Tutorial 7

1. Suppose that if you are s minutes early for an appointment, then you incur the cost cs, and if you are s minutes late, then you incur the cost ks. Suppose also that the travel time from where you presently are to the location of your appointment is a continuous random variable having probability density function f. Determine the time at which you should depart if you want to minimize your expected cost.

2. Let X be uniformly distributed over (α, β). Find (a) $E[X]$ and (b) $\mathrm{Var}(X)$.

3. Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, $7:15$, $7:30$, $7:45$, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and $7:30$, find the probability that he waits

   (a) less than 5 minutes for a bus;

   (b) more than 10 minutes for a bus.

4. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2) & ; \quad -1 < x < 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

   (a) What is the value of c?

   (b) What is the cumulative distribution function of X?

5. Consider the function

$$f(x) = \begin{cases} C(2x-x^3) & ; \quad 0 < x < \frac{5}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

   Could f be a probability density function? If so, determine C. Repeat if $f(x)$ were given by

$$f(x) = \begin{cases} C(2x-x^2) & ; \quad 0 < x < \frac{5}{2} \\ 0 & ; \quad \text{otherwise} \end{cases}$$

6. The probability density function of X, the lifetime of a certain type of electronic device (measured in hours), is given by

$$f(x) = \begin{cases} \frac{10}{x^2} & ; \quad x > 10 \\ 0 & ; \quad x \le 10 \end{cases}$$

   (a) Find $P\{X > 20\}$.

   (b) What is the cumulative distribution function of X?

   (c) What is the probability that, of 6 such types of devices, at least 3 will function for at least 15 hours? What assumptions are you making?