5.1 Equation

$$\arctan\left(x^2+1\right) \qquad x\epsilon[0,2] \tag{16}$$

3.5.2 Table

n	h	Tn	Sn
4	0.500000	2.177450	2.174418
8	0.250000	2.175061	2.174265
16	0.125000	2.174461	2.174261
32	0.062500	2.174311	2.174261
64	0.031250	2.174274	2.174261
128	0.015625	2.174264	2.174261
256	0.007812	2.174262	2.174261
512	0.003906	2.174261	2.174261

4 Forward and Central Difference at x=1

4.1 Q1

4.1.1 Equation

$$\arctan\left(x^2 - x + 1\right) \tag{17}$$

4.1.2 Table

h	ForwardDiff	CentralDiff	ErrorForward	ErrorCentral
0.10000	0.520855	0.495856	0.012500	-0.001042
0.05000	0.511460	0.498960	0.006250	-0.000260
0.02500	0.505990	0.499740	0.003125	-0.000065
0.01250	0.503060	0.499935	0.001563	-0.000016
0.00625	0.501546	0.499984	0.000781	-0.000004

4.2 Q2

4.2.1 Equation

$$\arctan\left(100x^2 - 199x + 100\right) \tag{18}$$

4.2.2 Table

h	ForwardDiff	CentralDiff	ErrorForward	ErrorCentral
0.10000	3.409790	0.200294	2.487500	-0.124792
0.05000	2.594051	0.390426	1.243750	-0.031198
0.02500	1.675666	0.469776	0.621875	-0.007799
0.01250	1.109328	0.492262	0.310938	-0.001950
0.00625	0.808388	0.498054	0.155469	-0.000487