Tutorial 4: Solution

AE4426-19 Stochastic processes and simulation, 2020-2021

1 Discrete-time Continuous-state Markov Chain

a)

$\mathbb{E}[X_t]=\mathbb{E}[X_{t-1}+V_t]=\mathbb{E}[X_{t-1}] = \mathbb{E}[X_{t-2}+V_{t-1}] = \cdots=\mathbb{E}[X_0]=0$

$\mathrm{Var}[X_t]= \mathrm{Var}[X_{t-1}+V_t] =\mathrm{Var}[X_{t-1}]+\mathrm{Var}[V_t]+2\mathrm{Cov}[X_{t-1},V_t]= ...= \mathrm{t}$

$\mathbb{E}[X_sX_t]=\min(s,t)$
Consider the case $s<t$. Then

$$\mathbb{E}[X_sX_t] = \mathbb{E}[X_s(X_{t-1}+V_t)] = \cdots = s$$

b)
$\mathbb{E}[X_{40}]=0$
$\mathrm{Var}[X_{40}]=40$

c)
$\mathrm{P}(X_{40}\in[-3,3])=0.37$
$\mathrm{P}(X_{50}\in[-5,5])=0.52$

d)
$\mathrm{P}(X_{40}>2.7)=0.37$ (n=100)
$\mathrm{P}(X_{40}>2.7)=0.33$ (n=10,000)

e) See Fig. 1

f) See Fig. 2

f) 95% CI $=[-0.11,0.12]$

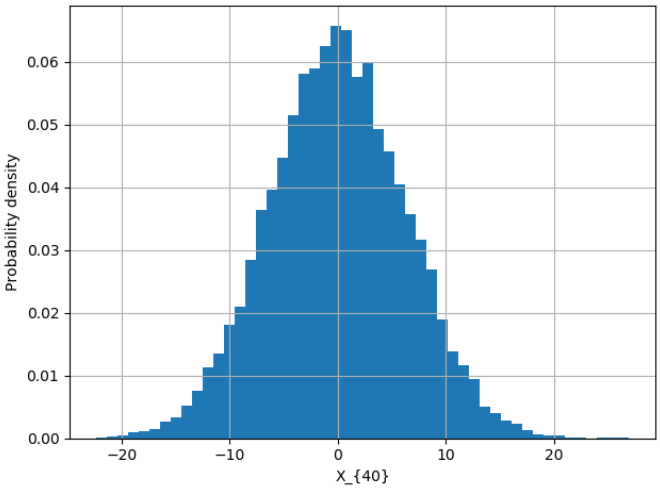

Figure 1: Pdf of X_{40}.

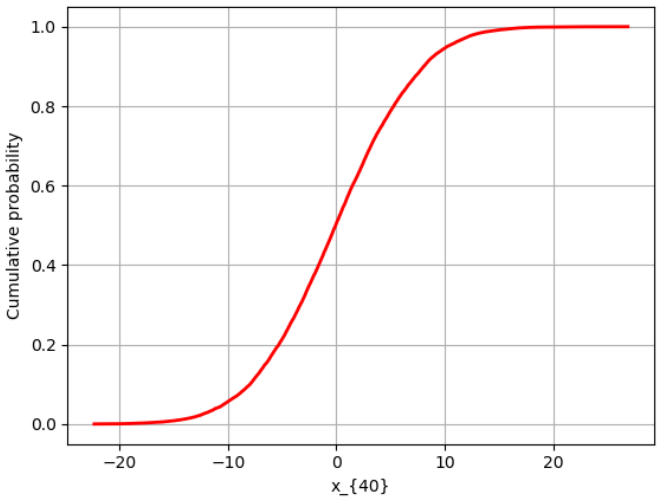

Figure 2: Cdf of X_{40}.