# Lecture III (Part IV) – Network and Fleet Development

Airline Planning & Optimization (AE4423)

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# Network and Fleet Modelling

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- Model 1: Point-to-point network model
- Model 2: Hub-&-spoke network model
- Model 3: Fleet & network model

Simplest model

Add connections

Add fleet

- Simplifications

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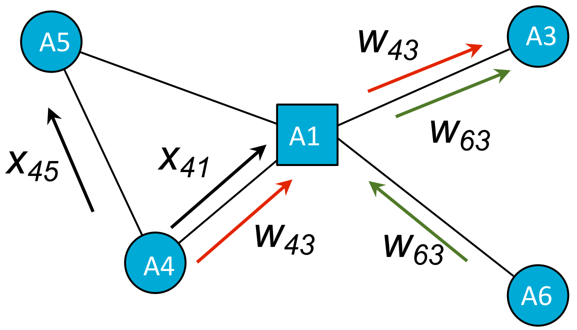
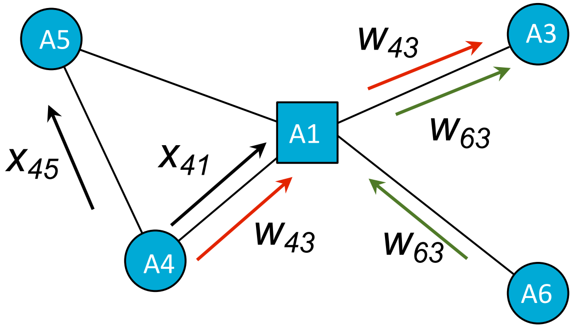
# Model 2: Hub-&-spoke network model

- Several airlines provide the possibility for passengers to connect at a hub airport:
  - In H&S networks
    - demand is forecast according to the OD market
    - the flight leg demand depends on our decisions
    - hub airports guarantee connections between flights
- Let's address the dichotomy between demand and supply:



# Model 2: Hub-&-spoke network model




- **Notation**

Sets

$N$: set of airports (where $h$ is the hub)

Decision variables

$w_{ij}$ : flow from airport $i$ to airport $j$ that transfers at the hub
$x_{ij}$ : direct flow from airport i to airport $j$
$z_{ij}$ : number of flights from airport $i$ to airport $j$

Parameters

$q_{ij}$ : travel demand between airport $i$ to airport $j$
$g_k = 0$ if a hub is located at airport $k$;
$g_k = 1$ otherwise
$d_{ij}$ : distance between airports $i$ and $j$
*Yield* : revenue per RPK flown (average yield)
$s$ : number of seats per aircraft
*CASK* : unit operation cost per ASK flown
$sp$ : speed of the aircraft

*LF* : average load factor
*AC* : number of aircraft
*LTO* : landing and take-off time
BT : aircraft avg. utilisation time

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# Model 2: Hub-&-spoke network model

$$Max\ Profit = \sum_{i \in N} \sum_{j \in N} \left[ Yield \times d_{ij} \left( x_{ij} + w_{ij} \right) - CASK \times d_{ij} \times s \times z_{ij} \right] \qquad \text{(OF)}$$

s.t.:

$$x_{ij} + w_{ij} \leq q_{ij} \quad , \forall\, i, j \in \boldsymbol{N} \qquad \text{(C1)}$$

All flow from each airport leave the airport, either through a hub or not

$$w_{ij} \leq q_{ij} \times g_i \times g_j \quad , \forall\, i, j \in \boldsymbol{N} \qquad \text{(C1*)}$$

Transfer passengers are only if the hub is not the origin or the destination

$$x_{ij} + \sum_{m \in N} w_{im} \times \left(1 - g_j\right) + \sum_{m \in N} w_{mj} \times \left(1 - g_i\right) \leq z_{ij} \times s \times LF \quad , \forall\, i, j \in \boldsymbol{N} \qquad \text{(C2)}$$

Capacity verification in each flight leg

$$\sum_{j \in N} z_{ij} = \sum_{j \in N} z_{ji} \quad , \forall\, i \in \boldsymbol{N} \qquad \text{(C3)}$$

Balance between incoming and outgoing flights at each node

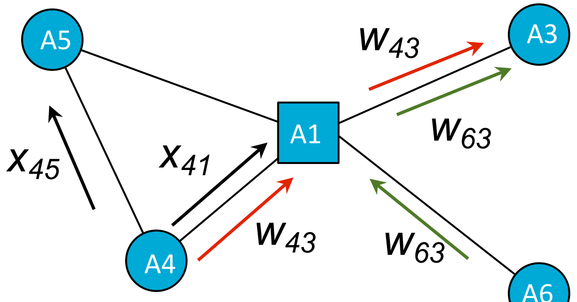
$$\sum_{i \in N} \sum_{j \in N} \left( \frac{d_{ij}}{sp} + LTO \right) \times z_{ij} \leq BT \times AC \qquad \text{(C4)}$$

Use of aircraft limited to the number of aircraft and the block hours associated

# Model 2: Hub-&-spoke network model

- How do constraints C2 work?

$$x_{ij} + \sum_{m \in \boldsymbol{N}} w_{im} \times (1 - g_j) + \sum_{m \in \boldsymbol{N}} w_{mj} \times (1 - g_i) \leq z_{ij} \times s \times LF \quad , \forall\, i,j \in \boldsymbol{N}$$




- For $i$ = 4 and $j$ = 1

$g_i$

$$x_{4,1} + [(w_{4,1} + w_{4,2} + w_{4,3} + \cdots) \times (1 - 0)] + \cancel{[(w_{1,1} + w_{2,1} + w_{3,1} + \cdots) \times (1 - 1)]} \leq z_{4,1} \times s \times LF$$

- For $i$ = 1 and $j$ = 3

$$x_{1,3} + \cancel{[(w_{1,1} + w_{1,2} + w_{1,3} + \cdots) \times (1 - 1)]} + [(w_{1,3} + w_{2,3} + w_{3,3} + \cdots) \times (1 - 0)] \leq z_{1,3} \times s \times LF$$

- For $i$ = 4 and $j$ = 5

$$x_{4,5} + \cancel{[(w_{4,1} + w_{4,2} + w_{4,3} + \cdots) \times (1 - 1)]} + \cancel{[(w_{1,5} + w_{2,5} + w_{3,5} + \cdots) \times (1 - 1)]} \leq z_{4,5} \times s \times LF$$

# Model 2: Hub-&-spoke network model

## Problem 2

The FlyAtlantic Airline is starting its operations with a hub in the Azores Islands (Portugal). The company wants to connect the Portuguese main airports (Lisbon, Oporto, Funchal and Ponta Delgada) with the North American cities of Boston and Toronto (distances in Table 1). For this, the airline needs to define the most profitable network and the frequency of flights. For these operations, the airline has 4 aircraft of the same type (see Table 2). The company forecasts an average revenue 0.16€/RPK. Propose a network based on the forecasted average weekly demand provided in Table 3.

Table 1

| | Distances (km) | | | | | |
|---|---|---|---|---|---|---|
| Airport | PDL | LIS | OPO | FNC | YTO | BOS |
| PDL | 0 | 1461 | 1536 | 975 | 4545 | 3888 |
| LIS | 1461 | 0 | 336 | 973 | 5790 | 5177 |
| OPO | 1536 | 336 | 0 | 1244 | 5671 | 5081 |
| FNC | 975 | 973 | 1244 | 0 | 5515 | 4851 |
| YTO | 4545 | 5790 | 5671 | 5515 | 0 | 691 |
| BOS | 3888 | 5177 | 5081 | 4851 | 691 | 0 |

Table 2

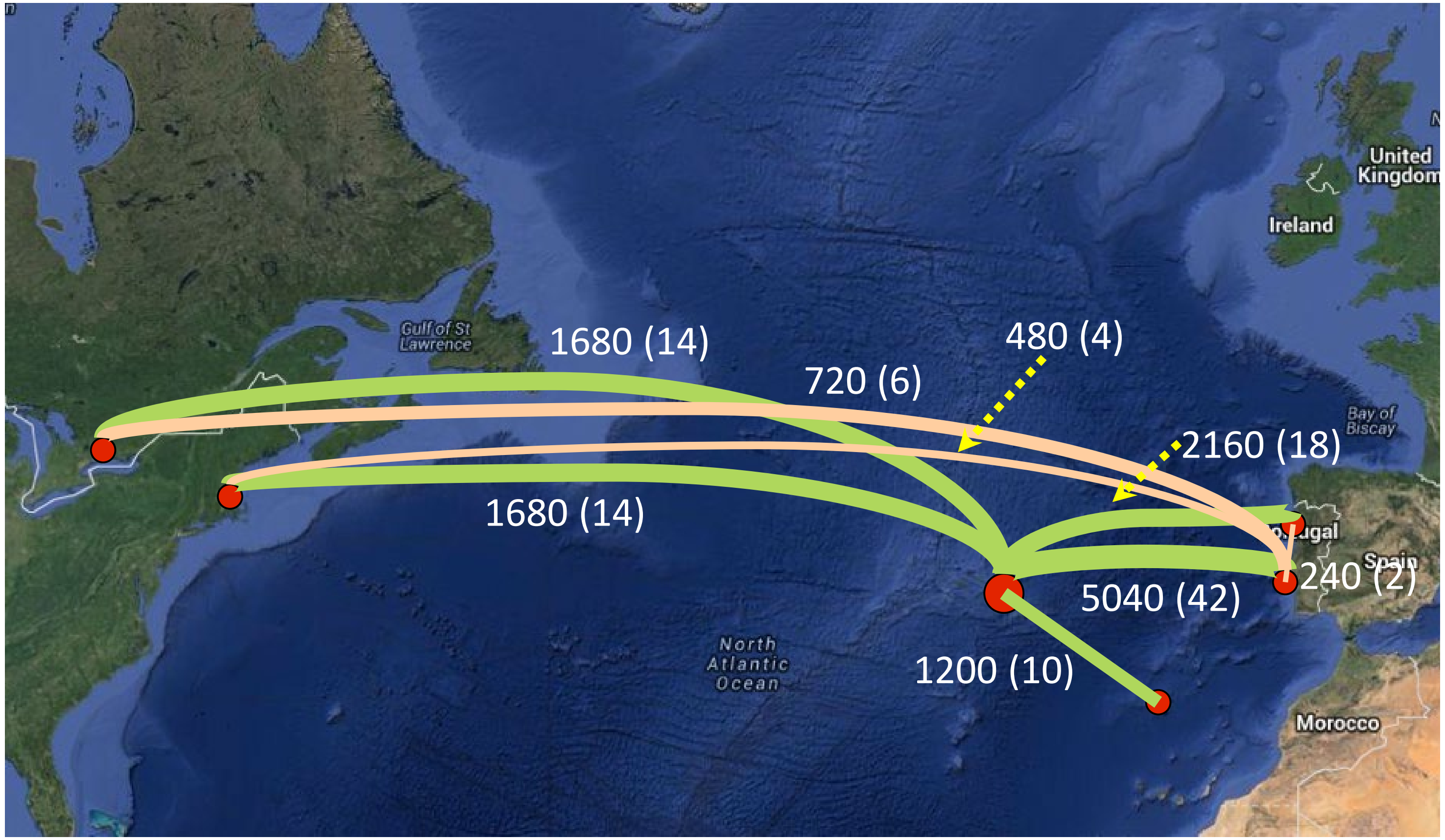
| Parameters | | |
|---|---|---|
| CASK | 0,12 | [$/ASK] |
| LF | 0,80 | [%] |
| Seats | 150 | [units] |
| Speed | 890 | [km/h] |
| LTO | 20 | [min] |
| BT | 13 | [h/day] |
| Fleet | 4 | [units] |

Table 3

| | Demand (pax/week) | | | | | |
|---|---|---|---|---|---|---|
| Airport | PDL | LIS | OPO | FNC | YTO | BOS |
| PDL | 0 | 2509 | 1080 | 558 | 770 | 713 |
| LIS | 2509 | 0 | 216 | 112 | 360 | 333 |
| OPO | 1080 | 216 | 0 | 78 | 46 | 43 |
| FNC | 558 | 112 | 78 | 0 | 32 | 30 |
| YTO | 770 | 360 | 46 | 32 | 0 | 70 |
| BOS | 713 | 333 | 43 | 30 | 70 | 0 |

# Model 2: Hub-&-spoke network model

- Problem 2 (solution)




**OF:**
316 821 €/week

**AC Utilisation:**
94,33%

**Legend:**
Pax (flights)

Note: Both ways flights and pax flows

# Model 3: Fleet & network model

- The problem:
  - given a set of airports from which an airline can operate;
  - knowing the demand in each specific flight-leg;
  - having a set of different aircraft types available, with specific characteristics (e.g., speed, seats, block hours);

  - define which flight legs to operate;
  - estimated number of passengers to transport;
  - define the flight frequency per leg; and
  - define the fleet size and composition.

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# Model 3: Fleet & network model

- Objective function
  - minimize costs (satisfying the demand)
  - maximize revenues
  - (OF) maximize profit
- Constraints
  - (C1) Demand verification: # pax ≤ demand
  - (C2) Capacity: # pax in each leg ≤ seats available per leg
  - (C3) Continuity constraint: #AC inbound = #AC outbound (per airport, per AC type)
  - (C4) AC Productivity: hours of operation ≤ BT * # AC
  - (C5) **Range Constraint:** Range ≤ distance → no flights
  - (C6) **Budget Constraint:** #AC * Cost ≤ Budget

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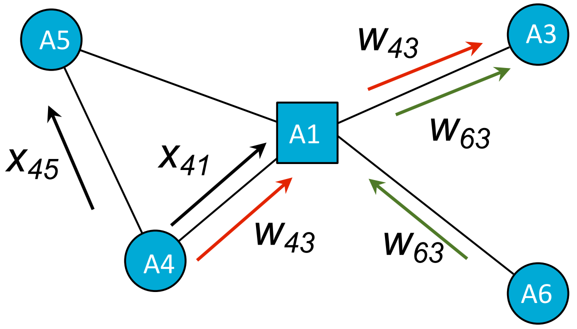
# Model 3: Fleet & network model




- **Notation**

Sets

$N$: set of airports (where $h$ is the hub)
$K$: set of aircraft types

Decision variables

$w_{ij}$ : flow from airport $i$ to airport $i$ that transfers at the hub
$x_{ij}$ : direct flow from airport i to airport $j$
$z^k_{ij}$ : number of flights from airport $i$ to airport $j$ with aircraft type $k$
$AC^k$ : number of aircraft type $k$

Parameters

$q_{ij}$ : travel demand between airport $i$ to airport $j$
$g_h$ = 0 if a hub is located at airport $h$;
$g_h$ = 1 otherwise
$d_{ij}$ : distance between airports $i$ and $j$
$Yield$ : revenue per RPK flown (average yield)
$s^k$ : number of seats per aircraft type $k$
$CASK^k$ : unit operation cost per ASK flown by aircraft type $k$
$sp^k$ : speed of the aircraft type $k$

$LF$ : average load factor
$LTO$ : landing and take-off time
$BT^k$ : aircraft type $k$ avg. utilisation time
$C^k$ : cost of purchasing aircraft type $k$

# Model 3: Fleet & network model

$$Max\ Profit = \sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N}} \left[ Yield \times d_{ij} (x_{ij} + w_{ij}) - \sum_{k \in \boldsymbol{K}} (CASK^k \times d_{ij} \times s^k \times z_{ij}^k) \right] \quad \text{(OF)}$$

s.t.:

$$x_{ij} + w_{ij} \leq q_{ij} \quad , \forall\, i,j \in \boldsymbol{N} \quad \text{(C1)}$$

$$w_{ij} \leq q_{ij} \times g_i \times g_j \quad , \forall\, i,j \in \boldsymbol{N} \quad \text{(C1*)}$$

$$x_{ij} + \sum_{m \in \boldsymbol{N}} w_{im} \times (1 - g_j) + \sum_{m \in \boldsymbol{N}} w_{mj} \times (1 - g_i) \leq \sum_{k \in \boldsymbol{K}} z_{ij}^k \times s^k \times LF \quad , \forall\, i,j \in \boldsymbol{N} \quad \text{(C2)}$$

$$\sum_{j \in \boldsymbol{N}} z_{ij}^k = \sum_{j \in \boldsymbol{N}} z_{ji}^k \quad , \forall\, i \in \boldsymbol{N}, k \in \boldsymbol{K} \quad \text{(C3)}$$

$$\sum_{i \in \boldsymbol{N}} \sum_{j \in \boldsymbol{N}} \left( \frac{d_{ij}}{sp^k} + LTO \right) \times z_{ij}^k \leq BT^k \times AC^k \ , k \in \boldsymbol{K} \quad \text{(C4)}$$

$$z_{ij}^k \leq a_{ij}^k \qquad \rightarrow a_{ij}^k = \begin{cases} 10000 & if\ d_{ij} \leq R^k \\ 0 & otherwise \end{cases} \quad \text{(C5)}$$

Aircraft range used to define matrix $a^k_{ij}$ and constrain frequency to range limits

$$\sum_{k \in \boldsymbol{K}} C^k \times AC^k \leq B \quad \text{(C6)}$$

Investment costs cannot be higher than the budget available

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# Model 3: Fleet & network model

## Problem 3

What if the FlyAtlantic Airline would have a fleet of different aircraft (see Table 1). How should we model a multi-fleet network model (aka, multi-class network model)? Would the results differ from the results obtained in Problem 2?

Table 1

| Parameters | A310 | A320 | |
|---|---|---|---|
| CASK | 0,12 | 0,11 | [$/ASK] |
| LF | 0,80 | 0,80 | [%] |
| Seats | 240 | 160 | [units] |
| Range | 9600 | 5400 | [km] |
| Speed | 900 | 870 | [km/h] |
| LTO | 20 | 12 | [min] |
| BT | 14 | 12 | [h/day] |
| Fleet | 2 | 2 | [units] |

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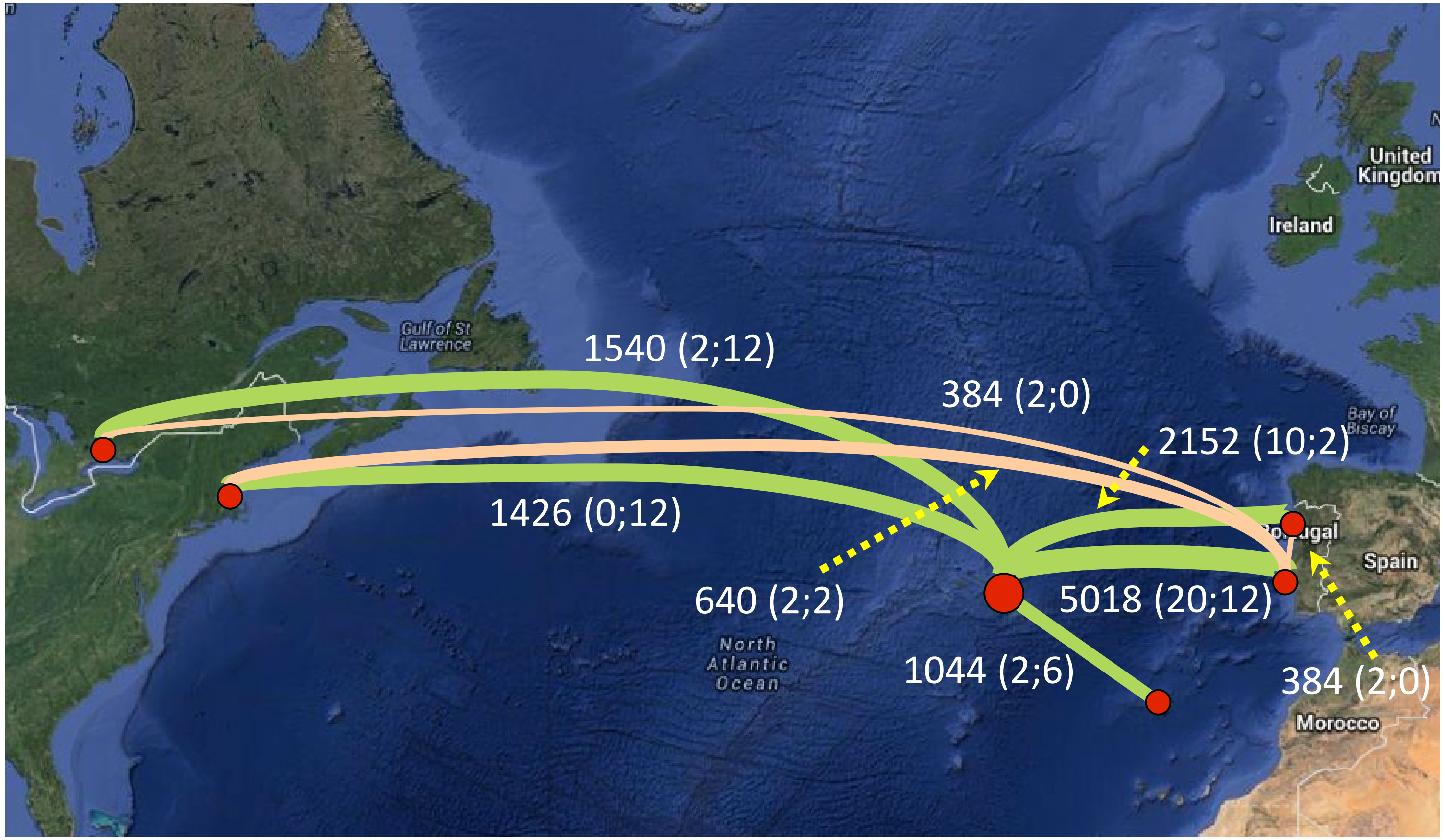
# Model 3: Fleet & network model

- Problem 3 (solution)




**OF:**
533 985 €/week

**AC Utilization:**
A310 - 51,15%
A320 - 99,90%

**Legend:**

Pax
(flights A310; A320)

Note: Both ways flights and pax flows

# Simplifications

The previous models represent an airline network development decision process in a simplified way. For instance:

- **demand is static** - demand does not vary with the solutions obtained (e.g., frequency, direct vs connecting flight);
- **no competition** - market share is assumed to be already considered when defining demand values;
- **no passengers choice** - demand is reacting to the decisions according to the objective of the airline (i.e., maximise profit);
- **possible aircraft routing discontinuity** - a total “time-budget” is considered for the entire fleet without checking routing feasibility;
- **single scenario for a static future** - the uncertainty of the future network is not considered and the model runs for a single time period in the future.

- … do you identify other simplifications?

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