Logistic Regression

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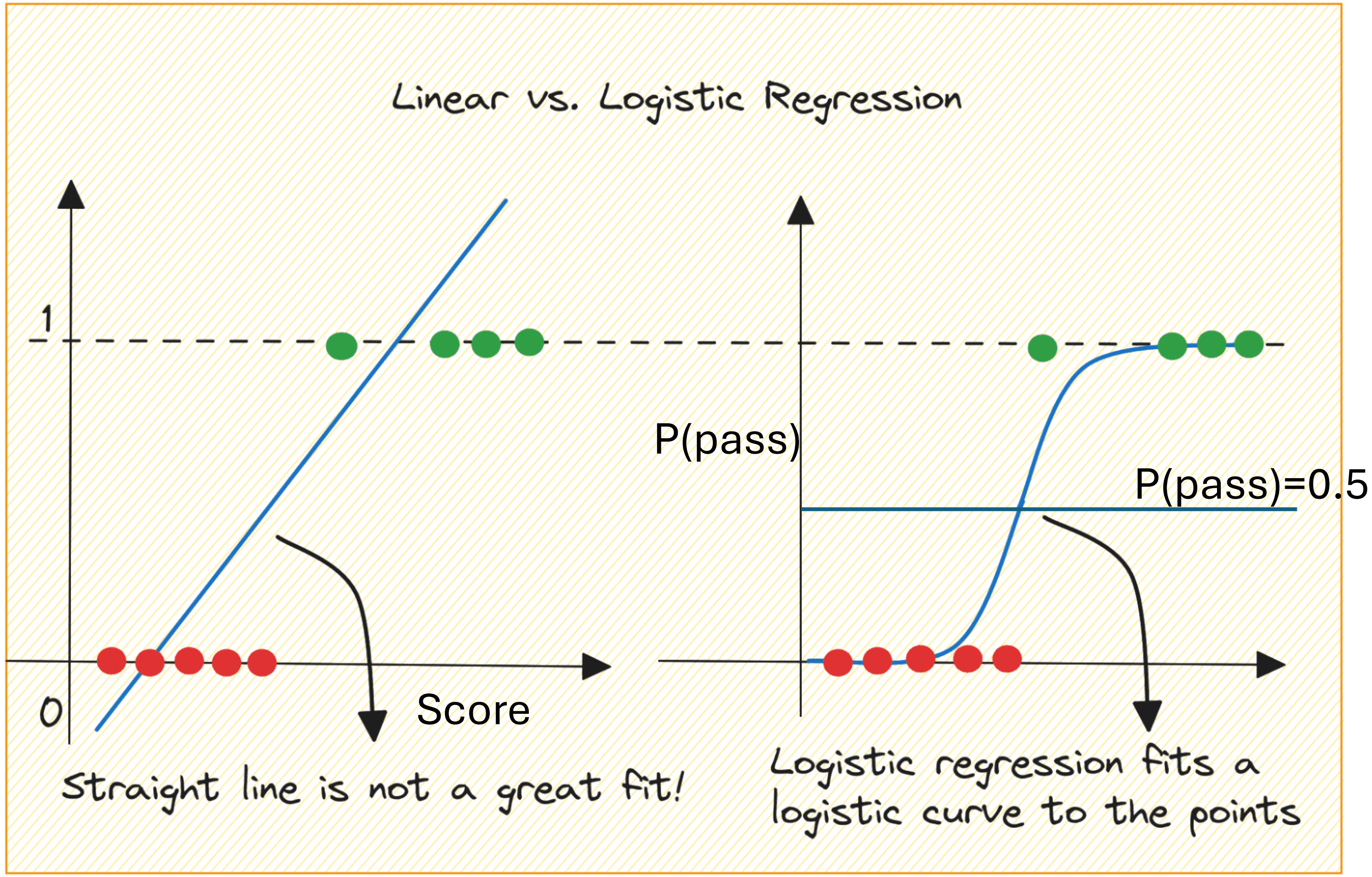
STUDY LOCALLY. LIVE GLOBALLY.

Linear vs Logistic Regression

Score	Label
10	Fail (0)
60	Pass (1)
80	Pass (1)
...	...
100	Pass (1)

Model a linear relationship between score and outcome (pass/fail).

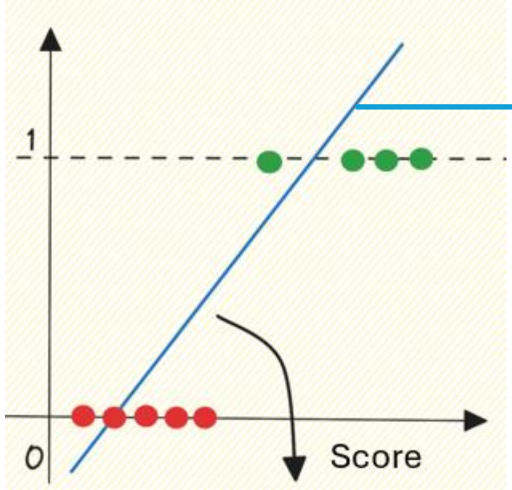


Sigmoid Function

- Use a sigmoid function to convert a real number (-inf,+inf) to a probability range (0,1).

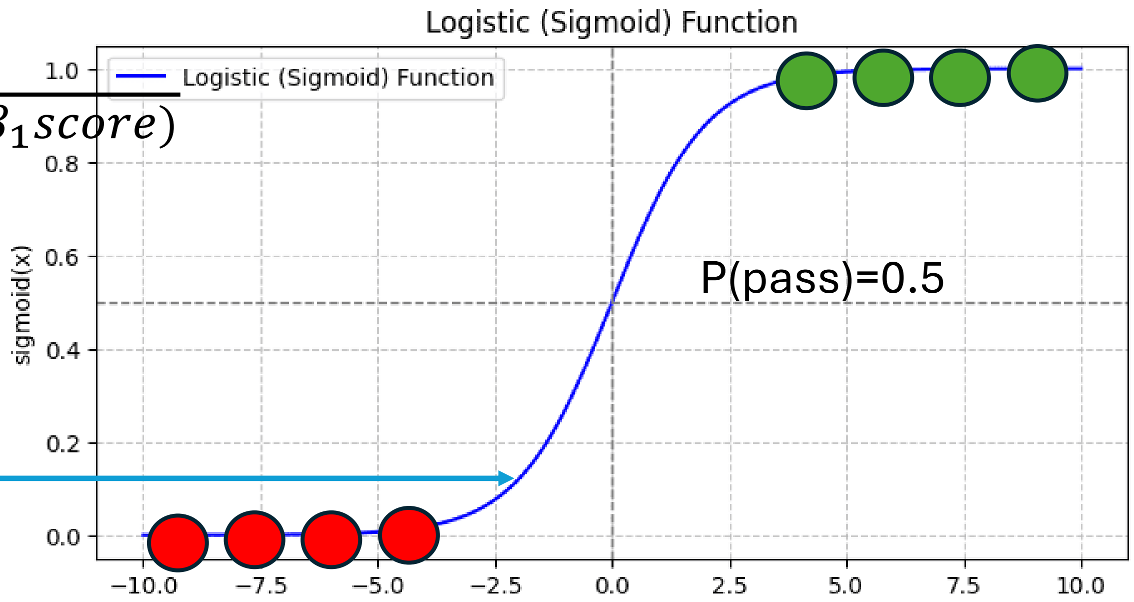
$$\sigma(x) = \frac{1}{1+e^{-x}}$$

$$\beta_0 + \beta_1 score$$



$$\frac{1}{1+e^{-(\beta_0+\beta_1 score)}}$$

P(pass)



$$\beta_0 + \beta_1 score$$

Connecting Probability, Odds, and Log Odds

- If the probability of passing the course is **70% (i.e., P(pass) = 0.5)**, **then P(fail) = 1.0 – P(pass) = 0.3**

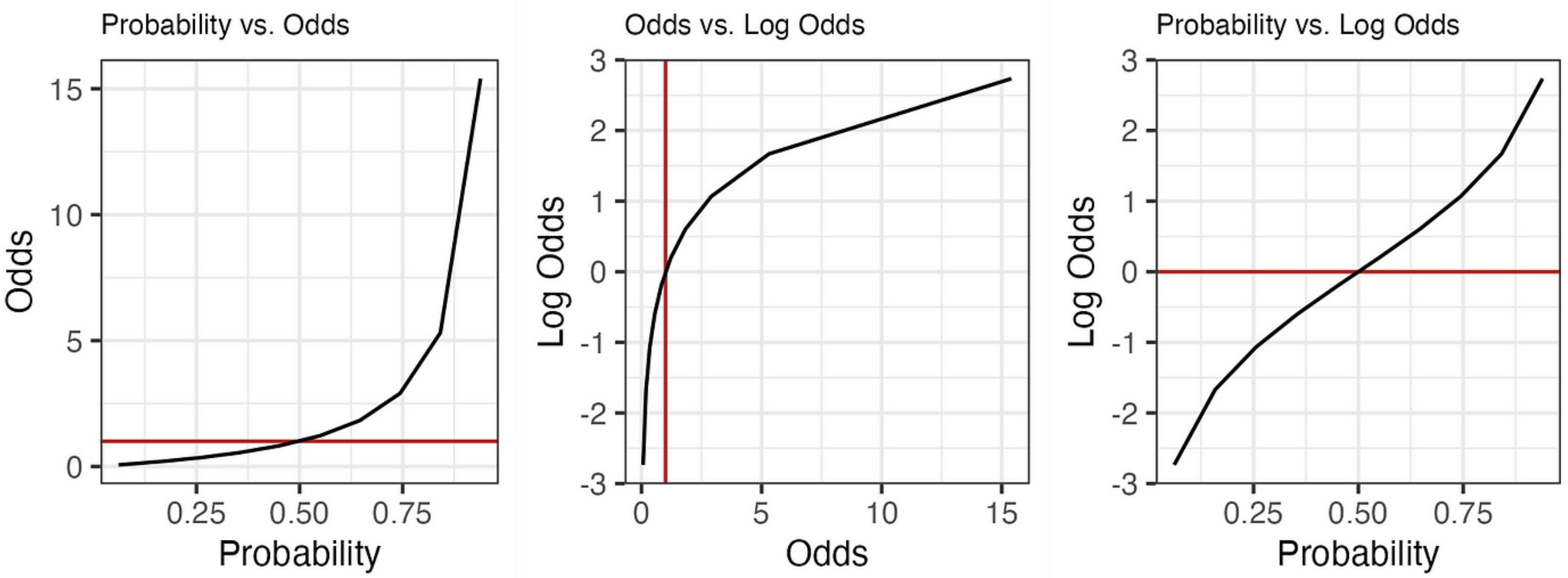
  0.7

- Alternatively, we can say, the odds ratio for passing the course is odds ratio = **0.7/0.3 = 2.3 or 2.3: 1**. Meaning, the chance of passing is 2.3 times that of failing. **(Comparatively)**

- log odds is just the logarithm of odd ratio : log(2.3)

Connecting Probability, Odds, and Log Odds

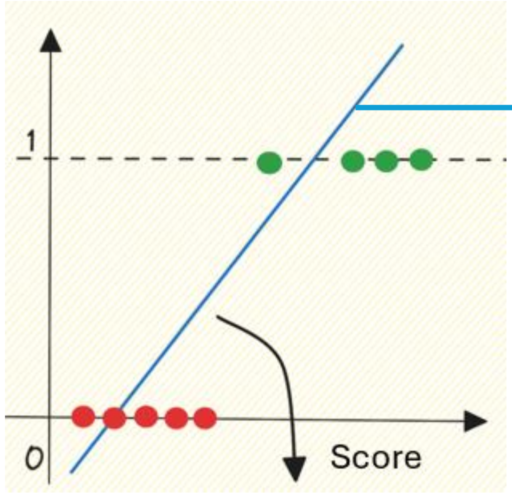
$$odds = \frac{p}{(1-p)} \qquad \log(odds) = log\left(\frac{p}{1-p}\right)$$



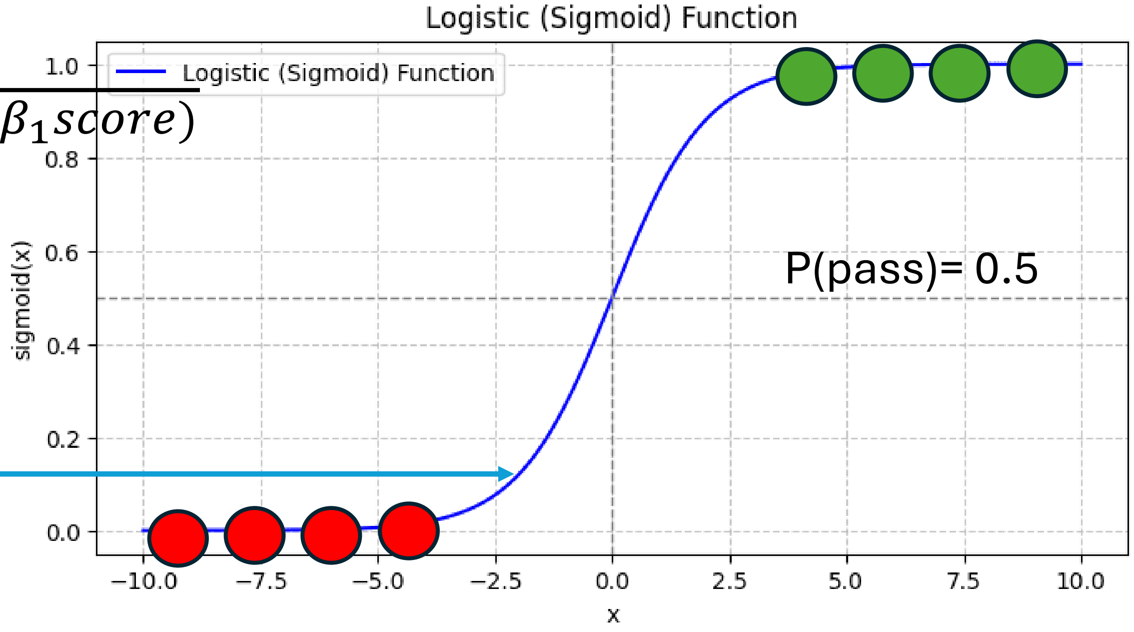
Connecting Probability, Odds, and Log Odds

$$\frac{1}{1+e^{-(\beta_0+\beta_1 score)}}$$

$\beta_0 + \beta_1 score$



P(pass)



$\beta_0 + \beta_1 score$

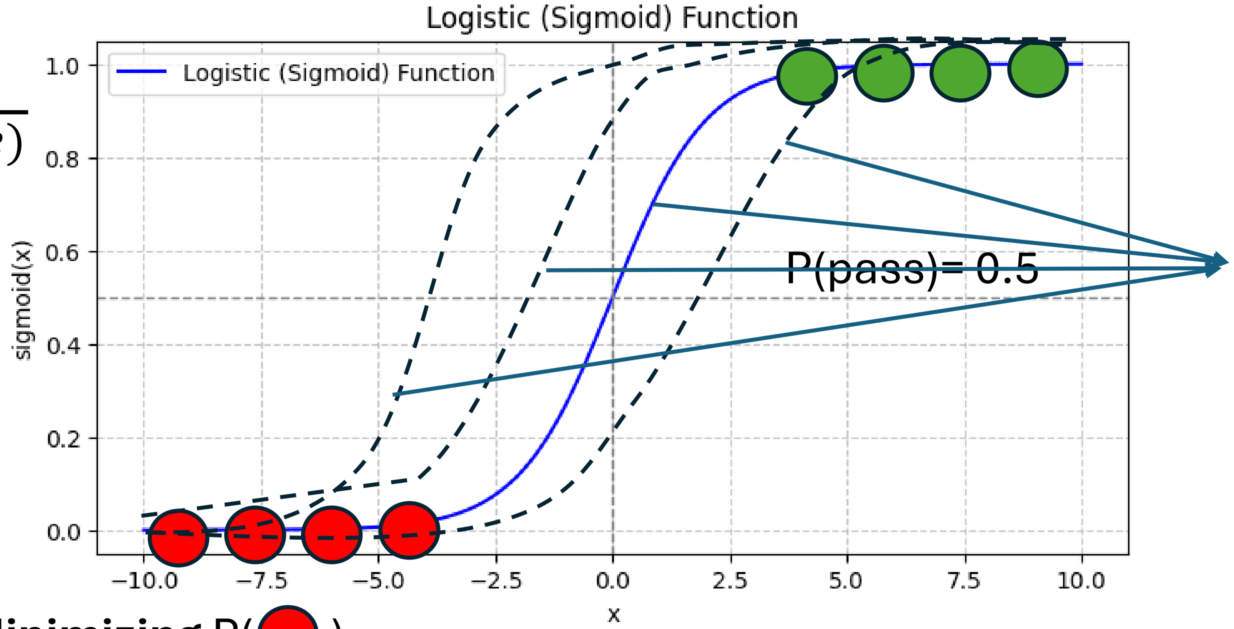
- $\beta_0 + \beta_1 score$ (-inf, inf) is log odds
- $e^{-(\beta_0+\beta_1 score)}$ (0, inf) is odds
- $\frac{1}{1+e^{-(\beta_0+\beta_1 score)}}$ (0,1) is probability

Model Training/Fitting

Maximizing P( )

$$\frac{1}{1+e^{-(\beta_0+\beta_1 score)}}$$

P(pass)



Parameterized by

β_0, β_1

Minimizing P( )

$\beta_0 + \beta_1 score$

Or **maximizing** 1 - P( )

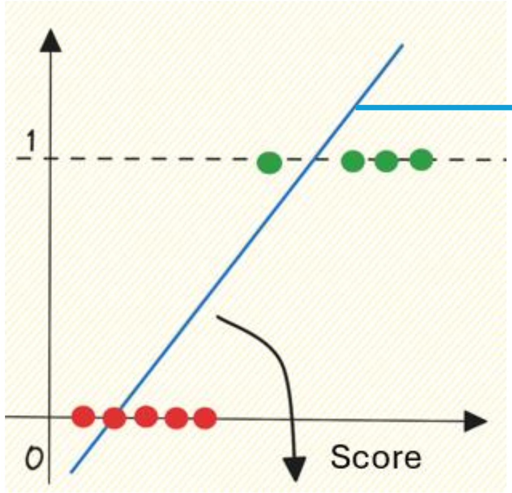
Maximum Likelihood Estimate

$$L(\beta) = \prod_{s \ in \ y_i = 1} p(x_i) \ * \prod_{s \ in \ y_i = 0} (1 - p(x_i))$$

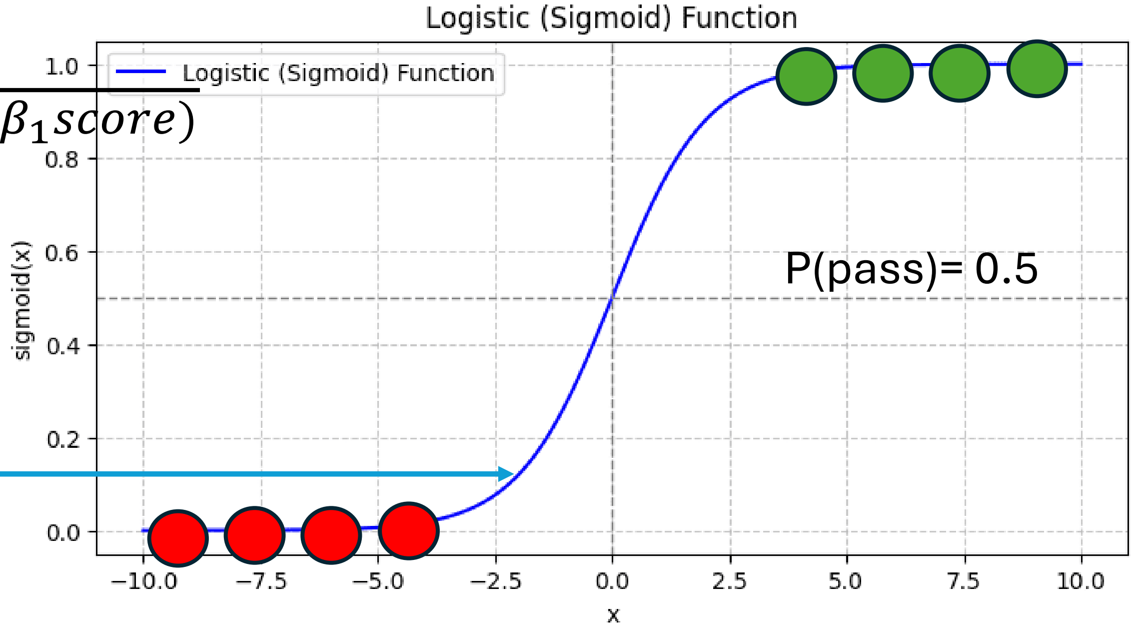
Interpreting Coefficients

$$\frac{1}{1+e^{-(\beta_0+\beta_1 score)}}$$

$\beta_0 + \beta_1 score$



P(pass)



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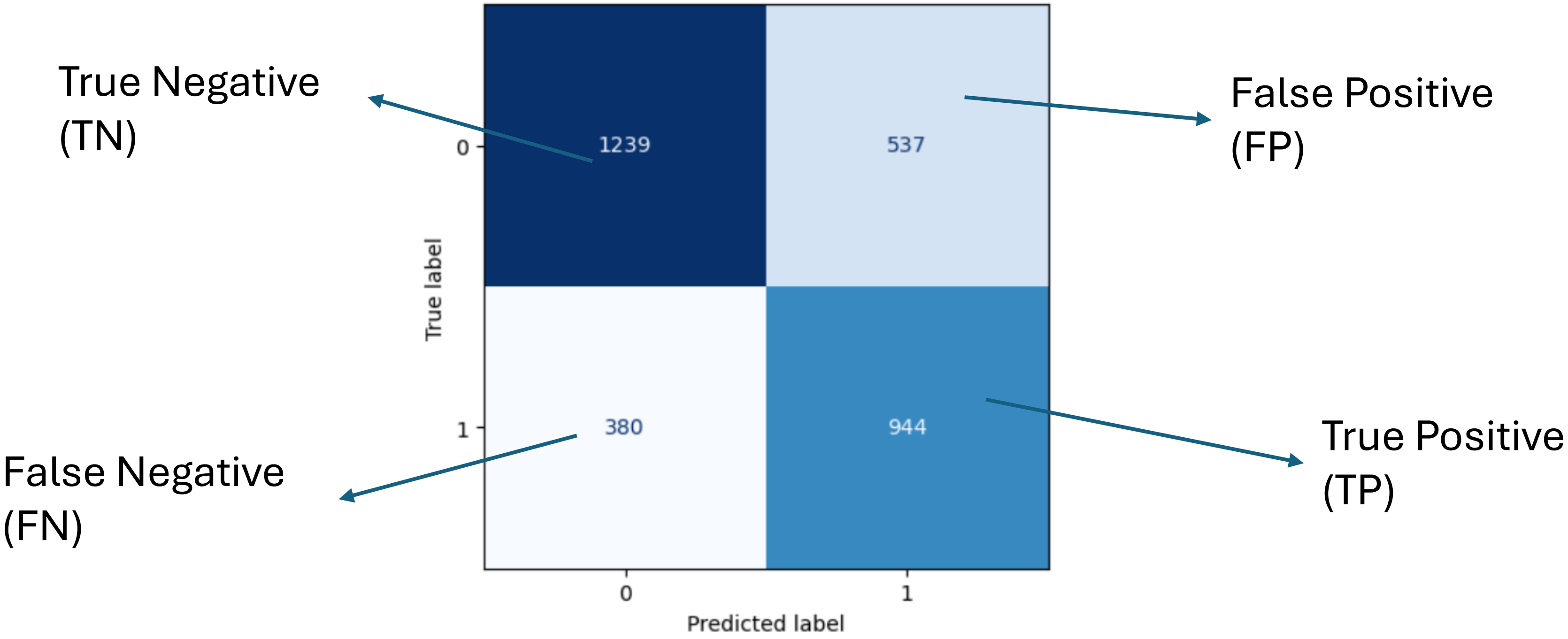
One unit change in $score$ results in β_1 change in log odds and $e^{-\log odds}$ in odds ratio.

Evaluating a LR Model – Confusion Matrix

- Confusion Matrix: A table used to describe the performance of a classification model. It presents the number of true positives, true negatives, false positives, and false negatives.



Evaluating a LR Model – Accuracy

- Accuracy is defined as the number of correct predictions over the total predictions:

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

Evaluating a LR Model – Accuracy

- Precision: It is the proportion of true positive predictions out of all positive predictions made by the model. Precision is useful when the cost of false positives is high (e.g., fraud).

$$Precision = \frac{TP}{TP + FP}$$

Evaluating a LR Model – Recall (Sensitivity)

- Recall: It is the proportion of true positive instances that were correctly identified by the model. Recall is useful when the cost of false negatives is high (e.g., Customer Churn Prediction).

$$Recall = \frac{TP}{TP + FN}$$

Evaluating a LR Model – F1 Score

- Recall: It is the harmonic mean of precision and recall. It provides a balance between precision and recall. It is a useful metric when there is an uneven class distribution.

$$F1\,Score = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

Practice