

Tutorial 4

AE4426-19 Stochastic processes and simulation, 2020-2021

1 Discrete-time continuous-state Markov process

Consider a Gauss-Markov process with the difference equation:

$$X_t = X_{t-1} + V_t, t \geq 1,$$

with $X_0 = 0$ and V_t independent and identically distributed random variables distributed according to the standard normal distribution $N(0, 1)$.

- a) Determine analytically $\mathbb{E}[X_t]$, $Var[X_t]$, $\mathbb{E}[X_s X_t]$, $s \neq t$.
- b) Determine analytically $\mathbb{E}[X_{40}]$, $Var[X_{40}]$.

Using Monte Carlo simulation

- c) Determine $P(X_{40} \in [-3; 3], X_{50} \in [-5; 5])$.
- d) Determine $P(X_{40} > 2.7)$. Discuss your result for $n = 100, 10.000$ simulation runs.
- e) Plot the empirical pdf distribution of X_{40} using 1) Dirac function representation and 2) histogram. Use this result to determine $P(-3 < X_{40} \leq 3)$.
- f) Plot the empirical cdf distribution of X_{40} . Use this result to determine $P(X_{40} > 2.7)$.
- g) Determine a 95% confidence interval (CI) for the mean value of the process at $t = 40$.