CS302: Modelling and Simulation
Lab: 02

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***Abstract*— This report tries to develop qualitative understanding of the Bass model using numerical analysis. Bass Model, the model used for modelling the diffusion of innovation has two parameters. The longevity and the speed at which the product is adopted in the market with various types of customer depends on these two parameters. Initially the market share is zero but in the due course of time different types of consumers play different role in the rate at which the product is adapted and at last the market share of the product saturates. This report tries to model the human population using a linear curve and a logistic curve and tries to develop an understanding about the fitting of both of these curves .**

I. Introduction

In this model, we model the diffusion of innovation using Bass model. There are two parameters which changes the longevity and the speed at which the product is adapted. We will be looking at the rate of change of adaption of product by varying the parameters. We will also be looking at the rate of change of adaption taking p as zero, q as zero and none of them as zero. More formally, we will be looking at the market share considering both internal and external influence, considering only external influence and considering only internal influence.

II. Model

Assumptions:

1) In model of innovation diffusion the initial market share of new product is zero. Considering $p = 0$ we will give zero market share for the whole timescale because initial market share is zero. So for the case of no external influence, we have considered p of the order of 10^{-16}.
2) In human population model, the carrying capacity and the rate of increase are considered constants and not function of time.

The mathematical model for the Bass Model is as follows:

Model of innovation diffusion:

$$\alpha(t) = p + \frac{qN(t)}{C}$$
$$\frac{dN}{dt} = -\alpha(t)(C - N(t))$$

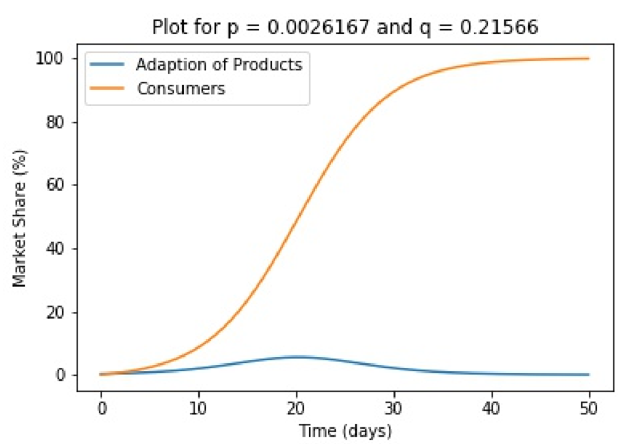
Model of human population:

$$\frac{dN}{dt} = rN(1 - \frac{N}{k})$$

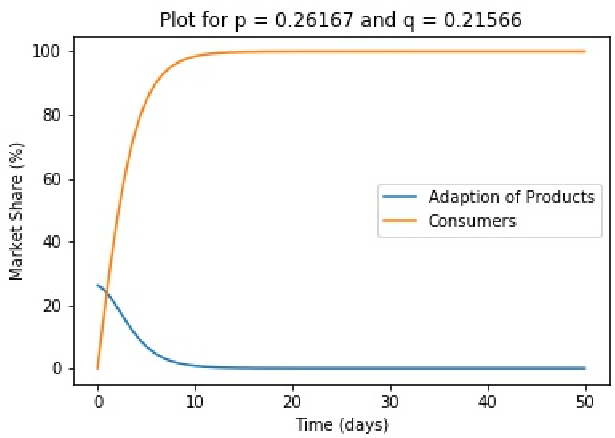
In the human population model, $N(t)$ is the human population at time t,r shows the rate of increase in the population and k is the carrying capacity. The data used for the study is is taken from the link
https://en.wikipedia.org/wiki/World_population_estimates
In the model of innovation diffusion $\alpha(t)$ is the coefficient of diffusion, C is the total population, N(t) is the total number of people who have adopted the product till time t. There will be three cases for this model:

1) External influence model in which $\alpha(t) = p$ where p is a constant and captures the innovators or people who adopt the product on their own without being influenced by others.
2) Internal influence model in which $\alpha(t) = \frac{qN(t)}{C}$ in which the rate $\alpha(t)$ now captures the adoption due to the effect of other users.
3) Mixed influence model $\alpha(t) = p + \frac{qN(t)}{C}$, which captures both the effects.

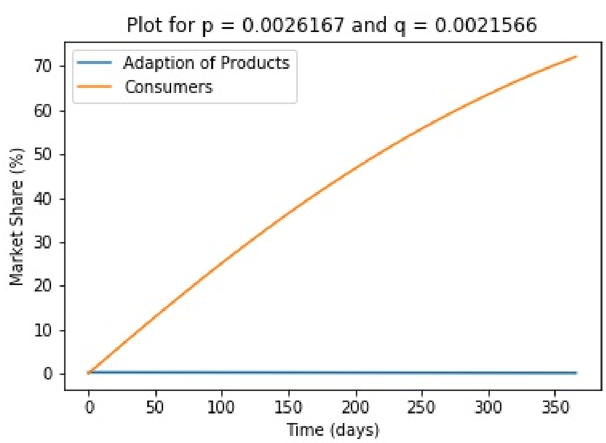
III. Results



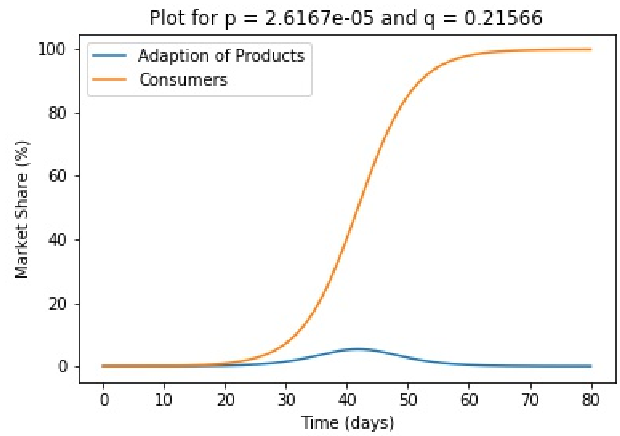
The initial increase in the graph of Adoption of product is due to the initial innovators who adopt the product without any influence. As time increases the the influence of innovators decreases and the imitators increase, the increase in market share reaches its maximum value and then the increase in market share(more formally the slope of the graph of adoption) keeps on decreasing. After a time, the market share saturates.



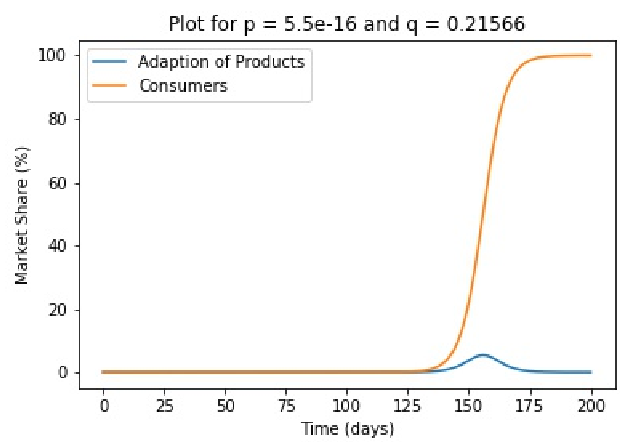
On increasing the value of p to a very high value, due to large external influence the value of market share will suddenly increase but the slope of the adoption graph i.e the increase in the market share keeps on decreasing and will saturate early compared to smaller vale of p. Here, for large value of p, the maximum number of consumers will be the innovators and so the curve of adoption keeps on increasing with decreasing slope.



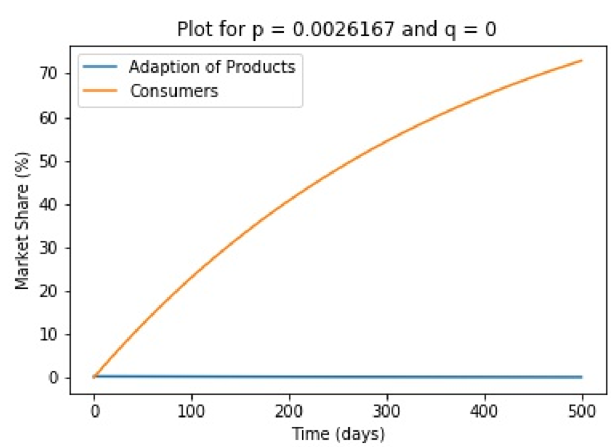
On decreasing the value of q, the time when the concavity changes and the time when the market share saturates increases. Thus the speed at which the product is adopted depends more upon the value of q while it depends on the value of p. Thus, same decrease in value of q and p in different cases, the change of the saturation time in the q case will more than it will be in p's case. As it was in the case of increasing the value of p, the market share will keep on increasing with decreasing slope.



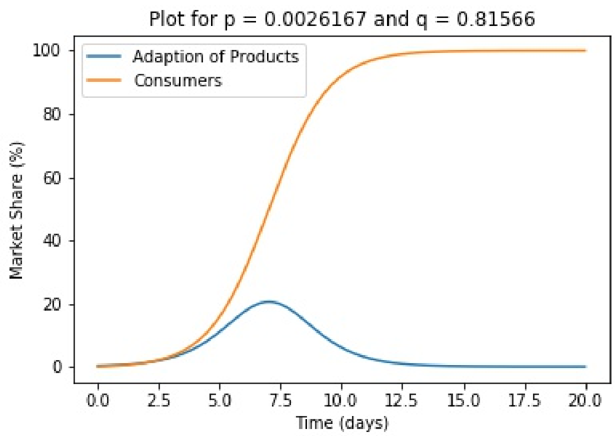
On decreasing the value of p, the time at which the market share curve changes concavity increases and the time at which it saturates also increase.



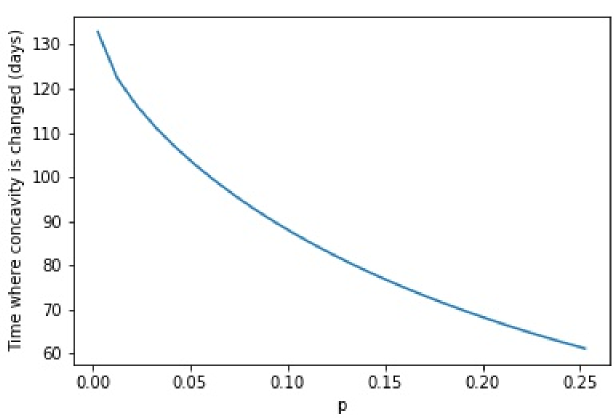
Further decreasing the value of p to almost zero(of the order of 10^{-16}), the saturation time and the time at which the market share curve changes concavity increases further. Here, the initial market share is taken as zero so we can't keep the value of p as zero else the value of market share on whole timescale will be zero. Thus, her we have studied the case using very small value of p.



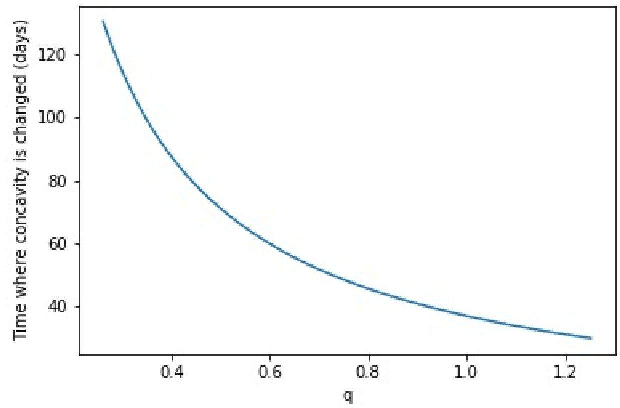
Similarly, further decreasing the value of q and taking it to zero, the market share curve will keep on increasing but the decrease in slope will be slower than the case of smaller values of q.



For, larger values of q, the time at which the curve of market share changes concavity decreases more significantly. Thus, on increasing the value of q, the speed at which the product is adopted increases significantly.

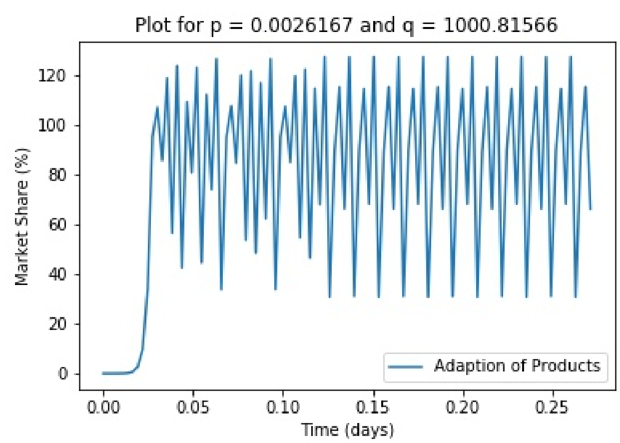


Plot of Time at which the curve of market share changes concavity vs the values of p

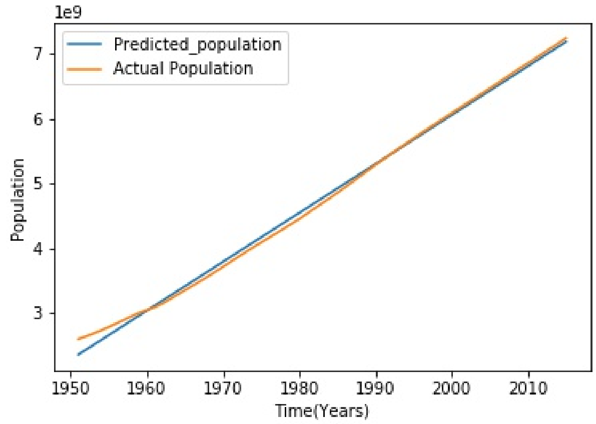


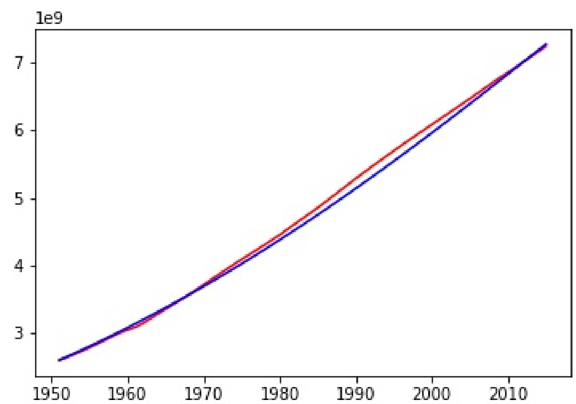
Plot of Time at which the curve of market share changes concavity vs the values of q

Comparing the above two curves, we can observe that the speed at which the product is adopted by consumers is more dependent on the values of q (More formally, increase in value of q leads to more increase in the speed of adoption than the increase brought by the increase in value of p). Thus, internal influence has more impact than the external influence on the speed at which the product is adopted and the time at which the curve of market share saturates.



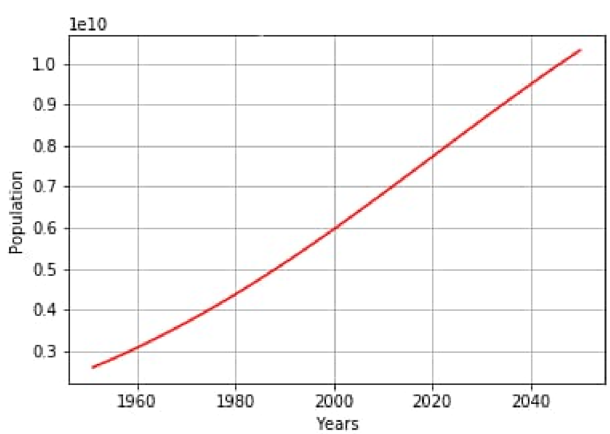
As seen in the above curve, after a threshold value of q(internal influence), the curve will become chaotic and it's period will keep on increasing. So, for very very large values of q the curve exhibits random oscillations as seen above.





Comparing the above two curves, we can observe that the logistic curve fits more better to the curve of human population(actual) than the linear curve. Predictions made by the logistic function are more better and it has more accuracy. As the human population saturates after long time and for larger time follows the logistic curve given by the equation stated in the model, it fits better.

Below is the curve plotted using the logistic equation and the calculated parameter values from the available data to provide an estimate for the US population in long run.



IV. Conclusion

Thus, to sum up our study, we modelled Bass model and analysed it by tweaking the values of it's parameters. Initially when a new product comes into the market, there are certain people called Innovators who buy the product. In the due course of time, the market share due to the Innovators decrease and market share due to the Imitators increase. In the end, the market share saturates to a value. Innovators define the external influence and Imitators define the internal influence and the effect due to external influencers is less than that of internal influence. And when we fit a curve to the curve of human population, Logistic curve fits better than linear as time increases there will be saturation in the population value but the linear curve will indefinitely increase.