Tutorial 4

1. Suppose that there are N distinct types of coupons and that each time one obtains a coupon, it is, independently of previous selections, equally likely to be any one of the N types. One random variable of interest is T, the number of coupons that needs to be collected until one obtains a complete set of at least one of each type. Derive $P\{T = n\}$.

2. A product that is sold seasonally yields a net profit of b dollars for each unit sold and a net loss of d dollars for each unit left unsold when the season ends. The number of units of the product that are ordered at a specific department store during any season is a random variable having probability mass function $p(i)$, $i \geq 0$. If the store must stock this product in advance, determine the number of units the store should stock so as to maximize its expected profit.

3.There are 2 coins in a bin. When one of them is flipped, it lands on heads with probability .6, and when the other is flipped, it lands on heads with probability .3. One of these coins is to be randomly chosen and then flipped. Without knowing which coin is chosen, you can bet any amount up to 10 dollars, and you then either win that amount if the coin comes up heads or lose it if it comes up tails. Suppose, however, that an insider is willing to sell you, for an amount C, the information as to which coin was selected. What is your expected payoff if you buy this information? Note that if you buy it and then bet x, you will end up either winning $x - C$ or $-x - C$ (that is, losing $x + C$ in the latter case). Also, for what values of C does it pay to purchase the information?

4.Teams A and B play a series of games, with the first team to win 3 games being declared the winner of the series. Suppose that team A independently wins each game with probability p. Find the conditional probability that team A wins

(a) the series given that it wins the first game;

(b) the first game given that it wins the series.