Assignment
CS374

Harsh Patel(201701021)
Viraj Patel(201701439)

Assigned by :

Prof. Arnab Kumar

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Contents

1 Question 1: Binary Search and Information Entropy

1.1 Part A

$$
\begin{aligned}
< I > &= -k \sum_i P_i \log_2 P_i \\
&= -kp \log_2 p - k(1-p) \log_2 (1-p)
\end{aligned}
$$

For finding maximum; equating derivative to zero;

$$
\begin{aligned}
\frac{d < I >}{dp} &= 0 \\
-k + k - k \log_2 p + k \log_2 (1-p) &= 0 \\
\log_2 p &= \log_2 (1-p) \\
p &= 1 - p \\
p &= \frac{1}{2}
\end{aligned}
$$

1.2 Part B

$$p = \frac{1}{2} + \epsilon \qquad \epsilon \ll \frac{1}{2}$$

$$< I > = -k(\frac{1}{2} + \epsilon)\frac{\ln(1+2\epsilon) - \ln 2}{\ln 2} - k(\frac{1}{2} - \epsilon)\frac{\ln(1-2\epsilon) - \ln 2}{\ln 2}$$

Considering $\ln(1+x) \approx x$ for $x \ll 1$

$$< I > \approx \frac{-k}{\ln 2} \times (4\epsilon^2 - \ln 2)$$

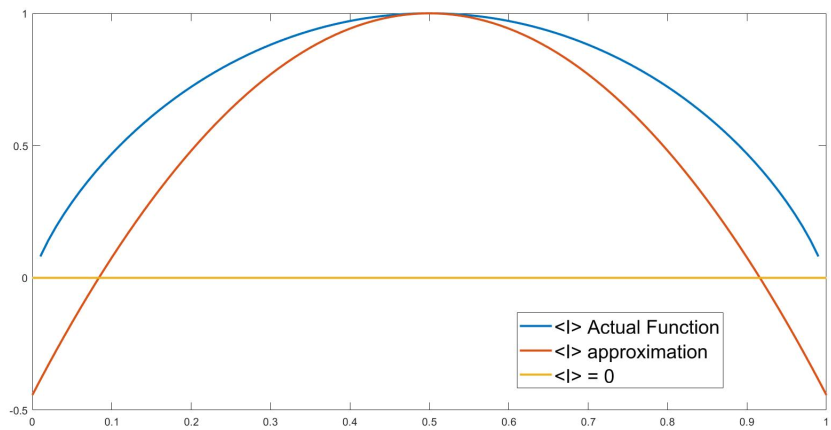
$$< I > \approx k - \frac{4k\epsilon^2}{\ln 2}$$

$$< I > \approx a - b\epsilon^2$$

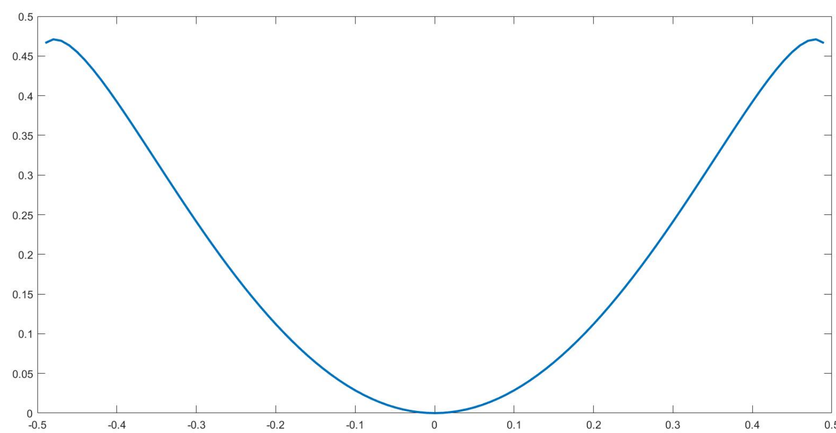
$$a = k$$

$$b = \frac{4k}{\ln 2}$$

1.3 Part C



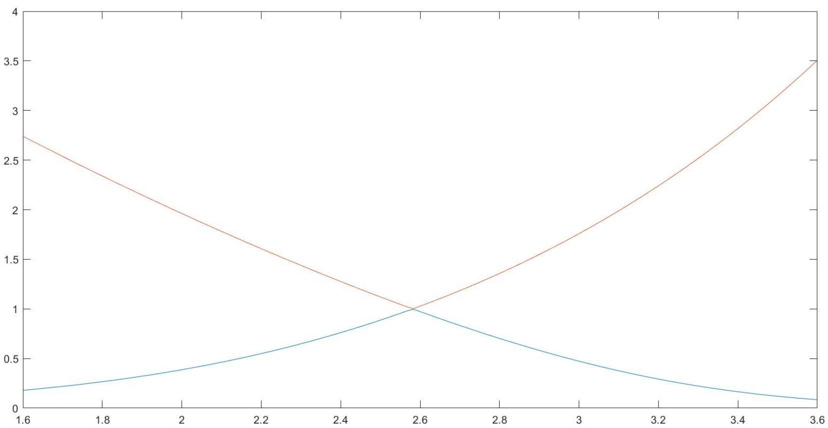
Plot of $< I >$ vs. P



Plot of Error between Actual $< I >$ and approximate $< I >$ vs. ϵ

2 Question 2 : An Astronomical Inflow

2.1 Graph



3 Question 3 : A Nuclear Outflow

$$xyR^2 = 1 \tag{1}$$

$$y^2 + 3x^2 - 4x = B \tag{2}$$

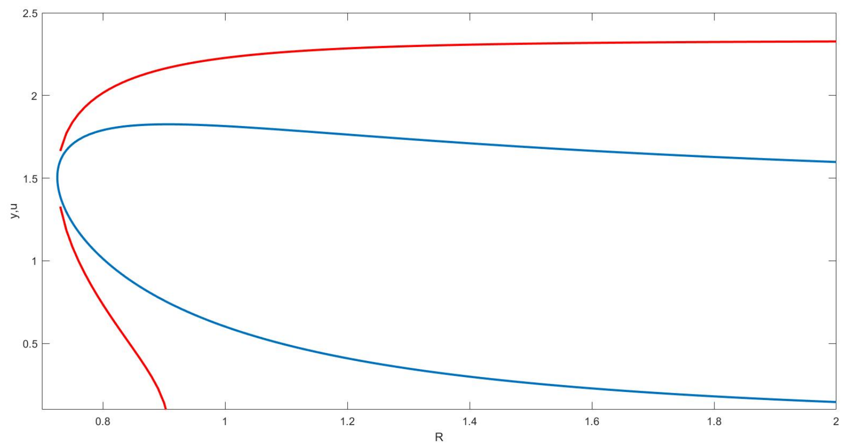
3.1 Singular and Turning Points

$$y^4 - 2y^2R^4 - 4yR^2 + 3 = 0 \tag{3}$$

$$\frac{dy}{dR} = \frac{2(2y + By^2R^2 - 4y^4R^2)}{2y^3R^3 - ByR^3 - 2R} \tag{4}$$

1. Denominator of Equation 3 is a cubic equation, So using Cardan's method we can analyze the equation.
2. $D = \frac{Q^2}{4} + \frac{P^3}{27}$
3. If $D > 0$ then there will be only one singular point.
4. If $D = 0$ then there will be two singular points as the equation would have three roots, two of which are equal
5. If $D < 0$ then there will be one singular point.

3.2 Graph



4 Question 4 : The Hydraulic Jump

$$4H - H^4 = 3(X - D) \tag{5}$$

4.1 $\frac{dH}{dX}$ for $X >= 0$

$$\frac{dH}{dX} = \frac{3}{4(1 - H^3)} \tag{6}$$

4.1.1 Implications

1. For $H = 1$ there will be vertical tangent as $\frac{dH}{dX} = \infty$.
2. For $H > 1$, the slope will be negative as $\frac{dH}{dX} < 0$.
3. For $H < 1$, the slope will be positive as $\frac{dH}{dX} > 0$.

4.2 Solve for $H = H(X)$

4.2.1 Descartes Method

$$x^4 + Qx^2 + Rx + S = (x^2 + kx + l)(x^2 - kx + m)$$

$$m = \frac{Q + k^2 + \frac{R}{k}}{2}$$

$$l = \frac{Q + k^2 - \frac{R}{k}}{2}$$

$$lm = S$$

$$k^6 + 2Qk^4 + (Q^2 - 4S)k^2 - R^2 = 0$$

4.2.2 $H = X = 0$

$$4H - H^4 = 3X \tag{7}$$

$$R = -4$$

$$S = 3X$$

$$Q = 0$$

4.2.3 $H = 1$ and $X = 2$

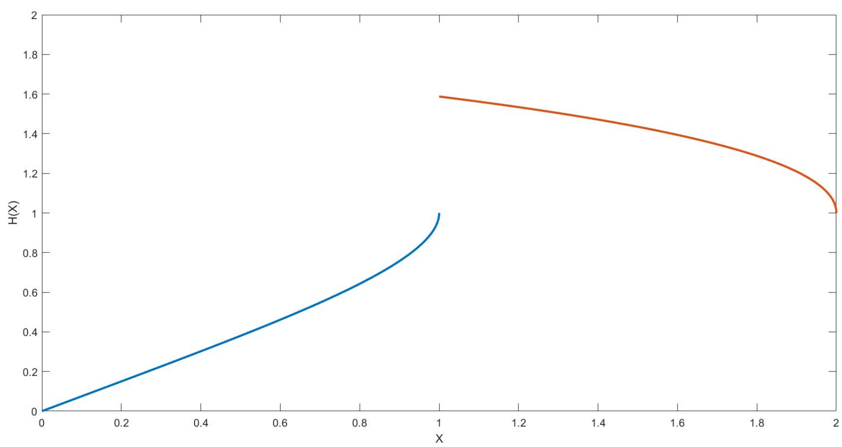
$$4H - H^4 = 3(X - 1) \tag{8}$$

$$R = -4$$

$$S = 3X - 3$$

$$Q = 0$$

4.2.4 Graph



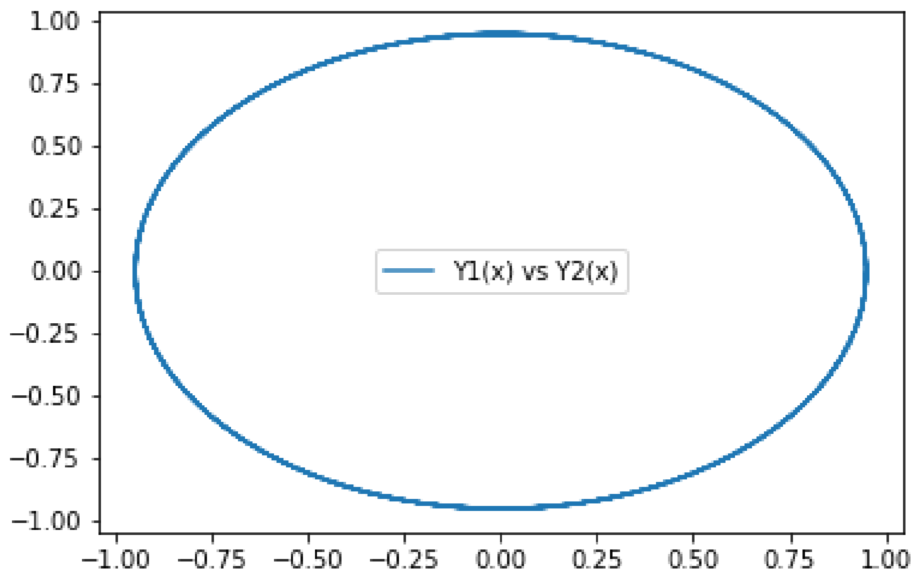
Plot of $H(X)$ vs X for both conditions

5 Question 5 : The Lienard System

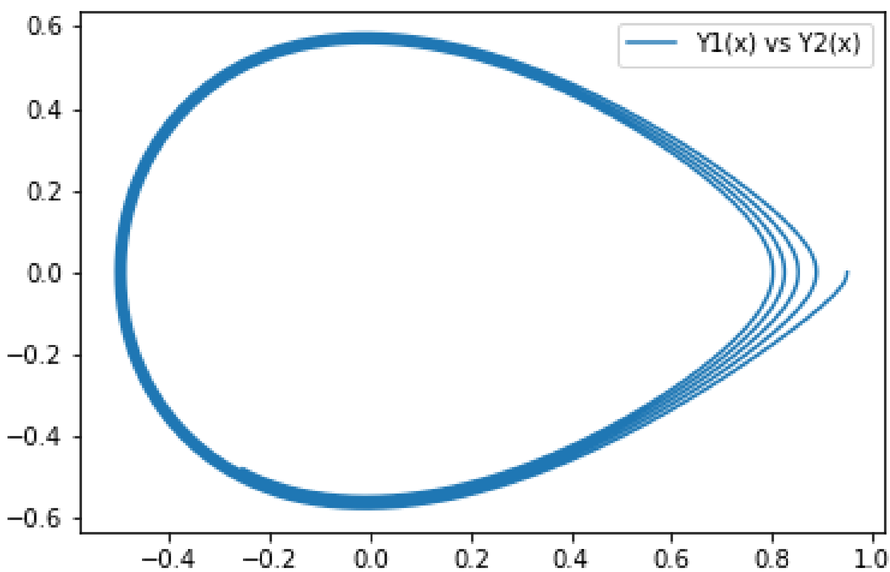
$$\phi' = \nu$$

$$\nu' = -\epsilon(A\phi + B\nu)\nu - (C\phi + \epsilon\phi^2 D)$$

5.1 Graph



Plot for $\epsilon = 0$



Plot for $\epsilon = 1$