Assignment 1

AE4426-19 Stochastic processes and simulation, 2020-2021

Deadline: 7th December 2020, 21:00

1 eVTOLs arriving at a hub (20pcts)

At a vertiport, eVTOLs from 6 surrounding regions arrive. Between 9:30-11:30 in the morning, approximately 20 eVTOLs are expected to arrive at this hub. The arrival times of these eVTOLs are distributed uniformly $U(9{:}30,\ 11{:}30)$. The probability that an arrival is from region i, $i \in \{1, 2, \ldots, 6\}$ is given in Table 1.

Table 1: Probability distribution of type of region.

Region i	1	2	3	4	5	6
$p_{\text{Region}}(i)$	0.03	0.25	0.3	0.4	0.01	0.01

The eVTOLs land at the vertiport according to a FCFS (First-Come-First Serve) sequence. Each eVTOL requires 2min to land at the vertiport, i.e., the minimum time between two consecutive landings is 2min. To ensure this minimum separation, incoming eVTOLs are delayed.

Knowing that at 9:30 there are no eVTOLs at the vertiport, use Monte Carlo simulation to determine:

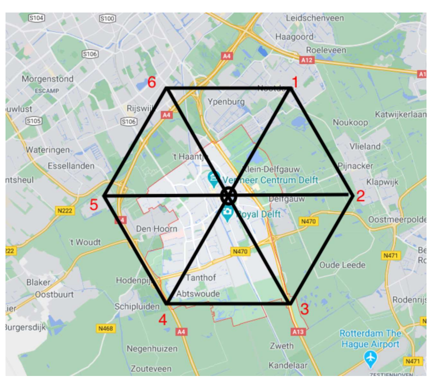

Figure 1: eVTOLs approaching from 6 regions.

a) the probability that at least 6 eVTOLs arrive between 9:45-10:15.

b) the expected number of eVTOLs arriving from region $i = 3$ between 9:30-11:30.

c) the total expected delay and the variance of the total delay during 9:30-11:30.

d) a 95% confidence interval of the total expected delay.

2 Aircraft maintenance (40pcts)

Consider an aircraft brake whose condition degrades after every flight cycles. Let X_i denote the degradation level of the brake after i^{th} flight cycle. $X_i = 0$ implies that the brake has no degradation (is new). As soon as $X_i \geq 1$, the degradation level of the brake exceeds a predefined threshold $D = 1$, the brake becomes inoperable and it must be replaced. After each flight cycle, the degradation level increases following the difference equation:

$$X_{i+1} = X_i + \nu$$

where ν is a random variable following a Gamma distribution with shape parameter α and scale parameter β, i.e., $\nu \sim Gamma(\alpha, \beta)$. The probability density function (pdf) of $Gamma(\alpha, \beta)$ is:

$$f(\nu) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \nu^{\alpha-1} e^{-\frac{\nu}{\beta}}$$

where $\Gamma(\alpha) = \int_0^\infty s^{\alpha-1} e^{-s} ds$ is the Gamma function.

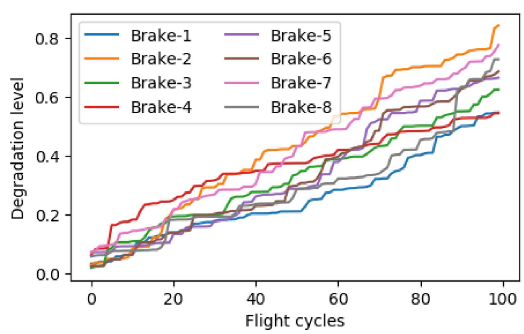

Figure 2: Degradation level of 8 identical, independent brakes. (*data.csv*)

a) In the file *data.csv* you are given historical data on the degradation of same-type 8 brakes of an aircraft during 100 flight cycles. Figure 2 plots this data. Your brake is of the same type as the 8 brakes. You know that the brakes degrade following $Gamma(\alpha, \beta)$.

Estimate parameters α and β using the data in *data.csv* and a maximum likelihood estimator (MLE). In the *data.csv* file, each column corresponds to one brake. Assume that all brake follows $Gamma(\alpha, \beta)$ with the same parameters. MLE of a Gamma distribution is explained in **Additional Document MLE for Assignment 1** (Brightspace $\rightarrow$ Assignment 1 $\rightarrow$ Additional Document MLE) and in **Tutorial 2**.

With the estimated α and β, use Monte Carlo simulation to determine the expected number of flight cycles that the component can make before $X_i \geq 1$ (we call this the MeanTimeToFailure).

For the next exercises, use the parameters $\alpha = 0.3$ and $\beta = 0.02$.

b) Determine the expected degradation level of a brake after 50 flight cycles, given that $X_0 = 0.1$. Use Monte Carlo simulation with 100, 10.000, 1.000.000 simulation runs. Determine the confidence interval for these expected degradation levels. What do you observe?

c) We replace the considered brake after it completes 160 flight cycles since its last replacement, i.e., scheduled replacement. If the brake becomes inoperable after i^{th} flight cycle ($X_i \geq 1$), we replace the brake immediately, i.e., unscheduled replacement. Because an unscheduled replacement causes delays and extra maintenance cost, it is not desired. Use Monte Carlo simulations to determine the following properties: (1) the mean number of flight cycles completed by a brake before it is replaced (both scheduled and unscheduled), (2) the ratio between the number of unscheduled replacements and the number of all replacements (both scheduled and unscheduled) during 1.000 flight cycles. Assume that we have a new brake at the beginning, i.e., $X_0 = 0$.

Hint for Python users:

- numpy.random.gamma(shape=α, scale=β) : to generate a random variable following the Gamma distribution.
- scipy.special.gamma(x) : to get the value of Gamma function $\Gamma(x)$.
- pandas.read_csv(open(*file_directory*, 'r')).values : to read csv file.

Hint for MATLAB users:

- gamrnd(α, β) : to generate a random variable following the Gamma distribution.
- gamma(x) : to get the value of Gamma function $\Gamma(x)$.
- readtable(*file_directory*) : to read csv file.

3 Surveillance drone (40pcts)

We consider a building with 8 distinct rooms (see Fig. 3). A drone flies inside the building, checking one room at a time for 1 minute, then moves to a new, accessible, neighboring room with equal probability. An intruder visits a room $i, i \in \{1, 2, ..., 8\}$ for an amount of time exponentially distributed with parameter

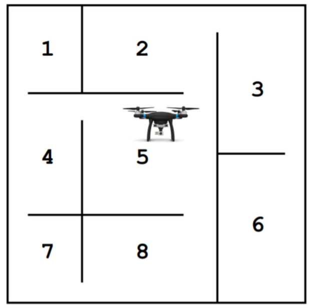

Figure 3: Building layout with 8 rooms.

$\lambda_i = \sqrt{(i+1)/8}$, then moves to a new, accessible, neighboring room with equal probability.

Derive analytically (use mathematical notation and write first the equations, then the numerical results):

a) Formulate the surveillance of the rooms by a drone as a discrete-time Markov chain $\{X_t\}$ and the intruder visiting the rooms as a continuous-time Markov chain $\{Y_t\}$. Explain why your formulation is a discrete/continuous time Markov chain and specify the defining elements (state space, transition matrix).

b) Determine $p_{X_5|X_6,X_3}(7|4,5)$.

c) Determine $p_{X_{1000},X_{1001}}(5,2)$ using the stationary distribution. Explain your result.

d) Knowing that the intruder is at the beginning in room 3, i.e., $Y_0 = 3$, determine the probability that the intruder leaves room 3 in less than 0.9 minutes.

MC simulation. Please upload the code with the assignment.

Important for Python and Matlab: In order to sample a random variable from an exponential distribution with rate λ, the Python and Matlab code should be given as input the mean $1/\lambda$ as shown below:
Python: x = numpy.random.exponential(scale=$1/\lambda$)
Matlab: x = exprnd($1/\lambda$)

e) The drone always starts in room 8. The intruder starts in room 1. Simulate this system and determine the expected time until the drone finds the intruder, i.e., they are in the same room at the same time. Plot the distribution of the time when the drone finds the intruder and determine the variance of this time. Also show that the number of simulations is large enough.

f) When the drone enters a room, it must stay there for at least 1 minute. You are tasked with developing a strategy for the drone to visit the rooms, i.e., how should the drone visit the rooms, starting from room 8, such that it minimizes the time to find an intruder. Assume the intruder enters the building at $t = 0$ with an equal probability of entering any of the outside rooms by climbing through the window. You can consider adjusting the order in which the drone visits the rooms and/or the time it spends in a room.

Note that the transition from one room to another is instantaneous, i.e., the drone and the intruder to not meet each other while crossing a door at exactly the same time. Also, neither the drone, nor the intruder see through walls, i.e., the drone and the intruder see each other only when they are in the same room.

NOTE: For each simulation exercise, the grading of the simulation code is:

30% - Declaration of the variables used with explanation of their meaning; Clarity of the code and proper use of comments.

20% - Final numerical results are correct.

50% - Correct logic of the simulation. The simulation answers the question posed.