Assignment 6
CS374

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Assigned by :

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Contents

1 Lagrange Linear Interpolation and $y = \sqrt{x}$

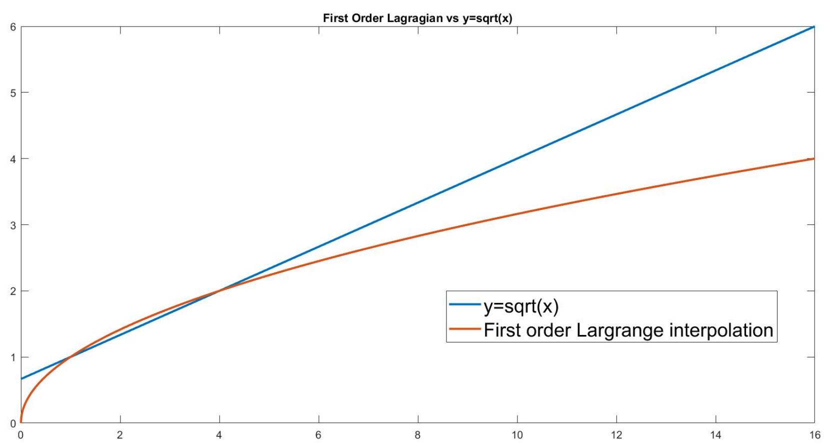
x	1	4
y	1	2

1.1 Equation

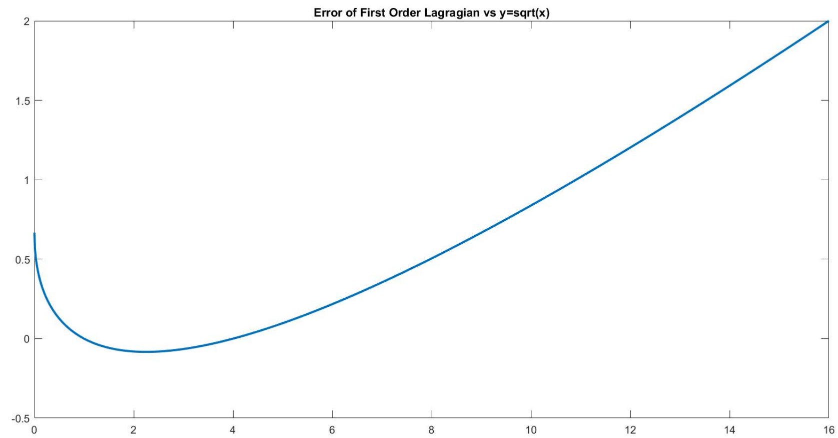
$$y = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1 \tag{1}$$

$$y = \sqrt{x} \tag{2}$$

1.2 Graph



Plot of $y = \sqrt{x}$ and Lagrange Linear Interpolation



Plot of Error between $y = \sqrt{x}$ and Lagrange Linear Interpolation

2 Lagrange Linear and Quadratic Interpolation

x	0.82	0.83	0.84
y	2.2705	2.293319	2.316367

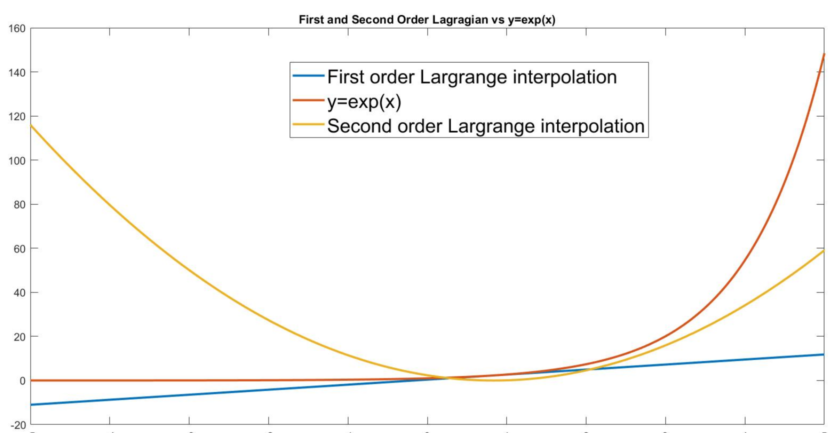
2.1 Equation

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2 \quad (3)$$

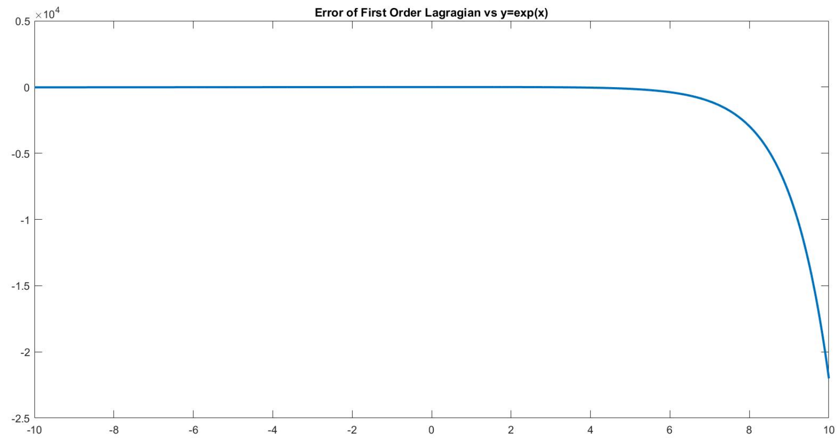
$$y = \frac{x-x_1}{x_0-x_1}y_0 + \frac{x-x_0}{x_1-x_0}y_1 \quad (4)$$

$$y = e^x \quad (5)$$

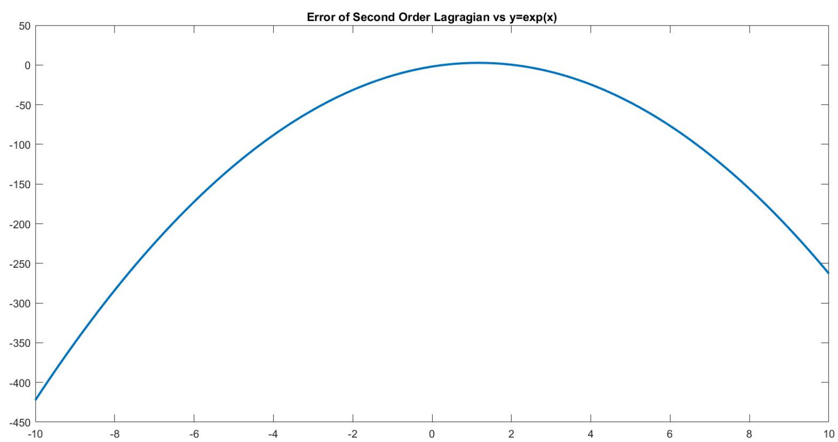
2.2 Graph



Plot of Lagrange Linear and Quadratic Interpolation and $y = e^x$



Plot of Error between Lagrange Linear Interpolation and $y = e^x$



Plot of Error between Lagrange Quadratic Interpolation and $y = e^x$

3 Lagrange Quadratic and Second order Newton Divided Difference

x	0	1	2
y	-1	-1	7

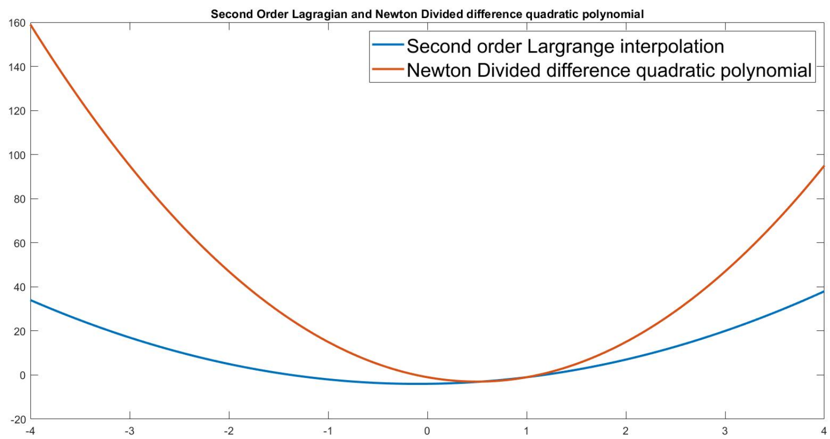
3.1 Equation

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2 \tag{6}$$

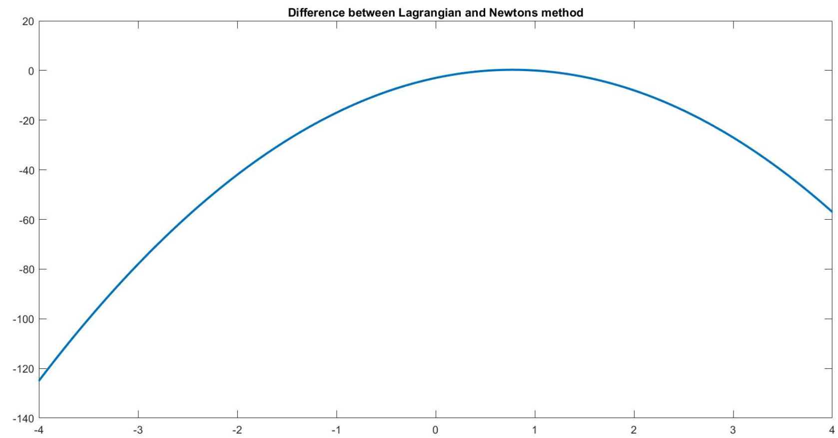
$$p_1(x) = y_0 + \frac{y_1-y_0}{x_1-x_0}(x-x_0) \tag{7}$$

$$p_2(x) = p_1(x) + (x-x_0)(x-x_1)f[x_0,x_1,x_2] \tag{8}$$

3.2 Graph



Plot of Lagrange Quadratic Interpolation and Newton Second Order Interpolation



Plot of Error between Lagrange Quadratic Interpolation and Newton Second Order Interpolation

4 Newton and Lagrange Interpolation

x	3.35	3.40	3.50	3.60
y	0.298507	0.294118	0.285714	0.277778

4.1 Equations

$$y = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1 \tag{9}$$

$$y = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \tag{10}$$

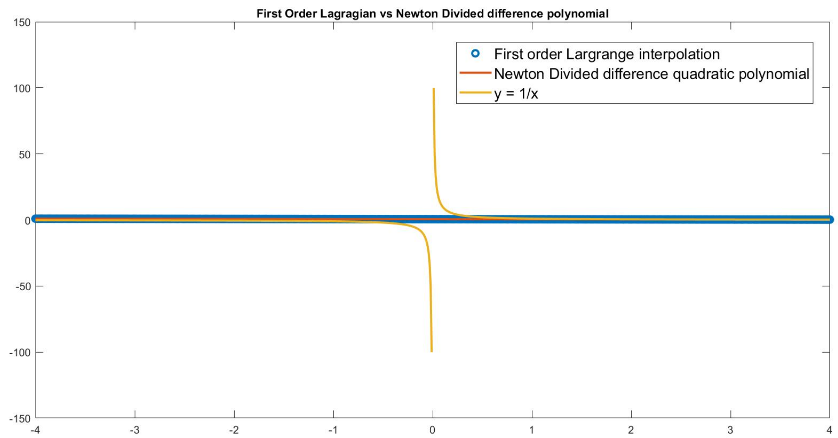
$$y = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3$$

$$p_1(x) = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) \tag{11}$$

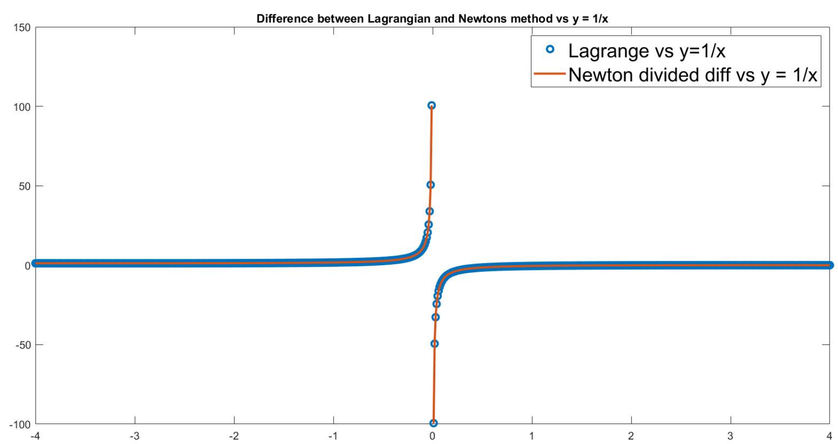
$$p_2(x) = p_1(x) + (x - x_0)(x - x_1) f[x_0, x_1, x_2] \tag{12}$$

$$p_3(x) = p_2(x) + (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \tag{13}$$

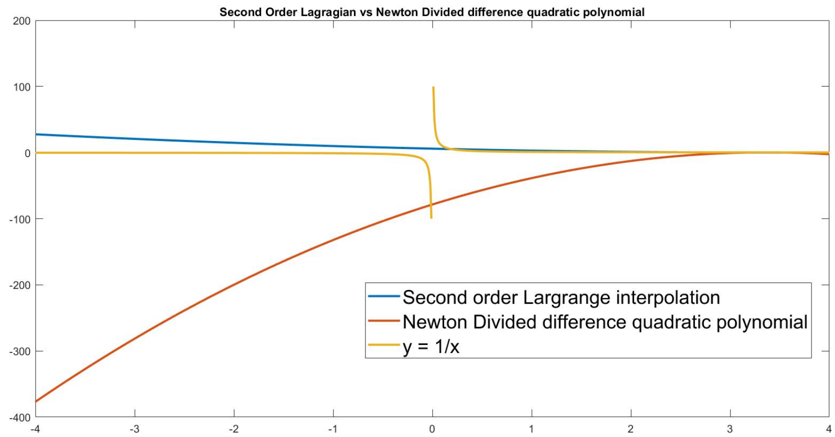
4.2 Linear Interpolation and $y = 1/x$



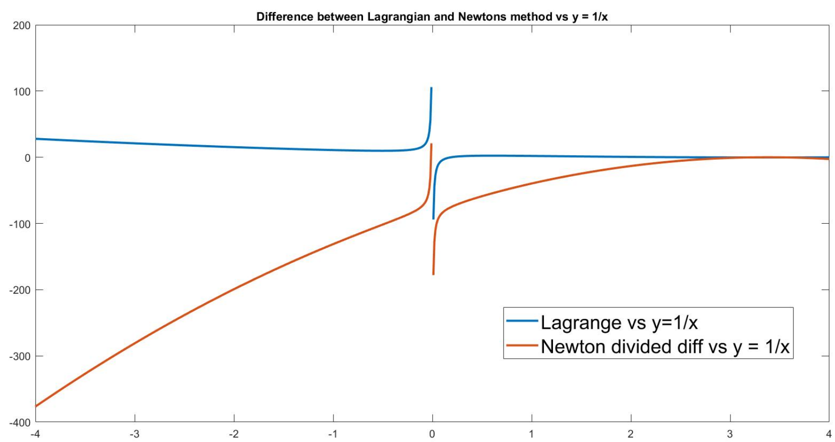
4.3 Error between Linear Interpolation and $y = 1/x$



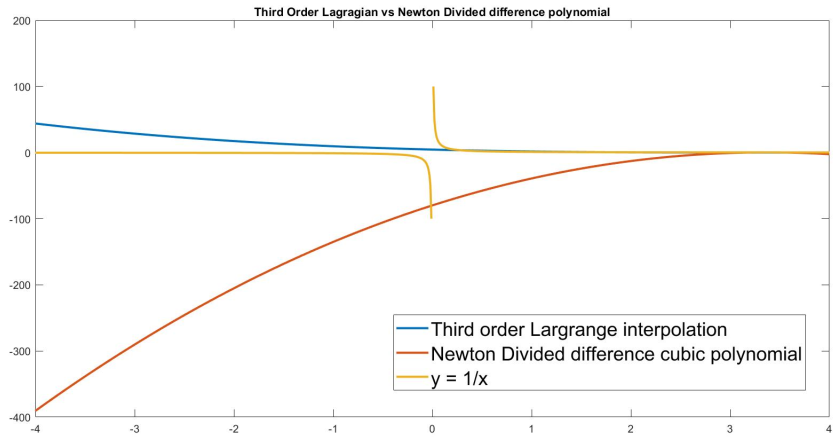
4.4 Linear Interpolation and $y = 1/x$



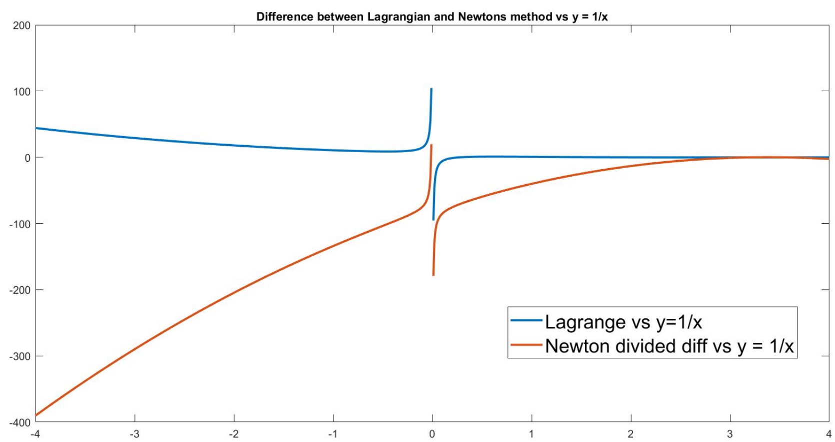
4.5 Error between Quadratic Interpolation and $y = 1/x$



4.6 Linear Interpolation and $y = 1/x$



4.7 Error between Cubic Interpolation and $y = 1/x$



4.8 Conclusion

1. Lagrange Interpolation is computationally more efficient as it doesn't require memory for saving prior values.

2. As Newton Divided Difference Interpolation requires the prior values to be stored, so it uses more memory and is computationally less efficient.

5 Interpolation in Sub - Intervals

x	0	1	2	2.5	3	3.5	4
y	2.5	0.5	0.5	1.5	1.5	1.125	0

5.1 Equations

$$y = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1 \tag{14}$$

$$y = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x2)} y_0 + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \tag{15}$$

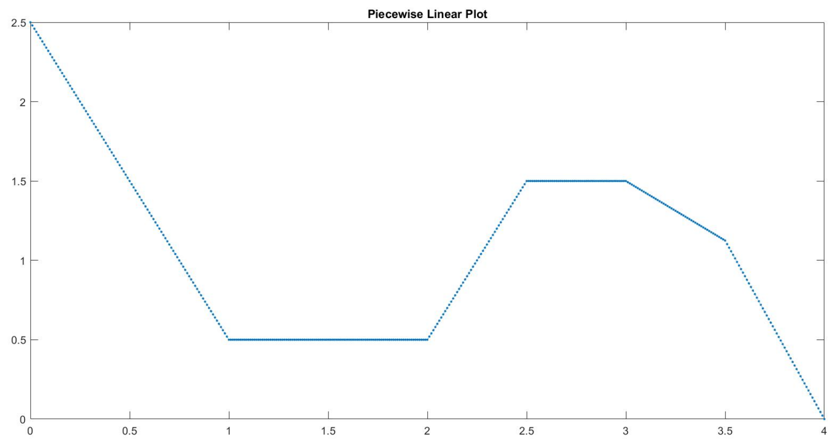
$$y = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3$$

$$p_1(x) = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) \tag{16}$$

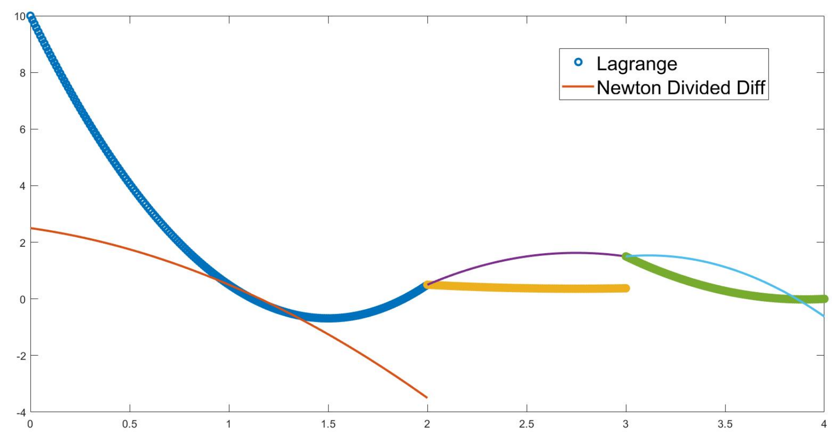
$$p_2(x) = p_1(x) + (x - x_0)(x - x_1) f[x_0, x_1, x_2] \tag{17}$$

$$p_3(x) = p_2(x) + (x - x_0)(x - x_1)(x - x_2) f[x_0, x_1, x_2, x_3] \tag{18}$$

5.2 Piecewise Linear Interpolation



5.3 Newton and Lagrange Quadtratic Interpolation in subintervals



5.4 Error between Newton and Lagrange Inerpolation