

# Assignment 4

## CS374

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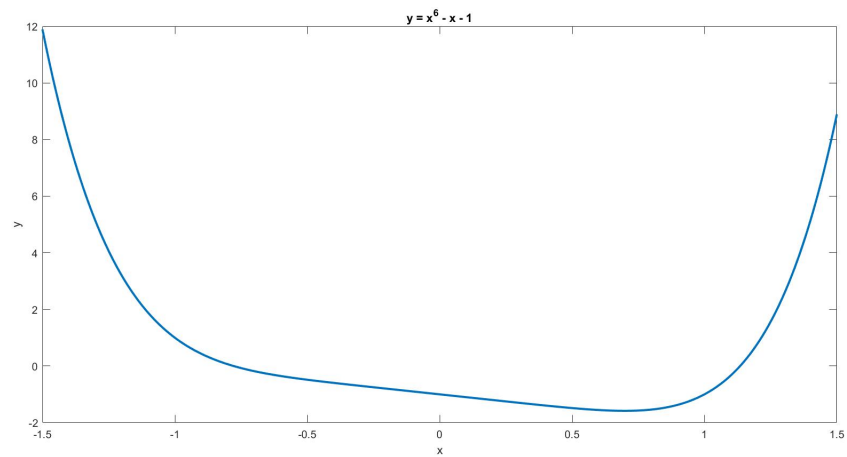
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# 1 Part A

## 1.1 Equation

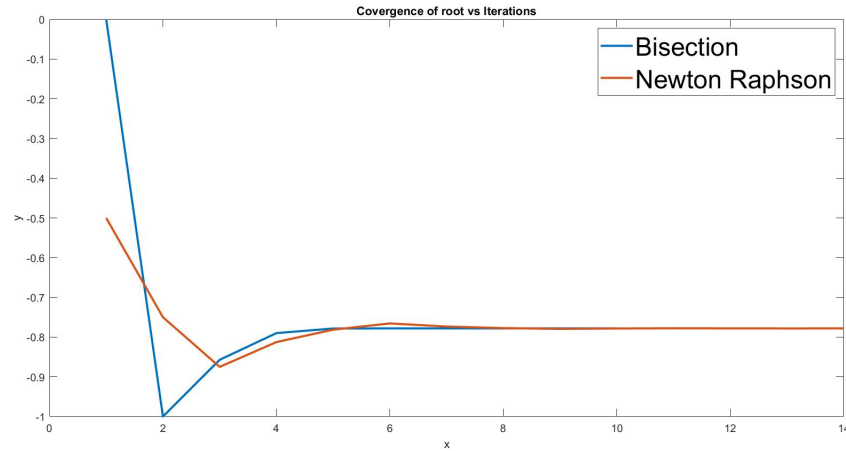
$$y = x^6 - x - 1 \quad (1)$$

## 1.2 Graph



- So, from the above graph we can observe that there are two real roots of these equations.

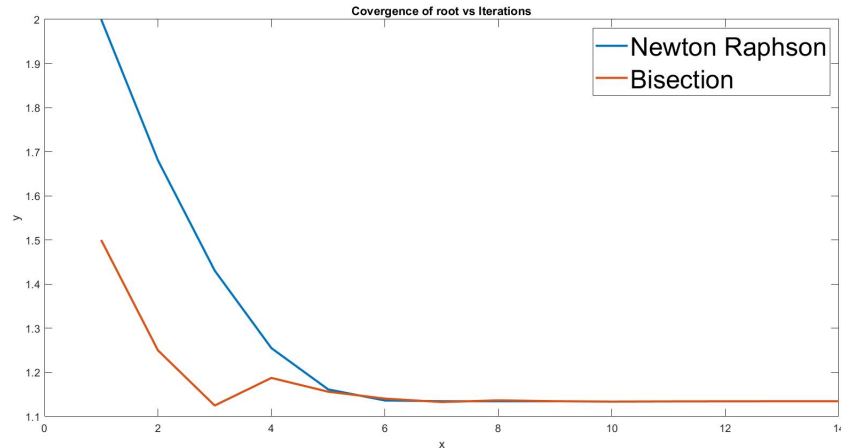
### 1.2.1 First Root



- Above shown plot is of convergence of first root vs iterations
- Value of first root is 1.1347.
- Root value is obtained in 9 iterations while it took 14 iterations using Bisection method.

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	2	61	191	1.68063	0.319372
2	1.68063	19.8529	79.4469	1.43074	0.249889
3	1.43074	6.1468	34.9711	1.25497	0.175768
4	1.25497	1.65166	17.6775	1.16154	0.0934325
5	1.16154	0.29431	11.6858	1.13635	0.0251852
6	1.13635	0.0168261	10.3689	1.13473	0.00162275
7	1.13473	6.57384e-05	10.2879	1.13472	6.38984e-06
8	1.13472	1.01541e-09	10.2876	1.13472	9.87017e-11
9	1.13472	-8.88178e-16	10.2876	1.13472	0

### 1.2.2 Second Root



- Above shown plot is of convergence of second root vs iterations.
- Value of second root is  $-0.7781$ .
- Root value is obtained in 9 iterations while using Bisection method took 14 iterations.

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	0	-1	-1	-1	1
2	-1	1	-7	-0.857143	-0.142857
3	-0.857143	0.253712	-3.77599	-0.789952	-0.067191
4	-0.789952	0.0329504	-2.84567	-0.778373	-0.0115791
5	-0.778373	0.000768014	-2.71431	-0.77809	-0.00028295
6	-0.77809	4.40606e-07	-2.7112	-0.77809	-1.62514e-07
7	-0.77809	1.45217e-13	-2.71119	-0.77809	-5.35127e-14
8	-0.77809	2.22045e-16	-2.71119	-0.77809	-1.11022e-16
9	-0.77809	-1.11022e-16	-2.71119	-0.77809	0

## 2 Part B

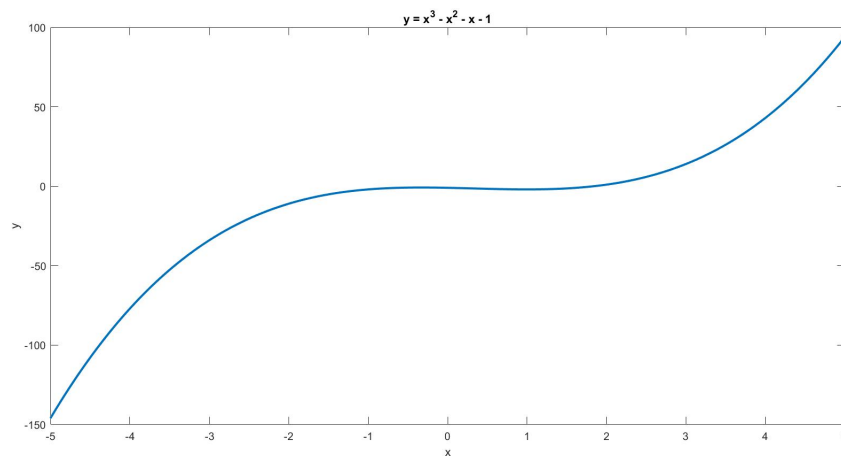
### 2.1 Question 1

#### 2.1.1 (A)

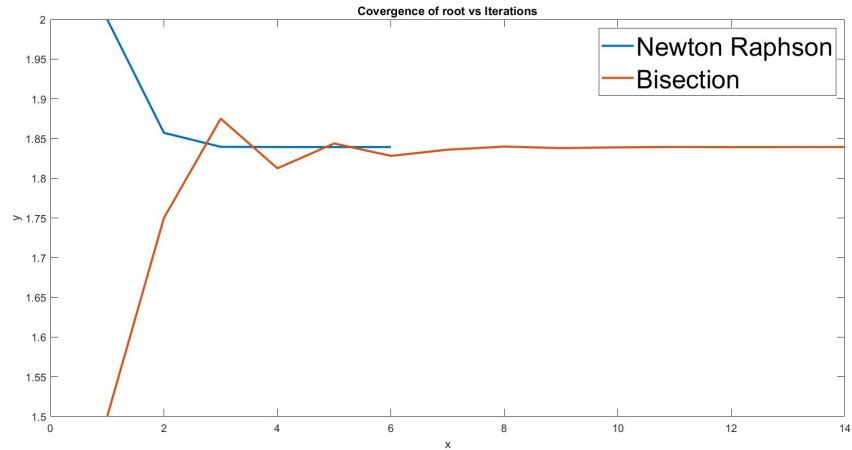
Equation:

$$y = x^3 - x^2 - x - 1 \quad (2)$$

Graph:



- So, from the above graph we can observe that there is one real root of these equation.



- Above shown plot is of convergence of first root vs iterations.
- Value of first root is 1.8393.
- Root value is obtained in 6 iterations while it took 14 iterations in Bisection method.

	Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
0	1	2	1	7	1.85714	0.142857
1	2	1.85714	0.0991254	5.63265	1.83954	0.0175983
2	3	1.83954	0.00141033	5.47268	1.83929	0.000257703
3	4	1.83929	3.0007e-07	5.47035	1.83929	5.48539e-08
4	5	1.83929	1.37668e-14	5.47035	1.83929	2.44249e-15
5	6	1.83929	2.22045e-16	5.47035	1.83929	0

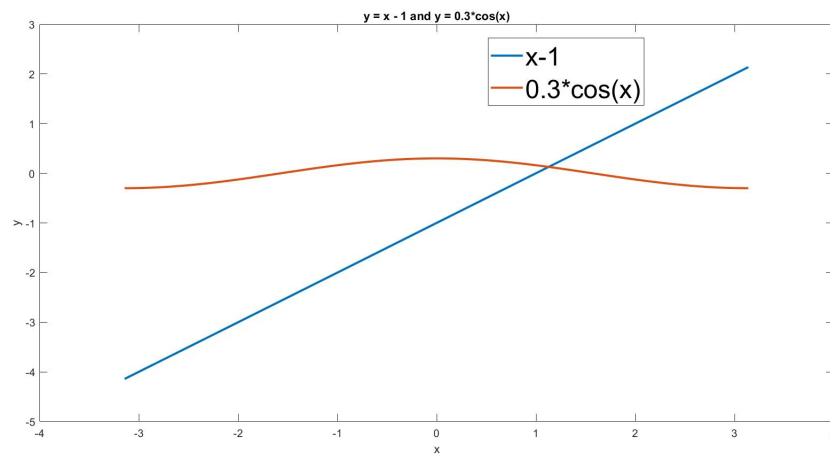


**2.1.2 (B)**

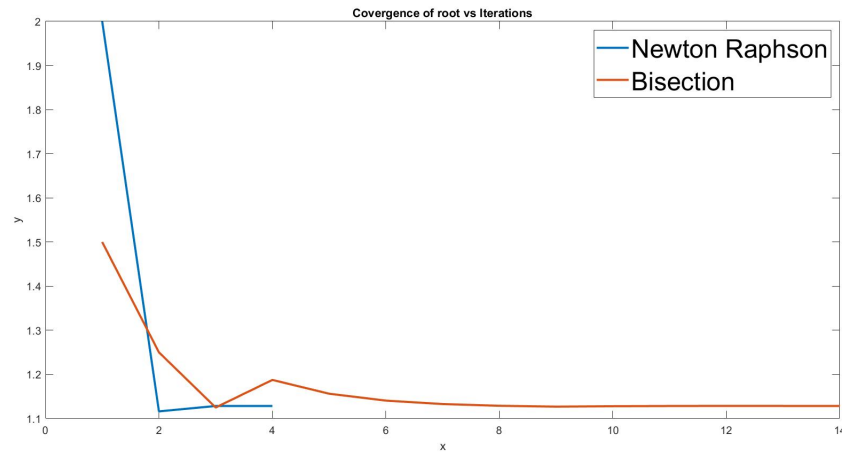
Equation:

$$x = 1 + (0.3) \cos x \quad (3)$$

Graph:



- So, from the above graph we can observe that there is one real root of these equation between 1 and 2 as both curves intersect between these two points.



- Above shown plot is of convergence of first root vs iterations.
- Value of first root is 1.1284.
- Root value is obtained in 4 iterations while Bisection method took 14 iterations.

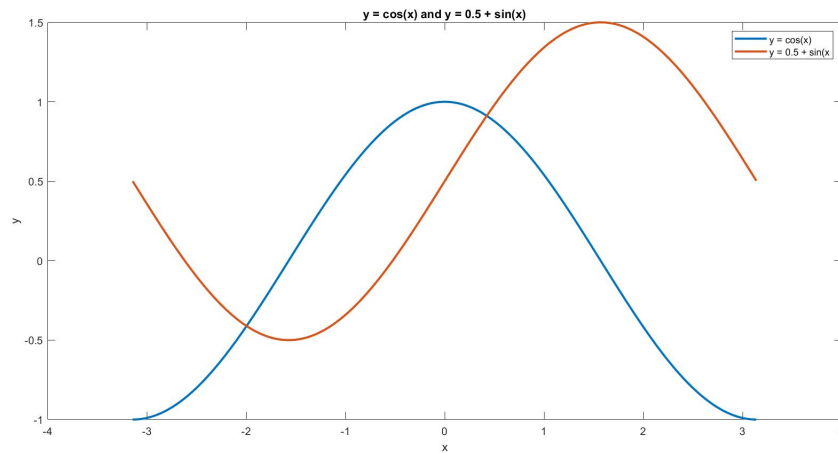
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	2	1.12484	1.27279	1.11624	0.883763
2	1.11624	-0.015483	1.26954	1.12843	-0.0121958
3	1.12843	9.71417e-06	1.27112	1.12843	7.64219e-06
4	1.12843	3.75017e-12	1.27112	1.12843	2.95031e-12

### 2.1.3 (C)

Equation:

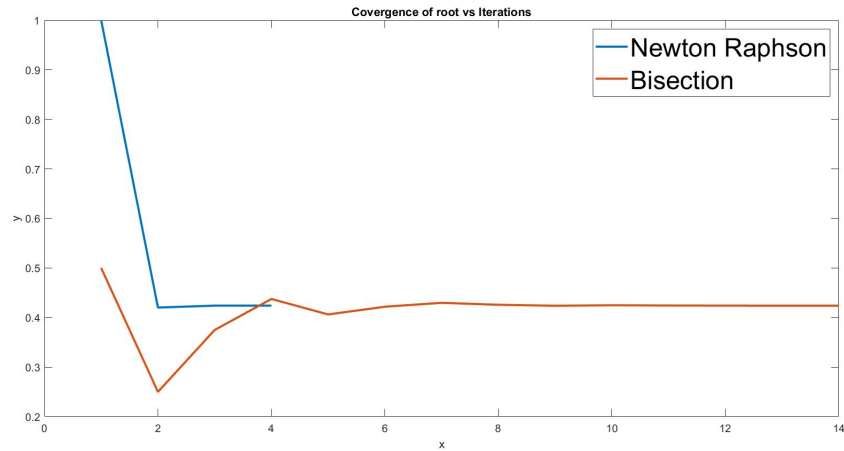
$$\cos x = 0.5 + \sin x \quad (4)$$

Graph:



- So, from the above graph we can observe that there is one positive real root i.e the smallest positive root of these equation between 0 and 1 as both curves intersect between these two points.

Root Between :  $a = 0$  and  $b = 1$



- Above shown plot is of convergence of first root vs iterations.
- Value of first root is 0.4240.
- Root value is obtained in 4 iterations while Bisection method took 14 iterations.

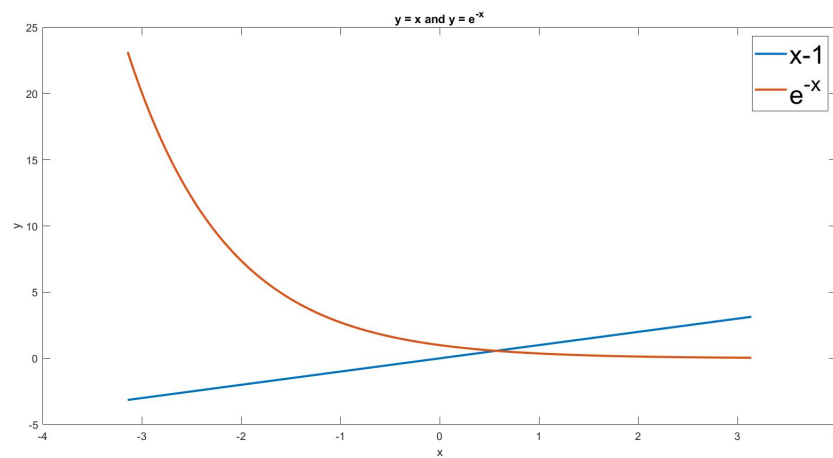
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	1	-0.801169	-1.38177	0.420188	0.579812
2	0.420188	0.0050801	-1.32094	0.424034	-0.00384581
3	0.424034	-3.72261e-06	-1.32288	0.424031	2.81403e-06
4	0.424031	-1.97969e-12	-1.32288	0.424031	1.49653e-12

### 2.1.4 (D)

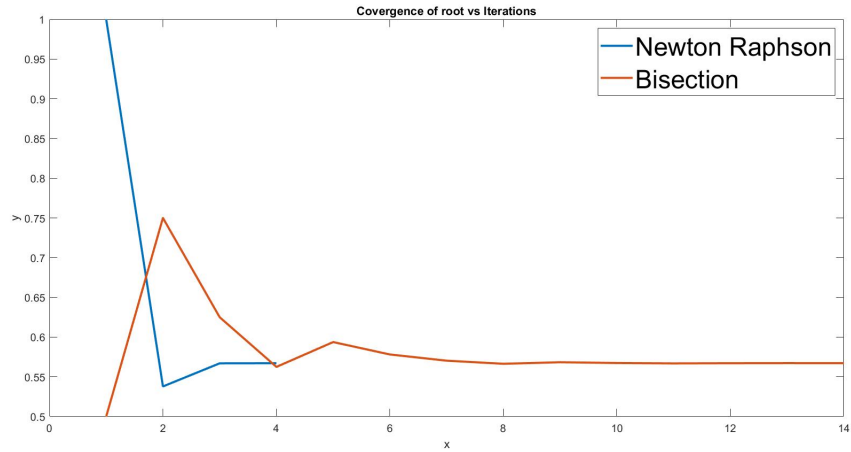
Equation:

$$x = e^{-x} \quad (5)$$

Graph:



- So, from the above graph we can observe that there is one real root of these equation between 0 and 1 as both curves intersect between these two points.



- Above shown plot is of convergence of first root vs iterations.
- Value of the root is 0.5671.
- Root value is obtained in 14 iterations in Bisection method while it is obtained in 4 iterations using Newton Raphson method.

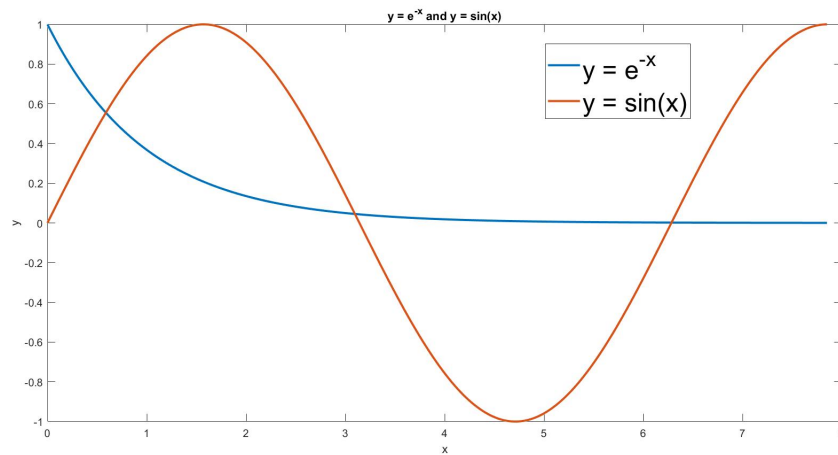
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	1	0.632121	1.36788	0.537883	0.462117
2	0.537883	-0.0461005	1.58398	0.566987	-0.0291041
3	0.566987	-0.00024495	1.56723	0.567143	-0.000156295
4	0.567143	-6.92781e-09	1.56714	0.567143	-4.42066e-09

**2.1.5 (E)**

Equation:

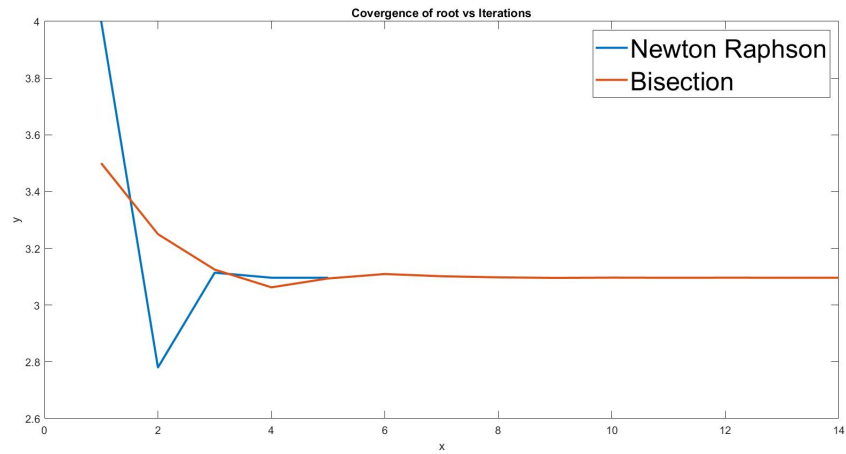
$$e^{-x} = \sin(x) \quad (6)$$

Graph:



- So, from the above graph we can observe that the two smallest real roots of these equation lie between 0 and 1 and another one between 3 and 4.

First Root( Between 0 and 1 )

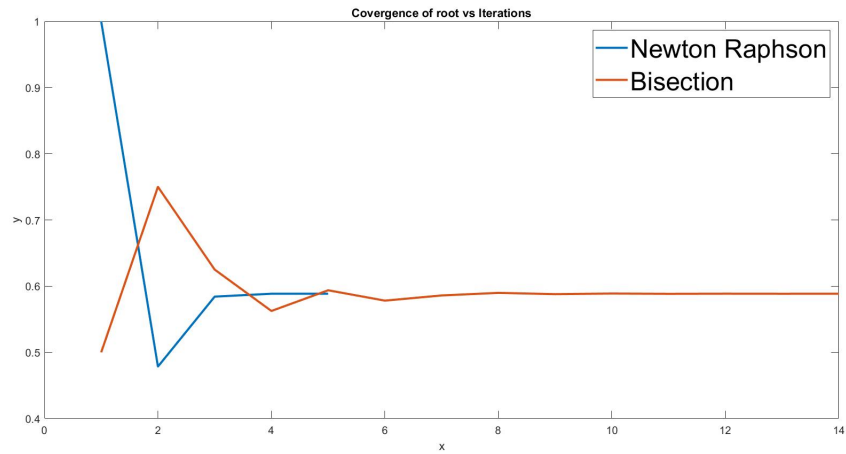


- Above shown plot is of convergence of first root vs iterations.
- Value of first root is 0.5885.
- Root value is obtained in 5 iterations while Bisection method took 14 iterations.

Count	x <sub>n</sub>	f(x)	f(x).1	x <sub>n+1</sub>	Error
1	1	0.473592	0.908182	0.478528	0.521472
2	0.478528	-0.159222	1.50737	0.584157	-0.105629
3	0.584157	-0.00607931	1.39175	0.588525	-0.00436809
4	0.588525	-1.05845e-05	1.38691	0.588533	-7.63175e-06
5	0.588533	-3.23334e-11	1.3869	0.588533	-2.33135e-11



First Root( Between 3 and 4 )



- Above shown plot is of convergence of first root vs iterations.
- Value of second root is 3.0964.
- Root value is obtained in 14 iterations with  $\epsilon = 0.0001$ .

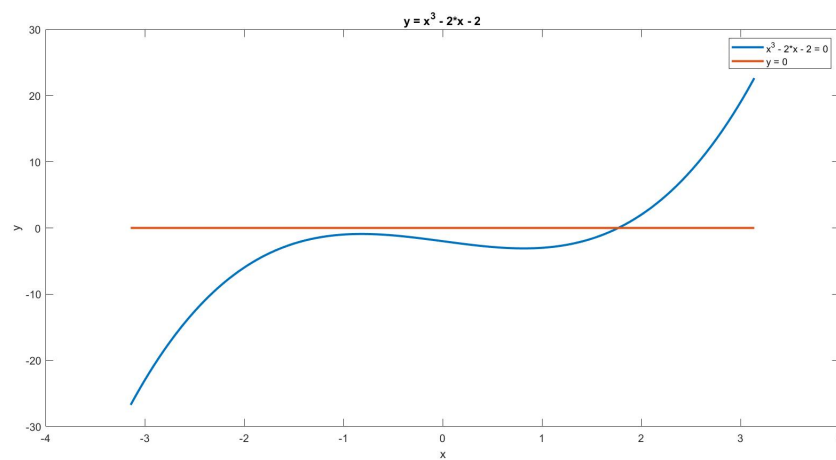
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	4	-0.775118	-0.635328	2.77997	1.22003
2	2.77997	0.291751	-0.873284	3.11406	-0.334084
3	3.11406	-0.0168872	-0.9552	3.09638	0.0176792
4	3.09638	-1.22063e-05	-0.953765	3.09636	1.2798e-05
5	3.09636	-7.40472e-12	-0.953764	3.09636	7.76357e-12

**2.1.6 (F)**

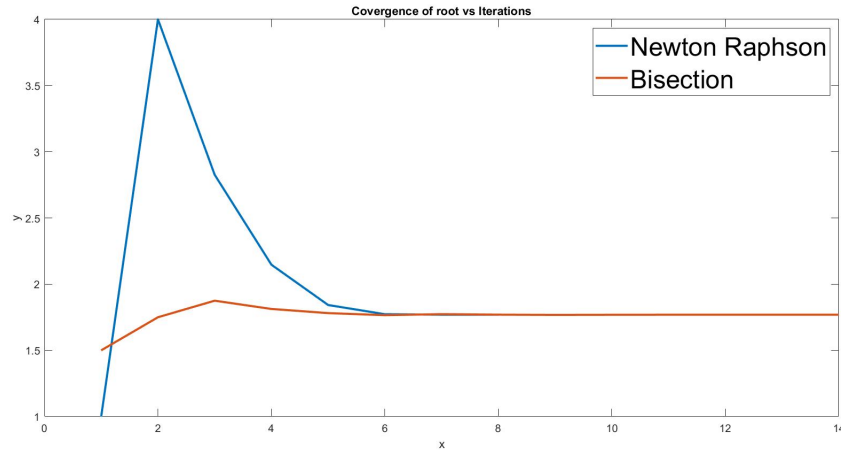
Equation:

$$y = x^3 - 2x - 2 \quad (7)$$

Graph:



- So, from the above graph we can observe that there is one real root of these equation between 1 and 2.



- Above shown plot is of convergence of first root vs iterations.
- Value of first root is 1.7693.
- Root value is obtained in 8 iterations while Bisection method took 14 iterations.

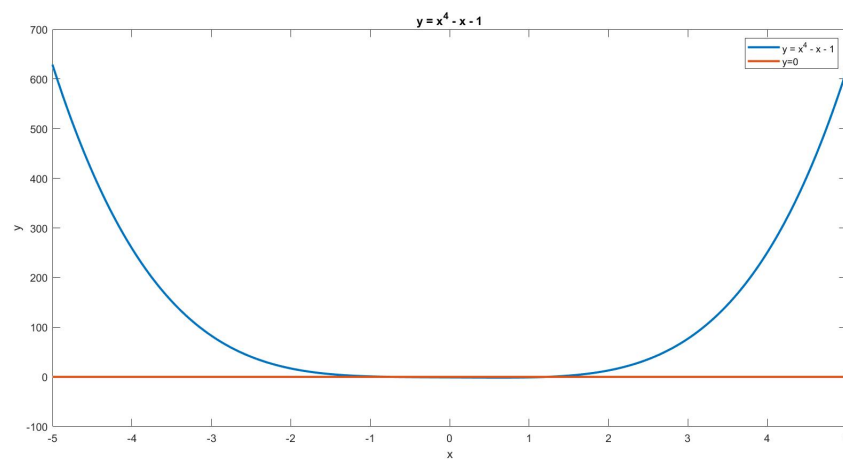
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	1	-3	1	4	-3
2	4	54	46	2.82609	1.17391
3	2.82609	14.9191	21.9603	2.14672	0.679368
4	2.14672	3.59951	11.8252	1.84233	0.304393
5	1.84233	0.568509	8.1825	1.77285	0.0694786
6	1.77285	0.0263449	7.42897	1.7693	0.00354624
7	1.7693	6.68404e-05	7.39128	1.76929	9.04314e-06
8	1.76929	4.34071e-10	7.39119	1.76929	5.87281e-11

**2.1.7 (G)**

Equation:

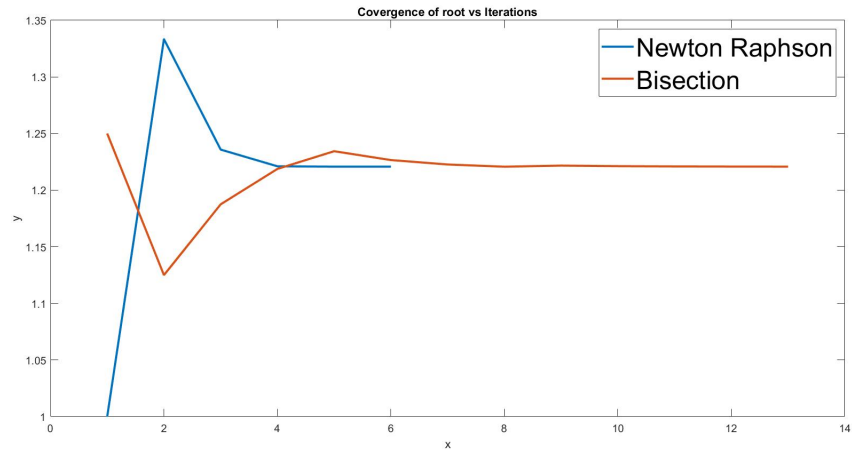
$$y = x^4 - x - 1 \quad (8)$$

Graph:



- So, from the above graph we can observe that there are two real roots of this equation between  $(-1,0)$  and  $(1,2)$ .

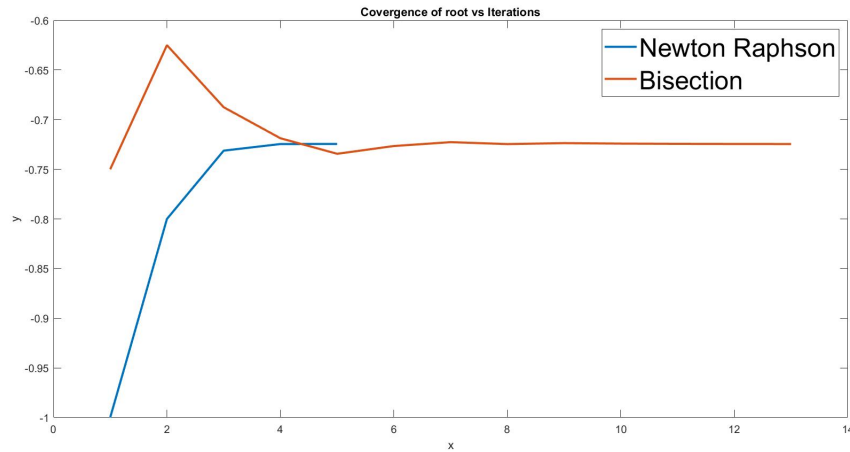
Root Between  $a = 1$  and  $b = 1.5$



- Above shown plot is of convergence of positive root vs iterations.
- Value of first root is 1.2207.
- Root value is obtained in 6 iterations while Bisection method took 13 iterations.

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	1	-1	3	1.33333	-0.333333
2	1.33333	0.82716	8.48148	1.23581	0.0975255
3	1.23581	0.0965963	6.54941	1.22106	0.0147489
4	1.22106	0.00197748	6.28232	1.22074	0.000314769
5	1.22074	8.86202e-07	6.27669	1.22074	1.41189e-07
6	1.22074	1.78968e-13	6.27669	1.22074	2.84217e-14

Root Between  $a = -1$  and  $b = -0.5$



- Above shown plot is of convergence of negative root vs iterations.
- Value of first root is -0.7245.
- Root value is obtained in 5 iterations while Bisection method took 13 iterations.

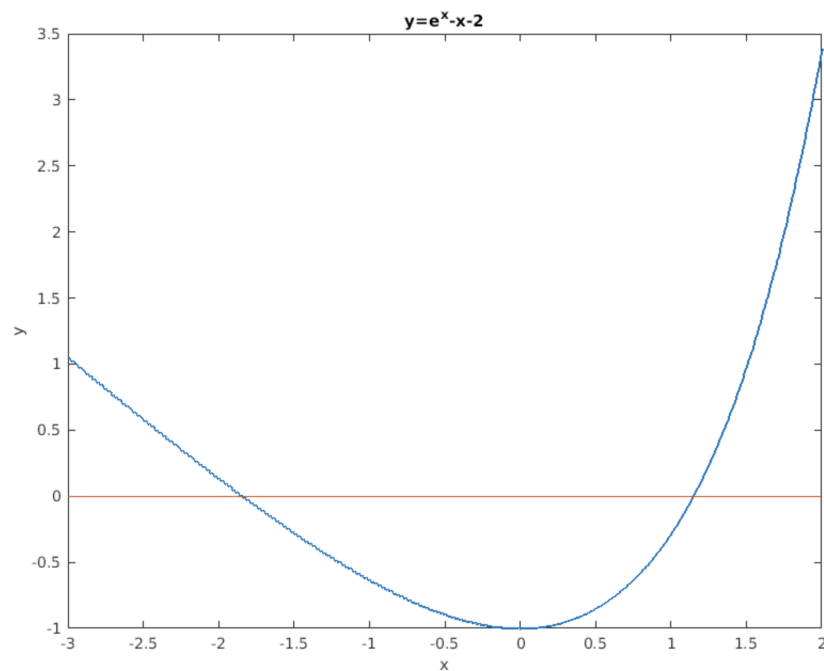
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	-1	1	-5	-0.8	-0.2
2	-0.8	0.2096	-3.048	-0.731234	-0.0687664
3	-0.731234	0.0171404	-2.56397	-0.724548	-0.00668512
4	-0.724548	0.000142506	-2.52147	-0.724492	-5.6517e-05
5	-0.724492	1.00606e-08	-2.52111	-0.724492	-3.99053e-09

## 2.2 Question 2

### 2.2.1 Equation:

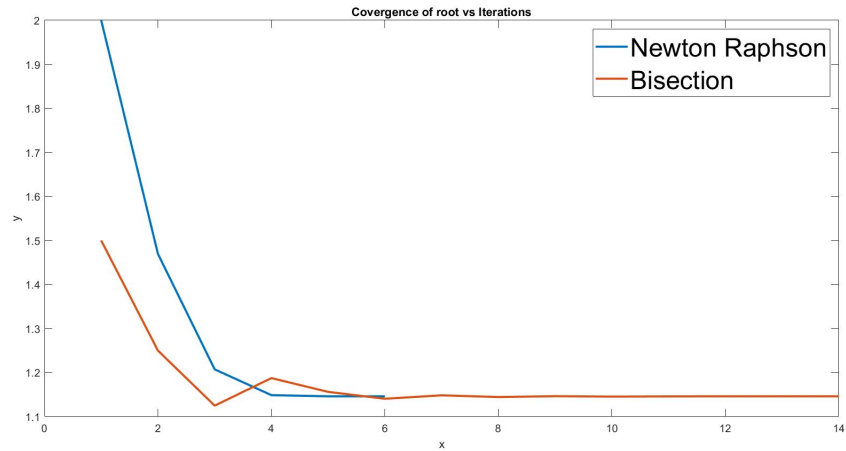
$$y = e^x - x - 2 \quad (9)$$

### 2.2.2 Graph:



- So, from the above graph we can observe that there are two real roots of this equation between  $(-2, -1)$  and  $(1, 2)$ .

Root between :  $a = 1$  and  $b = 1.5$

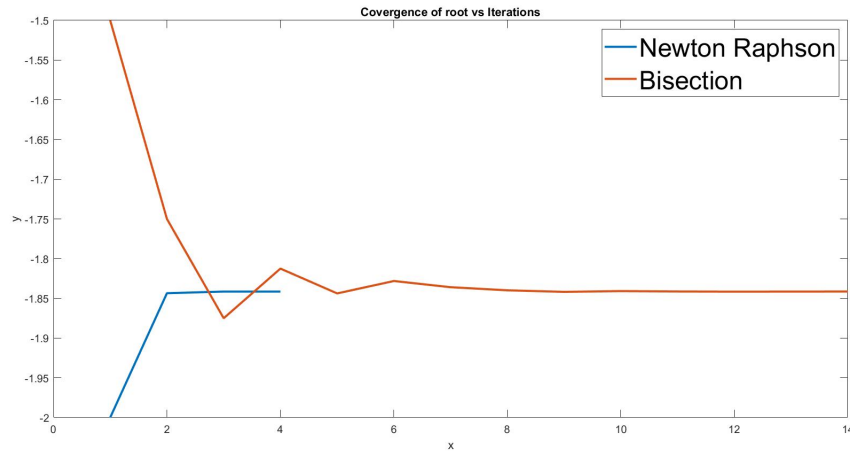


- Above shown plot is of convergence of first root vs iterations.
- Value of positive root is 1.1462.
- Root value is obtained in 6 iterations while Bisection method took 14 iterations.

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	2	3.38906	6.38906	1.46955	0.530447
2	1.46955	0.877738	3.34729	1.20733	0.262223
3	1.20733	0.137212	2.34454	1.14881	0.0585239
4	1.14881	0.00561748	2.15442	1.1462	0.00260742
5	1.1462	1.07135e-05	2.14621	1.14619	4.99185e-06
6	1.14619	3.91989e-11	2.14619	1.14619	1.82643e-11



Root between :  $a = -1$  and  $b = -0.5$



- Above shown plot is of convergence of first root vs iterations.
- Value of negative root is -1.8414.
- Root value is obtained in 4 iterations while Bisection method took 14 iterations.

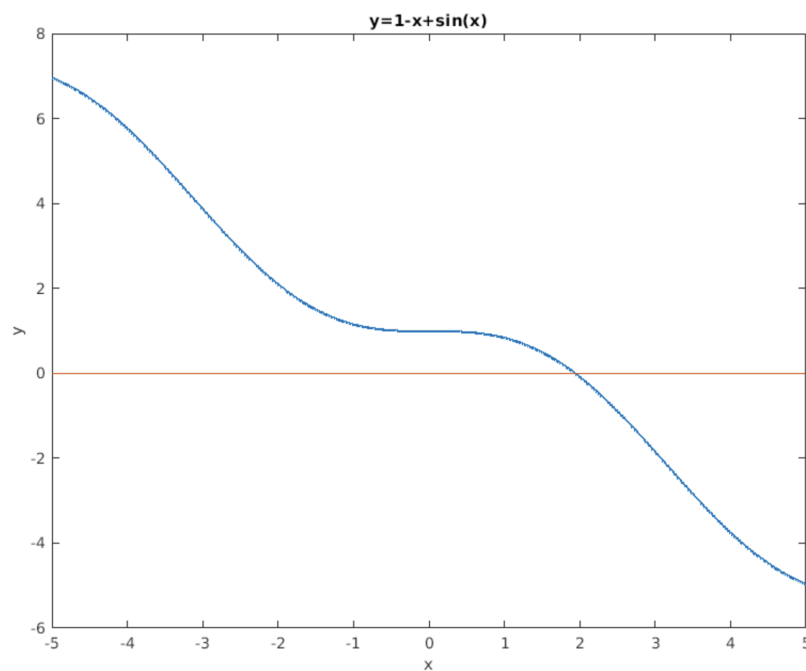
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	-2	0.135335	-0.864665	-1.84348	-0.156518
2	-1.84348	0.00174769	-0.841735	-1.84141	-0.00207629
3	-1.84141	3.41376e-07	-0.841406	-1.84141	-4.05721e-07
4	-1.84141	1.28786e-14	-0.841406	-1.84141	-1.53211e-14

## 2.3 Question 3

### 2.3.1 Equation:

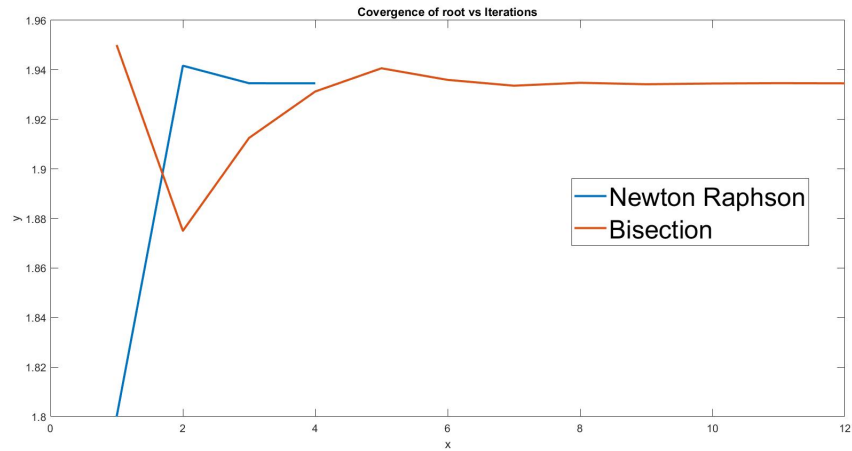
$$y = 1 - x + \sin(x) \quad (10)$$

### 2.3.2 Graph:



- So, from the above graph we can observe that there is one real root of these equation between 1 and 3.

Root Between :  $a = 1.8$  and  $b = 2.1$



- Above shown plot is of convergence of first root vs iterations.
- Value of smallest positive root is 1.9346.
- Root value is obtained in 4 iterations while Bisection method took 12 iterations.

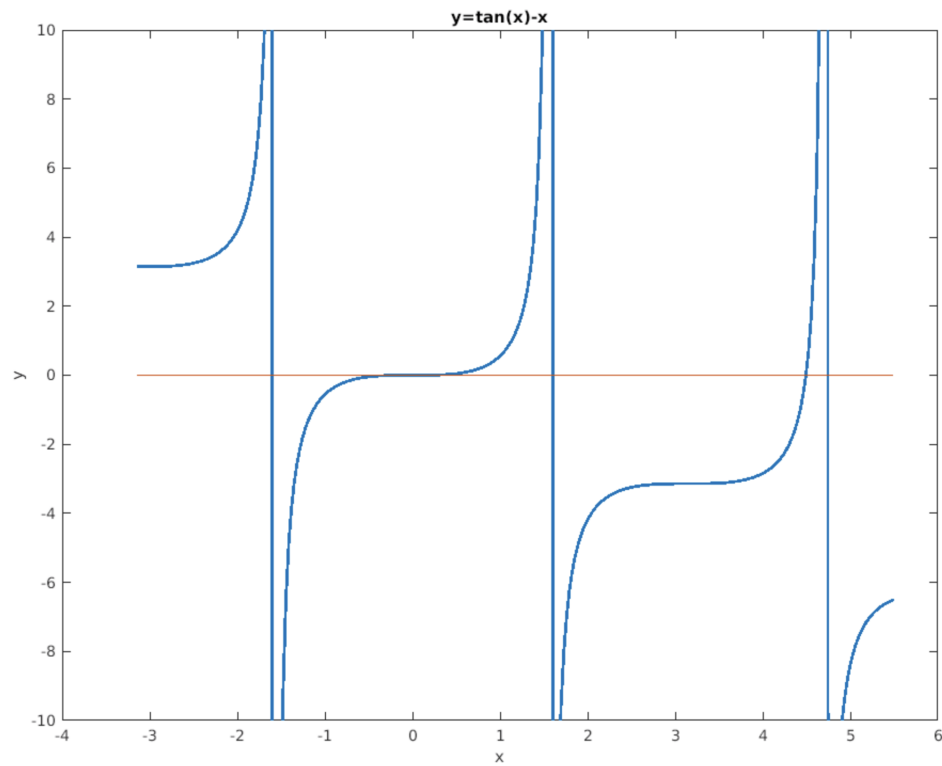
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	1.8	0.173848	-1.2272	1.94166	-0.141662
2	1.94166	-0.00964774	-1.36242	1.93458	0.00708132
3	1.93458	-2.33893e-05	-1.35581	1.93456	1.72511e-05
4	1.93456	-1.39063e-10	-1.3558	1.93456	1.02569e-10

## 2.4 Question 4

### 2.4.1 Equation:

$$y = \tan(x) - x \quad (11)$$

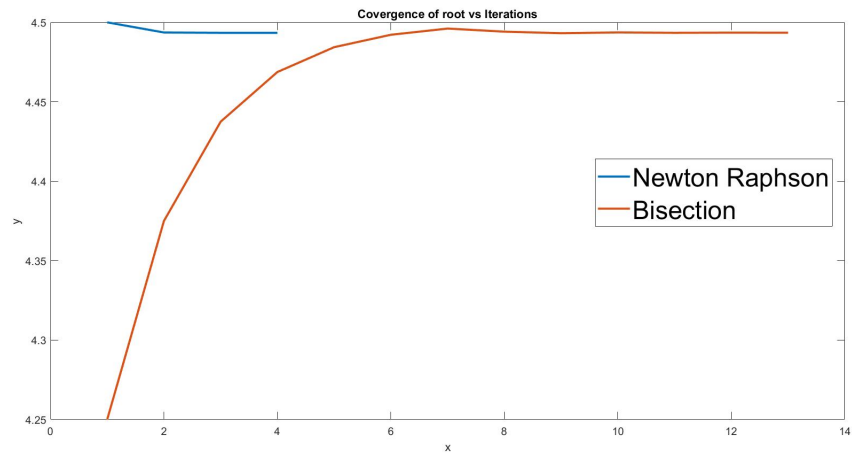
### 2.4.2 Graph:



- So, from the above graph we can observe that there are infinitely many positive real roots possible for this equation, smallest of which lies between 4 and 5.

### 2.4.3 Smallest positive real root

Assumed values :  $a = 4$  and  $b = 4.5$



- Above shown plot is of convergence of first root vs iterations.
- Value of smallest positive root is 4.4934.
- Root value is obtained in 4 iterations while Bisection method took 13 iterations.

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	4.5	0.137332	21.5048	4.49361	0.0063861
2	4.49361	0.00413187	20.2297	4.49341	0.000204248
3	4.49341	3.97968e-06	20.1908	4.49341	1.97104e-07
4	4.49341	3.69482e-12	20.1907	4.49341	1.82965e-13

#### 2.4.4 Positive real root near $x = 100$

Assumed values :  $a = 98.5$  and  $b = 99.5$

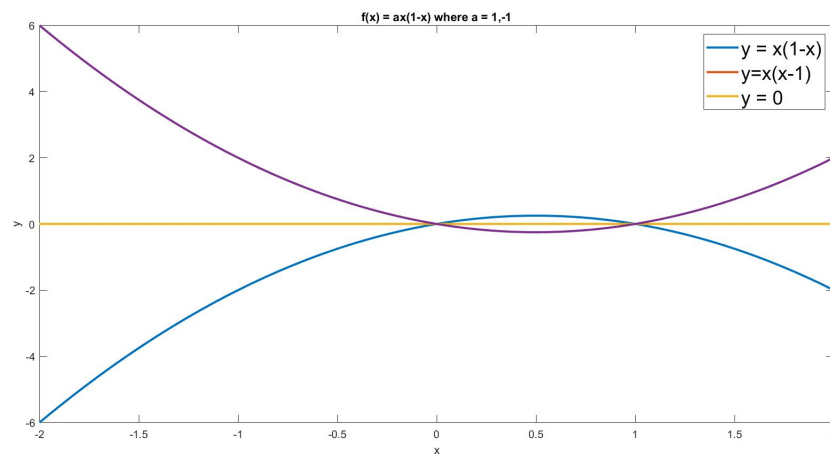
- Above shown plot is of convergence of approximate root vs iterations.
- The value of  $\tan(x)$  is infinite at 97.38 which is in the range of assumed values hence we can't find the exact root using Bisection method.
- Root value is obtained only if we take  $x_n = 98.95$ . in the first iteration itself else it is not obtained due to derivative being  $\infty$ .

## 2.5 Question 5

### 2.5.1 Equation:

$$y = ax(1 - x) \quad \text{where } a=1,-1 \quad (12)$$

### 2.5.2 Graph:

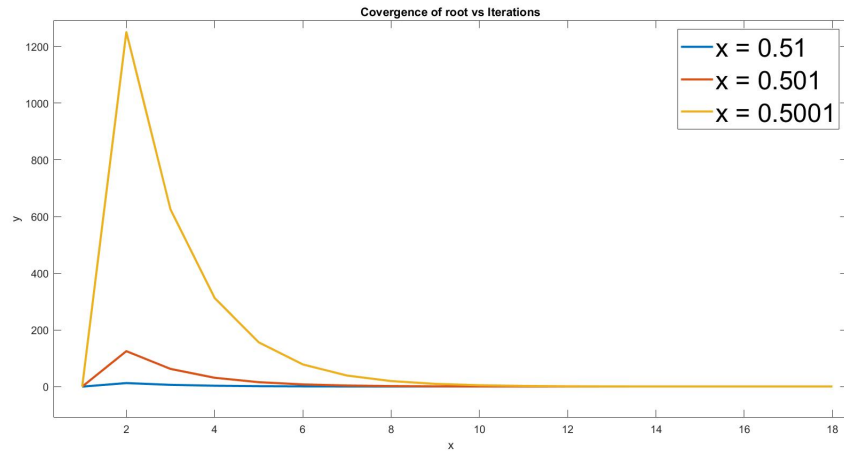


- So, from the above graph we can observe that there are two real roots at  $x = 0$  and  $x = 1$ .

### For Root $x = 0$

- Function  $f(x)$  has turning points at  $x = 0.5$ . So if we take the initial value greater than  $x = 0.5$ , Newton Raphson method converges  $x = 1$  root instead of  $x = 0$ .
- So, for all the given initial conditions, Newton Raphson method will give the root  $x = 1$  instead of  $x = 0$  as shown in the graph above and below table.

- For  $a = -1$ , similar behaviour is observed with just changes in sign of  $f(x)$  and  $f'(x)$ .

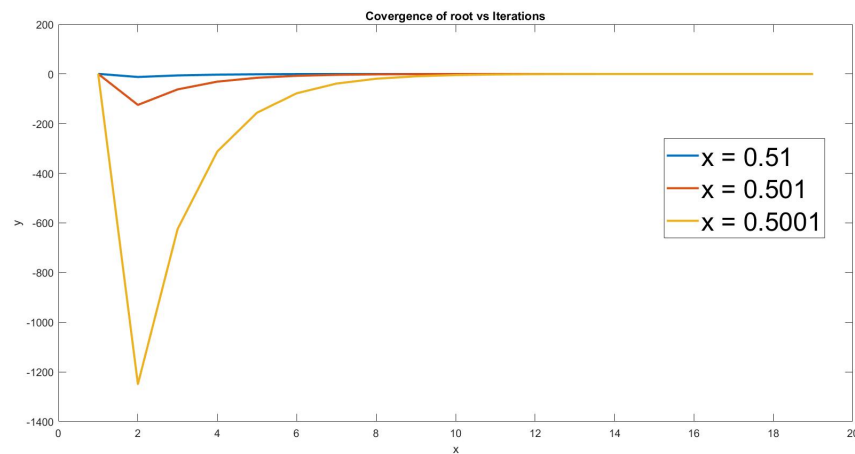


Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	0.5001	0.25	-0.0002	1250.5	-1250
2	1250.5	-1.5625e+06	-2500	625.5	625
3	625.5	-390625	-1250	313	312.5
4	313	-97656.2	-625.001	156.751	156.25
5	156.751	-24414	-312.501	78.6261	78.1245
6	78.6261	-6103.43	-156.252	39.5646	39.0614
7	39.5646	-1525.8	-78.1293	20.0355	19.5291
8	20.0355	-381.386	-39.071	10.2742	9.76136
9	10.2742	-95.2841	-19.5483	5.39987	4.87429
10	5.39987	-23.7587	-9.79973	2.97544	2.42442
11	2.97544	-5.87783	-4.95089	1.78822	1.18723
12	1.78822	-1.40951	-2.57644	1.24114	0.547076
13	1.24114	-0.299292	-1.48228	1.03923	0.201913
14	1.03923	-0.0407687	-1.07846	1.00143	0.0378027
15	1.00143	-0.00142905	-1.00285	1	0.00142498
16	1	-2.03057e-06	-1	1	2.03056e-06
17	1	-4.12315e-12	-1	1	4.12315e-12
18	1	0	-1	1	0



## For Root $x = 1$

- Function  $f(x)$  has turning points at  $x = 0.5$ . So if we take the initial value less than  $x = 0.5$ , Newton Raphson method converges  $x = 0$  root instead of  $x = 1$ .
- So, for all the given initial conditions, Newton Raphson method will give the root  $x = 0$  instead of  $x = 1$  as shown in the graph above and below table.
- For  $a = -1$ , similar behaviour is observed with just changes in sign of  $f(x)$  and  $f'(x)$ .



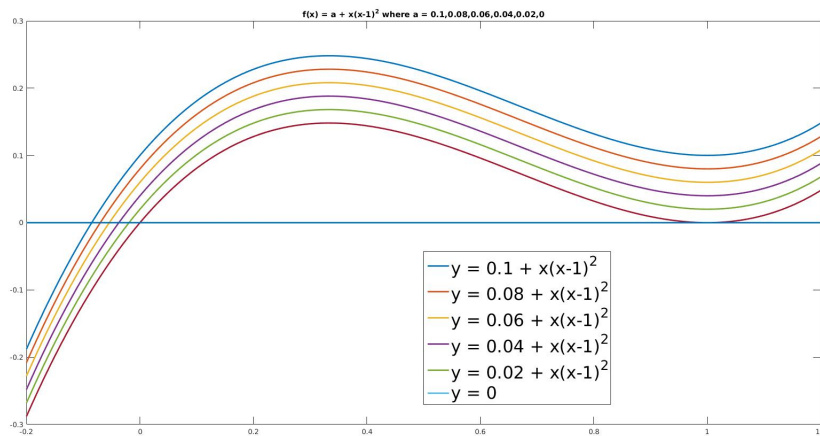
Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	0.4999	0.25	0.0002	-1249.5	1250
2	-1249.5	-1.5625e+06	2500	-624.5	-625
3	-624.5	-390625	1250	-312	-312.5
4	-312	-97656.2	625.001	-155.751	-156.25
5	-155.751	-24414	312.501	-77.6261	-78.1245
6	-77.6261	-6103.43	156.252	-38.5646	-39.0614
7	-38.5646	-1525.8	78.1293	-19.0355	-19.5291
8	-19.0355	-381.386	39.071	-9.27416	-9.76136
9	-9.27416	-95.2841	19.5483	-4.39987	-4.87429
10	-4.39987	-23.7587	9.79973	-1.97544	-2.42442
11	-1.97544	-5.87783	4.95089	-0.788218	-1.18723
12	-0.788218	-1.40951	2.57644	-0.241142	-0.547076
13	-0.241142	-0.299292	1.48228	-0.0392297	-0.201913
14	-0.0392297	-0.0407687	1.07846	-0.00142701	-0.0378027
15	-0.00142701	-0.00142905	1.00285	-2.03056e-06	-0.00142498
16	-2.03056e-06	-2.03057e-06	1	-4.12316e-12	-2.03056e-06
17	-4.12316e-12	-4.12316e-12	1	-1.70008e-23	-4.12316e-12
18	-1.70008e-23	-1.70008e-23	1	0	-1.70008e-23
19	0	0	1	0	0

## 2.6 Question 6

### 2.6.1 Equation:

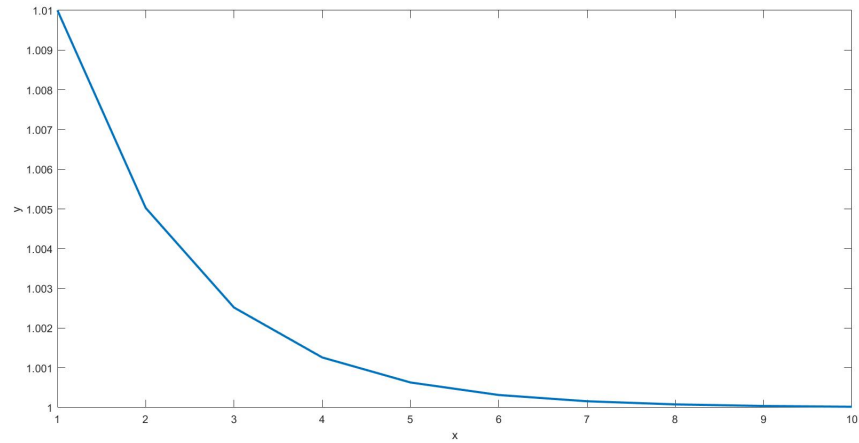
$$y = a + x(x - 1)^2 \quad \text{where } 0 \leq a \leq 0.1 \quad (13)$$

### 2.6.2 Graph:



- For all the values except  $a = 0$ , there is a turning point between the negative real root and the initial point considered, thus we won't be able to find the root using Newton Raphson method.
- For  $a = 0$ , also Newton Raphson method won't converge to negative real root due to the turning point but will converge to  $x = 1$ .

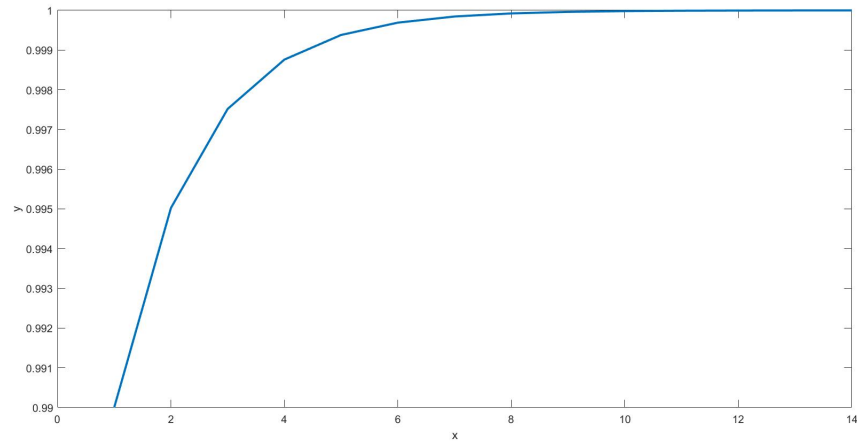
For  $a = 0$ ; Initial value  $x = 1.01$



Convergence towards root  $x = 1$

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	1.01	0.000101	0.0203	1.00502	0.00497537
2	1.00502	2.53738e-05	0.010125	1.00252	0.00250605
3	1.00252	6.35922e-06	0.00505619	1.00126	0.00125771
4	1.00126	1.5918e-06	0.00252651	1.00063	0.000630038
5	1.00063	3.982e-07	0.00126286	1.00032	0.000315316
6	1.00032	9.95812e-08	0.000631329	1.00016	0.000157733
7	1.00016	2.48992e-08	0.00031564	1.00008	7.8885e-05
8	1.00008	6.2253e-09	0.000157814	1.00004	3.94472e-05
9	1.00004	1.55639e-09	7.89052e-05	1.00002	1.97248e-05
10	1.00002	3.89104e-10	3.94522e-05	1.00001	9.86267e-06

For  $a = 0$ ; Initial value  $x = 0.99$



Convergence towards root  $x = 1$

Count	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	Error
1	0.99	9.9e-05	-0.0197	0.995025	-0.00502538
2	0.995025	2.46237e-05	-0.009875	0.997519	-0.00249354
3	0.997519	6.14047e-06	-0.00494369	0.998761	-0.00124208
4	0.998761	1.5332e-06	-0.00247338	0.999381	-0.000619881
5	0.999381	3.83063e-07	-0.00123707	0.999691	-0.000309652
6	0.999691	9.5736e-08	-0.000618633	0.999845	-0.000154754
7	0.999845	2.39303e-08	-0.000309341	0.999923	-7.73591e-05
8	0.999923	5.98211e-09	-0.000154676	0.999961	-3.8675e-05
9	0.999961	1.49547e-09	-7.73396e-05	0.999981	-1.93364e-05
10	0.999981	3.7386e-10	-3.86702e-05	0.99999	-9.66792e-06
11	0.99999	9.34641e-11	-1.93352e-05	0.999995	-4.83389e-06
12	0.999995	2.33659e-11	-9.66762e-06	0.999998	-2.41693e-06
13	0.999998	5.84147e-12	-4.83381e-06	0.999999	-1.20846e-06
14	0.999999	1.46036e-12	-2.41691e-06	0.999999	-6.04229e-07