Intro. to Deep Learning

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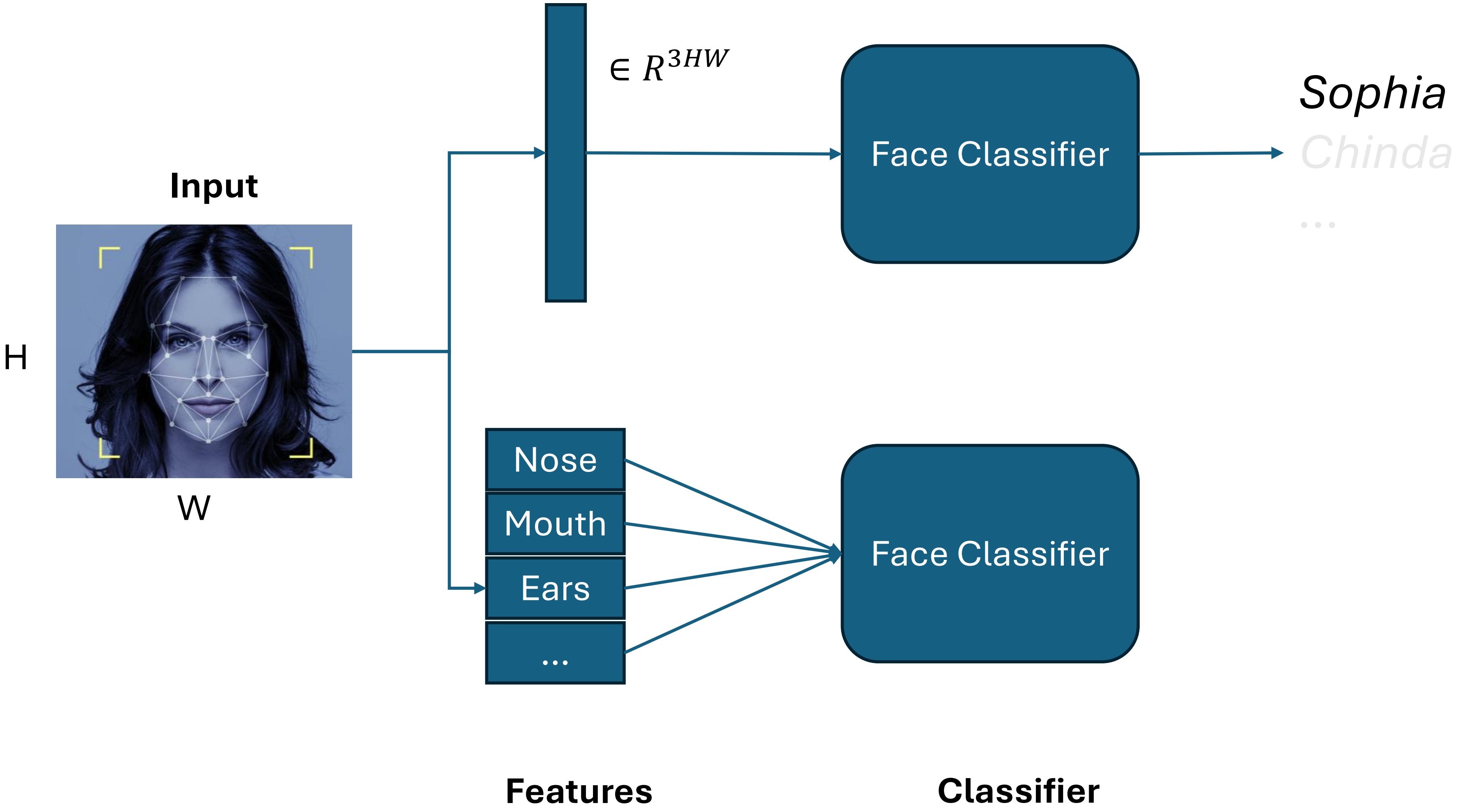
AMERICAN UNIVERSITY OF PHNOM PENH

STUDY LOCALLY. LIVE GLOBALLY.

Why Deep Learning ?



Why Deep Learning ?

- Hand-engineered features are designed characteristics that help in distinguishing different faces.

- Geometric Features
  - landmark points & distances and angles
- Texture Features
- Appearance Features
- Color and Illumination Features

Hand-Engineered Features

very hard to make

Why Deep Learning ?

Hand engineered features are time consuming, brittle, and not scalable in practice

Can we learn the **underlying features** directly from data?

Low Level Features | **Mid Level Features** | **High Level Features**

Lines & Edges | Eyes & Nose & Ears | Facial Structure

Why Now ?

- 1952 — Stochastic Gradient Descent
- 1958 — Perceptron
  - Learnable Weights
- ⋮
- 1986 — Backpropagation
  - Multi-Layer Perceptron
- 1995 — Deep Convolutional NN
  - Digit Recognition
- ⋮

Neural Networks date back decades, so why the dominance?

1. Big Data

- Larger Datasets
- Easier Collection & Storage

IMAGENET

WIKIPEDIA
The Free Encyclopedia

2. Hardware

- Graphics Processing Units (GPUs)
- Massively Parallelizable

3. Software

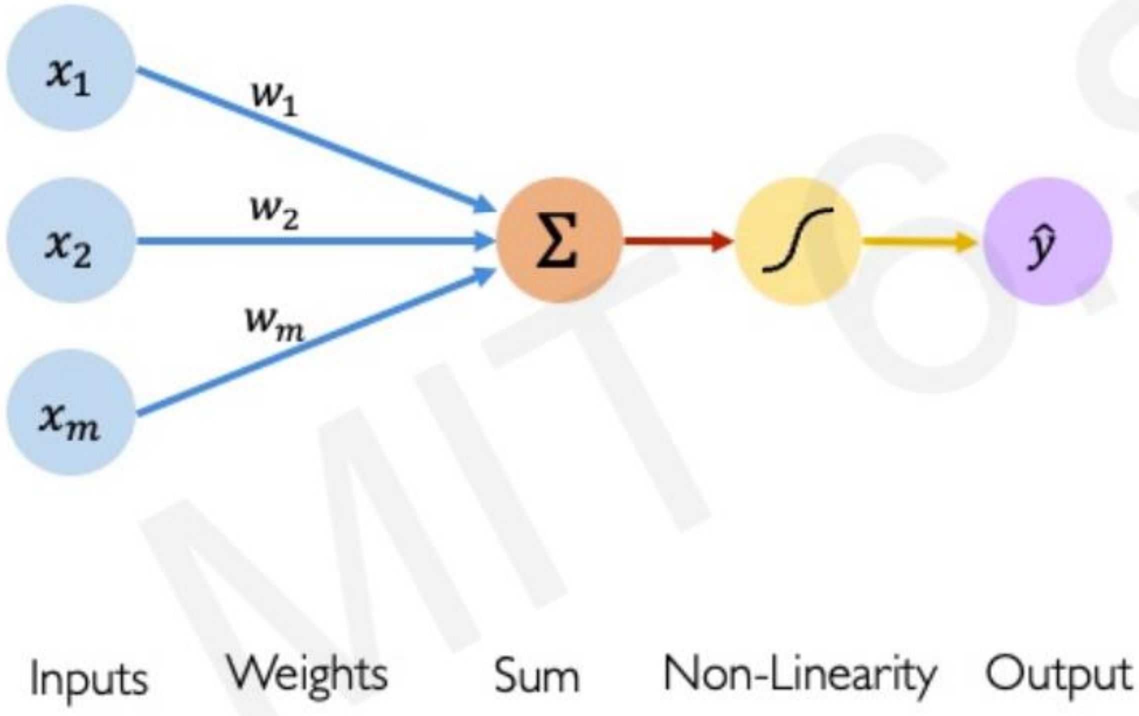
- Improved Techniques
- New Models
- Toolboxes

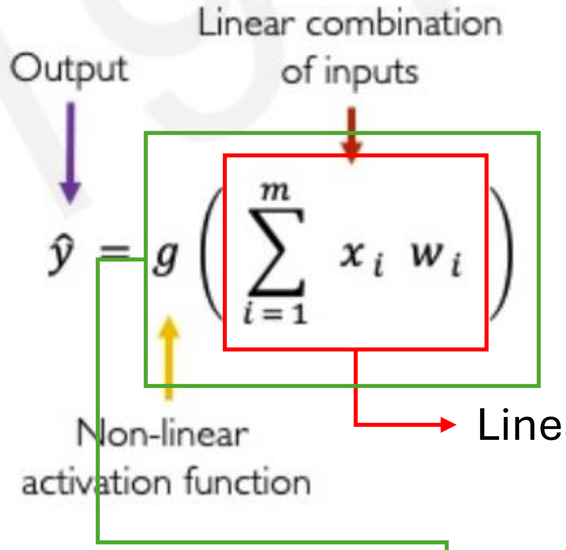
TensorFlow

The Perceptron

The Brick of Deep Learning

The Perceptron - Output Computation (i.e., Forward Pass)

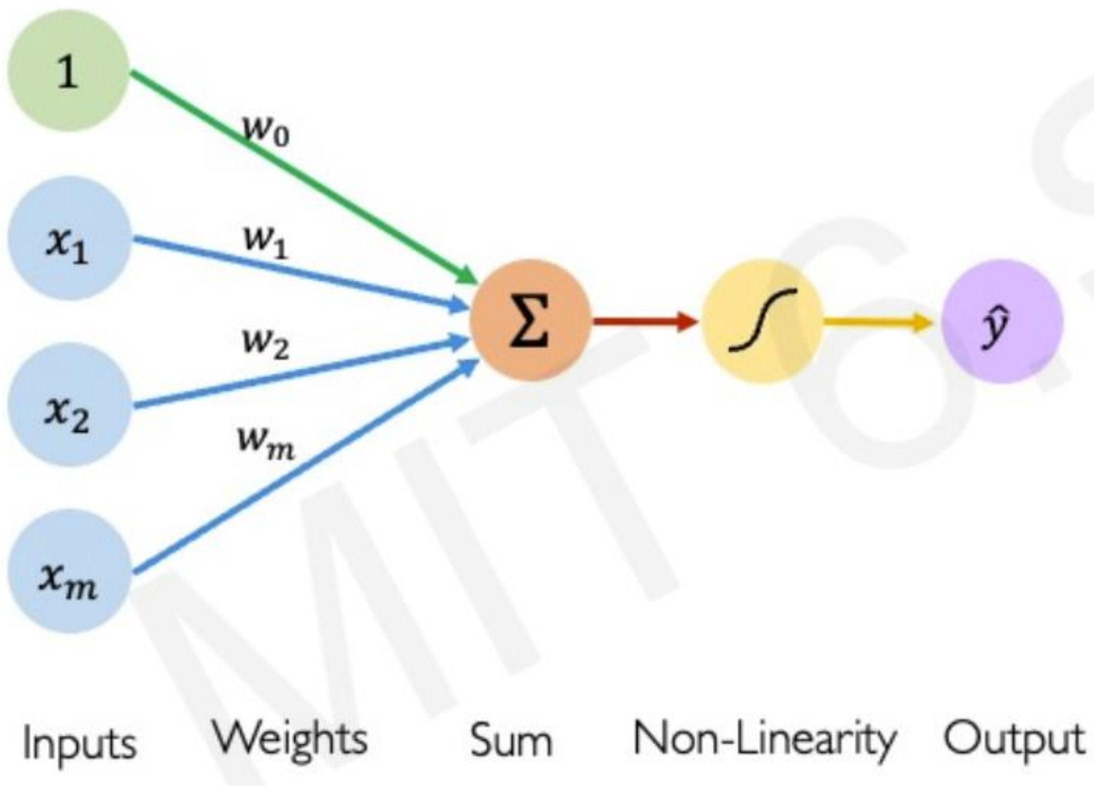


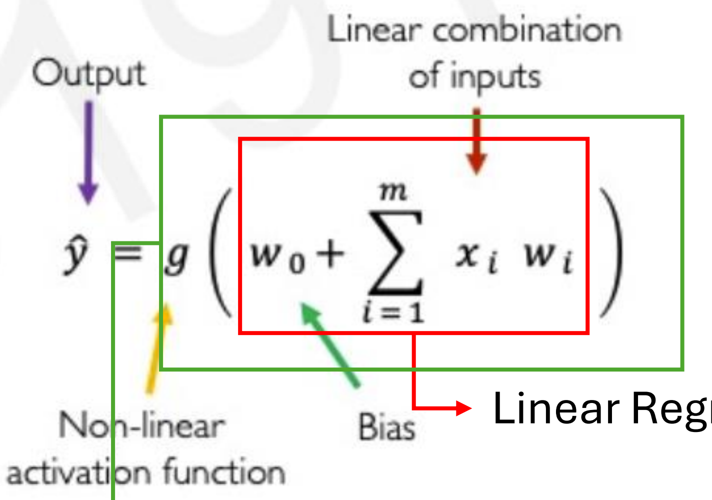


Linear Regression

Logistic Regression
(if $g()$ is sigmoid)

The Perceptron - Output Computation (i.e., Forward Pass)





$$\hat{y} = g\left(w_0 + \sum_{i=1}^{m} x_i\, w_i\right)$$

Linear Regression

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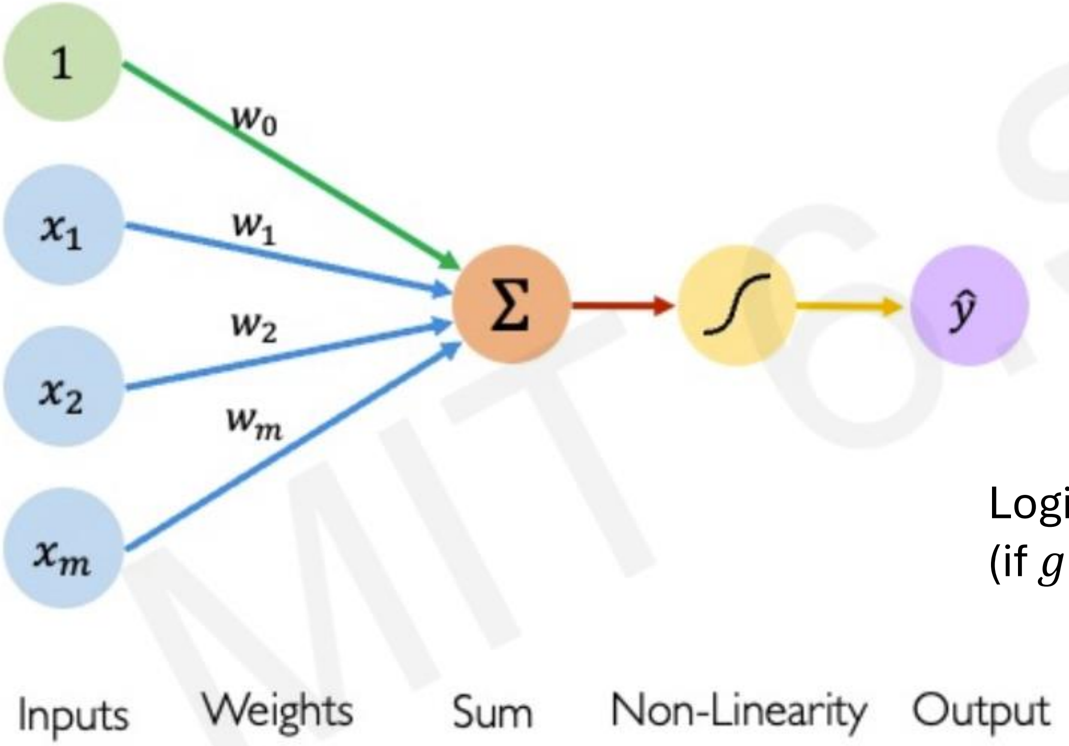
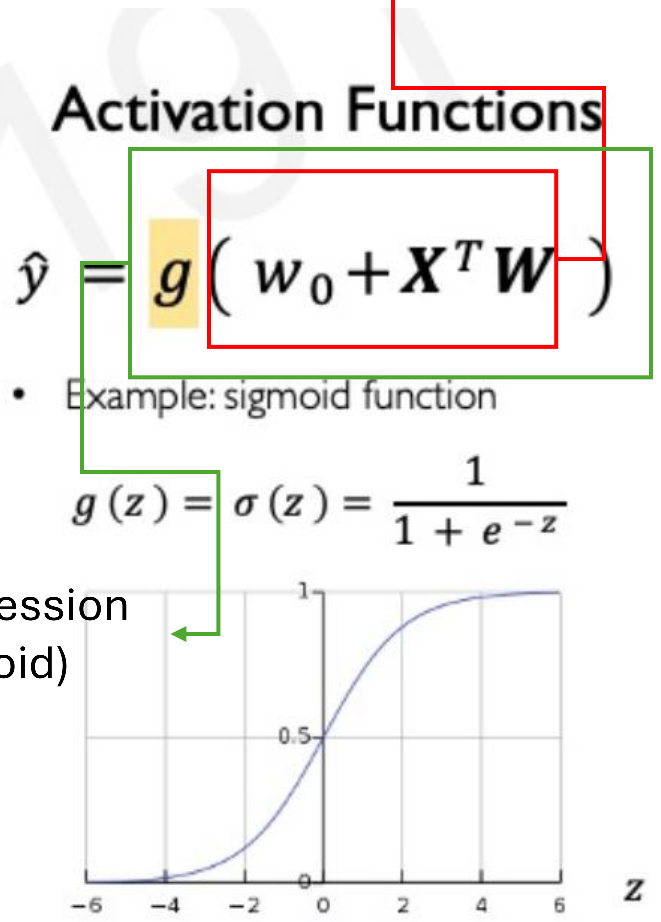
(if $g()$ is sigmoid)

The Perceptron - Output Computation (i.e., Forward Pass)

Linear Regression

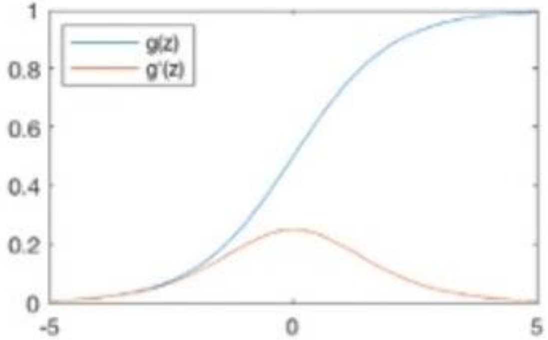




Logistic Regression
(if $g()$ is sigmoid)

Beyond Sigmoid - Activation Functions

Sigmoid Function

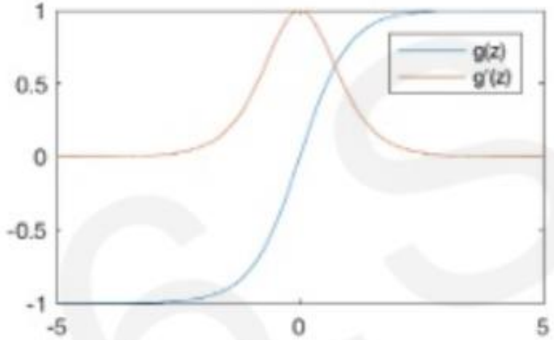


$$g(z) = \frac{1}{1 + e^{-z}}$$

$$g'(z) = g(z)(1 - g(z))$$

`tf.math.sigmoid(z)`

Hyperbolic Tangent

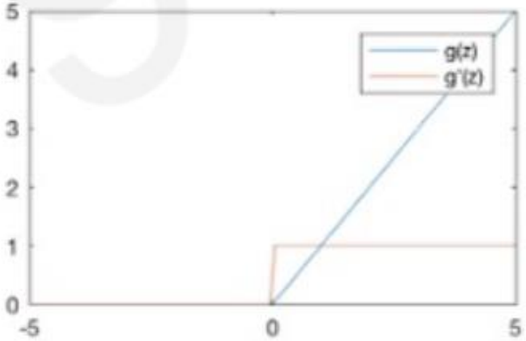


$$g(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}}$$

$$g'(z) = 1 - g(z)^2$$

`tf.math.tanh(z)`

Rectified Linear Unit (ReLU)



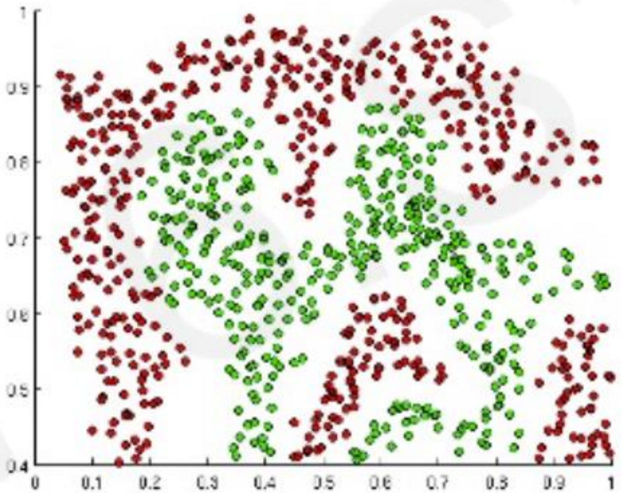
$$g(z) = \max(0, z)$$

$$g'(z) = \begin{cases} 1, & z > 0 \\ 0, & \text{otherwise} \end{cases}$$

`tf.nn.relu(z)`

Activation Functions

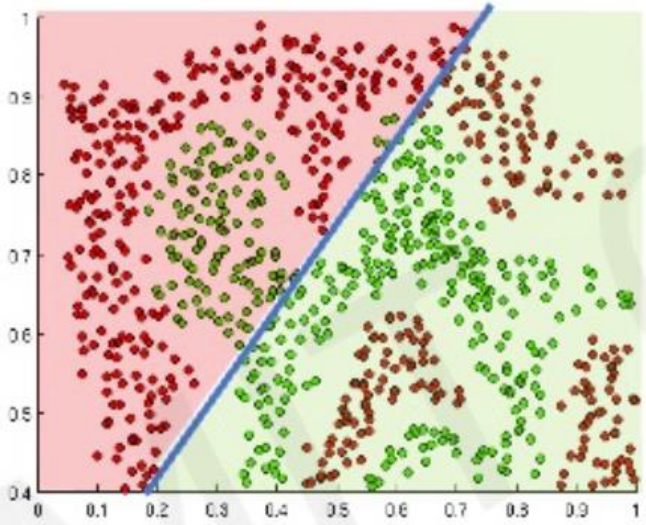
The purpose of activation functions is to ***introduce non-linearities*** *into the network*



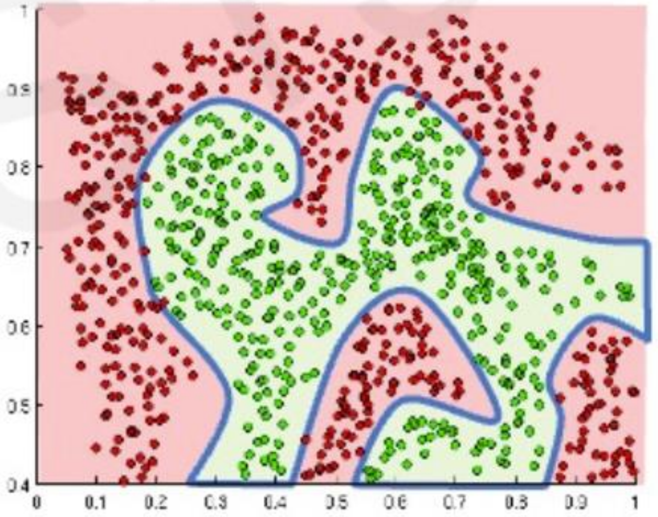
What if we wanted to build a neural network to distinguish green vs red points?

Activation Functions

The purpose of activation functions is to ***introduce non-linearities*** *into the network*

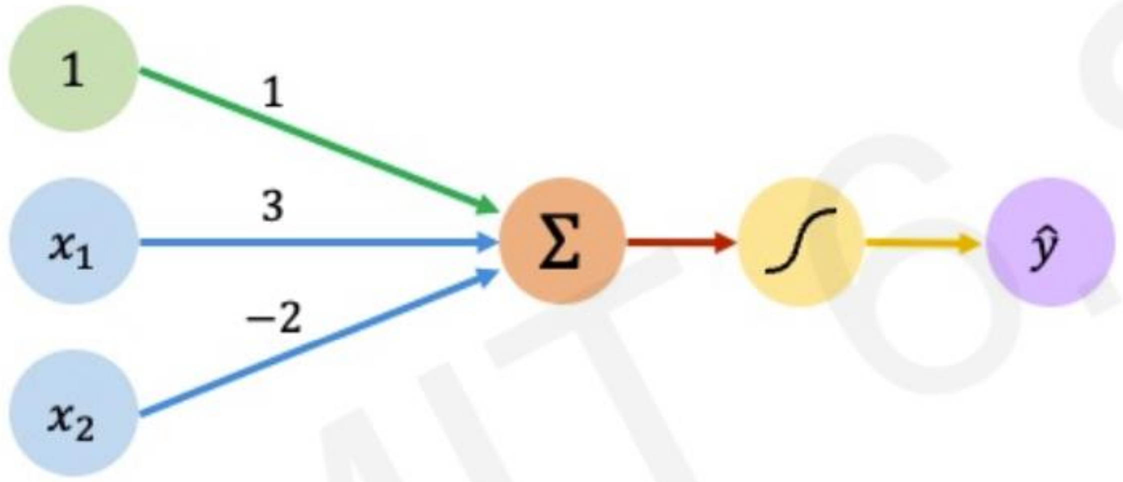


Linear activation functions produce linear decisions no matter the network size



Non-linearities allow us to approximate arbitrarily complex functions

The Perceptron – An Example of 2 Features



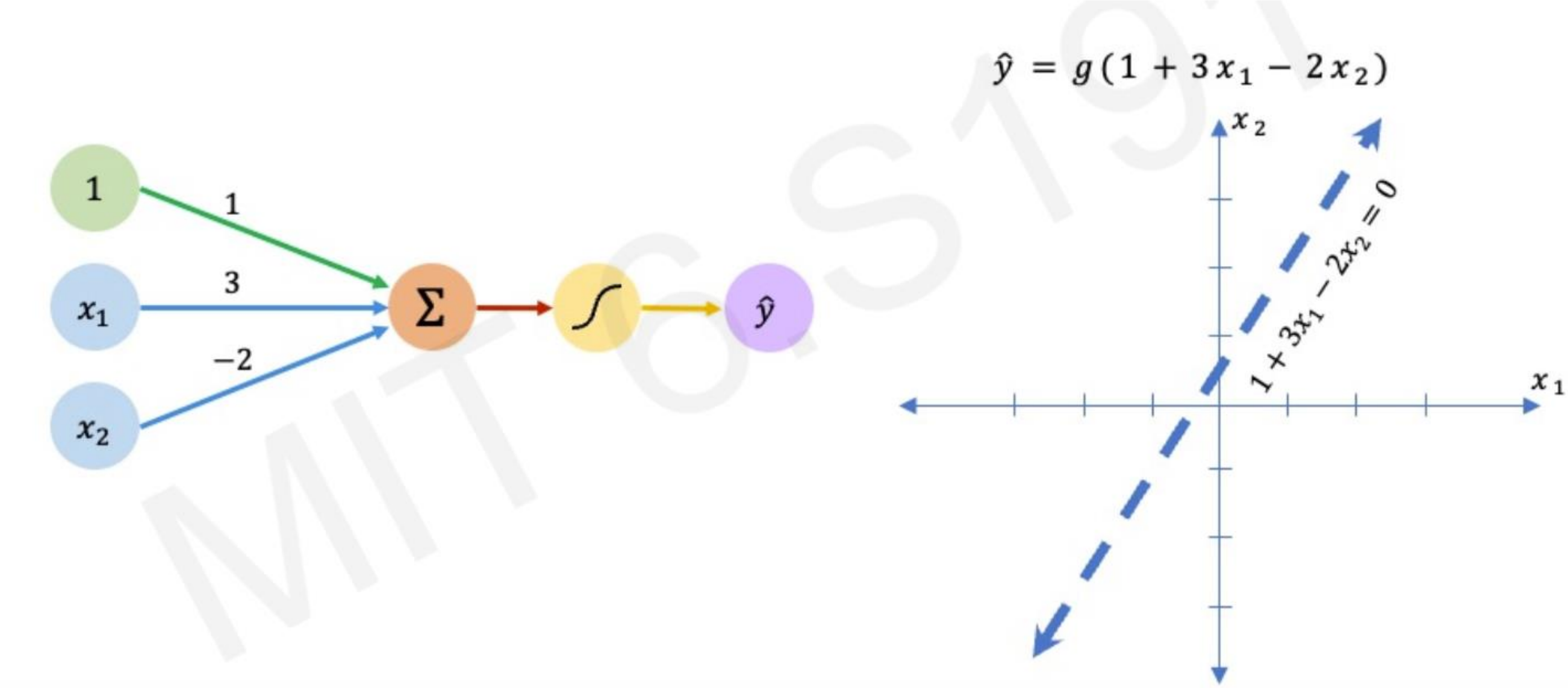
We have: $\boldsymbol{w}_0 = 1$ and $\boldsymbol{W} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$

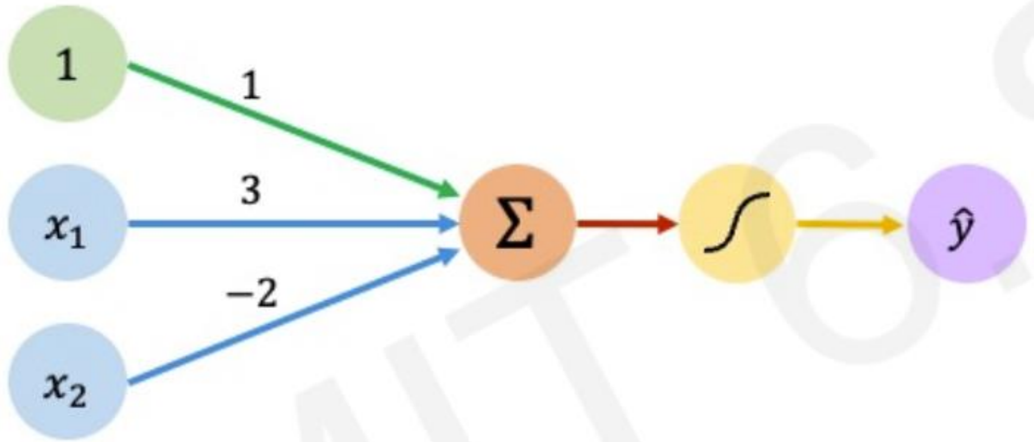
$$= g\left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right)$$

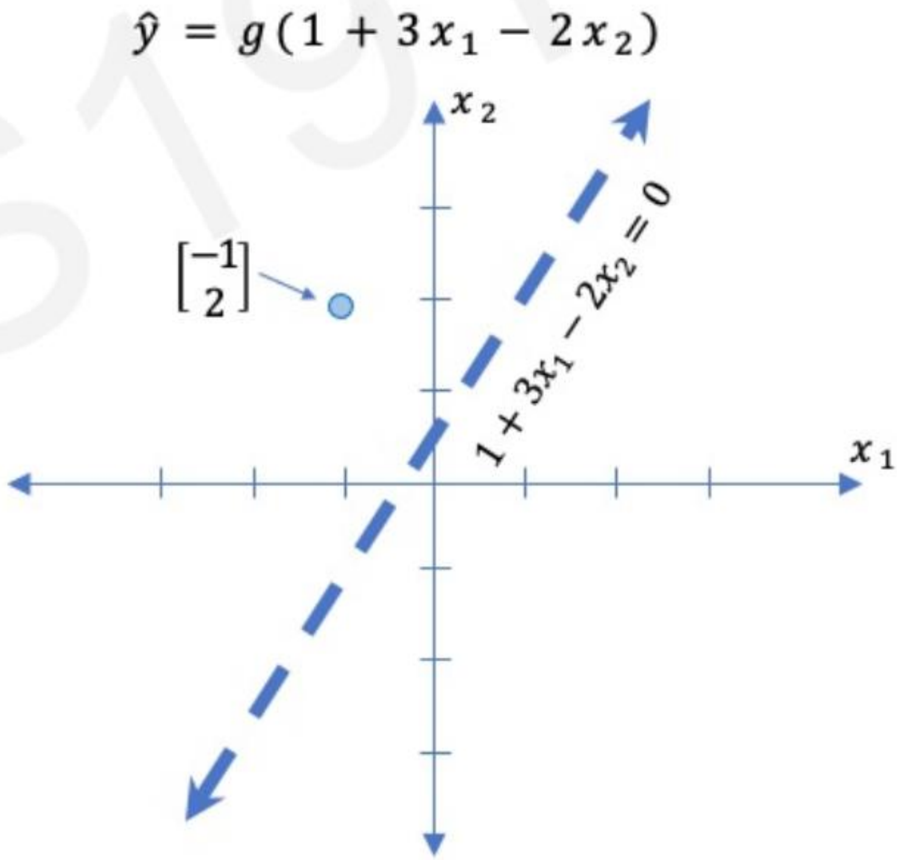
$$\hat{y} = g(\underbrace{1 + 3x_1 - 2x_2}_{\text{This is just a line in 2D!}})$$

The Perceptron – An Example of 2 Features



The Perceptron – An Example of 2 Features

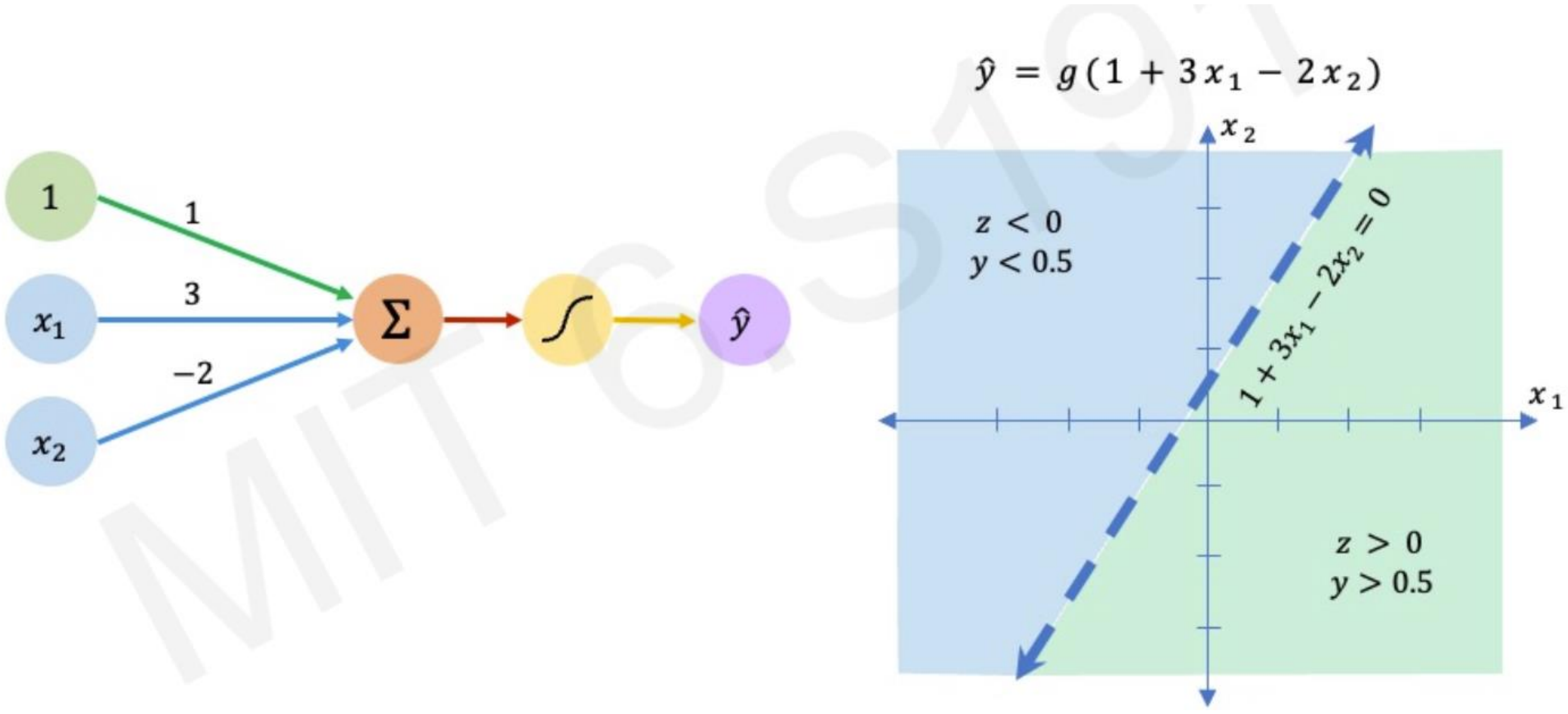




Assume we have input: $\boldsymbol{X} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$\begin{aligned} \hat{y} &= g\big(1 + (3 * -1) - (2 * 2)\big) \\ &= g(-6) \approx 0.002 \end{aligned}$$

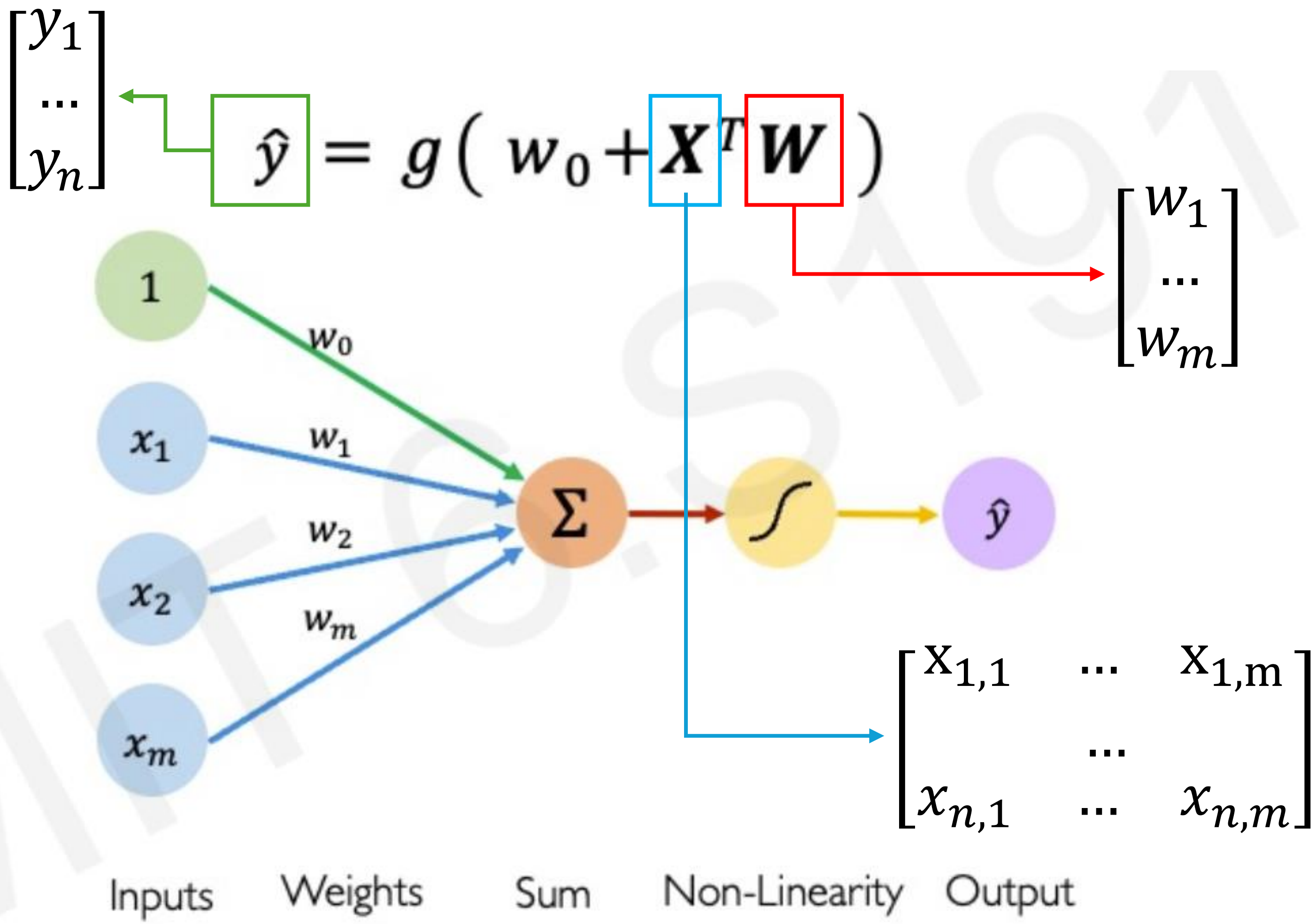
The Perceptron – An Example of 2 Features



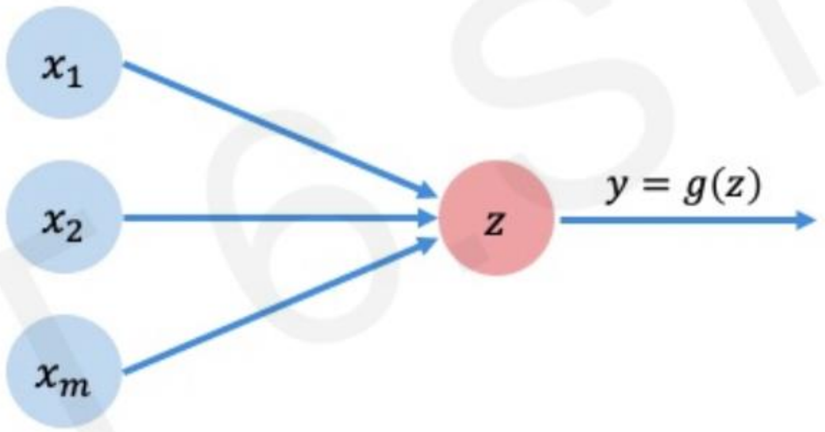
From Perceptron to Neural Network

The Perceptron in Matrix Form

$$\hat{y} = g\left(w_0 + \boldsymbol{X}^T \boldsymbol{W} \right)$$



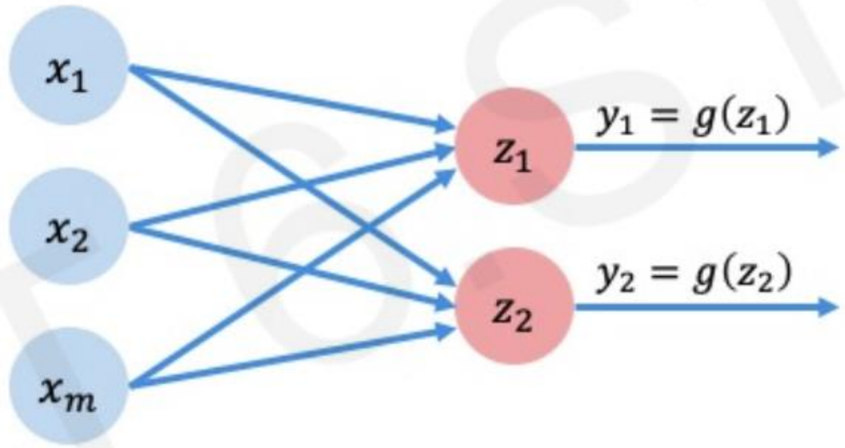
The Perceptron: Compact Notation



$$z = w_0 + \sum_{j=1}^{m} x_j \, w_j$$

Beyond One Output – Multivariate Case

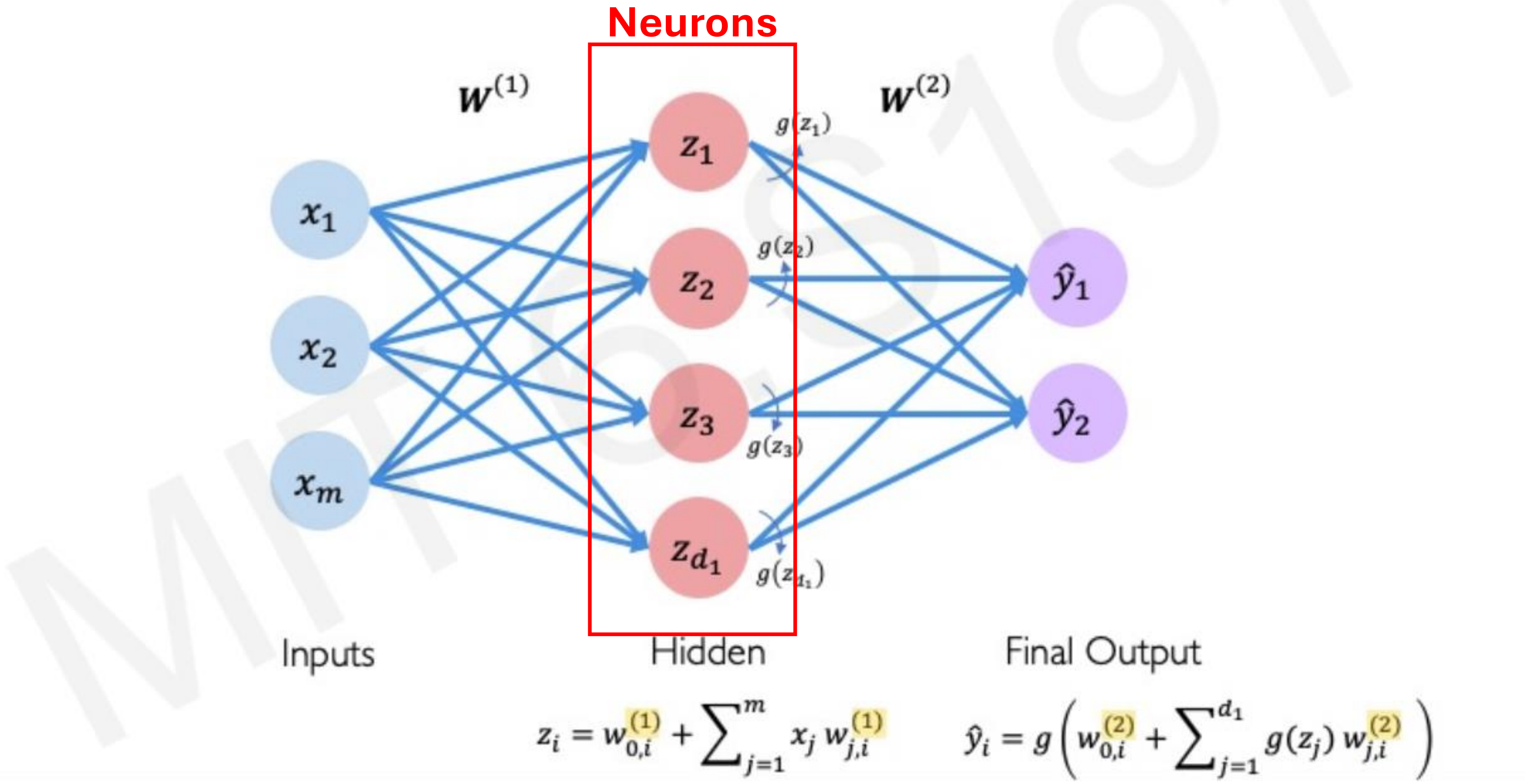
Because all inputs are densely connected to all outputs, these layers are called **Dense** layers



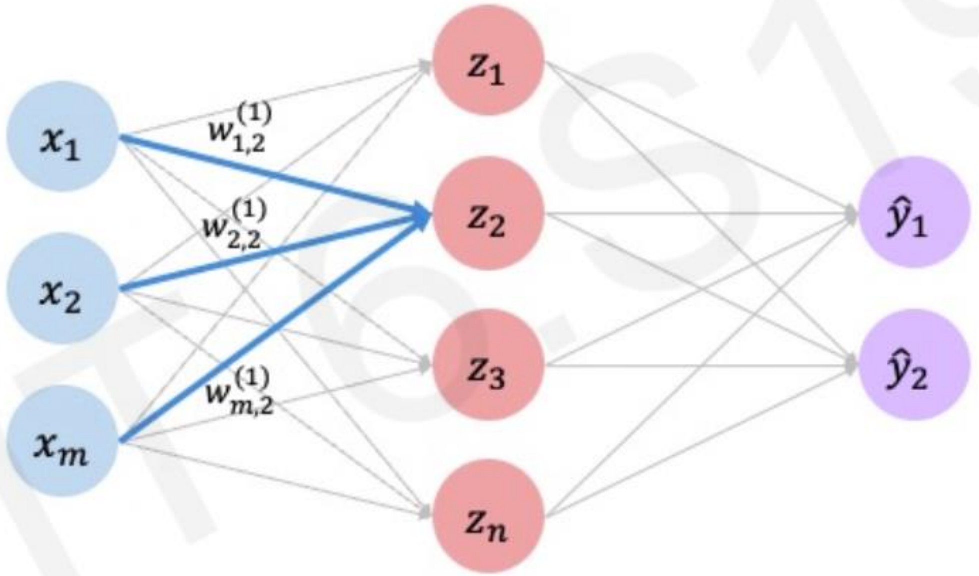
$$z_i = w_{0,i} + \sum_{j=1}^{m} x_j \, w_{j,i}$$

Now, Neural Network

Called a single layer neural network because there is only one layer of hidden neurons.

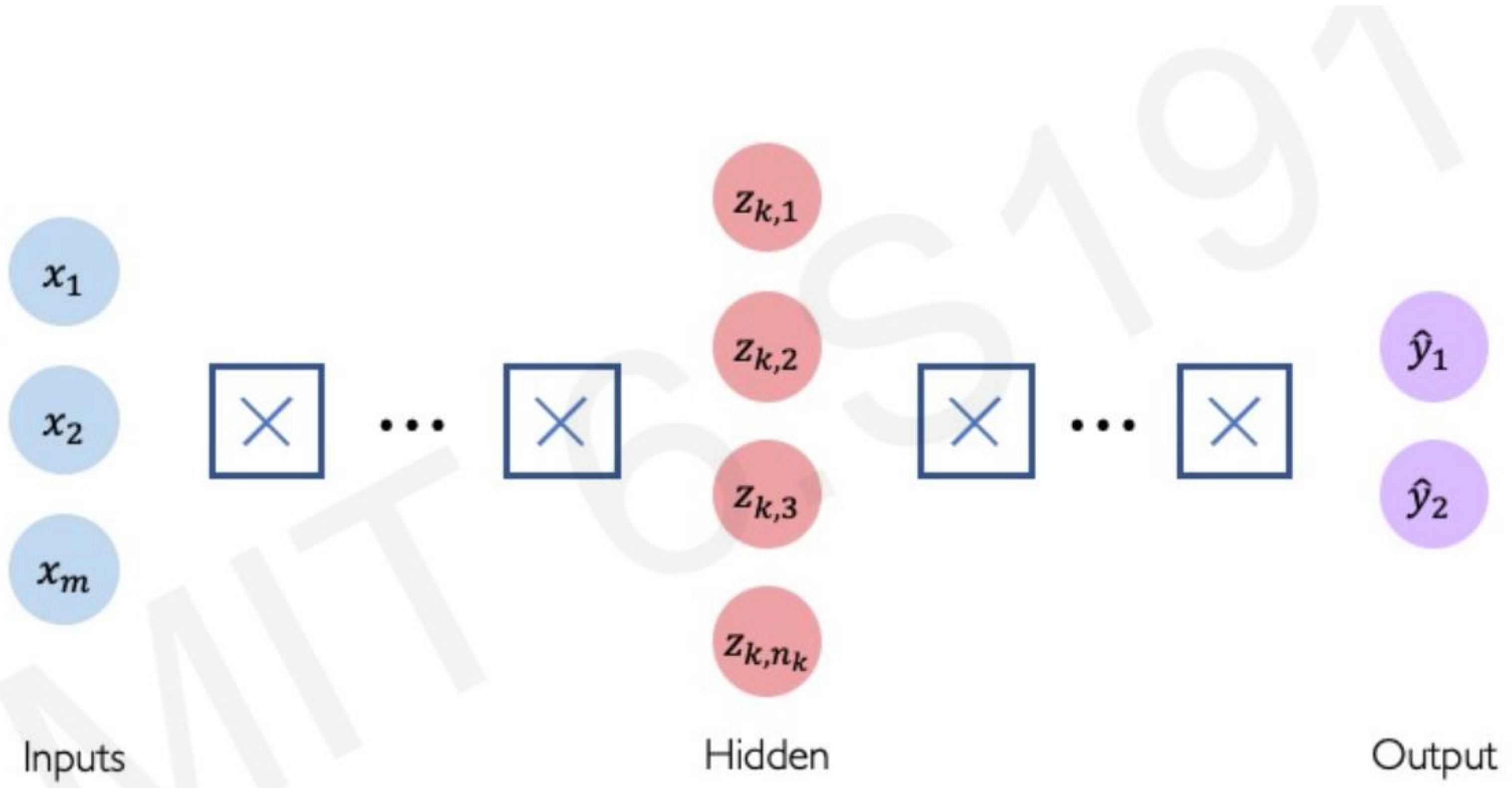


Now, Neural Network



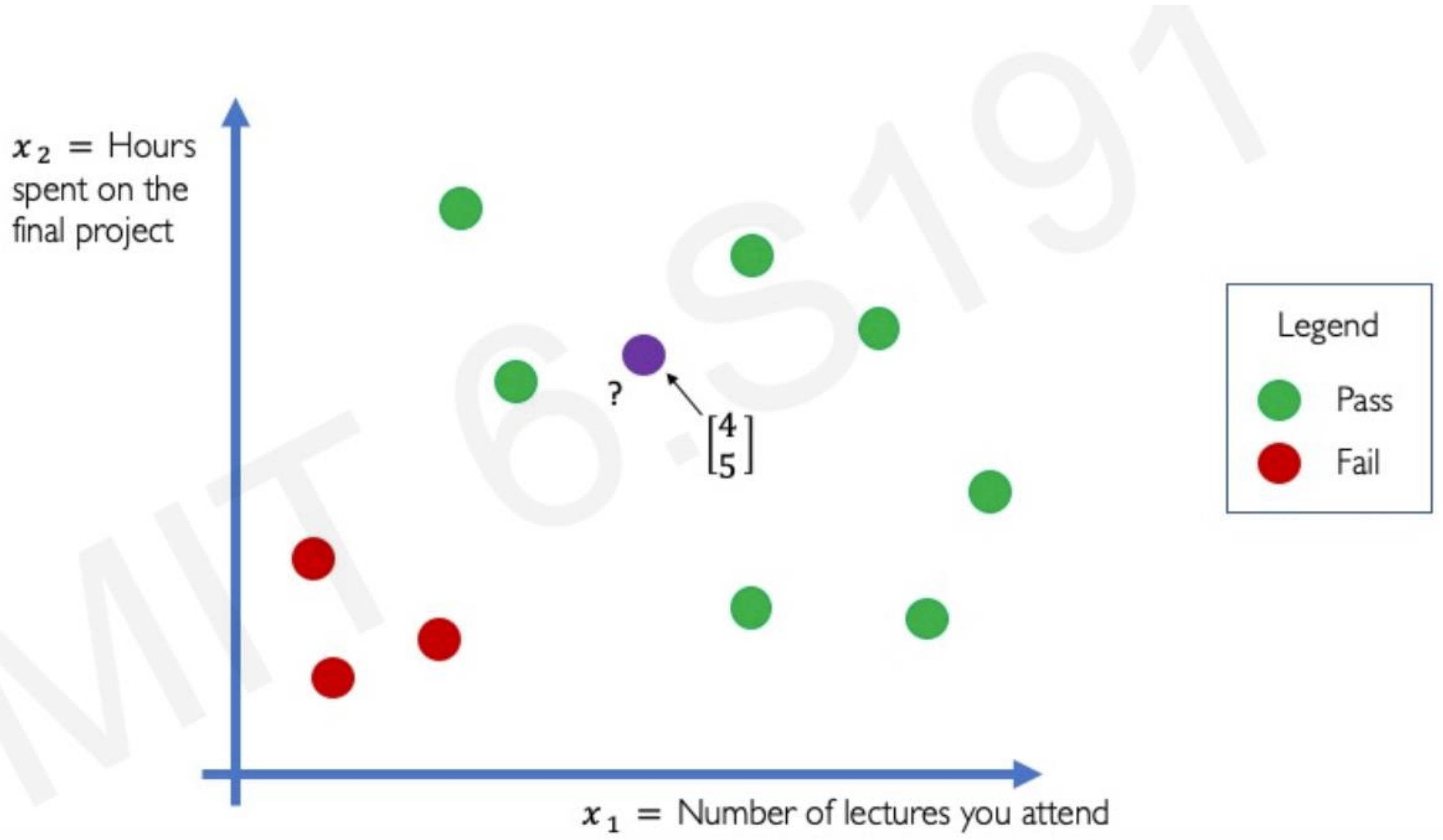
$$
\begin{aligned}
z_2 &= w_{0,2}^{(1)} + \sum_{j=1}^{m} x_j\, w_{j,2}^{(1)} \\
&= w_{0,2}^{(1)} + x_1\, w_{1,2}^{(1)} + x_2\, w_{2,2}^{(1)} + x_m\, w_{m,2}^{(1)}
\end{aligned}
$$

Beyond Single Layer - Deep Neural Network

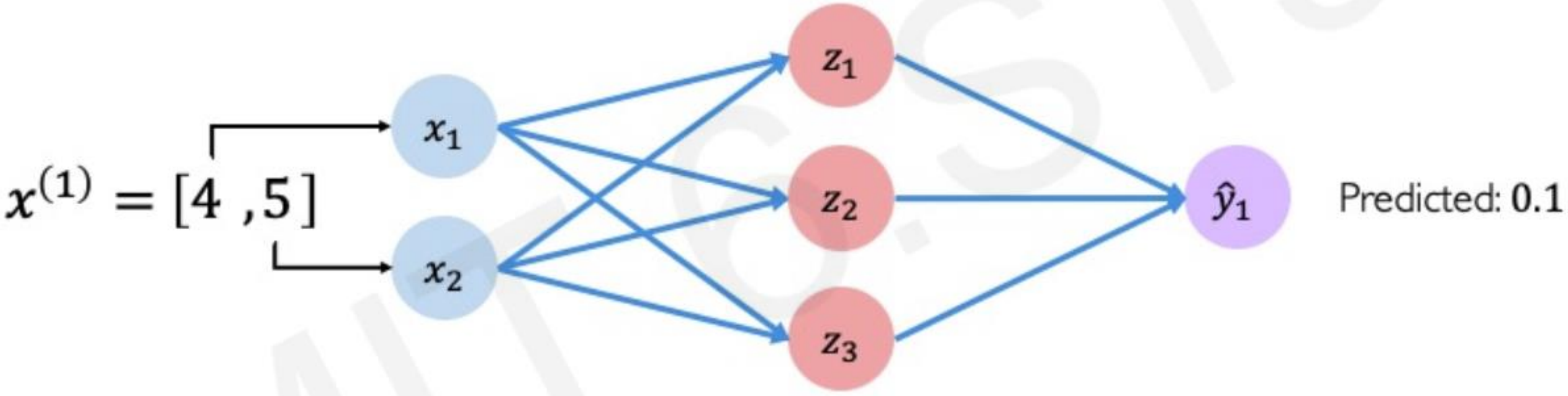


$$z_{k,i} = w_{0,i}^{(k)} + \sum_{j=1}^{n_{k-1}} g(z_{k-1,j})\, w_{j,i}^{(k)}$$

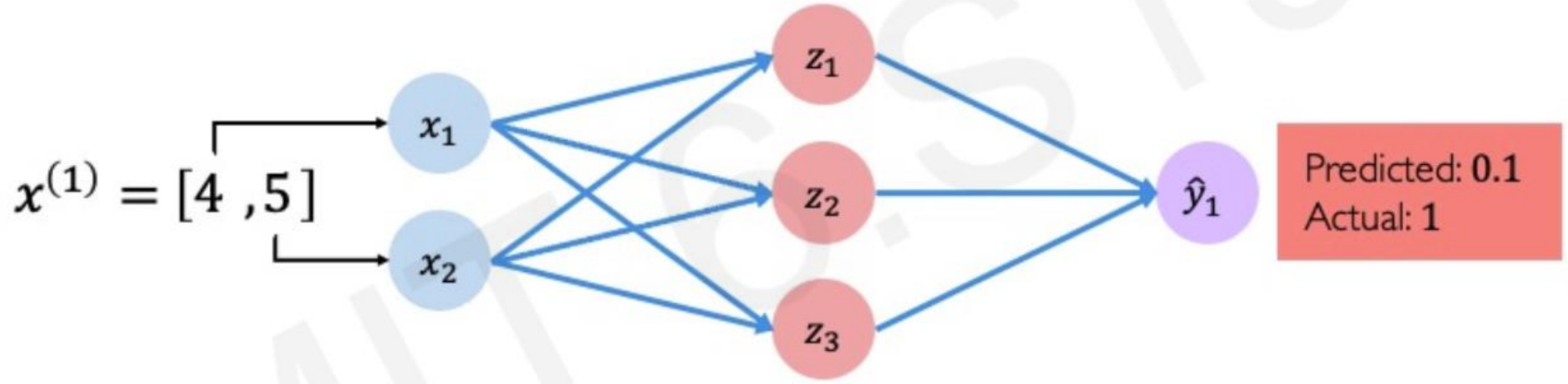
Example Problem



Example Problem



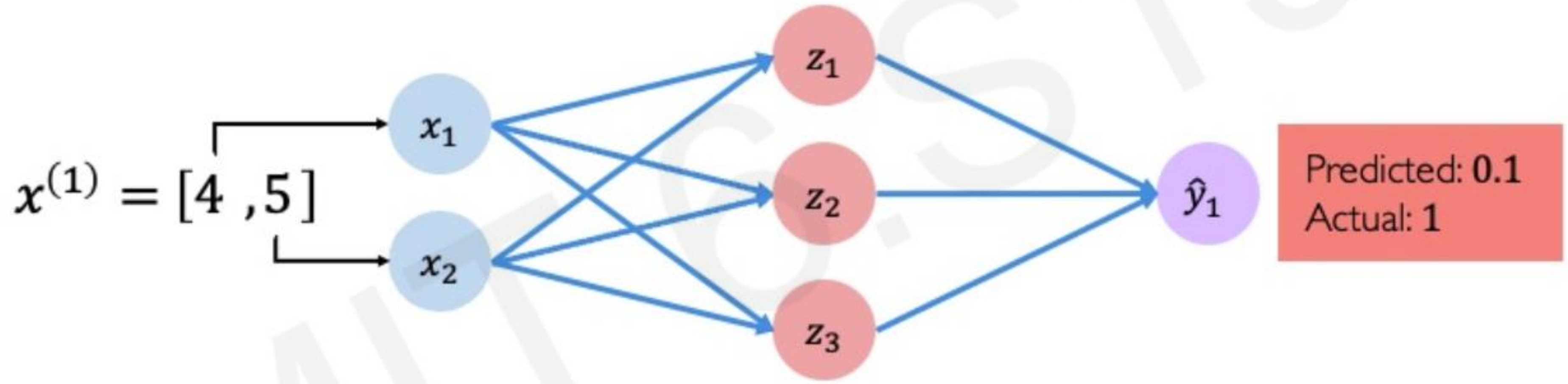
Example Problem



Computing Errors

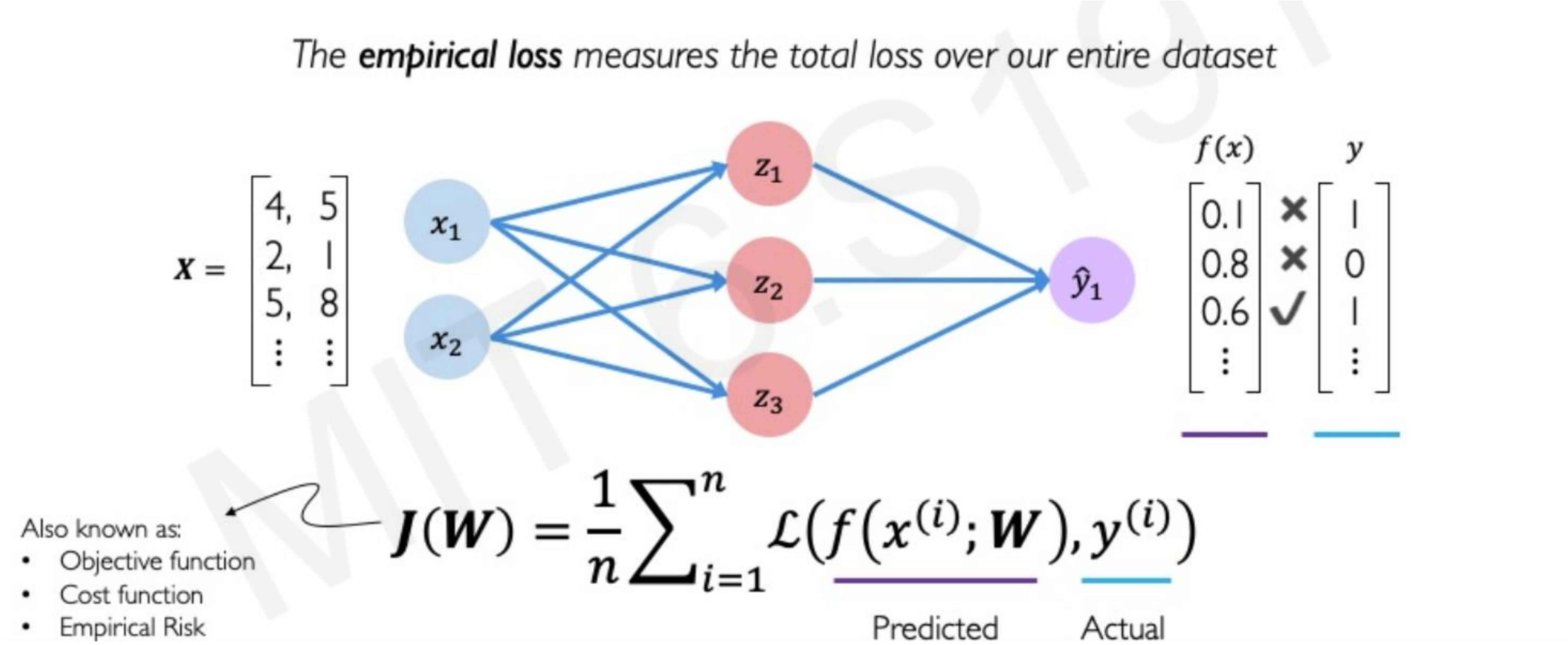
Computing Loss (i.e., Error)

*The **loss** of our network measures the cost incurred from incorrect predictions*



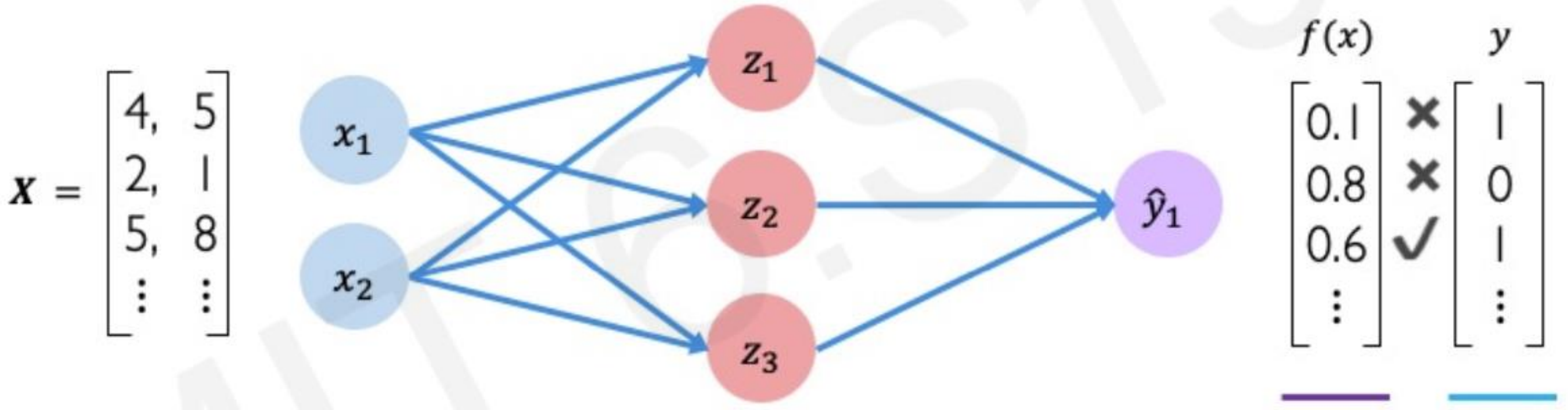
$$\mathcal{L}\big(\underbrace{f\left(x^{(i)}; \boldsymbol{W}\right)}_{\text{Predicted}}, \underbrace{y^{(i)}}_{\text{Actual}}\big)$$

Empirical Loss



Binary Classification Case – Recall Logistic Regression

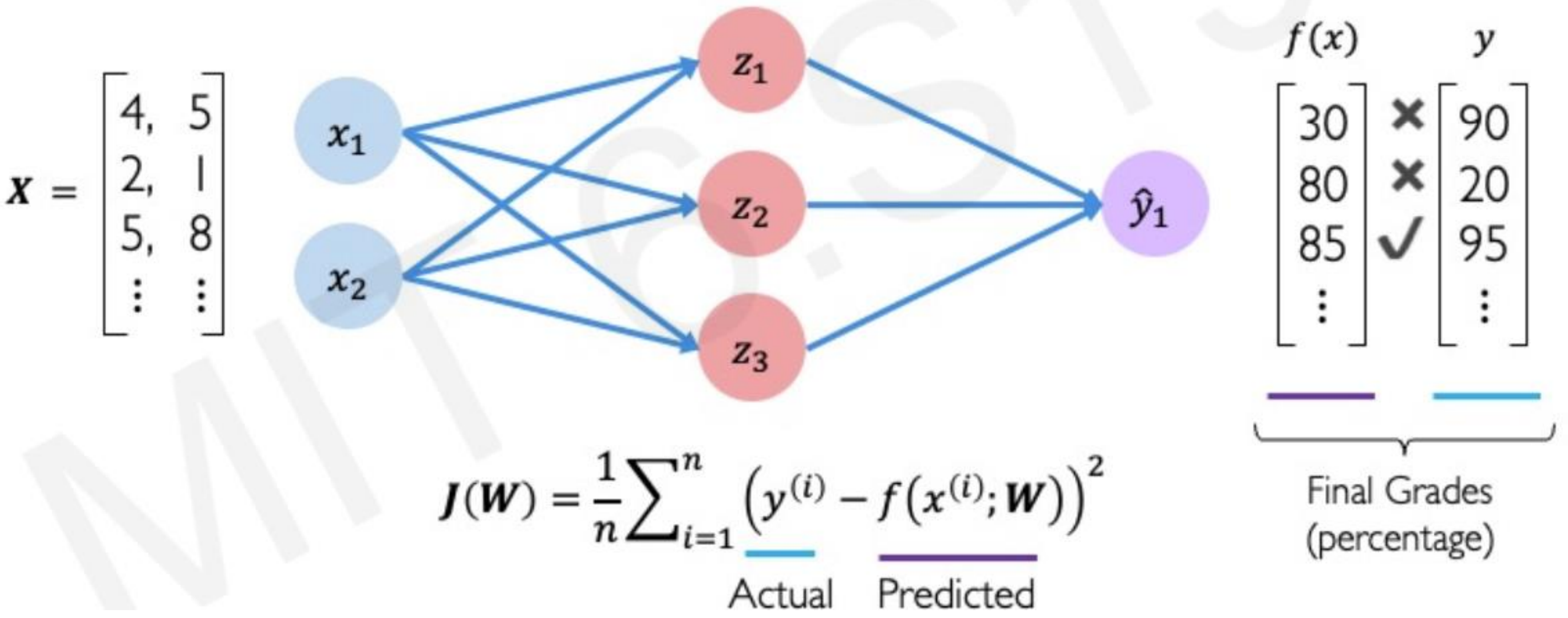
Cross entropy loss *can be used with models that output a probability between 0 and 1*



$$J(\boldsymbol{W}) = -\frac{1}{n}\sum_{i=1}^{n} \underbrace{y^{(i)}}_{\text{Actual}} \log\Big(\underbrace{f\big(x^{(i)};\boldsymbol{W}\big)}_{\text{Predicted}}\Big) + (1 - \underbrace{y^{(i)}}_{\text{Actual}}) \log\Big(1 - \underbrace{f\big(x^{(i)};\boldsymbol{W}\big)}_{\text{Predicted}}\Big)$$

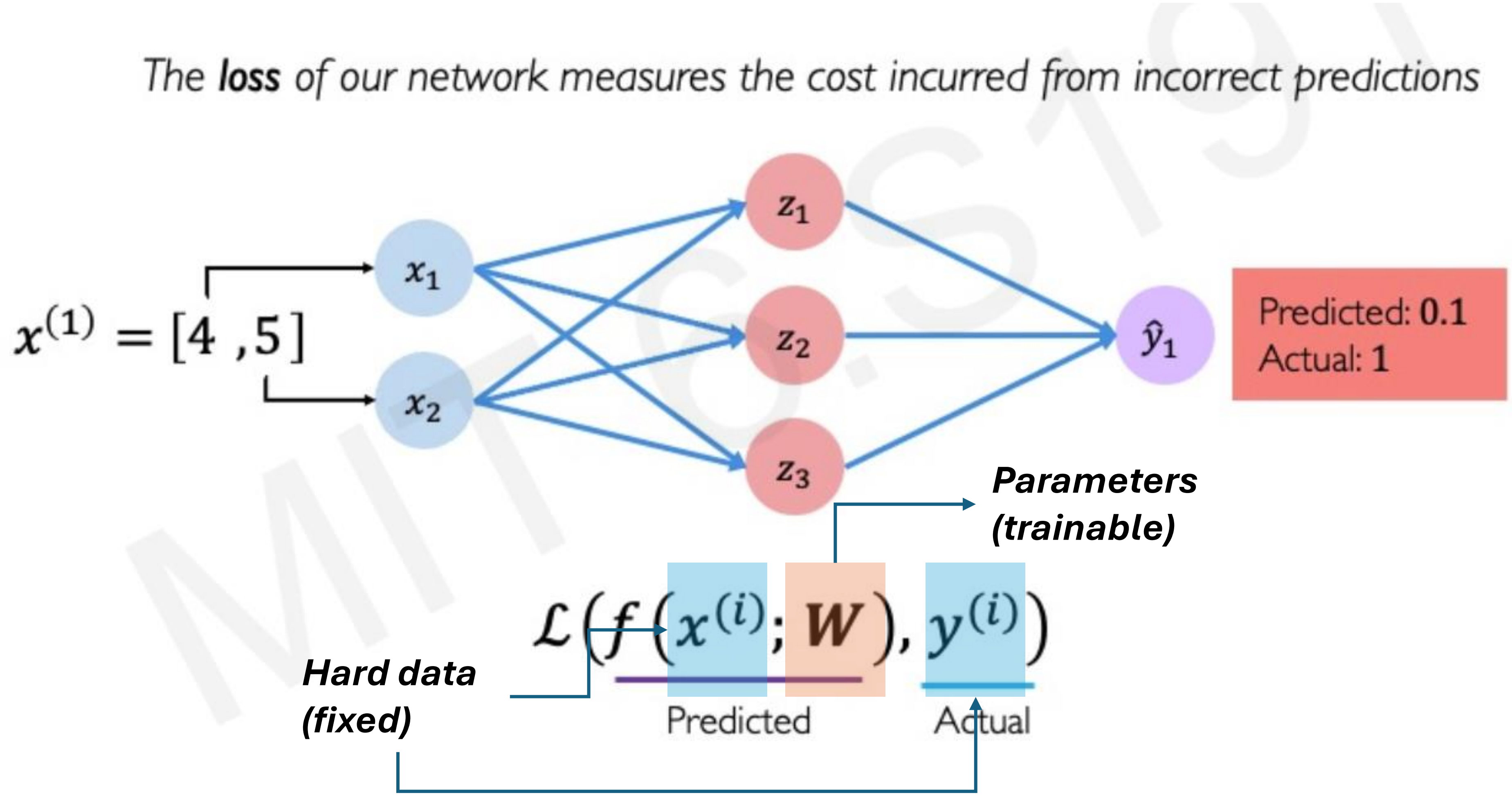
Regression Case – Recall Linear Regression

Mean squared error loss *can be used with regression models that output continuous real numbers*



Model Training

Model Training



How to match network outputs ($\hat{y}$) to actual values (y) ?

Model Training

We want to find the network weights that **achieve the lowest loss**

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}\left(f\left(x^{(i)}; \boldsymbol{W}\right), y^{(i)}\right)$$

$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}}\, J(\boldsymbol{W})$$

Model Training

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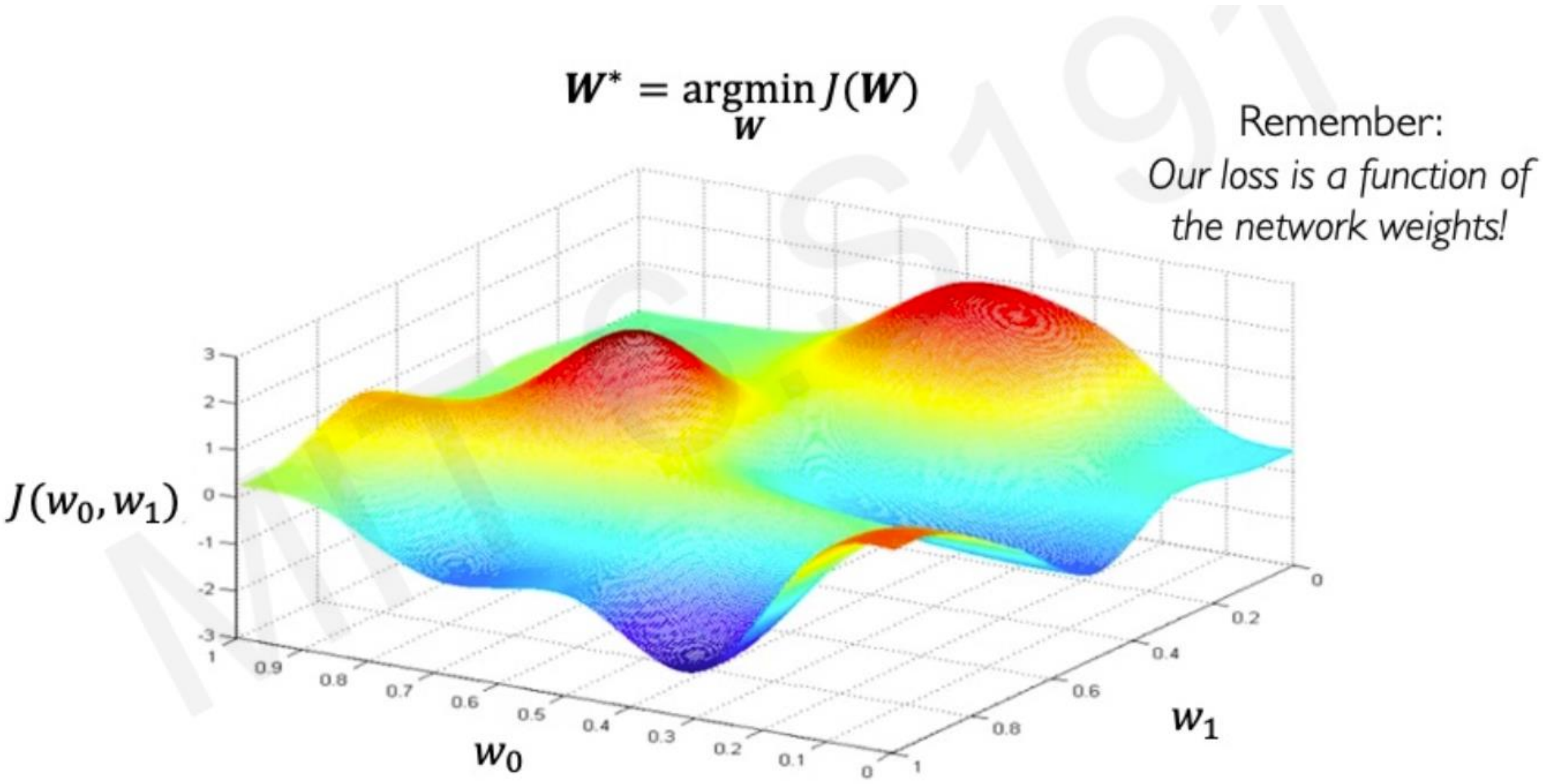
$$\boldsymbol{W}^* = \underset{\boldsymbol{W}}{\operatorname{argmin}}\, J(\boldsymbol{W})$$

Remember:

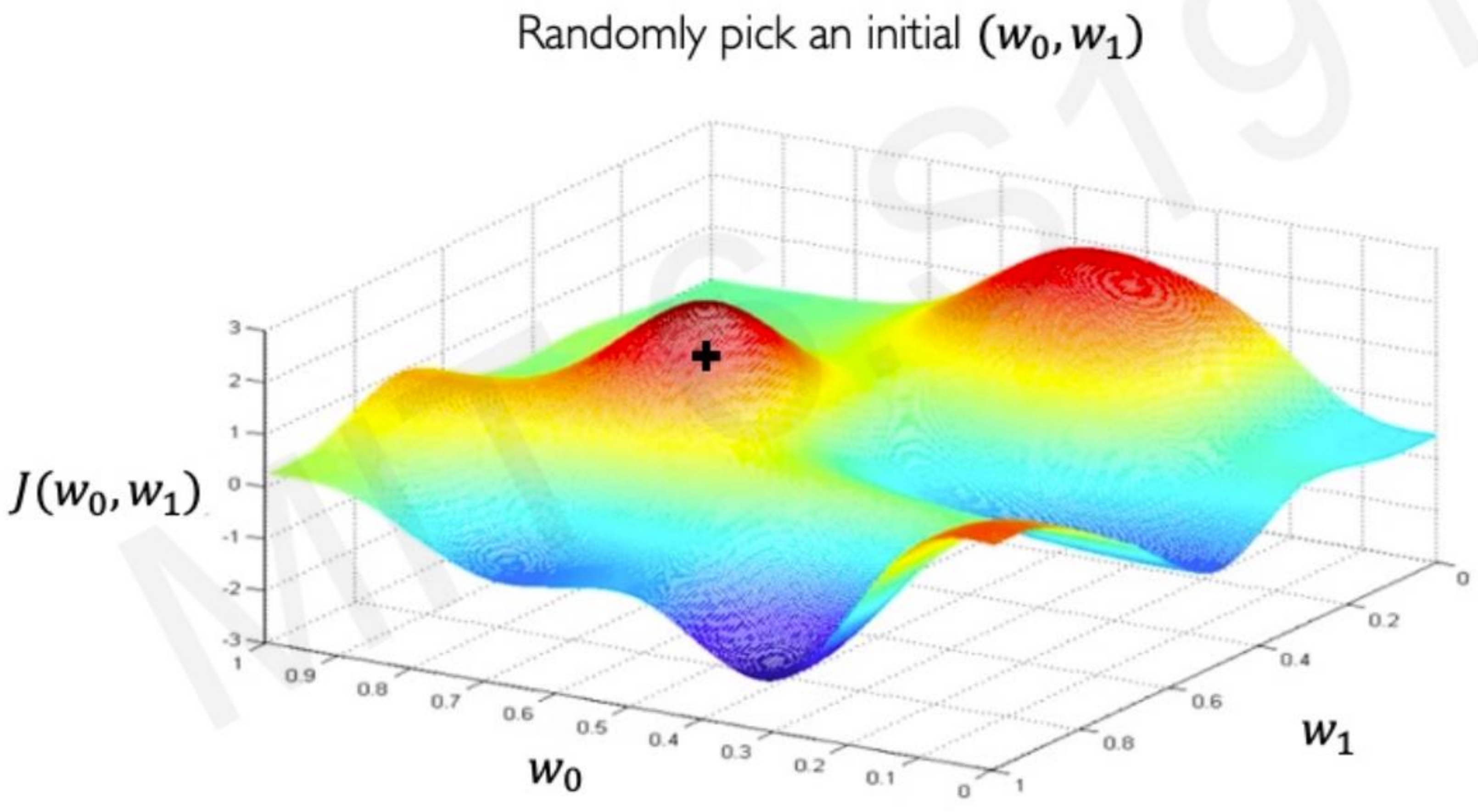
$$\boldsymbol{W} = \left\{\boldsymbol{W}^{(0)}, \boldsymbol{W}^{(1)}, \cdots\right\} \Rightarrow \begin{bmatrix} w_1 \\ \dots \\ w_m \end{bmatrix}$$

Loss Surface

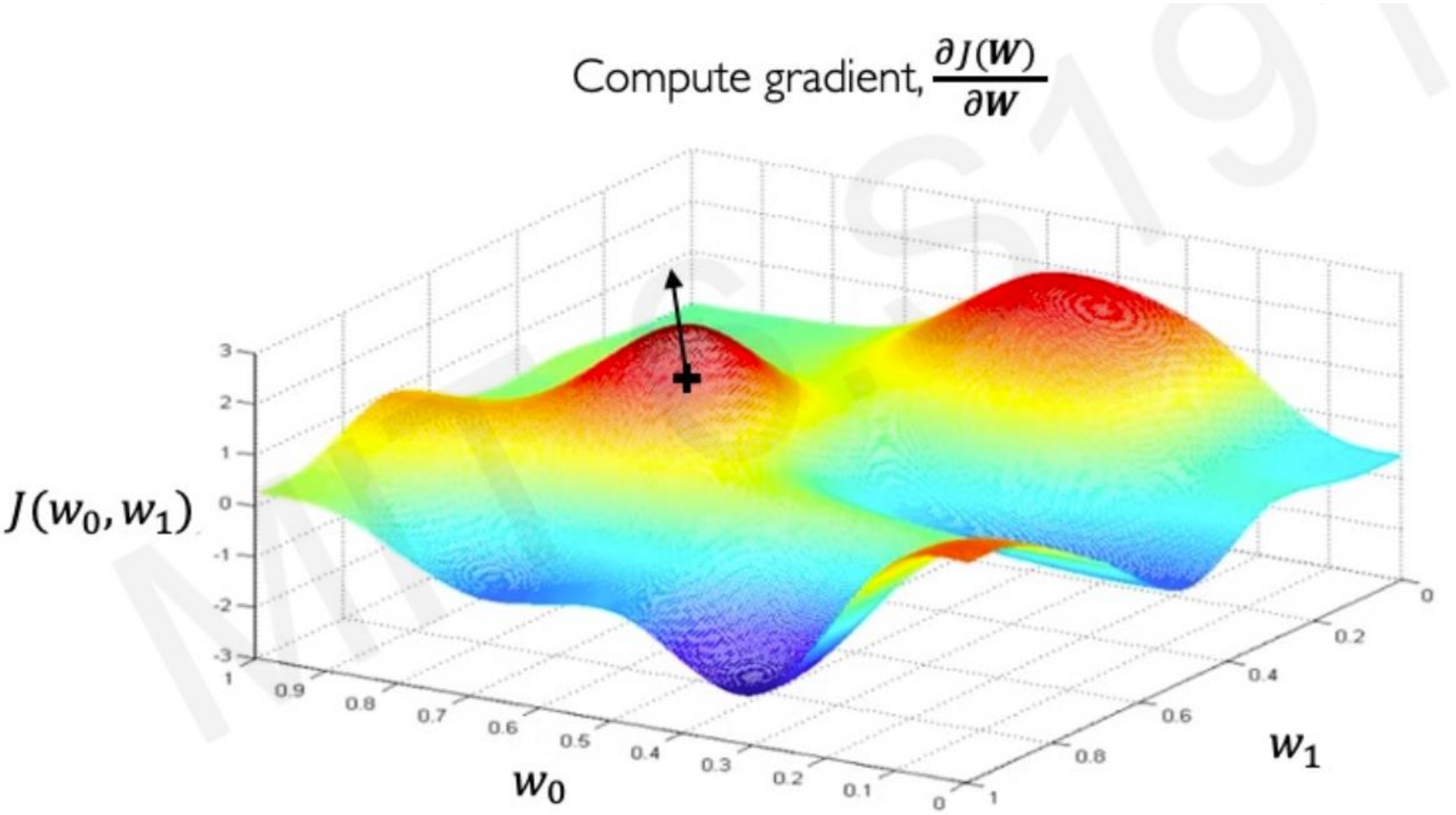
If we try all possible of parameters (W), here is the loss surface.



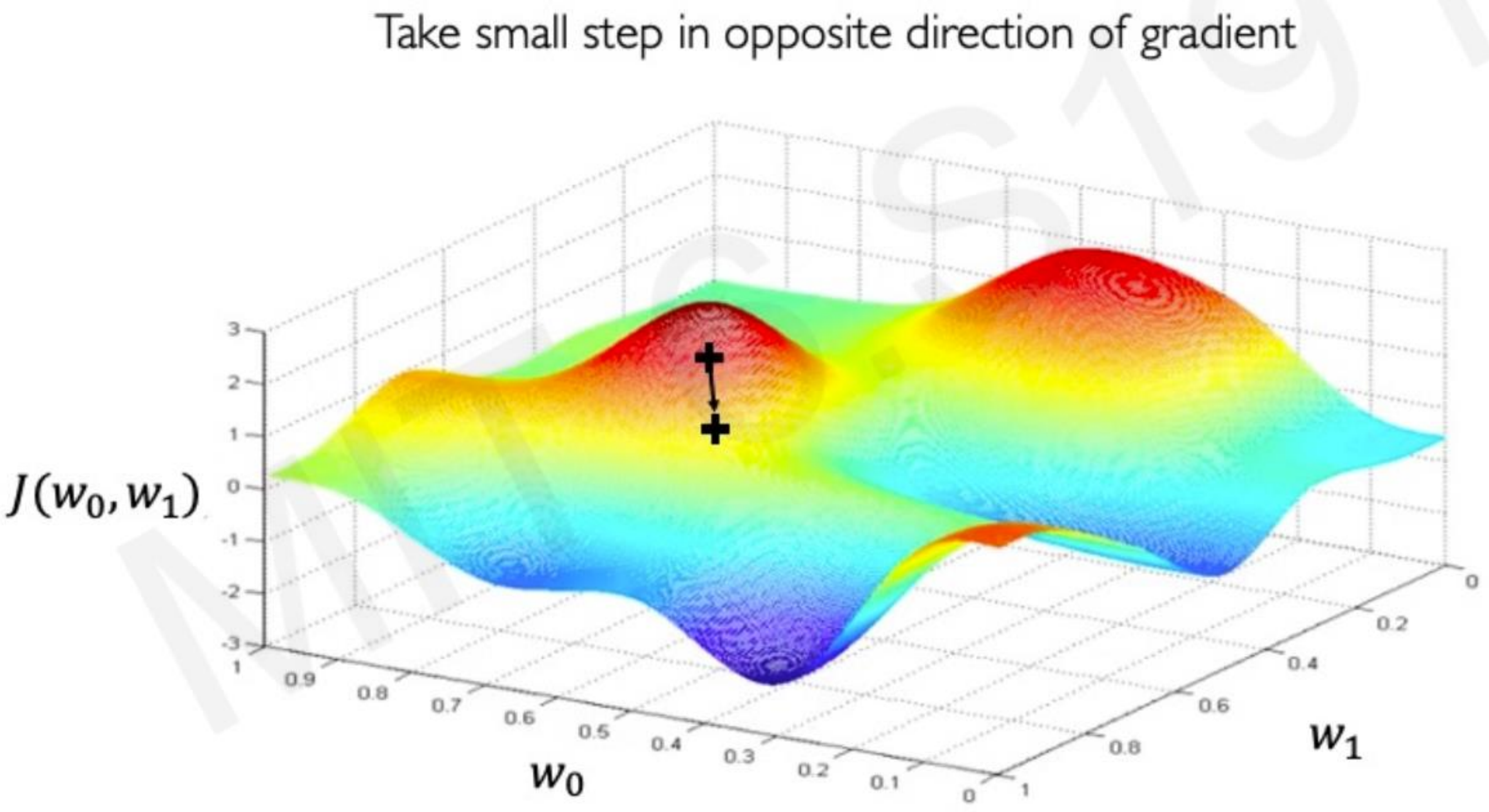
Gradient Descent



Gradient Descent



Gradient Descent



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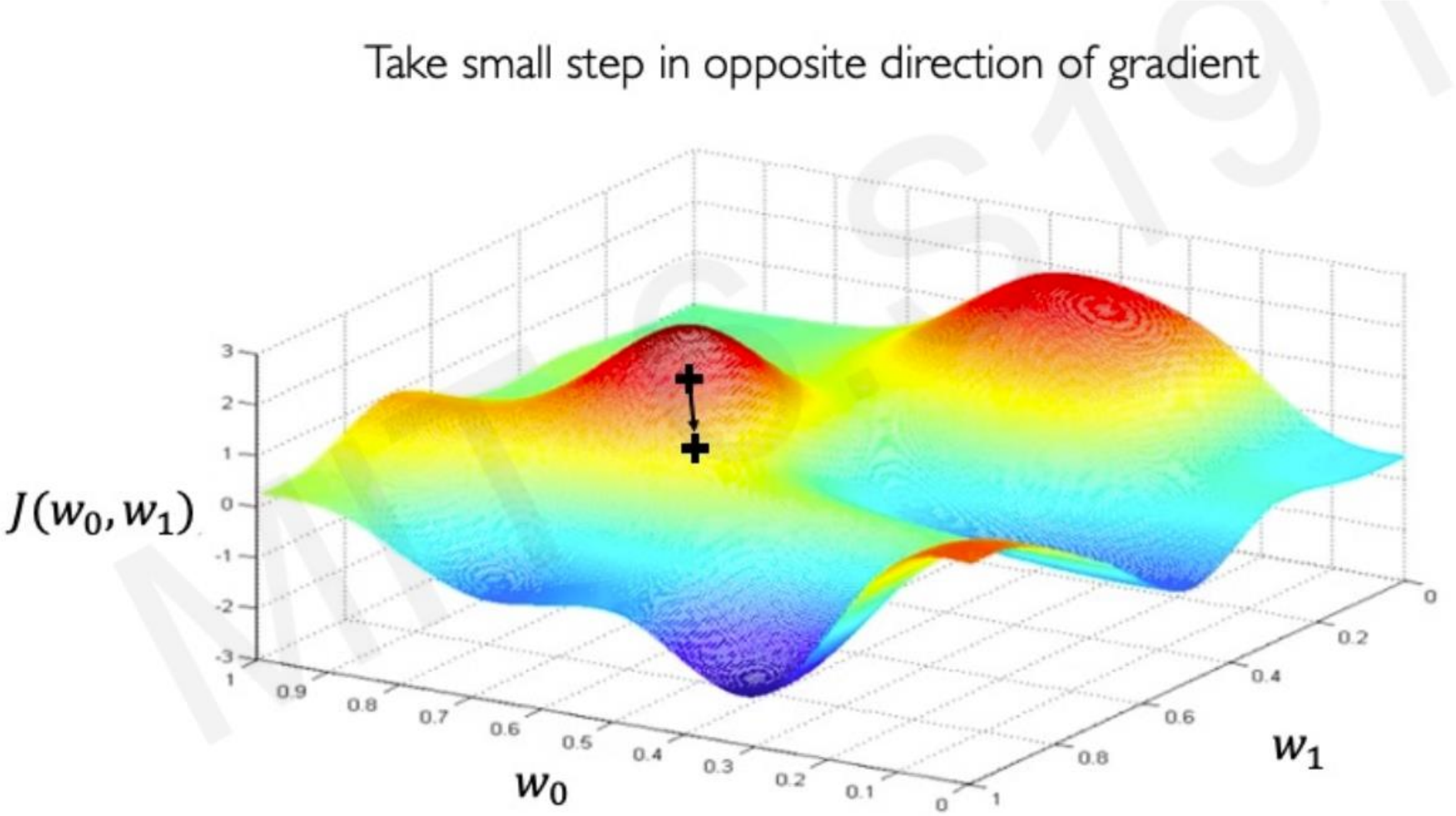
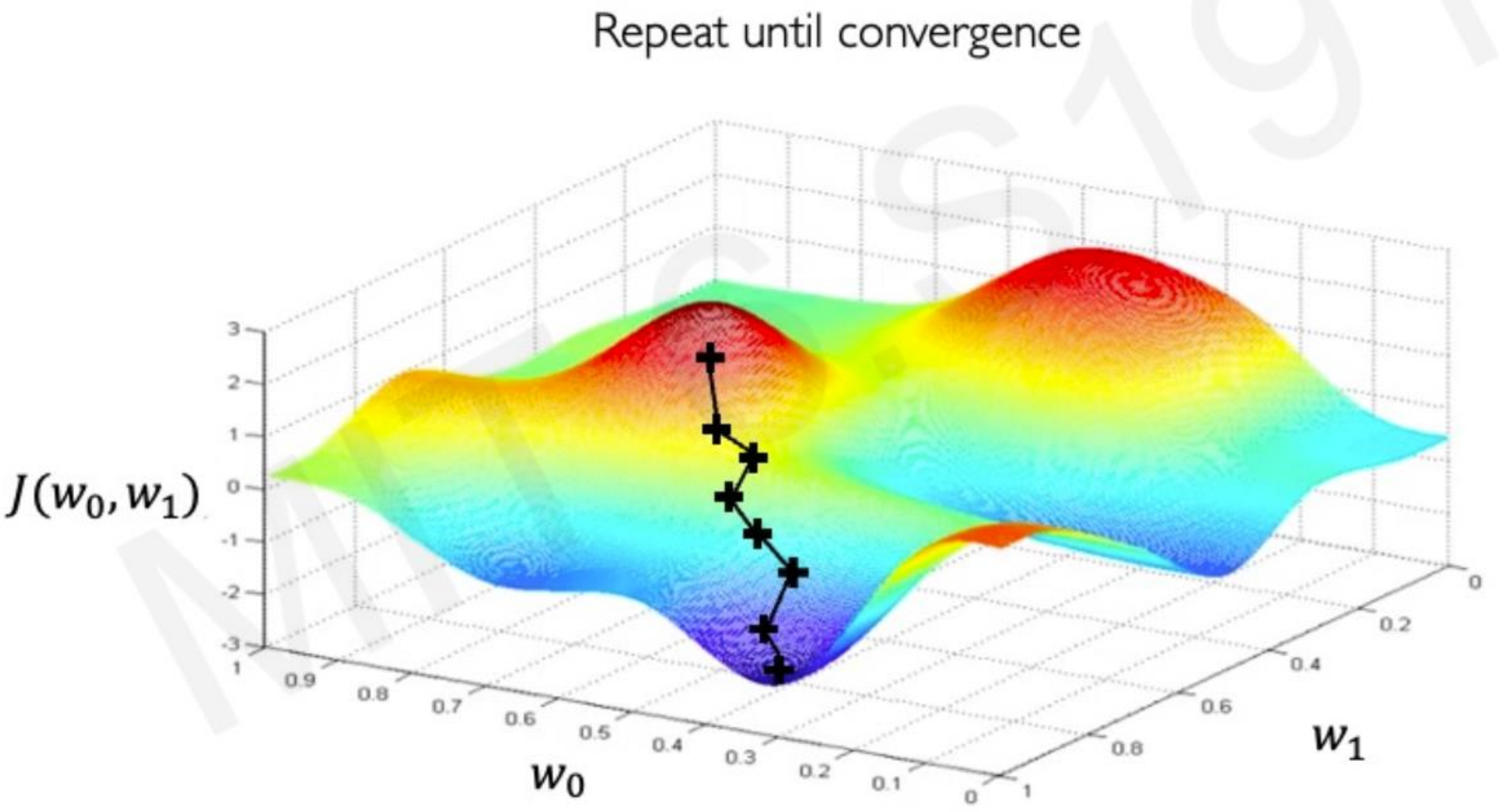


Gradient Descent



Gradient Descent

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0, \sigma^2)$
2. Loop until convergence:
3. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
4. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Return weights

Computing Gradients

Gradient Computations

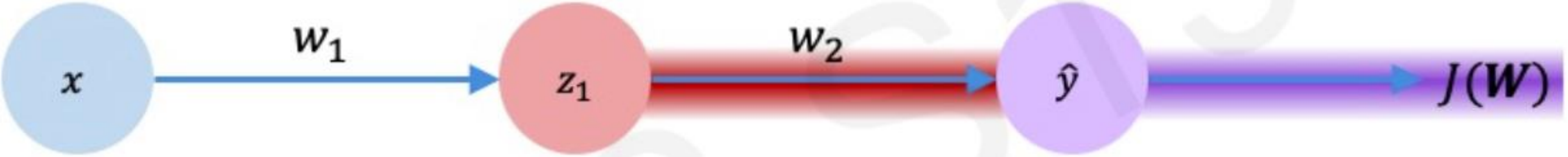
How does a small change in one weight (ex. $\mathbf{w_2}$) affect the final loss $J(\boldsymbol{W})$?

Gradient Computations

$$\frac{\partial J(\boldsymbol{W})}{\partial \boxed{w_2}} =$$

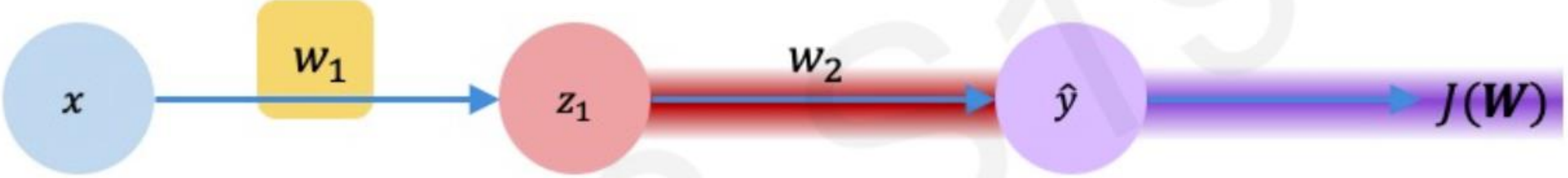
Let's use the chain rule!

Gradient Computations



$$\frac{\partial J(\boldsymbol{W})}{\partial w_2} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_2}$$

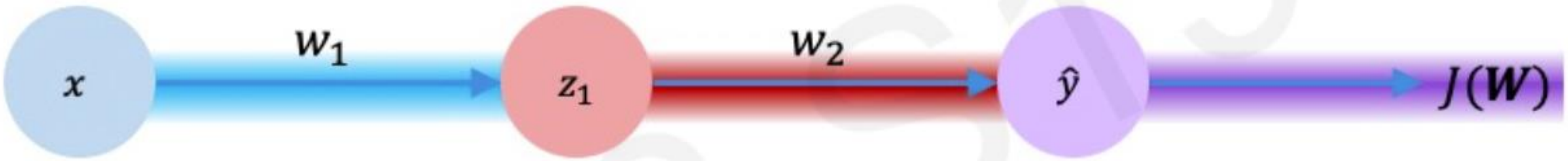
Gradient Computations



$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial w_1}$$

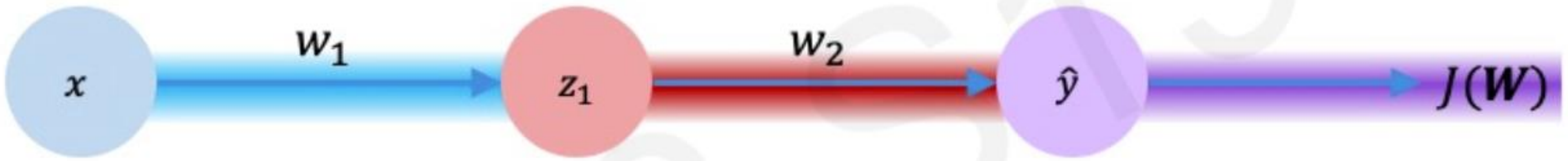
Apply chain rule! Apply chain rule!

Gradient Computations



$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

Gradient Computations



$$\frac{\partial J(\boldsymbol{W})}{\partial w_1} = \frac{\partial J(\boldsymbol{W})}{\partial \hat{y}} * \frac{\partial \hat{y}}{\partial z_1} * \frac{\partial z_1}{\partial w_1}$$

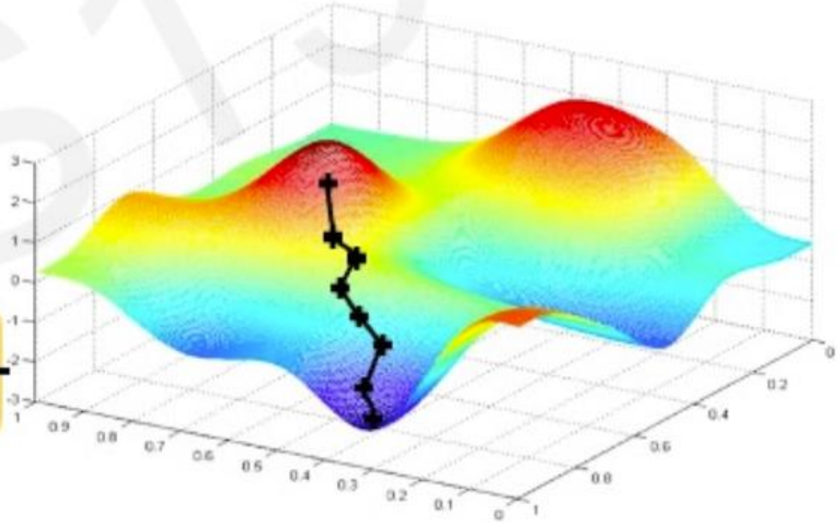
Repeat this for **every weight in the network** *using gradients from later layers*

Model Training Techniques

Mini-Batch Training

Algorithm

1. Initialize weights randomly $\sim\mathcal{N}(0,\sigma^2)$
2. Loop until convergence:
3. Pick batch of B data points
4. Compute gradient, $\frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}} = \frac{1}{B}\sum_{k=1}^{B} \frac{\partial J_k(\boldsymbol{W})}{\partial \boldsymbol{W}}$
5. Update weights, $\boldsymbol{W} \leftarrow \boldsymbol{W} - \eta \frac{\partial J(\boldsymbol{W})}{\partial \boldsymbol{W}}$
6. Return weights



Fast to compute and a much better estimate of the true gradient!

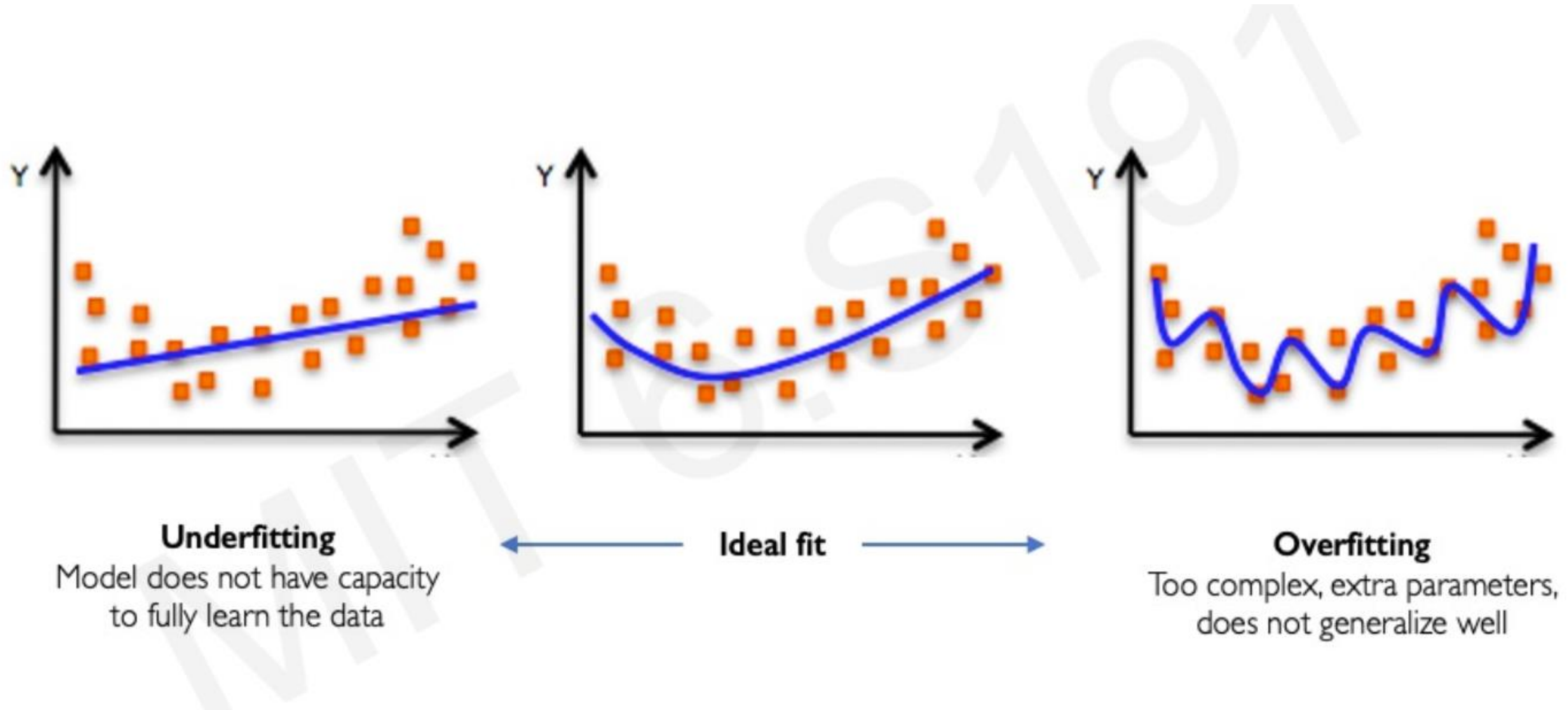
Adaptive Learning Rates

- Learning rates are no longer fixed
- Can be made larger or smaller depending on:
  - how large gradient is
  - how fast learning is happening
  - size of particular weights
  - etc...

GD Variants

Algorithm	TF Implementation	Reference
• SGD	`tf.keras.optimizers.SGD`	Kiefer & Wolfowitz. "Stochastic Estimation of the Maximum of a Regression Function." 1952.
• Adam	`tf.keras.optimizers.Adam`	Kingma et al. "Adam: A Method for Stochastic Optimization." 2014.
• Adadelta	`tf.keras.optimizers.Adadelta`	Zeiler et al. "ADADELTA: An Adaptive Learning Rate Method." 2012.
• Adagrad	`tf.keras.optimizers.Adagrad`	Duchi et al. "Adaptive Subgradient Methods for Online Learning and Stochastic Optimization." 2011.
• RMSProp	`tf.keras.optimizers.RMSProp`	

Dealing with Model Overfitting



Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Regularization

What is it?

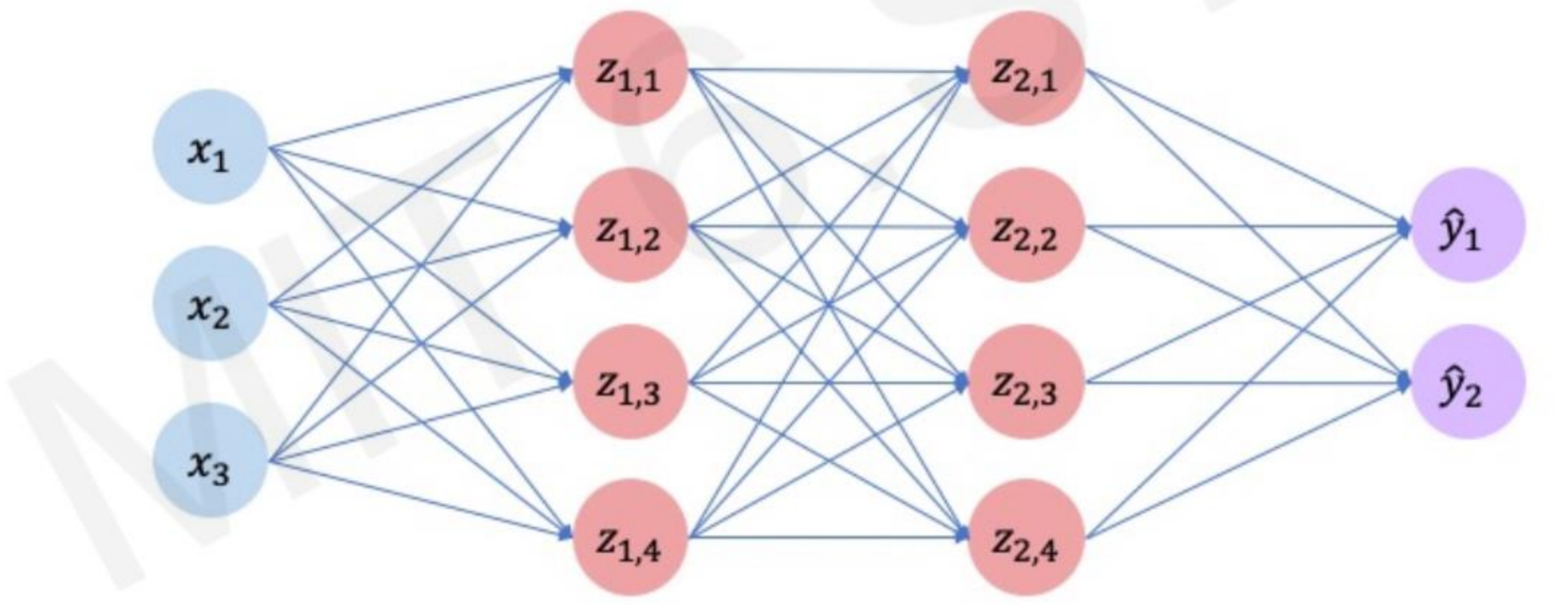
Technique that constrains our optimization problem to discourage complex models

Why do we need it?

Improve generalization of our model on unseen data

Dropout

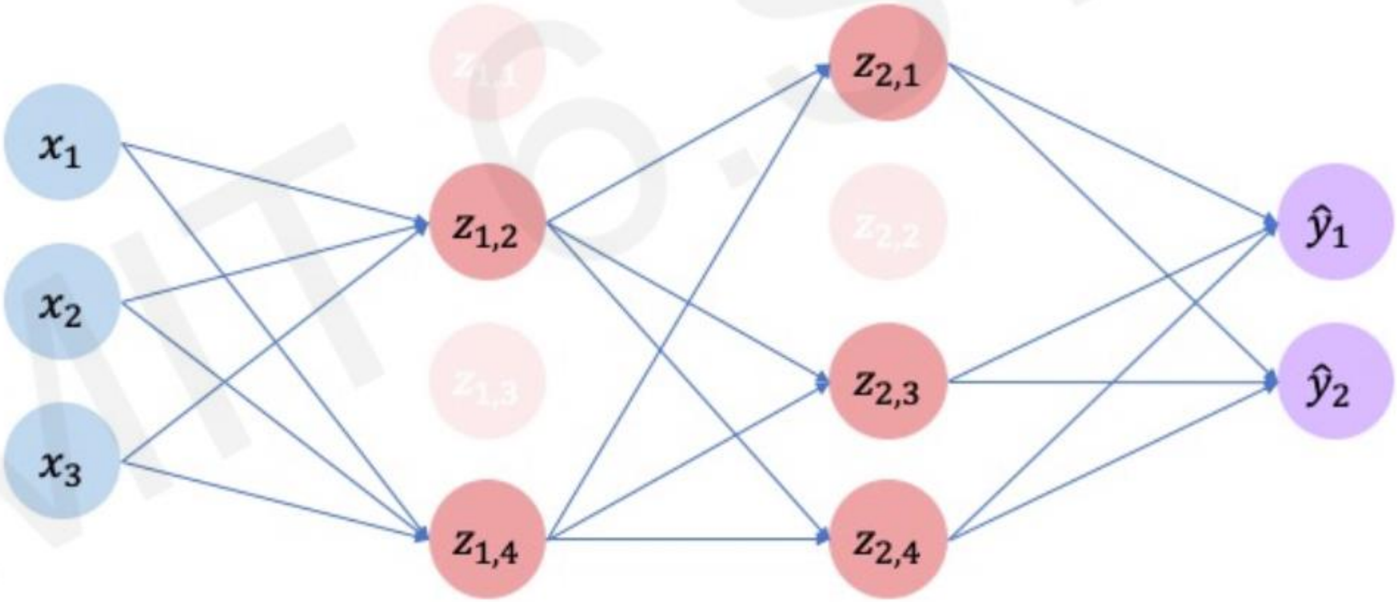
- During training, randomly set some activations to 0



Dropout

- During training, randomly set some activations to 0
  - Typically 'drop' 50% of activations in layer
  - Forces network to not rely on any 1 node

```
tf.keras.layers.Dropout(p=0.5)
```



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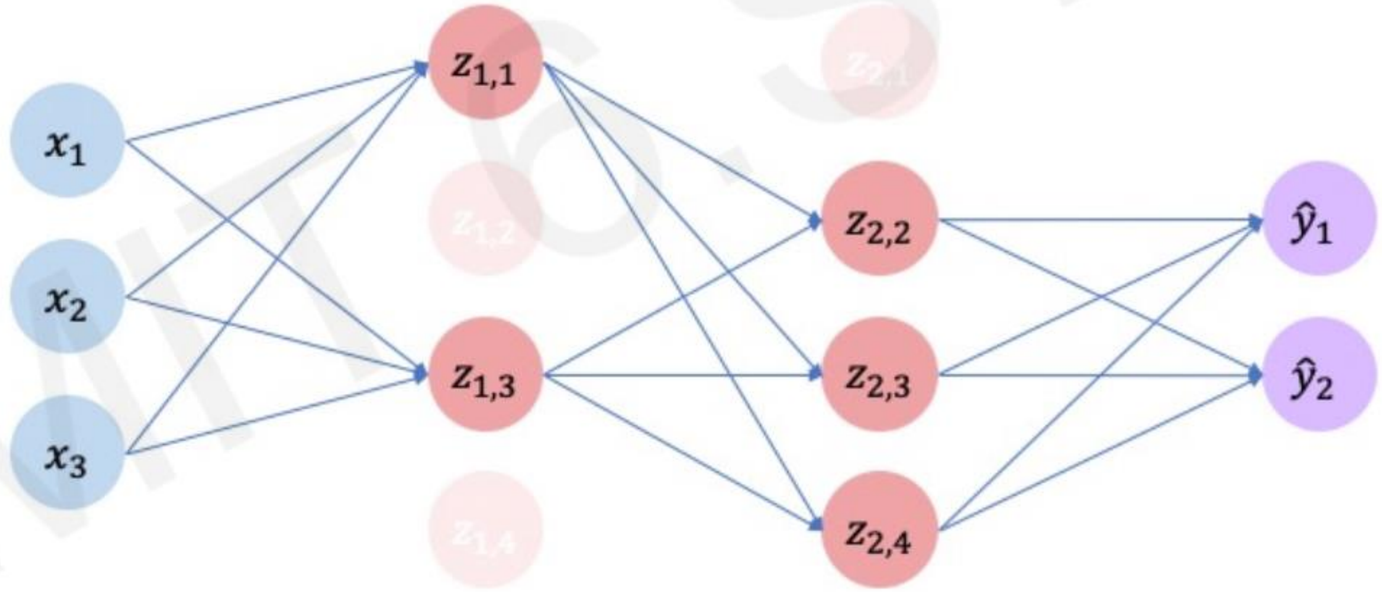
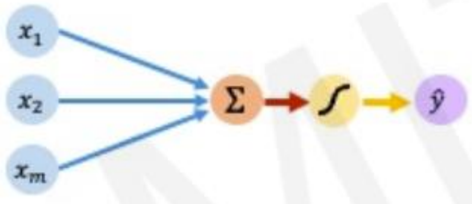


Early Stopping

- Stop training before we have a chance to overfit

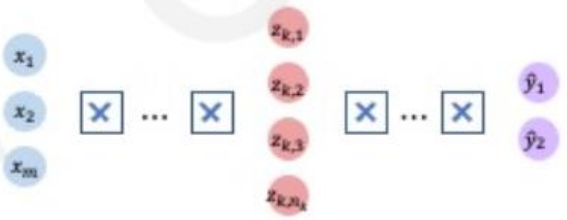
The Perceptron

- Structural building blocks
- Nonlinear activation functions



Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation



Training in Practice

- Adaptive learning
- Batching
- Regularization